THE

DOCTRINE

OF

Combinations, Permutations,

AND

COMPOSITIONS,

OF

QUANTITIES,

Clearly and ſuccinctly demonſtrated.

In tenui labor ——— VIRG.

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ADVERTISEMENT.

THIS small Tract was intended to make a part of the Treatise of Algebra; but other matters had swelled the book to too large a size, to admit of this; which therefore was postponed to a further opportunity.

The Subject of it, which is the Doctrine of Combinations and Alternations, is a very entertaining and curious speculation; by which, a great many very delightful problems may be resolved; which will appear strange and surprizing to them that are not versed with these sorts of calculations. The Doctrine thereof is here compleatly delivered, and in a little compass; and the rules demonstrated in the plainest and easiest manner, and therefore I hope it will prove beneficial and acceptable to the reader.

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The Doctrine of Combinations, &c.

Chronology; or the Art of Reckoning Time.

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Combinations, Permutations, &c.

DEFINITIONS.

DEFIN. I.

THE *Combination* of quantities, is ſhewing how oft a leſs number of quantities or things can be taken out of a greater, and combined together, without conſidering their places, or the order they ſtand in. This is alſo called *Election* or *Choice.*

Here every parcel muſt be different from all the reſt, and no two is to have preciſely the ſame quantities or things.

DEF. II.

Permutation, ſhews how many ſeveral ways, the places of any given number of things may be changed. This is alſo called *Variation*, *Alternation*, and *Changes*.

Here the only thing to be regarded is the order they ſtand in; for no two parcels are to have all the quantities placed in the ſame ſituation.

DEF. III.

Compoſition, is the taking a given number of quantities, out of as many equal rows of different quantities, one out of every row; and combining them together.

Here no regard is had to their places; and it differs from combination, in which there is but one row of things.

DEF. IV

Combinations of the *ſame form*, are thoſe wherein are the ſame number of quantities, and the ſame repetitions. As *abcc*, *bbad*, *cffg*, &c. are of the ſame form. Alſo AB^3C^2, DA^2E^3, C^3ED^2, are of the ſame form; putting the numbers 2, 3 for the repetitions. But ABBC, AB^3, AACC, are of different forms.

Here obſerve, when any quantity is written with an index at the top, it ſignifies ſo many repetitions of that quantity, as a^2 ſignifies *aa*; a^3 ſignifies *aaa*; and b^4 ſignifies *bbbb*, &c.

PROP. I. *Prob.*

To find the number of Permutations or Changes of m things, all different from one another.

RULE.

Multiply continually, $1 \times 2 \times 3 \times 4 \times 5$, &c. to *m* terms; and the product is the number of changes.

For 1 thing *a*, is capable but of one poſition *a*. And if there be 2 things *a* and *b*; they are only capable of theſe 2 variations *ab*, *ba*; whoſe number is 1×2.

If there be 3 things *a*, *b*, *c*; then any 2 of them, leaving out the 3d, will have 1×2 variations; therefore when theſe 3 are taken in, there will be $1 \times 2 \times 3$ variations.

And if there be 4 things, every 3, leaving out the 4th, will have $1 \times 2 \times 3$ variations. Then taking in ſucceſſively theſe four left out, and there will be $1 \times 2 \times 3 \times 4$ variations. And ſo on as far as you pleaſe.

Cor. 1. *$m \times$ changes of $m-1$ things $=$ changes of m things.*

Cor. 2. *The changes of $m-1$ things out of $m =$ changes of m things.*

For the changes of *abc* $= 1 \times 2 \times 3$, and taking in *d*, the changes of any 3 of *abcd* $= 1 \times 2 \times 3 + 3 \times 1 \times 2 \times 3 = 1 \times 2 \times 3 \times 4 =$ changes of *abcd*. And the ſame of others.

Example 1.

How many changes may be made of the words in this verſe? Tot tibi ſunt dotes, virgo, quot ſydera cœlo.

Anſ. $8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1 = 40320$.

Example 2.

How many changes may be rung on 6 bells?

Anſ. $1 \times 2 \times 3 \times 4 \times 5 \times 6 = 720$ changes.

Examp. 3.

For how many days can 7 perſons be placed in a different poſition at dinner?

Anſ. $1 \times 2 \times 3 \times 4 \times 5 \times 6 \times 7 = 5040$; near 14 years.

PROP. II.

The number of permutations of m things taken n and n, or n at a time; is equal to m changes of $m-1$ things taken $n-1$ at a time.

For ſuppoſe theſe 5 things *abcde*, and firſt leave out *a*; then we ſhall have the four *bcde*, out of which let there be taken all the 2's, *bc*, *bd*, &c. and put $v =$ number of all the variations of every 2, out of the four quantities *bcde*.

Now let *a* be put in the firſt place of each of them, which will make *abc*, *abd*, &c; then each

 will

will consist of three letters; and the number of changes will remain the same, whilst a possesses the first place; that is, v = number of variations of every 2 out of the 5 *abcde*, when a is first.

In like manner, if b, c, d, e be successively left out, the number of the variations of all the 2's will also be v; and, putting b, c, d, e in the first place to make 3 quantities out of 5, there will still be v variations, as before. But these are all the variations that can happen of 3 things out of 5, when a, b, c, d, e are successively put first; and therefore the sum of all these is the sum of all the changes of 3 things out of 5. But the sum of these is so many times v as is the number of things; that is, $5v$ or mv = all the changes of 3 things out of 5. And it is evident, the same way of reasoning may be applied to any numbers, m, n.

PROP. III. *Prob.*

Any number m of different things being given; to find how many permutations or changes can be made out of them, taken n by m, or n quantities at a time.

RULE.

Multiply continually $m \times \overline{m-1} \times \overline{m-2} \times \overline{m-3}$ × &c. to n terms, for the changes.

Suppose any number of things *a b c d e f g*; here $m = 7$, and let $n = 3$. Substract 1 from 3 and there remains 2, subtract 2 from 7 and there remains 5. Then set the numbers as follows,

n		m
1	——	5
2	——	6
3	——	7

Here it is evident, that the number of changes, made 1 by 1 out of 5 things, will be $5 = a$.

Then

Then (Prop. II.) when $m = 6$, $n = 2$, the number of changes $= mv = 6 \times 5 = v$, a second time.

Again, when $m = 7$, $n = 3$, the number of changes $= mv = 7 \times 6 \times 5$, that is, $= m \times \overline{m - 1} \times \overline{m - 2}$, continued to 3 or n terms. And the like may be shewn for any other numbers.

Cor. 1. *If* V $=$ *all the variations or changes of m things, taken n at a time; then,* $\overline{m - n} \times V =$ *all the changes of m things, taken $n + 1$ at a time.*

For when $m = 7$, $n = 3$, $V = 7 \times 6 \times 5$, and $m - n = 4$. But by this Prop. making $n = 4$, then the changes will be $= 7 \times 6 \times 5 \times 4$; that is, $= V \times \overline{m - n}$. And so for all other numbers.

Cor. 2. $n\,V = m \times$ *changes of n things out of m — the changes of $n + 1$ things out of m.*

For $mV = m \times$ changes of n things out of m; from which subtracting $\overline{m - n}$. V (Cor. 1.) gives n V as above.

Ex. 1.

How many changes can be rung with three bells out of eight?

Ans. $8 \times 7 \times 6 = 336$.

Ex. 2.

How many words can be made with 5 letters of the alphabet, admitting all consonants?

Ans. $24 \times 23 \times 22 \times 21 \times 20 = 5100480$.

PROP. IV. *Prob.*

Any number of things m, being given; whereof there are p things of one sort, and q things of another, &c. To find how many changes can be made out of them all?

RULE.

Multiply and divide according to this formula.

$$\frac{1 \times 2 \times 3 \times 4 \times 5 \ldots \text{ to } m}{1 \times 2 \times 3 \times \ldots \text{ to } p \times 1 \times 2 \times 3 \times \ldots \text{ to } q \text{ \&c.}}$$

For suppose *ab* two quantities, then there will be two changes, *ab*, *ba*. But if for *b*, you put *a*, then *aa* admits but of one alternation $= \frac{2.1}{2.1} = 1$. For *ab* and *ba*, being two variations, become *aa* and *aa*, which therefore will be but one.

