Fig. 2. Or a circle may be deſcribed on the plane with a pencil, by extending the thread; and then the center of this circle may be found, by drawing two parallel lines thro' it, and drawing a third line thro' the middle of theſe two; for if that line be biſected, the middle point will be the center.

PROB. VI.

To meaſure the perpendicular height of any point above a line or plane.

RULE.

Take the neareſt diſtance from that point to the line or plane, with a ſtring; and then meaſure its length.

If the perpendicular falls within a ſolid body; you muſt lay a ruler from that point, parallel to the baſe, and take the neareſt diſtance from the ruler to the baſe; and this is eaſily done, if the body ſtands upon a plain floor; but if not, then a board muſt be applied to the plane of its baſe, and a line drawn parallel to the board through the top of the body, which muſt be held ſo as to be every where equidiſtant; and then that diſtance is the height required.

If it be ſo ſituated, that a line or ruler cannot be applied, ſo as to be made parallel to the baſe. Then if it has a ſtreight ſide, the angle that the ſide makes with the baſe, and the length of the ſide, muſt be found; from whence the perpendicular height may be had.

PROB. VII.

To divide a line AB into any number of equal parts.

RULE.

Draw any line *ab* parallel to AB, and set off your number of any equal parts from *a* to *b*. Draw the lines *a*A and *b*B, to intersect in C. From C draw C*f*, C*e*, C*d*, C*c*; to intersect AB in 1, 2, 3, 4; which are the divisions required. 3.

The line AB may also be divided by trials, by running a pair of compasses from one end to the other; and increasing or decreasing the divisions, till they just answer to the whole.

If the distance be large, this way cannot be practised; then the length of the line AB must be measured in feet, yards, &c. and the whole length divided by the number of parts, to get the length of one part; which done, that length must be set off from B to 1, from 1 to 2, from 2 to 3, &c. to A.

PROB. VIII.

To describe a circle thro' three points given, A, B, F.

RULE.

Open the compasses, and setting one foot in C, 4. try if the other turned about will reach all the three points; if not, shift the center, and try again, till it do: or you may use a thread; then describe the circle ABF.

Otherwise.

From two of the points draw lines to the third, as AB, BF. Raise two perpendiculars DC, EC, upon the middle of the lines AB, BF; to intersect in C. From the center C describe the circle ABF.

PROB.

Fig.

PROB. IX.

To find the center of a circle.

1 RULE.

4. The center C may be found by trials; or thus; draw any line BF, and on the middle point E, erect the perpendicular GEH. Bisect GH in C, and C is the center.

2 *Or thus.*

Thro' the circle, draw any two lines BF, *bf*, parallel to one another; bisect these lines at E and *e*, thro' E, *e*, draw the line GH, which will be a diameter; therefore if GH be bisected in C, then C will be the center.

PROB. X.

To divide the semicircle ADB *into any number of equal parts.*

RULE.

5. Make AC and BC equal to AB. Divide AB into the same number of equal parts, at *b*, *c*, *d*, *e*. And let the divisions at the center rather be bigger than less than those near the circumference at A and B. Then draw the lines C*b*, C*c*, C*d*, C*e*, to intersect the circle at 1, 2, 3, 4, which will divide the semicircumference into equal parts as required.

The semicircle, or any other arch, may also be divided into any number of equal parts by trials, running a pair of compasses from one end to the other, and increasing or decreasing the extent till it is found to be too little or too big. Where note, the length of the radius is just the extent of the third part of the semicircle ADB.

Fig.

PROB. XI.

To make a square equal to a given circle.

RULE.

Thro' the center C draw two diameters AB, DE 6.
perpendicular to one another. And divide each of the quadrants into three equal parts at 1, 2. So that the middle division of each be rather bigger, than less, than the other two. Thro' 1, 1, draw the lines *ab*, *cd*; and thro 2, 2, the lines *ac*, *bd*; and the square *abdc* is equal to the circle AEBD.

2 *Otherwise.*

Thro' the center draw two diameters AC, BD, 7.
perpendicular to each other. And divide one diameter AC into eight equal parts, and set one of these parts (rather more than less) from A to *a*, and from B to *b*, C to *c*, and D to *d*. Draw *ab*, *bc*, *cd*, *da*. And *abcd* is a square equal to the circle ABCD.

3 *Otherwise.*

Make the diameter AB 14 parts, and AD 11; 9.
raise the perpendicular DE. Draw AE, and the square AEFG is equal to the whole circle, whose diameter is AB.

By the Sliding Rule.

Set 1 on A to .282 on B, and against the circumference on A, is the side of the square on B; whose area is equal to the circle.

For .282 is the side of a square equal to the circle, whose circumference is 1.

Fig.

PROB. XII.

To describe an Oval.

RULE.

8. Draw the transverse AB, and conjugate CD, at right angles to it. Describe two circles AFEK, and EGBL; then take two centers H, I, any way in D, equidistant from E; and from these centers describe the arches KL, MN, to touch the circles KF and GL.

The two circles may be drawn otherwise than to touch at E; for they may be made to pass thro' each other's center, or at a distance from one another, &c.

An ellipsis is easily drawn, by setting up two pins in two centers *n*, *o*; about which putting a thread *nop*, and knotting it at *p*; the point *p* carried about, will describe an ellipsis DPKAMCNBLD, by help of a pencil.

PROB. XIII.

To reduce any irregular figure, or any curve or part of a curve, into a rectangle.

RULE.

10. Let ACB be the figure. Erect any number of perpendiculars upon the base AB at equal distances; measure their lengths, and add them together, and divide the sum by their number, the quotient is the mean breadth AD, which is the breadth of the parallelogram ADEB, equal to the fig. ACB: the length being the same; or both of them having the common base AB.

Fig. 10.

Example.

Erect seven perpendiculars, whose lengths measured and marked in the figure, are 0, 2, 4, [illegible] 2, [illegible]. Their sum is 17, and 17 divided by [illegible], gives $2\frac{3}{7}$ for the mean breadth, which is equal to BE or AD, the breadth of the parallelogram ABED, equal to the figure ACB.

Note, this Prob. is of use in measuring irregular planks or boards.

PROB. XIV.

To find the hypothenuse of a right angled triang[illegible]

RULE.

From any scale lay down the length of the base AB. Draw BC perp. to it, of the length given, by the same scale. Draw AC, which, measured on the scale, gives its length. 11.

Otherwise.

To the square of the base add the square of the perpendicular; extract the square root of the sum, and you have the hypothenuse.

Examp.

Let the base AB *be* 22, *the perpendicular* BC 13.

22	13	653(25.55	the hypothenuse AC.
22	13	4	
44	39	45)253	
44	13	225	
484	169	505) 2800	
	484	2525	
	653	275	

PROB.

Fig.

PROB. XV.

To find either leg of a right angled triangle.

1 RULE.

11. Draw the given side AB, of it due length; raise BC perpend. to it; with the length of the hyp. in your compasses, set one foot in A, and with the other describe an arch cutting CB in C; then CB is the other side, which measured on the scale, gives its length.

2 *Or thus.*

From the square of the hyp. subtract the square of the given side; extract the square root of the remainder, and you have the other side.

Examp.

Let the hypothenuse AC *be* 25.55, *the base* AB 22.