In like manner *abc* affords six variations, but if it be reduced to *aaa* or a^3, then these six variations become but one $= \frac{1 \times 2 \times 3}{1 \times 2 \times 3} = 1$. And if *abc* be reduced to *aac*, the six variations will be reduced to only these three, *aac*, *caa*, *aca*; but half so many as before $= \frac{1 \times 2 \times 3}{1 \times 2} = 3$.

In like manner, if all these be different *abcd*, there will be 24 variations; but if they be all the same, as *aaaa* or a^4, then all these 24 will be but one $= \frac{1.2.3.4}{1.2.3.4}$. And if they be reduced to *aaab*, three of them being the same, there will but be four variations *aaab*, *aaba*, *abaa*, and *baaa* $= \frac{1 \times 2 \times 3 \times 4}{1 \times 2 \times 3} = 4$. And if the four quantities be *aabc*, the 24 will be reduced to $12 = \frac{1 \times 2 \times 3 \times 4}{1 \times 2} = 12$.

$= 12$; for it was shewn before that aab is reduced to 3, and since c may have four positions among the quantities aab; therefore the variations of all four $aabc$, will be 4×3 or 12.

After the same manner if we have $abbccc$, if they are all different, they will admit of 720 changes, but because b occurs twice, and c thrice, it must be divided by 2, and then by 6, and the quotient will be $60 = \frac{1\times2\times3\times4\times5\times6}{1\times2\times1\times2\times3}$, as is easily tried. And the like may be shewn of any other such quantities.

Cor. *Hence if any index p occurs twice or oftener, the series must be divided by* 1.2.3 ... *to p twice, or as oft as that index occurs, in different quantities.*

For in that case we shall have a^3b^3, or $aaa\,bbb$, where p is 3. Or a^4b^4, where p is 4.

Examp. 1.

How many variations may be made of the letters in the word Bacchanalia?

Ans. It will be in this form $abcdeffgggg$, and the changes will be

$$\frac{1\times2\times3\times4\times5\times6\times7\times8\times9\times10\times11}{1\times2\times1\times2\times3\times4.} =$$

831600.?

Ex. 2.

How many different numbers will the figures following make, 1220005555?

Ans. $\frac{1\times2.\,3\times4\times5\times6\times7\times8\times9\times10}{1\times2\times1\times2\times3\times1\times2\times3\times4}$

$= 12600.$

PROP. V. *Prob.*

To find the number of permutations of m things, taken n at a time; in which there are p things of one sort, q things of another, &c.

RULE.

First find all the different forms of combination of these m things, taken n at a time: and then how many combinations there are of each form.

Then by Prop. IV. find the number of changes in any form, which multiply by the number of combinations in that form.

Repeat the same for every distinct form; and the sum of all the products gives the whole sum of the changes required; as is evident from the nature of the Problem.

All the difficulty here is to find the combinations required; and this is easily done, when the quantities are but few, by joining together every two, every three, &c. till you come at the number n; and then reckoning how many different forms there are, having n at a time, and how many in each form.

But when the quantities are many and often repeated, this method would be impracticable; and then we must have recourse to Prop. VIII. following to find these combinations.

Examp. 1.

How many alternations or changes can be made of every four letters out of these eight, aaabbbcc?

Set them down as if you were going to multiply them, and all the products accordingly; omitting such as happen to be the same as some of the foregoing, and such as are out of the question.

$$\begin{array}{llllllll}
a & b & c \\
a & b & c \\
\hline
aa & ab & ac & bb & bc & cc \\
a & b & c \\
\hline
aaa\ aab & aac & abb & abc & acc & bbb & bbc & bcc \\
a & b & c \\
\hline
aaab & aaac & aabb & aabc & aacc & abbb & abbc \\
abcc & bbbc & bbcc.
\end{array}$$

Here we have four of this form *aaab*, whose number of changes is 4, by Prop. IV.

And three of this form *aabb*, number of changes 6.

And three of this form *aabc*, number of changes 12.

$$\begin{array}{rl}
\text{Therefore} & 4\times\ 4 = 16 \\
& 3\times\ 6 = 18 \\
& 3\times 12 = 36 \\
\hline
\text{Number of changes} & \quad 70
\end{array}$$

Examp. 2.

How many changes can be made of eight letters out of these ten; $a^4\ b^2\ c^2\ de$?

Here the sum of the indices must always be 8. And all the forms being found by Prop. VIII. following, and the combinations in each; put them down as in the following table, writing the indices of the quantities for the repetitions. Then find the changes (by Prop. IV.) for every eight, in these different forms, and multiply them severally by the number of combinations in each form, as in the last column.

Forms.	Numb. Combi.	Chang.	Chang. multip.
422	1	420	420
4211	6	840	5040.
41111	1	1680	1680
3221	2	1680	3360
32111	2	3360	6720
22211	1	5040	5040
Sum of all the Changes, 22260			

Ex. 3.

How many different numbers can be made out of 1 unit, two two's, 3 three's, 4 four's, and 5 five's; taken five at a time.

Put a, b, c, d, e for 5, 4, 3, 2, 1; then the things given will be $a^5\ b^4\ c^3\ d^2\ e$. Then by Cor. Prop. VIII. following, find all the different forms, theſe five things are capable of, and by that Prop. the number of combinations in each form; which ſet down as below. Then the changes in each form, being found by Prop. IV. muſt be multiplied into theſe combinations, as follows.

Indices.	Forms.	Combinations.	
54321	5	1	
$m = 15$	41	8	
$n = 5$	32	9	
$L = 5$.	311	18	
	221	18	
	2111	16	
	11111	1	

Then

Then the number of alternations or changes will be,

1 form, $\frac{1 \cdot 2 \cdot 3 \cdot 4 \cdot 5}{1 \cdot 2 \cdot 3 \cdot 4 \cdot 5} \times 1 = 1 \times 1 = 1$

2 form, $\frac{1 \cdot 2 \cdot 3 \cdot 4 \cdot 5}{1 \cdot 2 \cdot 3 \cdot 4 \cdot 1} \times 8 = 5 \times 8 = 40$

3 form, $\frac{1 \cdot 2 \cdot 3 \cdot 4 \times 5}{1 \cdot 2 \cdot 3 \times 1 \cdot 2} \times 9 = 10 \times 9 = 90$

4 form, $\frac{1 \cdot 2 \cdot 3 \cdot 4 \cdot 5}{1 \cdot 2 \cdot 3 \times 1 \times 1} \times 18 = 20 \times 18 = 360$

5 form, $\frac{1 \cdot 2 \cdot 3 \cdot 4 \cdot 5}{1 \cdot 2 \times 1 \cdot 2 \times 1} \times 18 = 30 \times 18 = 540$

6 form, $\frac{1 \cdot 2 \cdot 3 \cdot 4 \cdot 5}{1 \cdot 2 \times 1 \times 1 \times 1} \times 16 = 60 \times 16 = 960$

7 form, $\frac{1 \cdot 2 \cdot 3 \cdot 4 \cdot 5}{1} \times 1 = 120 \times 1 = 120$

Number of all the changes, 2111

Note, if 10 be to be taken at a time, the number of the forms of combination would be the ſame; but the combinations themſelves would be different.

PROP. VI.

If C *be the number of combinations of m different things taken n at a time; then* $\frac{m-n}{n+1}$C *will be the number of combinations, taken n* + 1 *at a time.*

Let *abcde* be the different things propoſed. Then all the different combinations, taken by one's, by two's, by three's, &c. will be as follows.

COMBINATIONS.

a	*ab*	4	*abc*	3	*abcd*	*abcde*
b	*ac*		*abd*		*abce*	—
c	*ad*		*abe*		*abde*	1
d	*ae*		*acd*	2	*acde*	
e	*bc*	3	*ace*		*bcde*	
—	*bd*		*ade*	1.	—	
5	*be*		*bcd*	2	5	
	cd	2	*bce*			
	ce		*bde*	1.		
	de	1	*cde*	1.		
	—		—			
	10		10			

1. Here the combinations of all the five simple letters, is evidently 5 or *m*.

2. Then all the quantities *abcde*, being combined with all the same 5, the whole number of combinations of two letters, will be 5 times 5 or 25, but the 5 squares are to be rejected, *aa*, *bb*, *cc*, *dd*, *ee*. Then there remains 20, which is 4 × 5 or $m \times \overline{m - 1}$, but out of these 20, every one is repeated twice over, as *ab* and *ba*, *ac* and *ca*, &c. Therefore $m \times \frac{m-1}{2}$, or $C \times \frac{m-1}{n+1}$ will be the just number of the combinatious of 2 things, n being $= 1$.

3. Then if the last 10 or C, be combined with the 5 *abcde*, there will be produced 5 × 10 or 50 combinations of 3 letters. And out of these there must be rejected 4 with *aa*, and as many with *bb*, *cc*, *dd*, *ee*; that is, 5 × 4 or 20 = 2C. Then there remains 30 combinations, or $\overline{m - n} \times C$. But each combination is thrice repeated, for three letters; for we shall have *abc*, *bac*, *cab*; and *abd*, *bad*, *dab*, &c. that is, they are as often repeated as there are letters; therefore taking a third part, $C \times \frac{m-2}{3}$

or $C \times \frac{m-n}{n+1}$ = the just number of combinations, in this case = 10.

4. Again, if these last be again combined with the five, we shall have 5×10, or 50 combinations of four letters, in this example. But out of these, there will be six with *aa*, and as many with *bb*, *cc*, *dd*, *ee*; that is, 6×5 or 30, to be rejected. Then there remains $20 = 2 \times 10 = m-n.C$. But these contain as many repetitions of each, as there are letters, that is, 4 or $n+1$. For amongst them we have *abcd*, *bacd*, *cabd*, and *dabc*; and so of the rest, therefore we must take only $\frac{1}{4}$ of these; and then we shall have $\frac{m-n}{4}C$ for four quantities combined, which in this case is $5 = \frac{m-n}{n+1}C$.

5. Then if these five be combined with *abcde*, we shall have 5×5, or 25 of all; but among them there will be four with *aa*, and as many with *bb*, *cc*, *dd*, *ee*; that is, 4×5, or 20 to be thrown out; then there remains 5, but all these 5 are but the repetition of the same letters differently placed; and therefore we must take the 5th part, or $\frac{5}{5} = 1$, for the number of combinations $= \frac{m-n}{5}C = \frac{m-n}{n+1}C$, in our example.

And it is evident, that the same reasoning may be applied to any number of distinct things whatsoever. And therefore if C be any number of combinations of n quantities, then $\frac{m-n}{n+1}C$ will be combinations for $n+1$ quantities.

PROP.

PROP. VII. *Prob.*

To find the number of Combinations of m things, all different from one another, taken n at a time.

RULE.

Multiply continually $\frac{m}{1} \times \frac{m-1}{2} \times \frac{m-2}{3} \times \frac{m-3}{4}$ &c. to n terms, for the number of combinations.

For let m be the number of the quantities $abcdef$, &c. then it is plain, that m will be the number of all the one's.

Then let $n = 1$, and $C = m$, and by the last Prop. the combinations of all the two's will be $C \times \frac{m-1}{2} = \frac{m}{1} \times \frac{m-1}{2}$.

Again, let $n = 2$, $C = \frac{m}{1} \times \frac{m-1}{2}$; then by the last Prop. the combinations of all the three's will be $C \times \frac{m-2}{3} = \frac{m}{1} \times \frac{m-1}{2} \times \frac{m-2}{3}$.

Also if $n = 3$, $C = \frac{m}{1} \times \frac{m-1}{2} \times \frac{m-2}{3}$; then the combinations of all the four's will be $C \times \frac{m-3}{4} = \frac{m}{1} \times \frac{m-1}{2} \times \frac{m-2}{3} \times \frac{m-3}{4}$. And so forward as far as you will.

Cor. 1. *The number of combinations of n things, taken out of m, is equal to the uncia or coefficient of the $\overline{n+1}^{th}$ term of a binomial, raised to the m^{th} power.*

For

For $\frac{m}{1} \times \frac{m-1}{2} \times \frac{m-2}{3}$ &c. to n terms is the $\overline{n+1}^{\text{th}}$ term of a binomial; by Cor. 1. Prob. V. Sect. I. Algebra.

Cor. 2. *The number of all the possible combinations, of all the one's, two's, three's, &c. is one less than the sum of all the unciæ of a binomial raised to the m^{th} power, and that is equal to $2^m - 1$. This is the number of all the elections.*

For the sum of the coefficients of the m^{th} power = sum of the coefficients of $\overline{1+1}^m = \overline{1+1}^m = 2^m$.

Cor. 3. *There is the same number of combinations of n things out of m,* as of *m — n things out of m.*

For there is the same variety of taking n things, as of leaving n things; that is, of taking $m - n$ things.

Cor. 4. *There is but one combination of m things out of m; whether they be all different, or several of one or more sorts.*

Cor. 5. *The number of permutations of m things taken n at a time = the like number of combinations × into* 1.2.3.4.5 &c .. *to n terms.*

This is plain from this Prop. and Prop. III.

Ex. 1.

How many combinations can be made of six letters out of ten?

Ans. $\frac{10 \times 9 \times 8 \times 7 \times 6 \times 5}{1 \times 2 \times 3 \times 4 \times 5 \times 6} = 210.$

Ex. 2.

How many combinations can be made with three letters of the alphabet?

Ans. $\frac{24 \times 23 \times 22}{1 \times 2 \times 3} = 2024.$

SCHOL.

SCHOL.

The number of divisors of m quantities, *abcd*, &c. is $2^m - 1$. And the number of aliquot parts, $2^m - 2$. For the number of all the combinations of the one's, two's, three's, &c. is the same as the number of the divisors. And the number of aliquot parts is one less; because *abcd* is a divisor, but not a part of itself.

PROP. VIII. *Prob.*

To find the number of combinations of m things, taking n at a time; in which there are p things of one sort, q things of another sort, &c.

Here when any quantity is often repeated, as *aaa*, *bbbb*, &c. they are written thus a^3, b^4, &c. where the index shews the number of repetitions, of any quantity, or how often it is to be taken. In the former cases of combination where the quantities are all distinct, the solution is had by a single rule, or a single operation. But this cannot be done where any of the quantities are repeated, for this requires several operations, for the different forms of combination, that n things are capable of, one for each form; whereas if they were all distinct, there could be only one form for all.

To proceed regularly, we must first find how many different forms n things will admit of, and what they are; and then how many combinations are in each form. And the sum of all these combinations will be the number required.

RULE, *for the number of forms.*

1. Place the m things so, that the greatest indexes may be first, and the rest decrease in order. Then begin at the first letter, and join it with the second,

second, third, fourth, &c. to the last; then take the second letter and join it with the third, fourth, &c. and then take the third letter and proceed the same way; and then the fourth, and the fifth, &c. till all be done; always taking care to reject such combinations as you have had before. Thus you have the combinations of all the two's.

Then for all the three's, join the first letter to every one of the two's, successively; and then the second, and third, &c. to the last; taking care to leave out such combinations as are had before. Then you have all the combinations of the three's.

Proceed the same way to get the combinations of all the four's, all the five's, &c. till you come at n, whose combinations are required. So you will at last get all the several forms of combination of n things; and how many there is in each form.

But in particular cases, shorter ways than this may often be found out.

2. To find the number of combinations in any form; let the quantities, both in the m things, and n things, be ranged according to the indexes, setting the greatest first, as has been said. Then if the quantities are not many, the number of combinations may be found out, and reckoned up, after a like manner, as their different forms were found out before; beginning at the first, and going gradually thro' them all. As if you have $a^3\ b^3\ c^2\ d$; and you would know how many are contained in it, of this form $aabc$. Here aa may be connected with bc, bd, cd; and there are three such squares which may be connected in like manner, viz. aa, bb, and cc; therefore 3×3 or 9 will be the number of combinations in that form.