22	25.55	168(12.96,	the perpendicular, CB.
22	25.55	1	
44	5110	22)68	
44	1277	44	
	128		
484	13	249)2400	
		2241	
	652.8		
	484	159	
	168.8		

Fig.

PROB. XVI.

Having the three sides of any triangle ACB, *to find the perpendicular* CD.

1 RULE.

Draw the base AB of a proper length, by any scale; and with the length of AC in your compasses, and one foot in A, describe a small arch; then with BC in your compasses, and one foot in B, cross that arch at C. Then draw AC, BC; and the triangle is constructed. 14. 15.

Then from C let fall a perpendicular upon AB, as CD, for the perpendicular. Or take the nearest distance from C to AB, which apply to your scale.

2 RULE.

Find one segment as $AD = \frac{AC^2 + AB^2 - CB^2}{2AB}$.

Then find $CD = \sqrt{CA^2 - AD^2}$.

3 RULE.

Take half the sum of the three sides, and subtract each side from it, and you have three remainders; then multiply that half sum, and the three remainders together continually. Extract the square root, and divide it by half the base, and it gives the perpendicular.

Cor. *Hence either segment of the base is had thus*; To the square of the adjoining side add the square of the base, from which subtract the square of the opposite side; and then divide by twice the base.

Fig.

PROB. XVII.

To find the circumference of a circle, from the diameter.

1 RULE.

23. Say as 7 to 22, fo the diameter to the circumference.

2 *Or more exactly.*

As 113 to 355, fo the diameter to the circumference.

3 *Or thus.*

Multiply the diameter by 3.1416, the product is the circumference.

Examp.

Suppose the diameter be 32.

	Or thus.
7 : 22 : : 32 :	3.1416
22	32
64	62832
64	94248
7)704(100.6	100.5312

1 *By the Sliding Rule.*

Set 7 on A, to 22 on B, then againft the diameter (32) on A, is the circumference (100.6) on B.

2 *Or thus.*

Set 1 on A, to 3.14 on B; then againft the diameter (32) on A, ftands (100.6) the circumference on B.

PROB.

Fig.

PROB. XVIII.

To find the diameter of a circle from the circumference.

1. RULE.

Say as 22 to 7, or as 355 to 113, or as 3.1416 to 1; so the circumference to the diameter. 23.

2. *Or thus.*

Multiply the circumference by .3183, the product is the diameter.

Examp.

Suppose the circumference be 50.

```
     50
      7
    ———
22)350(15.91 the circumference.
    22
    ———
    130
    110
    ———
     200
     198
     ———
       2
```

By the Rule.

Set 22 on A, to 7 on B, and against the circumference (50) on A, is the diameter (15.9) on B.

Fig.

PROB. XIX.

The base and height of the segment of a circle being given; to find the diameter.

RULE.

12. As height of the segment DF :
to half the chord or base AF or FB : :
So the same half :
to a fourth GF ;
then this fourth proportional GF, added to the height of the segment FD, gives the diameter GD.

Examp.

Suppose DF 2 feet; AF, 3. Then $2 : 3 :: 3 : \frac{9}{2} = 4\frac{1}{2} =$ FG, then adding DF, we have GD $6\frac{1}{2}$.

PROB. XX.

Having the radius of a circle; to find the side of a regular poligon inscribed in it.

RULE.

Multiply the given radius, by the multiplier, over against the number of sides in this table; the product is the side of the poligon.

Numb. of sides.	Multipliers.
3	1.732051
4	1.414214
5	1.175570
6	1.000000
7	0.867767
8	.765367
9	.684040
10	.618034
11	.563465
12	.517638

Ex. 1.

Suppose the radius of a circle 5, what is the side of an inscribed equilateral triangle?

1.73205
5
―――
8.66025 Ans.

Fig.

Ex. 2.

If the radius of a circle be 315, *what is the side of an inscribed octagon?*

$$
\begin{array}{r}
.76536 \\
315 \\
\hline
382680 \\
76536 \\
229608 \\
\hline
231.08840
\end{array}
\quad \text{Answer.}
$$

PROB. XXI.

To find the length of the arch of a circle, ADB, *by having the chord* AB.

1 RULE

Divide the chord AB into four equal parts, and set one part from B to E, upon the arch; and from A to H upon the chord; and draw HE; then 2HE is the length of the arch ADB, the lesser segment. 12.

2. *Otherwise.*

Make arch AD = DB, and draw the chord AD; then $\frac{8AD - AB}{3}$ = length of the arch ADB.

3. *Otherwise.*

Make AD = DB, and draw the diameter DFG. Then $\frac{DG - .16DF}{DG - .82DF} \times AB$ = length of the arch ADB.

Note, if the arch is greater than a semicircle; find the length of half of it by this rule, and double it.

 This

Fig. 12. This Prop. may more easily be resolved, by putting a string round ADB, and then measuring the string.

Examp.

Suppose AB *be* 7; DF, 2. Then by Prob. 18. DG = $8\frac{1}{8}$.

.16	8.125	8.125
2	.32	1.640
.32 = .16DF	7.805	6.485
.82	7	
2	6.485)54635(8.42	
1.64 = .82DF	51880	
	2751	
	2594	
	161	

So the length of ADB is 8.42.

PROB. XXII.

To find the length of the quadrant of a circle, AB.

RULE.

13. Divide the semicircle AGC into three equal parts at G, H; which is done by setting CO from C to H, and from A to G; and draw BG, BH. Make DF = DE; then BF is equal to the quadrant BA or BC.

Or thus.

$1\frac{1}{9}$ the chord AB = quadrant AIB.

Examp.

Let the radius AO *be* 20;
Then by Prob. 14, AB is 4.24;

9)4.34(.47	4.24
36	.47
64	4.71 = quad. AIB.

PROB.

Fig.

PROB. XXIII.

To find the periphery of an ellipsis.

1. RULE.

Multiply half the sum of the transverse and conjugate diameters, by 3.1416; the product is the periphery.

2. *Or thus.*

To twice the transverse add $1\frac{1}{7}$ the conjugate for the periphery.

Examp.

Let the transverse be 3, and the conjugate 2.

	Otherwise.
Then $1\frac{1}{7} \times 2 = 2\frac{2}{7} = 2.285$	3 3.1416
6.	2 $2\frac{1}{2}$
Anf. 8.28	$2)5(2\frac{1}{2}$ 62832
	15708
	Anf. 7.8540

PART III.

The Meaſuring of Areas and Surfaces.

PROB. I.

To find the area of a triangle ABC.

1 RULE.

14. MULTIPLY the baſe AB, by the perpen-
15. dicular CD, half the product is the area.

2. *Or thus.*

Take half the ſum of the three ſides, and ſubtract each ſide from it; then multiply that half ſum and theſe three remainders together; the ſquare root of the laſt product is the area.

3. *Or thus.*

Find the nat. ſine of the angle A, and multiply that ſine, and the ſides AC, and AB, together; half the laſt product is the area. Or elſe uſe logarithms.

If CD be not perpendicular, but drawn to make any angle you will at D. Then add the logarithms of AB, CD, and ſine $\angle$D, together; abate 10. Then find the number belonging; half of it is the content.

Fig. 14. 15.

Examp.