But when there are many terms, we must proceed to calculation in this manner. Let the things given whose number is m, be those $a^p\ b^q\ c^r\ d^s$, &c. where p, q, r, s are the indices or repetitions, no

matter

matter whether equal or unequal. And let $a^t\ b^v$ $c^w\ d^x$, &c. be the form to be combined, the indices being $t, v, w, x,$ &c. either equal or unequal. And when a letter is but once expreſſed, its index will be 1. Then for the

RULE *for any ſingle form.*

Let A, B, C, D, &c. be the number of different letters in the m things expoſed, whoſe indices are equal to, or greater than ($t, v, w, x,$ &c. reſpectively) the indexes in the n things.

Then the number of combinations in that form is $\dfrac{A \times \overline{B-1} \times \overline{C-2} \times \overline{D-3}, \text{\&c.}}{RS, \text{\&c.}}$ continued to as many terms as there are different letters in the form propoſed; where R $= 1 \times 2$, or $1 \times 2 \times 3$, or $1 \times 2 \times 3 \times 4$, &c. when any index in the form is twice, thrice, or four times, &c. repeated; and the like for S, &c.

3. The ſame operation is to be repeated for every form; and then the ſum of all, gives the number of combinations; being all that n things out of m, can admit of.

For ſuppoſe any number of things as $m = a^5 b^3 c^2 d$, and the form propoſed, $\overline{}$ $a^3 b^2 c$.

It is evident 3 (the index of a^3) is contained in 5 and 3 (the indexes of $a^5\ b^3$), which gives 2 caſes, where a cube can be taken out of the m things, here A $= 2$.

Then, 2 (the index of b^2) is contained in 5, 3, 2 (the three indexes of $a^5\ b^3\ c^2$); here B $= 3$. So there are three caſes where a ſquare can be taken, conſidered ſingly. But as ſome cube goes along with it, that takes away one caſe, and there remains B $-$ 1 or two caſes only. Now for every one of theſe, there will be A caſes, where the cube and ſquare are both contained in the m things: that

is,

is, the number of combinations with a cube and a ſquare will be $A \times \overline{B-1}$.

Again, 1 (the index of c) is contained in 5, 3, 2, and 1, (the indices of all the quantities $a^5\ b^3\ c^2\ d$); whence $C = 4$; ſo that there will be four caſes, where (c) any ſimple quantity can be taken. But ſince a cube and a ſquare is to go along with it; theſe take up two places, and deſtroy two of the caſes; ſo that the number of caſes where a ſimple quantity can be connected with a ſquare and a cube (whoſe places are fixt, or ſpecies determined), will be $C-2$. And for every one of theſe, there will be $A \times \overline{B-1}$ caſes by varying the ſpecies of the cube and ſquare; ſo the number of theſe three combinations, will be $A \times \overline{B-1} \times \overline{C-2}$. And in like manner, if there had been a fourth quantity (d) to be combined, we ſhould have $D =$ number of the ſimple caſes, and $D-3$ the number of caſes, when going along with the other three. And then the whole number of combinations of quantities, with four different indices, would be $A \times \overline{B-1} \times \overline{C-2} \times \overline{D-3}$, for d muſt be ſuppoſed to have a different index from any of the reſt.

It is evident, this way of reaſoning is general, and may be applied to any collections of quantities whatever; where the indices of all the quantities n, in the propoſed form, are all different. But it matters not, whether the indices in the whole collection of quantities m, which are expoſed, be the ſame or different.

Now when any index in the propoſed form happens to be repeated ſeveral times, the former number muſt be divided by a correſpondent number, after a like manner as was ſhewn in Prop. IV. for the changes of things.

Let $m = a^2b^2c^2$, the form a^2b, then $A = 3$, $B = 3$, and $A \times \overline{B-1} = 6$. But if the form be

a^2b^2 or ab, in either case A and B remain the same, and $\frac{A \times \overline{B - 1}}{1.2} = 3$, which shews all the cases that two squares, or two single letters, can be taken out of m. Here R $= 1 \times 2$.

Again, let $m = a^3b^3c^3d^3$, and the form a^3b^2c; then A $= 4$, B $= 4$, C $= 4$, and $4.3.2 = 24$, the number of combinations of a cube and a square and a single letter. But if the form be a^2b^2c; A, B, C will be the same as before, and $\frac{4.3.2}{1.2} = 12$ combinations of two squares and a single letter, as is easily tried. And if the form be $a^2b^2c^2$; A, B, C, will still be the same, and then R $= 1 \times 2 \times 3$, whence $\frac{4.3.2}{1.2.3} = 4$, the combinations of three squares; and it is evident by inspection, there can be no more. And there would be the same number for the form abc. And the same will hold in all such quantities, where the indexes are alike. For a repetition of the same index causes a certain number of repetitions among the quantities combined; which ought to be reduced in proportion. And what is said of the repetition of one index, is equally true for the repetition of any other index, and then S comes into the calculation.

Cor. *Hence it appears, that before the required number of combinations can be found; all the different forms of combination must be had. And these may be obtained by Art. 1. before; or rather, when the quantities are many, by the following method.*

Here we must work by the indices. Put L = number of different letters.

Then set down all the indexes contained in the m quantities, taking the greatest first. Then out of these

these must be made so many rows of figures as there are different forms, and the sum of the figures in each row must make n, and no row must exceed L places. Now to get all these forms, set down the greatest index first, and the next lesser towards the right hand, and so on. Then proceed from one row to another, by breaking the least numbers (those towards the right) still into lesser numbers, till they be units. Thus proceed gradually figure by figure, from the right towards the left, till all the numbers be dimininished as low as they will admit of.

Ex. 1.

Let the things proposed be $a^3b^2c = m$, to find the number of combinations made of every 3 (n) of these quantities.

The indexes 3 2 1 . L = 3.

	Forms	combin.
	3	1
	2 1	4
	1 1 1	1
Sum of the combinations,		6

In this example there are 3 forms; 1. a cube; 2. a square and a single one; 3. three single ones. The first admits of one combination only; the second of 4, and the third of 1, whose sum is 6.

Ex. 2.

Let $a^3b^3c^2$ be proposed; to find the number of combinations, taken 4 at a time?

Indexes.	Forms.	Combin.
3 3 2	3 1	4
L = 3.	2 2	3
	2 1 1	3
Sum of the combin.		10.

Ex. 3.

How many Combinations, in $a^4b^2c^2de$; *taking* 8 *at a time?*

Indexes.	Forms.	Combinations.
42211	422	1
$n = 8$	4211	6
$L = 5$	41111	1
	3221	2
	32111	2
	22211	1
	Combinations	13

In this example there is no threes to be combined with the four.

Ex. 4.

Suppose $a^5b^5c^4d^4e^4f^3g$ *be proposed; to find all the combinations, taken* 10 *at a time.*

Indices.	Forms.	Combinations.
5544431	55	1
$n = 10$	541	40
$L = 7$	532	40
	5311	100
	5221	80
	52111	100
	511111	12
		373

Forms.

Forms.	Combin.	Forms.	Combin.
442	40	3331	80
4411	110	3322	90
433	50	33211	360
4321	400	331111	75
43111	250	32221	180
4222	50	322111	240
42211	300	3211111	30
421111	125	22222	6
4111111	5	222211	45
		2221111	20
	1320		1126
			1320
			373
		Combinations in all	2819

To give account of our proceedings in this Example, it may be obſerved, that all the figures decreaſe towards the right. In the firſt place, we take the greateſt 5 and 5, and then break the latter 5 into 4 and 1, or into 3 and 2, or into 3 and 1,1. Having done with 5,3, we take 5,2, and fill up (10) with 2,1, or with 1,1, 1.

Having done with 5, we take 4, and fill up with the greateſt numbers, not exceeding 4; as with 4,2, and 4,1,1. For if we had taken a bigger number than 4, we ſhall find it diſpatched before; and therefore we muſt always take an equal or a leſs number. In the next place we take 4,3, and fill up with 3, or 2,1, or 1,1,1; and thus we go on till all the right hand numbers are diſſolved into 1's.

Having finiſhed thoſe with 4, we take 3, and put to it 3,3,1, the greateſt numbers that exceed not 3; and proceed as before; but we have now

 no

additional numbers, to do with, above 2. And being done, we go to 2, and the whole collection ends with 2221111; for we cannot break 2 into 1 and 1, for the number of places would then exceed 7, which it cannot do.

All these forms being thus had, their several combinations are obtained, by the second article, and set against them in the table. And this example may serve as a specimen for other more difficult cases.