The base AB *is* 24, *the perpendicular* CD, 13.

```
 13
 12
---
 26
13
---
156
```

By the Rule.

Set 1 on A, to half the base (12) on B; then against the perpendicular (13) on A, is the content (156) on B.

PROB. II.

To find the area of a square ABCD.

RULE.

Multiply any side by itself; the product is the area. This is called squaring the side AB. 16.

Examp.

Suppose the side of the square 23 *inches.*

```
      23
      23
     ---
      69
     46
     ---
Anf. 529
```

By the Rule.

Set 1 on A, to the side 23 on B, then against 23 on A, is the area 529 on B.

Fig.

PROB. III.

To find the area of a right angled parallelogram ABCD.

RULE.

17. Multiply the longer ſide by the ſhorter; the product is the area.

Examp.

Suppoſe the ſides 16 *and* 35.

```
 35
 16
----
210
35
----
560 area.
----
```

By the Rule.

Set 1 on A to the length (35) on B; and againſt the breadth on A (16), is the area (560) on B.

Scholium.

To find the number of tiles to cover a houſe.

Find the ſquare yards in the floor or baſe, which multiply by 23, if a ſquare roof. Or by 25 if ſharper; or by 32 if true pitch. Generally a quarter of the ſquare yards, ſhews how many hundred are required.

PROB. IV.

To find the area of a Rhomboides, or oblique Parallelogram, ABOD.

RULE.

18. Multiply the length DO or AB, by the breadth, or perpendicular AE; the product is the area.

Examp.

Fig. 18.

Examp.

The length AB 27 *feet, the breadth* AE 18.

27
18
———
216
27
———
486. area.

By the Rule.

Set 1 on A, to the length (27) on B; then against the breadth (18) on A, is the content (486) on B.

PROB. V.

To find the area of a Trapezoid ABCD.

RULE.

Multiply the ſum of the parallel ſides AB, DC, by the perpendicular diſtance between them AE; half the product is the area or content. 18.

Ex.

Suppoſe the length AB 30 *feet,* DC 57; *and the breadth* AE 18; *to find the area.*

30	87
57	18
87	696
	87
	2)1566.
	783 area.

By

Fig.

By the Rule.

Set 1 on A, to half the sum of the parallel sides ($43\frac{1}{2}$) on B; and against the breadth (18) on A, is the area (783) on B.

PROB. VI.

To find the area of a Trapezium ABCD.

RULE.

19. 20. Measure the diagonal AC, and the perpendiculars thereon BE, DF. Then multiply the diagonal, by the sum of these perpendiculars; half the product is the area.

Ex.

$$
\begin{array}{rr}
AC = & 28 \\
\hline
BE = & 13 \\
DF = & 25 \\
\hline
 & 38 \\
 & 28 \\
\hline
 & 304 \\
 & 76 \\
\hline
 & 1064 \\
\text{Area,} & 532 \\
\hline
\end{array}
$$

By the Rule.

Set 1 on A to 28 on B, then against 19 (half of 38) on A, is the content (532) on B.

Fig.

PROB. VII.

To find the content of any irregular plain figure.

RULE.

Divide it into triangles and trapezia, by diagonal lines. And measure all these triangles and trapezia separately, by the former Rules; and the sum of them all is the area or content. 21.

If any side be crooked, draw a streight line, that shall leave as much out, as it takes in.

Examp.

In the figure annext, there are eight sides, and it is divided into two trapeziums, ABGH, and BDEC; and two triangles BCD, GEF. In the trapezium ABGH, AG = 12, BI = 4, HK = 4.25; In the trapezium BDEG, GD = 167, BM = 8.6, EN = 5.7. In the triangle BCD, BD = 10½, CO = 4.71. In the triangle GEF, GE = 12,36, LF = 3.25.

BI = 4.	BM = 8.6
HK = 4.25.	EN = 5.7
8.25.	14.3, then by Logarithms,

½AG,

Fig.				
21.	½AG,	6	0.77815	
	Sum. per.	8.25	0.91645	
	Area	49.5	1.69460	49.5
	½GD,	8.35	0.92168	119.4
	Sum. per.	14.3	1.15533	24.72
				20.08
	Area	119.4,	2.07701	213.70 whole area.
	½BD	5.25,	0.72015	
	CO	4.71	0.67302	
	Area	24.72	1.39317	
	½GE	6.18	0.79098	
	LF	3.25	0.51188	
	Area	20.08	1.30286	

PROB. VIII.

To find the area of a regular Poligon.

1. RULE.

22. Measure the perpendicular distance CP from the center of the figure to one side AG.

Then multiply the perpendicular CP by the circumference of the poligon ABDEFG; half the product is the area.

2. *Other-*

Fig. 22.

2. *Otherwiſe.*

Multiply the given ſide of the poligon by itſelf, and then by the multiplier over againſt the number of ſides, in this Table; the product is the area.

number of ſides.	Multipliers.
3	0.433013
4	1.000000
5	1.720477
6	2.598076
7	3.633912
8	4.828427
9	6.181824
10	7.694209
11	9.365640
12	11.196152

Examp.

Let the ſide of a hexagon be 12 inches.

12	2.598
12	144
24	10392
12	10392
	2598
144	
area,	374.112

By the Rule.

Set 1 on A to 144 on B, and againſt 2.6 on A is the area 374 on B.

PROB. IX.

To find the area of a circle ABCD.

1. RULE.

Multiply half the circumference ABCD, by half the diameter AC; and the product is the area. 23.

2. *Or thus.*

Make as 14 to 11; ſo the ſquare of the diameter AC, to the area.

3. Or

Fig. 23.

3. Or thus.

Multiply the ſquare of the diameter by .7854, the product is the area.

4. Or thus.

Say as 88 to 7, ſo the ſquare of the circumference, to the area.

5. Or thus.

Multiply the ſquare of the circumference by .07958; the product is the area.

You may uſe any one of theſe rules, according as the data happens to be.

Ex. 1.

Let the diameter be 26, to find the area.

26	.7854
26	676
156	47124
52	5498
	471
676	
	530.93 area.

1. By the Rule.

Set 14 on A, to 11 on B; then againſt the ſquare of the diameter on A (676), is the area on B (531).

2. Or thus by the Rule.

Set 1	on D, to .785 on C;	then againſt the
or ſet 1.13	on D, to 1 on C;	diameter (26)
or ſet 11	on D, to 95 on C;	on D,
or ſet 8	on D, to 50 on C;	is the area (531)
or ſet 12	on D, to 113 on C;	on C.

Ex.

Fig. 23.

Ex. 2.

Suppose the circumference be 5, what is the area?

```
 5        .0796
 5           25
---      -------
25         3980
---        1592
         -------
     1.9900, or 2 the area.
```

3. *By the Rule.*

Set 88 on A, to 7 on B, and against the square of the circumference (25) on A, is the area (1.99) on B.

4. *Or thus by the Rule.*

Set 10 on D, to 7.96 on C;
or set 11.3 on D, to 10 on C;
or set 5 on D, to 2 on C;
or set 11 on D, to 9.6 on C;
} then against the circumference (5) on D, is the area (1.99) on C.

Cor. *To measure the area of a ring or annulus.*

1. RULE.

Find the areas of the outer and inner circles, and their difference is the area of the annulus.