SCHOLIUM.

When n, the number of things to be taken, is very great; you may first take the number of $m-n$ things, and find how many combinations there will be for them; for there will be just as many for one as for the other. And then having the combinations of $m-n$ things; by taking these severally out of the whole, you will have the combinations of n things required. As if $m = a^3b^2c$, and $n = 4$, then $m - n = 2$, and all the two's will be aa, ab, ac, bb, bc. These taken severally out of a^3b^2c, will leave $abbc$, $aabc$, $aabb$, a^3c, a^3b; which are all the combinations of the four's. But the number of forms will not be the same in both, there being only two forms in the two's; and three forms in the four's. Neither will the number of combinations, in the correspondent forms, be the same.

PROP. IX.

The number of compositions of n things taken out of n rows, each row consisting of m things, is m^n, or the n^{th} power of m.

Let there be any number of rows as these annext. It is plain the number of single things as a, b, c, d, is m or m^1.

a	b	c	d
e	f	g	h
i	k	l	m

Then

Then the number of combinations of every 2, is had by joining each quantity in the second row to all the quantities in the first, which will make as many times m as there are things in the second row, or m times m, that is m^2; for all the two's.

Again, taking in, the third row; there will be as many times m^2, as there are things in the third row, that is m times mm, or m^3; for the composition of three things.

After the same manner if a fourth row was taken in; all the combinations of every four, would be m^4; and so on. And therefore universally when n rows are taken in, the number of combinations will be m^n, which is the number of compositions.

Cor. 1. *The number of compositions of all the one's, two's, three's, &c. to n, is* $\frac{m^n - 1}{m - 1} m$.

For $\frac{m^4 - 1}{m - 1} = m^3 + m^2 + m + 1$.

And $\frac{m^4 - 1}{m - 1} m = m^4 + m^3 + m + 1$, as will appear by division, or in general, $\frac{m^n - 1}{m - 1} m = m^n + m^{n-1}$, &c. to m = compositions of all the one's, two's, three's, &c. to n.

Cor. 2. *Hence, may be found the composition of n things out of m, as follows. Involve m to the n^{th} power for the answer?*

Examp. 1.

How many compositions, of 3 letters out of 20?

Ans. $20^3 = 8000$. But these should be here, 3 different alphabets.

Ex. 2.

How many changes are there on throwing four dice?

Ans. $6^4 = 1296$.

PROP.

PROP. X. *Prob.*

If there be m rows of quantities given, having the same quantities, and the same number of them, as a, b, c, d, &c. To find the number of compositions of m things taken out of the m rows; for any given form, of these quantities, as $a^t b^v c^w d^x$, &c.

RULE.

Put V = variations of the things $a^t b^v c^w d^x$,

And A = variations or alternations of the indexes, $tvwx$, both found by Prop. IV. Then will

AV = number of compositions required.

$a\ b\ c\ d$
$a\ b\ c\ d$
$a\ b\ c\ d$
$a\ b\ c\ d$
$a\ b\ c\ d$
$a\ b\ c\ d$
$a\ b\ c\ d$

For let the different rows in the last Prop. be made all alike, or the first row m times repeated, as here. And let the number of things proposed be $a^3 b^2 c$; and first let the index 3 be fixt to a, and 2 to b, &c. Now since a^3 may be taken out of any 3 rows, and b^2 out of any two remaining rows; and c out of the last remaining row; therefore there will be so many ways of taking these letters, as each of them can be placed in different rows, that is, in different places or situations; for the varying the rows is the same thing as varying the places of the letters; and consequently the number of different ways of taking them out of the several rows, is equal to the number of variations or permutations of these letters, to be found by Prop. IV. which number is = V. And this will hold as long as the indexes are fixt to these particular letters, and to none else.

But since in any one form as $a^3 b^2 c$, that form will take in as many cases, as there can be variations made in shifting the indices, from one letter to another; therefore there will be so many times V, as is the number of these variations. Therefore if

A =

A = number of variations or alternations, of the indices 3, 2, 1; or in general, of t, v, x, w. Then AV will be the whole number of compoſitions, that particular form will admit of.

Cor. 1. *Hence in any* $a^t b^v c^w d^x$ &c. *where the indices are fixt invariably to their particular letters; the number of compoſitions* V *will be,*

$$= \frac{1 \times 2 \times 3 \times 4 \ldots \text{to } m}{1.2.3 \ldots \text{to} \times 1.2.3 \ldots \text{to } v \times \&c}.$$

Cor. 2. *If* n = *number of different letters in any form* $a^t b^v c^w d^x$ &c. *Then the whole number of compoſitions for that form will be* =

$$\frac{1.2.3 \ldots \text{to } n}{\text{RS \&c.}} \times \frac{1.2.3 \ldots \text{to } m}{1.2 \ldots t \times 1.2 \ldots v \times \&c.}$$

where R = 1.2, or 1.2.3 &c. according as any index is twice or thrice, &c. repeated; and the like for S, and ſo on.

Here follow ſome examples.

Examp. 1.

How many compoſitions are in the form a^3b^2c?

Here $n = 3$, $m = 6$, $t = 3$, $v = 2$, $w = 1$. And here is no repetition of indexes. Therefore

Anſ. $1.2.3 \times \frac{1.2.3.4.5.6}{1.2.3 \times 1.2} = 6 \times 60 = 360.$

Ex. 2.

How many compoſitions in the form a^2b^2c?

Here $n = 3$, $m = 5$. $t = 2$, $v = 2$, $w = 1$. The index 2 is twice repeated. Therefore

Anſ. $\frac{1.2.3}{1.2} \times \frac{1.2.3.4.5}{1.2 \times 1.2} = 3 \times 30 = 90.$

Ex. 3.

To find the compoſitions in the form a^3b^3.

Here

Here $n = 2$, $m = 6$, $t,v = 3$, $w = 0$. The index 3 is twice repeated; and the letters a, b, thrice.

$$\text{Anf. } \frac{1.2}{1.2} \times \frac{1.2.3.4.5.6}{1.2.3 \times 1.2.3} = 1 \times 20 = 20.$$

Ex. 4.

To find the number of compofitions of the form $a^3b^3c^2d^2$.

Here $n = 4$, $m = 10$. $t,v = 3$, $w,x = 2$. The index 3 is twice repeated, as alfo the index 2. The letters, a, b, are thrice repeated; and c, d, twice.

$$\text{Anf. } \frac{1.2.3.4}{1.2 \times 1.2} \times \frac{1.2.3.4.5.6.7.8.9.10}{1.2.3 \times 1.2.3 \times 1.2 \times 1.2} =$$

$6 \times 25200 = 151200$.

Ex. 5.

How many compofitions in the form $a^5b^3c^3def$?

Here $n = 6$, $m = 14$; $t = 5$; $v,w = 3$, $x = 1$, &c. Here the index 3 is twice repeated, and the index 1 thrice. The letter a, 5 times repeated; and b and c thrice. Hence,

$$\text{Anf. } \frac{1.2.3.4.5.6}{1.2.3 \times 1.2} \times \frac{1.2.3.4.5.6.7.8.9.10.11.12.13.14}{1.2.3.4.5 \times 1.2.3 \times 1.2.3}$$

$= 60 \times 20180160 = 1210809600$. So prodigioufly do the numbers increafe in thefe operations.

SCHOL.

It may be obferved, that if the multinomial $A + B + C + D$ &c. be raifed to the m^{th} power, and the given form, whofe number of compofitions is fought, be found among the terms of that power; the numeral coefficient annexed to it, will always be the number of variations V, that we want. See my Algebra, Schol. to Prob. LIX. B. I. Thus for the form a^3b^2c, look in the fixth power, and you'll find $60A^3B^2C$; therefore $60 = V$. Alfo in the fifth

fifth power, you'll find $30A^2B^2C$, therefore $V=30$, for the variations of a^2b^2c. Alſo with a^3b^3 in the ſixth power, you'll find 20 for V; and ſo of others. And this being known, this doctrine may be ſerviceable for finding the unciæ of any term of a multinomial. As ſuppoſe you have the 4th term of the 7th power of $A+B+C$ &c. which is A^4B^3, A^5BC, and A^6D,

The variations of A^4B^3 is $\dfrac{1.2.3.4.5.6.7}{1.2.3.4\times1.2.3}=35.$

and the variations of $A^5BC=\dfrac{1.2.3.4.5.6.7}{1.2.3.4.5}=42;$

and the variations of $A^6D=\dfrac{1.2.3.4.5.6.7}{1.2.3.4.5.6}=7;$

therefore the complete term is $35A^4B^3+42A^5BC+7A^6D$. And the ſame may be done for any other term, if the letters that compoſe it be known. The doctrine of Combinations and Permutations, is likewiſe applicable to many other purpoſes, particularly to ſeveral problems of chance, which for brevity's ſake I omit.