2. *Or thus.*

Multiply the sum of the diameters by their difference, and the product by .7854.

SCHOL.

The reason of these different processes is founded on this, that the diameter is to the circumference, as 1 to 3.1416, or nearly as 7 to 22; and the square of the diameter to the area, as 1 to

3.1416

Fig. 23. $\frac{3.1416}{4}$ or .7854; that is nearly as 14 to 11. Likewise the square of the circumference, to the area; is as 1 to .07958, or as 22×22 to $\frac{7 \times 22}{4}$, or as 22×4 to 7, or 88 to 7. And upon the rule, the numbers on C are as the squares of the numbers on D. Therefore it will be, $1^2 : .785 :: \overline{1.13}^2 : 1 :: 11^2 : 95 :: 8^2 : 50$, which are all in the same proportion. For the same reason, $1^2 : .0796 :: \overline{11.3}^2 : 10 :: 5^2 : 2 :: 11^2 : 9.6$, are in the same proportion, nearly.

Likewise in the square which is equal to the circle, whose diameter is 1, the side is $.886227 = \sqrt{.7854}$. And the side of the inscribed square is $.707107 = \sqrt{\frac{1}{2}}$.

Also in the circle whose circumference is 1, the diameter is $.318310 = \frac{1}{3.1416}$; and the area $.079577 = \frac{.7854}{3.1416}$; and the side of the equal square, is $.282095 = \frac{.886227}{3.1416}$. And side of the inscribed square is $.225079 = \frac{.707107}{3.1416}$.

Whence as 7 to 11, so the inscribed square to the circle's area.

PROB. X.

To find the area of the sector of a circle ADC.

RULE.

24. Multiply the arch AD by the radius AC, and half the product is the area.

Ex.

Fig. 24.

Ex.

Let the radius AC *be* 10, *and the arch* AD 7.

$$\begin{array}{r} 10 \\ \underline{7} \\ 2)70(\ 35 \text{ area.} \end{array}$$

PROB. XI.

To find the area of the ſegment of a circle, ADB.

1 RULE.

Multiply the baſe AB by the height DE; and $\frac{2}{3}$ the product is the area, when the height is ſmall; or $\frac{3}{4}$ the product, if near a ſemicircle.

2. *Otherwiſe.*

Let DE be the height, and AE or EB half the baſe; then multiply the ſquare of DE, by .392, to which add the ſquare of AE. Extract the ſquare root of the ſum, which multiply by $\frac{4}{3}$ DE, for the area.

3. *Otherwiſe.*

Let D be the vertex, and draw the chord AD; to the ſquare of AB, add the ſquare of DE, extract the ſquare root of the ſum; to twice that ſquare root add AD, which multiply by $\frac{4}{15}$ DE for the area.

4. *Otherwiſe.*

Find d the diameter, then $\frac{80d - 39\text{DE}}{16d - 3\text{DE}} \times \text{AD} \times \frac{4}{15}\text{DE}$, is equal to the area.

5. *Other-*

Fig. 24.

5. *Otherwise.*

Find d the diameter, then $\frac{5d - 3DE}{5d - 4DE} \times \frac{2}{3}AB \times DE$ = the area of the segment.

Note, a zone adjoining to the center may be found by this Rule, by subtracting the segment from the semicircle.

6. *Otherwise.*

$\frac{4AD + 3AB}{5} \times \frac{2}{3}DE$ = area of the segment ADB.

Note, the 1, and 5th Rules, will also find the area of an elliptic segment.

7. *Or thus.*

Let DI = IE, then $\frac{AD + 4AI}{3} \times \frac{4}{5}DE$ = area of the segment.

8. *Or thus.*

Let arch AD + AE = p, arch AD — AE = q. Then $\frac{p \times DE^2 + q \times AE^2}{2DE}$ = area of the segment.

Examp.

Let the base AB *be* 32, *chord* AD 20, *height* DE 12. *Then by the 6th way,*

```
 20      32       12
  4       3        2
 ---     ---     ---
 80      96     3)24(8
         80     ------
        ---      35.2
      5)176(35.2    8
                -------
                281.6 the area.
                -------
```

When the segment is greater than a semicircle, find the lesser segment, which subtract from the whole circle.

PROB.

Fig.

PROB. XII.

To find the area of an ellipsis ACBD.

RULE.

Multiply the transverse AB by the conjugate axis CD; and the product by .7854, for the area. 8.

For an elliptic segment, see the last Problem.

Example.

Suppose the transverse be 32, and the conjugate 24.

32	.7854
24	768
128	62832
64	47124
	54978
768	603.1872 area.

By the Rule.

Find a mean 27.7 between 32 and 24. Then set 1 on D to .785 on C; and against 27.7 on D, is the content 603. on C.

PROB. XIII.

To find the area of a Parabola ABC.

RULE.

A parabola is made by cutting a cone by a plane parallel to its side. 25.

Multiply the base AC by the height BD; and $\frac{2}{3}$ that product is the area.

Fig. 25.

Examp.

Suppose the height BD = 100, *and base* AC = 68.

```
 68        6800
100           2
----     ------
6800     3)13600(4533, the area.
----
```

By the Rule.

Set 1 on A, to $\frac{1}{3}$ the height (33.3) on B, and against twice the base (136) on A, is the content (4533) on B.

PROB. XIV.

To find the segment AEFC *of a Parabola.*

RULE.

EF is parallel to AC, then $\frac{AC^3 - EF^3}{AC^2 - EF^2} \times \frac{2}{3}GD$ = the area.

Examp.

Let AC *be* 12, EF 8, GD 5, *to find the area.*

```
  12     8    144   80)1216(15.2    15.2
  12     8     64      80              5
----   ---   ----      --   ---     ----
 144    64     80     416           76.0
  12     8   ----     400              2
----   ---   1728     ---          -----
1728   512    512     160         3)152.0
----   ---   ----     160          50.66 area.
             1216     ---          -----
             ----       0
                      ---
```

Fig.

PROB. XV. 25.

To measure irregular Parallelograms, such as irregular planks, &c.

RULE.

Take the breadth in several equidistant places, the sum of which divide by the number of places, for the mean breadth; multiply the mean breadth by the length, for the area.

Examp.

Suppose a plank 16 *feet long, and the breadth in several places* 6, 7, 9, 9, 11 *inches.*

6	16
7	8.4
9	
9	64
11	128
5) 42 (8.4 mean breadth.	12)134.4(11.2 feet.

By the Rule.

Having found the mean breadth in inches; then set 12 on A, to the length 16 on B; and against the breadth 8.4 on A, is the content on B 11.2 feet.

PROB. XVI.

To measure any irregular figure, bounded by a curve.

1. RULE.

Draw AC for the base, then measure the three 26.
equidistant perpendiculars Aa, Bb, Cc; then

$$\frac{Aa + Cc + 4Bb}{6} \times AC = \text{area } AacC.$$

 2. *Other-*

Fig.

2. Otherwiſe.

27. Meaſure the four equidiſtant perpendiculars A*a*, B*b*, C*c*, D*d*; then

$$\frac{Aa + Dd + 3Bb + 3Cc}{8} \times AD = \text{area } AadD.$$

3. Or thus more exactly.

28. Meaſure the five equidiſtant perpendiculars A*a*, *Bb*, C*c*, D*d*, E*e*; then

$$\frac{7Aa + 7Ee + 32Bb + 32Dd + 12Cc}{90} \times AE = \text{area}$$

A*ae*E; but this is troubleſome to complete.