F I N I S.

HRONOLOGY:

Or, the ART of

ECKONING TIME.

DESCRIBING

eral Diviſions of TIME in common Uſe;
Nature and Original of CYCLES and PE-
, and the moſt remarkable AERA's; and
anner of Computing the Numbers of ſuch
s, of the Common Notes, for any Year;
oveable Feaſts; the Places of the Sun and
Moo; and the Nature of the Calendar.

Founded upon

ASTRONOMICAL PRINCIPLES.

To which is added,

Short CHRONOLOGICAL TABLE.

——— *res antiquæ laudis et artis*
ingredior, ——————— VIRG.

LONDON:
Printed for J. NOURSE, in the Strand;
Bookſeller in Ordinary to his MAJESTY.
MDCCLXX.

THE PREFACE.

I Suppose little or nothing need be said in praise of an art, which we stand in need of, and are forced to practice every day; and such is the art of Chronology. *Without this art we could not tell how time goes on, nor understand the difference of times and seasons. We daily want to know the rising and setting of the sun and moon, the increase or decrease and the length of days, the moon's changing and full, and such like things.*

There is such a near connection between Astronomy and Chronology, that some authors treat of them together; and in this Treatise I have laid down such astronomical problems as are always wanted in Chronology; which are here solved or rather reckoned, after a short way for common use. But to have them accurate, must be performed more laboriously by Astronomy.

As Chronology is of such service in reckoning time, and every body has some business or other with time; therefore every body should at least know the use of the Calendar, and the common Chronological notes; and be acquainted with so much Chronology, as to be able to point out the several times and seasons, as they are distinguished in the country where he lives. Therefore I have here put together the easiest elements of

Chronology, or those of most frequent use, being such as relate to the civil computations of time.

Chronology is absolutely necessary to be known by all such as want to understand history; without which he will wander in the dark, and be always at a loss. For as the History tells us the person that acts, and Geography gives us the place of action, so Chronology must give us the time of acting; without which we shall have very imperfect notions of what we read. For this reason, I have added a Chronological table, containing the most memorable actions that have happened, and the most remarkable events; and the times when the most famous men were alive.

It is true, the Chronology of ancient times is very obscure and uncertain; for though the facts are mostly true, yet these being annexed to wrong periods of time, has occasioned infinite perplexities and inconsistencies in Chronology, which stood in great need of amendment. For this reason Sir I. Newton *set about rectifying the times, which he did with wonderful sagacity; and has given us a table of ancient transactions, adjusted to their proper periods of time, as far as he had any* data *to work by, and perhaps as far as it is possible to be done in this age. By this means he has made a great many events in history, surprisingly to agree. This he effected by reasoning from the lengths of generations of men, and of the reigns of kings, and by astronomical observations, where they could be had; and these are infallible guides.*

For to confirm the time of any transaction, several characters, taken from Astronomy, are of especial use; as the eclipses of the sun and moon, the conjunctions of the superior planets, the acronical rising or setting of the stars, the occultation of the fixt stars by the moon, the places of the equinoctial points, and such like. These, when taken notice of by any historian, go a great way in determining the time of any action or event, then mentioned.

But

But ſome perverſe people, bigotted to another way, and reſolved to oppoſe all improvements; not content with what Sir I. Newton *had done, have ventured to write againſt him; tho' they cannot tell how to give a better ſyſtem, or clear the common ſyſtem of Chronology from its known inconſiſtencies. In what they advance, they only give us the chaff and the corn together; ſome truths mixt with endleſs falſhoods, abſurdity and nonſenſe; knowing no way to ſeparate them, and wanting Sir* Iſaac'*s philoſophical ſieve; ſo they blunder on in the old road, and ſtick faſt among their abſurdities, not knowing how to extricate themſelves. Some of them attempt to confute Sir* Iſaac'*s aſtronomical computations, which perhaps may go down with ſuch as are no judges; but will be laughed at by ſuch as are; for when things are examined, they appear to be moſt wretched computers. The way to confute him is to invalidate his proofs, which they can never do. Therefore the table I have annext is moſtly taken from his table, as being evidently the moſt conſiſtent ſcheme that has yet appeared.*

W. Emerſon.

CHRONOLOGY.

DEFINITIONS.

DEF. I.

CHRONOLOGY is the art of meaſuring time, and adjuſting all actions and events, to the ſeveral points of time in which they happened. And theſe parts of time are years, months, weeks, days, hours, minutes, ſeconds, &c.

DEF. II.

A *Year* is the time wherein the ſun (apparently) moves round the earth in its orbit, or the earth (really) moves round the ſun. This is the moſt obvious and ſenſible meaſure of large portions of time.

DEF. III.

The *aſtronomical* or *periodical Year*, is the time wherein the earth moves round the ſun, from any ſtar or fixt point in its orbit, to the ſame point again, and conſiſts of 365 days, 6 hours, 9 minutes, 14 ſeconds.

DEF. IV.

The *tropical Year* is the time wherein the earth (or ſun) performs its revolution thro' all the ſeaſons; which being ended, they all return again as before. It conſiſts of 365 days, 5 hours, 48 min. 57 ſec.

DEF. V.

The *Egyptian Year* is the ancient year uſed by the *Egyptians*, and now uſed by the *Perſians*, and is juſt 365 days continually. This being ſhorter than the tropical year by near 6 hours, a day is loſt in 4 years; and in about 1500 years, the beginning of the year moves backward thro' all the ſeaſons. This year is the moſt convenient for aſtronomical calculations, as it is always the ſame, having no intercalary days to interrupt it.

DEF. VI.

The *Julian Year* is the year inſtituted by *Julius Cæſar*, 46 years before Chriſt. It contains 365 days for three years, which are called *Common Years*; and 366 days, every fourth year, which is called *Leap Year*; whence the Julian Year is $365\frac{1}{4}$ days at a mean. But this year being longer than the tropical year by full 11 minutes, a day is gained in 130 years; or the ſeaſons come ſooner by a day in 130 years. And in 47540 years the beginning of the year would move forward, thro' all the ſeaſons of the year.

DEF. VII.

The *Gregorian Year*, is the year inſtituted by Pope *Gregory*, in 1582; it conſiſts of 365 days for three years, and 366 for the fourth, which is leap year, like the Julian. But then every hundredth year, that is, not diviſible by 4, muſt be only a common year, inſtead of a leap year.

This ſort of year is neareſt the tropical year of any, and agrees nearly with the apparent motion of the ſun, and is deſigned to keep pace with the tropical year. But it is very unfit for aſtronomical calculations, being perplext with ſo many intercalations.

It was contrived to be brought to the ſun's motion as it was at the time of the council of Nice, in 325; which was done by dropping 10 days out of the Calendar in the month of October. In the

laſt

last century it was 10 days before the Julian account; and in the present century, is 11 days before the Julian year; and gains one day more, every following century, except in such as are exactly divisible by 4. But it is strange that the Pope should bring back the beginning of the year, only to the Nicene council; when it ought to have been brought back to the time of Julius Cæsar, when the error was first begun; at which time the vernal equinox was on March 24 or 25. And then instead of 10 days, 13 or 14 should have been cut off.

This year was brought into use in England in the year 1752; before which, the Julian account was always used. It was effected by striking out 11 days in September, calling the third day the fourteenth, and so on. And the year to begin on Jan. 1.

DEF. VIII.

The *civil Year* is the year which is in common use in any nation. Thus the Julian year was used in England till 1752, and after that the Gregorian year was to be always used.