29. And the ſame holds, if the figure be curve on both ſides; drawing the baſe AC thro' it.

4. Otherwiſe.

30. Divide the baſe AI into any even number of equal parts, and

Let X equal ſum of the extreme ordinates A, I.

O = ſum of all the other odd ones, the 3d, 5th, 7th, &c.

E = ſum of all the even ones, 2d, 4th, 6th, &c.

L = length of two of theſe equal parts. Then

$$\frac{X + 2O + 4E}{6} \times L = \text{area of the curve.}$$ Where for every two ordinates, from the firſt; the curve ought to be concave the ſame way.

Examp.

26. *There is a curve line whoſe baſe is 67 feet, and three ordinates or perpendiculars erected,* A*a*, B*b*, C*c*, *are* 35, 40 *and* 28.

By the firſt Rule.

	35	223
	28	67
40	160	——
4	——	1561
——	223	1338
160	——	—— area.

6)14941(2490$\frac{1}{6}$.

PROB.

Fig.

PROB. XVII.

To find the ſurface of a vaulted roof, being like the arch of a bridge.

RULE.

Meaſure the length of the curve of the arch by a ſtring or otherwiſe; then multiply this length of the arch, by the length of the vault; the product is the ſurface.

Examp.

Suppoſe the arch be 30 yards, and the length of the vault $7\frac{1}{2}$.

$$
\begin{array}{r}
30\frac{1}{2} \\
7\frac{1}{2} \\
\hline
210 \\
15 \\
\hline
\end{array}
$$

Surf. 225 yards.

Cor. *If the arch be a ſemicircle, or any ſegment of a circle; its length may be found by* Prob. II. *without meaſuring.*

PROB. XVIII.

To find the ſurface of a pyramid, parallelopipedon, or of any ſolid bounded by planes.

RULE.

Meaſure every particular plane ſeparately; and the ſum of all is the ſurface.

Cor. *If it be an upright priſm; meaſure the circumference of the baſe, which multiply by the height of the ſolid.*

Fig.

PROB. XIX.

To measure the surface of a right cone, or a frustum of it.

RULE.

31. Multiply the circumference of the base by half the slant side, for the surface of the whole.

Or multiply half the sum of the circumferences of the top and bottom, by the slant side, for the frustum.

Or thus.

If AB be the side, and BC the Diameter. Then as 7 : 11 : : AB × BC : surface of the cone.

Example.

Let the side AB *be* 20; *diameter* BC 8.

```
   20
    8
  ---
  160
   11
  ---
  160
 160
 ----
7)1760
  251.4
Surface.
```

PROB. XX.

To find the surface of a right cylinder.

1. RULE.

32. Multiply the circumference of the base AB by the height AC, for the convex surface.

2. *Or thus.*

As 7 : 22 : : AB × AC : surface.

Examp.

Examp. Fig.

Let the side CA *be* 20, *diameter* AB 8. 32.

```
    20
     8
   ---
   160
    22
   ---
   320
  320
  ---  surface.
7) 3520(502.8
  -----
```

PROB. XXI.

To find the surface of a sphere.

1. RULE.

Multiply the circumference of the sphere ABCD, by the diameter AC, for the surface.

2. *Or thus.*

Find the area of a great circle of the sphere, which multiply by 4 for the surface. 33.

Or thus.

7 : 22 : : AC^2 : surface.

Examp.

Let AC = 9 *feet.*

```
 9    81
 9    22
--    --
81   162
     162
    ----
   7)1782
     254.6 surface.
```

Examp.

PROB. XXII.

To find the surface of the segment of a sphere.

1. RULE.

54. Multiply the circumference of the whole sphere by the height of the segment CD, for its surface.

2. Or thus.

To the square of the diameter of the base add 4 times the square of the height CD; the sum multiplied by .7854, gives the surface; or $3.1416 AC^2$ = surface.

3. Or thus.

As 7 to 22, so AC^2 to the surface ACB.

Cor. *Hence if the surface of any zone be required, work by the first rule,* viz. *multiply the circumference of the whole sphere, by the height of the zone.*

Examp.

If AC *be* 25 *feet, what is the surface* ACB?

25	625
25	22
125	1250
50	1250
625	7)13750(1964.3 feet.

PROB.

Fig. 35.

PROB. XXIII.

To find the surface of a cylindric ring.

1. RULE.

Add the thickness AB to the inner diameter BD, which sum multiply by the thickness AB, and the product by 3.1416^2, gives the surface.

2. *Or rather thus.*

The surface $= AB \times AD \times 9.87$.

3. *Or thus.*

From $10 AB \times AD$, subtract $\frac{1}{77}$ part thereof, for the surface.

Examp.

Let AB *be* 1, AD 12.

```
12        9.87
 1          12
--      ------
12        1974
          987
        ------
        118.44 surface.
        ------
```

PROB. XXIV.

To find the surface of a spheroid.

1. RULE.

Multiply the two axes AB CD together, and 36.
that product by 3.1416 for the surface of an oblong spheroid.

2. *Or*

Fig. 36.

2. *Or thus.*

To twice the tranſverſe (or axis of revolution) add the conjugate diameter, multiply the ſum by $\frac{1}{3}$ the conjugate; and the product by 3.1416 gives the ſurface.

3. *Otherwiſe.*

Meaſure the circumferences about AB and CD, which call P and Q. Then as 22 : 7 :: P × Q : ſurface of the ſpheroid; which is a ſmall matter too big, when the diameters AB, CD, are very unequal.

Examp.

Let AD *be* 10, CD 7.

10	3.1416
7	70
70	219.9120 or 220 the ſurface.

PROB. XXV.

To find the ſurface of any ſolid CAD, *generated by revolving about the axis* AB.

RULE.

37. Take with a ſtring the length of the arch CA or AD; and divide the ſtring into 3 equal parts at *e*, *f*; apply the ſtring again to the curve, and mark where the points *e*, *f*, fall: take the diameters there, which call P and Q, then $\frac{CD + 3P + 3Q}{8} \times$ 3.1416 A*fe*C = ſurface.

Note, If the curve does not come to the axis at A, you muſt take its diameter there, and add it to CD.

If

If the body be very long, divide it into two or more parts, and measure each part separately, by this Rule. Fig. 37.

Examp.

Suppose the three diameters at C, *e and f; that is* (D, P *and* Q, *to be* 12, 10, 7; *and the length of the curve* AfeC, 15.

10	7	12	3.1416	
3	3	30	63	
—	—	21	——	197.92
30	21	—	94248	15
—	—	63	188496	——
		—	——	98960
			197.9208	19792
			——	——
			8)	2968.80
				371.10
				Surface.

SCHOL.

Any proportion on the lines A and B on the ſliding rule, may be wrought on Gunter's line, with a pair of compaſſes; by extending from the firſt term to the ſecond; and ſetting that extent the ſame way, from the third to the fourth.

Or, extending from the firſt to the third; and ſetting that extent from the ſecond to the fourth.