Different nations begin the year at different times. The ancient *Egyptians* fixt its beginning in August; the *Athenians* and *Grecians*, at the summer solstice; some places in *Germany* and *Italy*, at the vernal equinox; and others at the autumnal equinox. But most of the *Europeans* begin it at the first of January; as England does by a late act of parliament.

The year is divided into 12 months; as also into four quarters, for the four seasons of the year; spring, summer, autumn, and winter.

DEF. IX.

The *lunar Year* consists of 12 lunations, or 354 days; it is used by the *Turks* and *Mahometans*. It runs thro' all the seasons in 32 years.

DEF.

D E F. X.

The *lunisolar Year*, is the moſt ancient year of all; it conſiſted of 12 lunations or months of 30 days each; which fell ſhort of the ſolar year, and therefore they were obliged to add a month, whenever they found it too ſhort for the return of the ſeaſons; and to omit a day or two in an intercalary month, when they found it too long for the courſe of the moon.

D E F. XI.

A *Year current* is a year running on, and not yet compleated. The ſame of days, &c.

D E F. XII.

A *Month* is the 12th part of the year.

D E F. XIII.

A *ſolar Month* is the time of the ſun's paſſing thro' a ſign of the zodiac.

D E F. XIV.

A *lunar* or *ſynodical Month*, is the time between one conjunction of the ſun and moon to another. It conſiſts of 29 days, 12 hours, 44 min. at a mean lunation.

D E F. XV.

Periodical Month, is the time of the moon's revolution, till it came again to the ſame point, or the ſame ſtar. It is 27 days, 7 hours, 43 min.

D E F. XVI.

Civil or *calendar Month*, is the length of 30 or 31 days, according as is ſet down in the Calendar.

D E F.

DEF. XVII.

A month of weeks; this consists of four weeks; and 13 of these months make a year, wanting a day.

DEF. XVIII.

A week is a collection of seven days, the 1st day is sunday, 2 monday, 3 tuesday, 4 wednesday, 5 thursday, 6 friday, 7 saturday; then they come about again in the same order. These days have their names from the seven planets.

DEF. XIX.

A day is the time wherein the sun seems to make one revolution. This is the most sensible part of time for measuring the small parts of it.

DEF. XX.

A natural day, is the time wherein the sun moves round the earth, from one meridian to the same again. This is longer by about four minutes, than the time of the earth's rotation.

Different people begin the day differently. The *Astronomers* and *Arabians* begin the day at noon the preceding day, and end it at noon again. The *Jews*, *Italians*, *Athenians*, *Austrians* and *Bohemians*, begin their day at sun-set the preceding evening, and reckon till sun-set again. The *Babylonians* begin and end their day at sun-rise. The ancient *Egyptians*, and most *Europeans*, begin the day at midnight preceding, and reckon to the midnight following.

DEF. XXI.

An *artificial or shining day*, is the time the sun is above the horizon, or the time between sun-rise and sun-set. And *night* is the time of the sun's absence.

DEF.

DEF. XXII.

An *hour*, is the 24th part of a natural day. The *Italians* reckon to 24 hours; moſt others reckon to twice 12.

DEF. XXIII.

Planetary hour, is the 12th part of the time of the ſun ſhining; as alſo the 12th part of the time of his abſence.

The Jews and ancient Romans divided the day into 12 hours, whether long or ſhort; and began to reckon their hours at ſun-riſe, and ended at ſun-ſet, when they began to reckon the hours of the night, which they alſo divided into 12 parts or hours. Whence the hours of their day were longer than the hours of their night in ſummer; and ſhorter in winter. The Turks uſe this way at preſent.

DEF. XXIV.

A *minute* is the 60th part of an hour; a *ſecond* the 60th part of a minute; and ſo of *thirds*, *fourths*, &c.

DEF. XXV.

A *period* or *cycle*, is a certain ſpace of time, or a revolution of a certain number of years, which being ended, it begins anew. Thus;

The *Julian period* conſiſts of 7980 years; the product of the numbers, 19, 28, 15. This period was 4713 at the beginning of the Chriſtian Era.

The *Dionyſian* or *Victorian period* is 532 years; being the product of 19 and 28. This period was 457 at the beginning of the Chriſtian Era. And.

DEF. XXVI.

An *Era* is a particular account or reckoning of time, from ſome remarkable point; to which account all events and memorable actions are referred

referred: This has a beginning but no ending, as a period has.

The following are the most remarkable Era's, according to the common account.

		Years before Christ.
Of the world,	——	4000
Noah's flood,	—	2294
Of the Olympiads (Greek)		776
Building of Rome (Roman)		753
Nabonassar,	——	747
Of Christ's birth,	—	2

	Years after Christ.
Dioclesian persecution,	284
Hegira, among the Turks,	622
Gesdegird, among the Persians,	632

DEF. XXVII.

Epocha is the first point of time, or the beginning of an Era.

DEF. XXVIII.

Stile, the method of reckoning time. The *Julian stile* is the *old stile*, in use from the time of Julius Cæsar. The *Gregorian stile*, is that begun by Pope *Gregory*, called the *new stile*; and used in *England* since 1752.

DEF. XXIX.

The *Calendar* is a day book for a year, divided into twelve parts for the twelve months, and each month into its proper number of days, all regularly numbered; which numbers are to shew the day of the month.

The names of the months from the beginning to the end of the year, and the days in each are, January 31, February 28, March 31, April 30, May 31, 31, June 30, July 31, August 31, September 30, October 31, November 30, December 31; in leap year February hath 29. The number of days in each month may be remembered by these lines,

30 days hath September,
April, June, and November,
February has 28 alone,
And all the rest have 31.

Against the days of the month in the Calendar are set all the remarkable days, holy days, festivals, term times, with such Phænomena of the sun and moon, as are useful for measuring and determining the several times and seasons of the year, but especially all the fixt days; for if the moveable days are inserted, it is more properly an *Almanack*, and lasts only for a year; whilst the Calendar is perpetual.

Every week is divided into seven days, which are denoted by the first seven letters of the alphabet, A, B, C, D, E, F, G; A is set at the first day of January, B at the second, C the third, &c. all following in order, and keep their places invariable. The letter that falls on a sunday is called the Dominical letter.

Since a common year has 365 days, or 52 weeks and one day; therefore A will stand against the last day of the year, as well as against the first. Therefore if any year begins on a sunday it will also end on a sunday; and therefore the next year will begin on a monday; and therefore the next year, after that, on a tuesday; and the next, on a wednesday, and so on till a leap year happens, when February has 29 days, and then that year goes forward, or leaps two days. And hence it follows, that the sunday letter for each succeeding year, moves one letter back, in this order, A, G, F, E, &c. till leap year, and then it moves two letters back.

In the yearly Diarys or Almanacks, the Sunday letter is commonly marked a red letter, to denote it belongs to Sunday. For all nations ſet apart one day in the week for publick worſhip. The *Chriſtians* obſerve the firſt day in the week, or Sunday, which they call the Lord's day. The *Grecians*, Monday; the *Perſians*, Tueſday; the *Aſſyrians*, Wedneſday; the *Egyptians*, Thurſday; the *Turks*, Friday; the *Jews*, Saturday, which they call the Sabbath.

Beſides the periods or cycles before mentioned, there have been ſeveral others in uſe among the ancients; as a *Jubilee* 49 years, uſed by the Jews. An *Olympiad* 4 years, uſed by the Greeks. A *Luſtrum* 5 years, uſed by the Romans; and the *Roman Judicature* 15 years. An *Age* or *Century* 100 years.

SCHOLIUM.

I ſhall here make ſome remarks concerning the calendar and the diviſion of the year; not only in their preſent form, but as formerly uſed.

Before the time of *Julius Cæſar*, the months conſiſted alternately of 29 and 30 days, by which means the new moon kept to the firſt day of every month, when once rightly adjuſted; and their year always begun with the new moon neareſt the vernal equinox. But afterwards, by the order of *Julius Cæſar*, the months were made to conſiſt of 30 and 31 days alternately, and the laſt of 29; which made up the year, or 365 days. And the year began at March. The months and days were as follows.

March	31	July	31	November	31
April	30	Auguſt	30	December	30
May	31	Septem.	31	January	31
June	30	October	30	February	29, and

30 in leap year.