Alſo, any of theſe proportions, which are wrought on the lines D and C, may be wrought on Gunter's line with compaſſes, thus. Extend from the firſt term to the third; and ſet twice that extent (the ſame way) from the ſecond term, and it will reach to the fourth.

Examp. If the diameter of a circle be 26 (*Ex.* 1. Prob. IX,); extend from 11 to 26, and that extent

Fig. 37. tent (turned twice over) will reach from 95 to the area 531.

And the ſame rule holds good for ſolids, by the proportions laid down in the following parts of this book.

Alſo if the diameter or ſide of a ſolid is known, and its content; the content of any other ſimilar ſolid may be found; whoſe diameter or ſide is given. Thus

Extend from the known ſide to the given ſide, and thrice that extent will reach from the known content to the content required.

PART

PART IV.

The Measuring of Solids.

PROB. I.

To find the solidity of a Cube ABCD.

RULE.

MULTIPLY the side AB by itself, and that product by the same side again; and the last product is the solid content, in the same sort of measure. 38.

This is called *cubing* the side AB.

Examp.

Suppose the side AB, *be two feet two inches.*

2 f. 2 in. = 2.166 feet.

2.166	4.691
	2.166
12996	
12996	28146
2166	28146
4332	4691
	9382
4.691556	
	10.160706
or	10.16 feet.

Fig. 38.

1. By the Rule.

Set 1 upon D, to the side (2.166) on C; and against the side (2.166) on D, is the content on C (10.16) feet, being the same sort of measure.

2. Or thus, by the Rule.

Set 12 upon D, to the height in feet (2.16) on C; and against the side in inches (26) on D, is the content in feet (10.16) on C.

PROB. II.

To find the content of a square prism, or parallelopipedon with a square base, ABCD.

RULE.

40. Square the side AB for the area of the base; which multiply by the height, gives the solidity.

Examp.

Suppose the base 2 feet 6 inches, and the height 19 feet.

2 f. 6 in. =	2.5	6.25
	2.5	19
	125	5625
	50	625
base	6.25	118.75 content.

1. By the Rule.

Set 1 upon D, to the height (19) upon C, and against the side of the base ($2\frac{1}{2}$) upon D, is the content (118.7) upon C, in the same measures.

2. Or

Fig.

2. *Or thus, by the Rule.*

Set 12 upon D, to the height in feet (19) upon C, and against the side of the base in inches (30) upon D, is the content in feet (118.7) upon C.

PROB. III.

To find the solidity of any sort of prism, ABCD.

RULE.

Find the area of the base AB, by some of the Problems in Part III. then multiply that base by the height AC, gives the solid content. 41, 42.

Exam.

Let the base AB, *be* 4.65 *feet, and height* AC 25 *feet.*

$$\begin{array}{r} 4.65 \\ 25 \\ \hline 2325 \\ 930 \\ \hline 116.25 \end{array}$$ feet.

1. *By the Rule.*

Set 1 upon A, to the base (4.65) upon B; then against the length (25) on A, is the content on B (116), in the same sort of measure.

2. *Or thus, by the Rule.*

Set 144 upon A, to the base in inches (669$\frac{1}{2}$) on B, then against the length (25) in feet on A, is the content in feet on B (116).

Fig.

PROB. IV.

To find the solid content of a cylinder, ABCD.

RULE.

39. Find the area of the base AB or DC, by Prob. 9. Part III. then multiply that base by the height AD, for the content.

Examp. 1.

Let the diameter of a cylinder be 14 *inches, and the height* 5 *feet.*

14 in. = 1.166 +	.785	1.067
1.167	1.36	5
8162	4710	5.335 content.
6996	2355	
1166	785	
1166		
	1.06760 base.	
1.360722		

1. *By the Rule.*

Set 1.13 upon D, to the height (5) upon C, and against the diameter (1.166) on D, is the content (5.33) on C, in the same measure.

2. *Or thus, by the Rule.*

Take $\frac{1}{4}$ the circumference for the girt. Then set .886 on D, to the height (5) on C; then against the girt (.917) on D, is the content (5.33) on C; in the same measure.

For here the diameter being 14 inches the circumference is 44, and the girt 11 or .917 feet.

3. Or

Fig. 39.

3. *Or thus, by the Rule.*

Set 3.54 upon D, to the height (5) on C; and against the circumference (3.66) on D, is the content (5.33) on C; in the same measure.

Ex. 2.

There is a cylinder whose height is 40 *feet, and diameter* 30 *inches, or circumference* 94.25 *inches; to find the content in feet.*

30 in. = 2.5 f.	.7854	4.908
2.5	6.25	40
125	47124	196.320 content.
50	1571	
	392	
6.25	4.9087	

4. *By the Rule.*

Set 13.54 on D, to the height in feet (40) on C; and against the diameter in inches (30) on D, is the content in feet (196.3) on C.

5. *Or thus, by the Rule.*

Take ¼ the circumference in inches for the girt. Then set 10.63 on D, to the height in feet (40) on C; and against the girt in inches (23.5) on D, is the content in feet (196.3) on C.

6. *Or thus, by the Rule.*

Set 42.54 on D, to the height in feet (40) on C; and against the circumference in inches (94.25) on D, is the content in feet (196.3) on C.

Fig. 39.

SCHOLIUM.

Obſerve here, that .886227 is the ſquare root of the area of a circle whoſe diameter is 1 $=\sqrt{.7854}$.

And .282095 is the ſide of a ſquare, equal to a circle, whoſe circumference is 1.

And $3.5450 = \frac{1}{.282095} = \frac{3.1416}{.8862}$, and is the circumference of a circle, whoſe area is 1.

And 10.63472 is the ſide of a ſquare, equal to a circle, whoſe diameter is 12 inches $= \sqrt{.7854 \times 144} = 12 \times .886227$.

And $42.538896 = 4 \times 10.634724 = 12 \times 3.5450$.

And $13.54055 = \frac{42.538896}{3.1416}$, and is the diameter of a circle, whoſe area is 144, and therefore is $= \frac{12}{.8862}$.

And 1.128381 is the diameter of a circle, whoſe area is 1, $= \frac{1}{.8862}$.

PROB. V.

To find the content of a cone or pyramid ADB.

RULE.

43. Meaſure the baſe, then multiply the baſe by one
44. third of the height DC; or elſe multiply the baſe by the height, and take $\frac{1}{3}$ the product, for the content.

Examp.

Fig.

Examp. 43.

There is a cone or pyramid whose height DC *is* 60, *and base* 51, *to find the content.* 44.

51	or	51
20		60
1020 content		3)3060
		1020 content.

By the Rule.

Set 1 on A, to $\frac{1}{3}$ the height (20) on B; and against the base (51) on A, is the content (1020) on B.

PROB. VI.

To find the content of the frustum of a cone or pyramid AD.

1. RULE

Find the areas of the two bases, at top and bottom AB, CD, and the height PQ; call the bases B and b. Then $\frac{B + b + \sqrt{Bb}}{3} \times PQ =$ content. 45. 46.

That is, multiply the two bases together, and extract the square root; to the sum of the bases add this square root, and multiply the third part thereof by the height.

2.° *Or thus.*

For a square pyramid, to the rectangle of the sides of the two bases, add $\frac{1}{3}$ the square of the difference; the sum multiplied by the height, gives the solidity of the frustum.