But

But this regular and uniform method was altered by *Auguſtus Cæſar*, who made Auguſt 31, and February 28, and the reſt as we have them now. But tho' the ſaid uniform method of computing, was altered at the pleaſure and humour of a Roman tyrant; yet it is ſtrange that none reſtored it again after his death, upon account of its ſimplicity.

This Julian year with its months thus diſpoſed, has ſeveral advantages; as 1. the year beginning at March; the intercalary day, in leap year, would come in at the end of the year, where it ought to be, and would diſturb nothing. Thus in leap year, Feb. 30 would be the laſt day of the year. 2. From this diſtribution of the months into days, it will follow, that if the day of the moon's changing in March, is known for any year, it will be known in all the following months, according to her mean motion, by continually adding a day for every month paſt March; becauſe each month taken one with another, is a day longer than the moon's ſynodic revolution. 3. The year beginning at March, it would be near the time when the ſun is in the vernal equinox, which is the beginning of all aſtronomical computations.

It is certain, the more ſimple any account of time is, the more eaſy and regular will any computation be, that depends upon it. But the year now in uſe is quite otherwiſe. For the beginning of the year is fixt to the firſt of January; and the intercalary or additional day, which comes in every fourth or leap year, is added at the end of February; which in effect makes two beginnings of the year. For at the beginning of January, there is a change of the dominical letter, and of every other thing depending on it. And on the firſt of March there is another change of the dominical letter, and of all other things relating thereto.

thereto. So that one fragment of the year is regulated by one rule; and another, by another rule; which is a manifest absurdity. For it must needs appear absurd to any body, to see an additional day thrust into the middle of a year, where there is no room for it. But this is not all, for in the times following *Julius Cæsar*, this intercalary day was absurdly thrust into the middle of a month; and the 24th of February was to be reckoned twice over; as if it were possible for one day to be two, or two days to be only one day.

It may be said for excuse, that most of Europe follows that way of reckoning, and therefore we in England, for the conveniency of corresponding with them, ought to follow the same. However, all nations that observe this sort of year, must labour under the like absurdities.

When Pope Gregory altered the civil year, and imposed it upon all Christendom to observe; he should have rectified it to the time of Christ's birth, and not to the time of the council of *Nice*. For our epocha is not dated from the Nicene council, but from the birth of Christ. And at that time the vernal equinox was not on the 21st of March, but on the 24th or 25th.

When the Protestant states of *Germany* came to a resolution to alter the Calendar, they would not receive the Pope's mandate, but did it in their own way, after this manner. They ordered that eleven days should be left out of February after the 18th day, in the year 1700. So that instead of writing Feb. 19, they should write March 1. And that the time of Easter for the future should be determined by astronomical calculation; which was to be the first sunday after the first full moon after the vernal equinox. Or the sunday after, when the full moon fell on a sunday.

As to our Calendar, as now ordered by the Stile act, it keeps the ſame form, yet it differs in this particular, that the golden numbers are ſet to the full moons in each month, and not to the change, as formerly; which variation ſeems to have no manner of advantage in it, but a manifeſt diſadvantage. For it is more material to know the moon's change, than the full, in every month except March.

To enquire particularly what ſort of a year, or what kind of computation, had been the moſt convenient; we ſhall mention ſeveral ways that it may, or rather might have been ordered. And,

1. As the year conſiſts of 365 days, 5 hours, 49 minutes very near. It might have been ordered to conſiſt of 365 days for three years, and 366 every fourth year, as the Julian year does. And placing the odd day at the end of December; beginning the year with January. Or,

2. The Julian account might be preſerved, and the odd day put at the end of February; and the year to begin at March. Then altering the dominical letters in January and February, ſo as to be continued from December in alphabetical order.

3. The year might always have begun on the ſame day of the week, as ſuppoſe on ſunday, which would have much facilitated any computation, by the day of the month. But it would have happened that we ſhould always have, at the end of the year, two ſaturdays, and ſometimes three. But I know of no inconvenience this could have been; and it would have had that advantage, that the firſt day of the week would alſo have been the firſt day of the year.

4. But this might at leaſt have been done, viz. continuing the Julian year as it was, only beginning the year at March 1. For the Julian year is very near the ſolar year, and is very commodious for calculation, being a mean between the aſtronomical

mical and tropical years. And is attended with no other inconvenience, but a ſmall alteration of the ſeaſons, the terms, fairs, &c. which might have been mended by putting them forward about a week in a thouſand years, by an act of parliament. Whilſt ſhifting the year to agree with theſe, is like ſhifting the fire to a man, to ſave him the labour of going to it. But be the convenience or inconvenience what it will, we muſt be forced to take up with what we have. However we know this, that we had but one account to follow before, and now we have two to follow.

PROB. I.

To find how many years it is after leap year.

By the Gregorian account, which is the account we alſo follow; every hundredth year that is diviſible by 4, is a leap year, and the others not. And moreover, every common year that is diviſible by 4, is leap year, and the reſt not. Whence,

RULE.

Divide the year of our Lord by 4, and the remainder ſhews how long it is after leap year: if 0 remains it is leap year.

This holds for both Julian and Gregorian accounts. But in the Gregorian, no even hundreds are leap years, but ſuch as can be divided by 4.

Example 2.

Let the year be 1769.

```
4)1769(442
  16
  ——
   16
   16
   ——
     9
     8
     —
     1 the firſt after leap year.
     —
```

PROB. II.

To find on what day of the week, a given day of the month falls on.

RULE.

The initial letters of the words in the following two lines, are the dominical letters for the firſt day of each month in order; whence it is eaſy to reckon any day by the dominical letter.

A dram did give Ben eaſe;
Good cheer, for all did freeze.

Examp.

On May 16, 1769; *the Sunday letter is* A, *and* B *(Ben) is May* 1*ſt, whence* A *is the* 7*th, therefore the* 7, 14, 21, 28 *are all Sundays; and therefore the* 16*th is Tueſday.*

PROB.

PROB. III.

To find the cycle of the ſun, or the number of it, for any year.

The cycle of the ſun, or rather of the Sunday letter, is a period of 28 years; in which time it ſhews all the variations of the dominical letter. And this period being ended, all the numbers and correſpondent letters, return again in the ſame order, for ever, by the Julian ſtile; but only till the century is out, by the Gregorian. Its principal uſe is to find the Sunday letter; but it is alſo an ingredient in the Julian period.

RULE.

Add 9 to the year of Chriſt, and divide the ſum by 28; the remainder is the number of the cycle. If 0 remains, take 28.

Examp.

Let the year be 1769.

$$\begin{array}{r l}
 & 1769 \\
 & \underline{9} \\
28) & 1778\ (63 \\
 & 168 \\
 & \overline{98} \\
 & 84 \\
\text{rem.} & \overline{14}\ \text{the sun's cycle.}
\end{array}$$

SCHOL.

The cycle of the ſun continues always the ſame, and alſo the connection of the numbers, with the letters, according to the Julian form. But the connection of the numbers with the letters, by the Gregorian ſtile, is variable; any cycle laſting but 100 years, and then a new cycle is to take place.

PROB. IV.

To find the dominical letter or letters, for any year.

1 RULE.

Add 17 to the year of the Lord, and divide the ſum by 28; the remainder being found in the old cycle below, ſhews the dominical letter, for the *new ſtile*. If 0 remain, take 28. This rule holds only till 1800.

For the *old ſtile*, ſeek the number of the ſun's cycle (found by the laſt Prop.) in the old cycle below, and againſt it is the dominical letter. This holds for ever.

Note, there are two letters in leap year; one holds till the end of February; the other to the year's end.

The old Cycle.

1 GF	5 BA	9 DC	13 FE	17 AG	21 CB	25 ED
2 E	6 G	10 B	14 D	18 F	22 A	26 C
3 D	7 F	11 A	15 C	19 E	23 G	27 B
4 C	8 E	12 G	16 B	20 D	24 F	28 A

To make this rule general for the new ſtile; inſtead of 17, between the beginning of 1800 and 1900, add 5, between 1900 and 2100, add 21; from 2100 to 2200, add 9, &c. And in general, add 16 more at every hundred years forward, except ſuch as are diviſible by 4. But after 7 additions of 16, the ſame numbers will return again; that is, after 8 hundred years.

Example