In a cone use the diameters instead of the sides; and the last product must be multiplied by .7854, for the frustum.

 3. *Other-*

Fig.

45. 3. *Otherwise.*

46. In a square pyramid, let D, *d*, be the sides of the bases, or diameters in a cone. Then $\frac{D^3 - d^3}{D - d}$ $\times \frac{1}{3}$ the height, = content of the square frustum; which frustum must be multiplied by .7854, for the content of the conic frustum.

Examp.

Suppose the greater diameter 36, *the lesser* 27, *and the height* 24.

36	36	972	23976 the square frustum,
27	27	27	.7854
252	9	999	167832
72	9	24	19181
			1198
972	3)81(27	3996	95
		1998	
			18830.6 the
		23976	conic frustum.

PROB. VII.

To find the content of a Prismoid or Pyramidoid GE.

47. This is a solid whose sides are streight, and the two bases parallel, but not similar figures; when they are poligons, 'tis called a *Prismoid*, or *Pyramidoid*; but if they are ellipses, 'tis called a *Cylindroid.* For this solid, when the bases are rectangles, or ellipses.

RULE.

To the bottom length (BE), add half the top length (AD); and multiply the sum by the bottom breadth (BC), for the first product.

To

To the top length (AD), add half the bottom Fig.
length (BE), and multiply the ſum by the top 47.
breadth (AG); for the ſecond product.

Multiply the ſum of theſe products by $\frac{1}{3}$ the height (AL) for the content.

When the body is a cylindroid, with elliptic baſes; you muſt uſe the diameters inſtead of the ſides parallel to them; and compute the ſolidity as before; and then multiply the laſt product by .7854, and it gives the content of the inſcribed cylindroid.

When the baſes are poligons, you may uſe the firſt rule in the laſt Problem.

Examp.

There is a ſquare priſmoid, whoſe dimenſions are BE = 40, AD = 36, CB = 32, AG = 12; *and height* AL = 28.

40	36	1856
18	20	672
58	56	2528
32	12	$9\frac{1}{3}$
116	112	22752
174	56	$842\frac{2}{3}$
1856	672	$23594\frac{2}{3}$ content.

But for the inſcribed cylindroid, multiply by .7854.

	23594.6
	.7854
	1651626
	188757
	11797
	943
Content,	18531.23

Fig. 48. 49.

PROB. VIII.

To find the content of any ungula, or hoof of a cone.

This is a ſolid made by cutting the fruſtum of a cone, by a plane paſſing through it diagonally, or touching the two oppoſite baſes, on contrary ſides.

1. RULE.

Meaſure the diameter of the baſe AB, and the conjugate diameter IO, of the ellipſis AICO; alſo the height CD. Then divide the difference of the cubes of AB, IO, by the difference of their ſquares; and multiply the quotient by .7854AB $\times \frac{1}{3}$ CD, for the content of either ungula.

Note, if d be = the diameter that is wanting; then you may take $\sqrt{AB \times d}$ inſtead of IO.

2. *Or thus.*

Let diameter AB = b, conjug. diameter IO = c, Then $\frac{bb + bc + cc}{b + c} b \times \frac{.7854}{3} \times$ AB, is the content.

Examp

Examp. Fig. 48.

Suppose AB 16, IO 9, CD 12; *to find the content.* 49.

```
   16    9    16         481
   16    9     9          16
   --   --   ---        ----
   96   81   144        2886
  16    --    81        .481
  --         256        ----
  256        ---     25)7696(307.8
  ---        481        75..
             ---        --
  307.8                 196
     12   1231.2        175
  -----    .7854        ---
   6156   ------         21
  3078     86184        ---
  -----    9849
3)3693.6    615
  1231.2     49
  ------  ------
          966.97 the content.
          ------
```

PROB. IX.

To measure irregular parallelopipedons, such as pieces of timber, &c.

1. RULE.

Girt it about in the middle with a ſtring, and meaſure it, and take a quarter of the girt in inches for the ſide of the ſquare; therefore ſquare it, and then multiply it by the length of the body; the product is the content.

2. *Otherwiſe.*

If the body is very irregular, divide it into two or more parts, and meaſure each part ſeparately, by this rule.

Or

Fig. Or else, gird it in several equidistant places, and add all the girths together, and divide the sum by the number of places, for the girt; then proceed as before.

Note, if the girt be taken in inches, and the length in feet, the last product, divided by 144, gives the content in feet.

Examp.

A piece of wood is 18 *feet long, and the girt round about the middle, is* 66 *inches; to find the content in feet.*

A quarter of the girt is $16\frac{1}{2}$ for the side of the square.

```
   16.5           272.25
   16.5               18
   ----          -------
    825          217800
   990           27225
  165            -------
  ------    144)4900.50(34.02 feet.
  272.25         432..
                 ---
                  580
                  576
                  ---
                   400
                   ---
```

3. *Otherwise by a Table.*

From this table, take the area corresponding to the side of the square in inches; and multiply it by the length in feet for the content. If the side of the square exceed the table, take half of it; and at last, take four times the content found.

A Table

Fig.

A Table for measuring Wood.

Side square	Area.	Side square	Area.	Side square	Area.
Inch.	Feet.	Inch.	Feet.	Inch.	Feet.
6	.250	12	1.000	18	2.250
6¼	.272	12¼	1.042	18½	2.376
6½	.294	12½	1.085	19	2.506
6¾	.317	12¾	1.129	19½	2.640
7	.340	13	1.174	20	2.777
7¼	.364	13¼	1.219	20½	2.917
7½	.390	13½	1.265	21	3.062
7¾	.417	13¾	1.313	21½	3.209
8	.444	14	1,361	22	3.362
8¼	.472	14¼	1.410	22½	3.516
[illegible]	.501	[illegible]	1.460	23	3.673
8¾	[illegible]	14¾	1.511	23½	3.835
9	.562	15	1.562	24	4.000
9¼	.594	15¼	1.615	24½	4.168
9½	.626	15½	1.668	25	4.340
9¾	.659	15¾	1.722	25½	4.516
10	.694	16	1.777	26	4.694
10¼	.730	16¼	1.833	26½	4.876
10½	.766	16½	1.890	27	5.062
10¾	.803	16¾	1.948	27½	5.252
11	.840	17	2.006	28	5.444
11¼	.878	17¼	2.066	28½	5.640
11½	.918	17½	2.126	29	5.840
11¾	.959	17¾	2.187	29½	6.044
				30	6.250

1. *By*

Examp.

If a piece of wood be 18 *feet long, and* 16½ *inches a quarter of the girt, as before.*

```
  1.890
     18
 ------
  15120
  1890
 ------
 34.020 the content as before.
```

1. *By the Sliding Rule.*

Meaſure the circumference in the middle, and take a quarter of it in inches, and call it the girt.

Then ſet 12 on D, to the length in feet (18) on C; then againſt the girt in inches (16½) on D, is the content on C (34) feet.

2. *Or thus, by the Rule.*

Take a quarter of the circumference, or the girt in feet, which here is 1.375; then,

Set 1 upon D, to the length (18) upon C; and againſt the girt (1.37) in feet on D, is the content (34) upon C.

Note, theſe are the ſame rules, as given in Prob. II.

SCHOL.

This is the common method of meaſuring wood, as practiſed in all raff yards, taking a quarter of the circumference in the middle, for the ſide of the ſquare. But it is objected, that it makes the content too little in proportion as 14 to 11; and that it ought to be meaſured as a cylinder. But to this it is anſwered, that before the wood can be ſquared and made fit for uſe, a great part of it goes to waſte in chips; and therefore the quantity of round timber ought

ought to be reckoned no more than what the in- Fig.
ſcribed ſquare will amount to, which ſhould be reduced in that proportion, that is as 11 to 7, which is more than 14 to 7. Another objection is, that in tapering timber, taking the ſide of the ſquare in the middle of the piece, makes the content alſo too little. But to this it is anſwered, that in moſt caſes, the great end is to be cut away, till it be of the ſame dimenſions as the leſſer end; or elſe it cannot be ſawn into proper ſtuff; and upon that account taking the ſquare in the middle makes too much of it.

On the other hand, when ſquare timber is meaſured, if one ſide of the ſquare is greater than the other, a quarter of the girt will make too much of it; ſo that this Rule will in ſome caſes give too much, and in others too little, by an inconſiderable quantity; and therefore may paſs well enough in common uſe. But if a round piece of wood is to be meaſured, every inch of which may be made uſe of, then I think it ought to be meaſured as a cylinder, or elſe as the fruſtum of a cone, if it be tapering, according to the Rules in Prob. 4th and 6th.

PROB. X.

To meaſure a heap of ſand, or earth; a pond of water, &c.

RULE.

Take the depth of it, in ſeveral places, at equal diſtances, as near as you can; add all theſe depths together, and divide by the number of them; the quotient is the mean depth. Then find the area of the baſe on which it ſtands, which multiply by the mean depth, and the product is the content.

If it happen, that the heap is ſo ſolid, that the depth cannot be taken, draw a ſtring over the top of it, parallel to the baſe, and meaſure the height

Fig. height of it above the baſe, for the greateſt depth of the heap. Then take ſeveral equidiſtant places upon the ſtring, and meaſure from the ſtring plumb down to the heap; then theſe being ſubtracted from the greateſt depth, give the depth in theſe ſeveral places. Then put the ſtring into a new poſition (or elſe draw another ſtring acroſs it), and take equal diſtances, and meaſure as before; and repeat the ſame, as oft as need be; till you get a ſufficient number of depths thro' the whole heap; all which muſt be ſet down and ſubtracted from the greateſt depth as before. And then you may apply the rule at firſt given.

Note, you may ſometimes reduce the heap into ſome ſort of a regular body, which will facilitate the meaſuring.

Examp.

There is a heap of ſtones, whoſe baſe is 15 yards; the greateſt depth 3, and the depth from lines drawn over it, taken in 9 places, are as in the table.

[illegible]	3.
$2\frac{1}{2}$	0.5
2	1
1	2
$1\frac{1}{2}$	1.5
2	1
1	2
$2\frac{1}{2}$	0.5
$1\frac{1}{2}$	1.5
	13.0

9)13(1.44
9
—
40
36
—
4

1.44
15
—
720
144
—
content, 21.60 yards.

PROB.

Fig.

PROB. XI.

To find the solidity of any of the five regular solids, from a side being given.

1. RULE.

Multiply the cube of the given side, by the number standing against the name of the body in this Table; the product is the solid content.

Tetraedron or Pyramid.	.1178511
Hexedron, or Cube.	1.000000
Octaedron.	.4714045
Dodecaedron.	7.6631188
Icosaedron.	2.1816951

2. Or thus.

Multiply the surface of the body by $\frac{1}{6}$ of the distance of two oppolite faces, when they are parallel.

Examp.

The side of an Octaedron is 27.

$$\begin{array}{r} .4714 \\ 27 \\ \hline 32998 \\ 9428 \\ \hline 12.7278 \end{array}$$ content.

PROB.

Fig.

PROB. XII.

To find the content of any irregular body bounded by planes.

RULE.

Suppose it divided into prisms and pyramids; then measure all these separately, and the sum of all is the content.

Examp.

50. The solid AHFD is divided into five parts, by planes drawn parallel to the sides. There is 1. The parallelopipedon KHGLNFIO. 2. The triangular prism AHKOIB. 3. The triangular prism EFNOIC. 4. The square pyramid IBDCO. 5. The prism GPF. And each of these may be computed by the foregoing Problems, and the sum will be the whole content. Part of this solid may be measured by Prob. VII. But when they are compounded they cannot, but must be cut in pieces, like this.

PROB. XIII.

To find the content of a cylindric ring.

RULE.

35. To the thickness of the ring, AB, add the inner diameter BD; multiply the sum by the area of the section AB, and the product by 3.1416 gives the solidity.

Examp.

Examp. Fig.

Let AB *the thickneſs be* 13, BD *the inner diameter* 35.
157, *to find the ſolid content* BAED.

157	.7854
13	169
170	70686
	47124
13	7854
13	
	area, 132.7326
39	170
13	
	92912820
169	1327326
	22564.54|20
	3.1416
	6769362
	225645
	90258
	2256
	1353
	Content, 70888.74

PROB. XIV.

To find the ſolidity of a ſphere or globe, ABCD.

1. RULE.

Multiply its ſurface by the radius AO; $\frac{1}{3}$ the 33.
product is the ſolidity.

2. *Otherwiſe.*

Say as 21 to 11, ſo the cube of the diameter, to the content: which is nearly as 2 to 1.

Fig. 33.

3. *Otherwise.*

Multiply the cube of the diameter by .5236; the product is the solidity.

4. *Or thus.*

Multiply the cube of the circumference by .0169, and you have the content.

5. *Or thus.*

Find the area of a great circle of the sphere, and multiply it by $\frac{2}{3}$ of the diameter.

Examp.

Suppose the diameter of a globe is 16 inches, or circumference 50.3.

16	256	4096	21)45056(2145½ content.
16	16	11	42...
96	1536	4096	30
16	256	4096	21
256	4096	45056	95
			84
			106
			105
			11

1. *By the Rule.*

Set 1.38 on D, to the diameter (16) on C, and against the diameter (16) on D, is the content (2140) on C, in the same measure.

2. Or

2. *Or thus, by the Rule.* Fig. 33.

Set 57.4 on D, to the diameter in inches (16) on C; and againſt the diameter (16) in inches, is the content (1.24) in feet.

3. *Or thus, by the Rule.*

Set 7.7 on D, to the circumference (50.3) on C, and againſt (50.3) on D, is the content (2140) on C, in the ſame meaſure.

4. *Or thus, by the Rule.*

Set 320 on D, to the circumference in inches (50.3) on C, and againſt the circumference (50.3) on D, is the content in feet (1.24) on C.

Cor. *To find the ſolidity of an Orb.*

This is only finding the ſolidities of the two ſpheres, and ſubtracting the inner from the outer.

SCHOLIUM.

The number .52359 is the ſolid content of a ſphere, whoſe diameter is 1.

And .0169 is the content, when the circumference is 1.

And $1.3819 = \sqrt{\frac{1}{.52359}}$.

And $57.444 = \sqrt{\frac{1728}{.52359}}$.

And $7.6923 = \sqrt{\frac{1}{.0169}}$.

And $319.76 = \sqrt{\frac{1728}{.0169}}$.