Fig. ther; that you may not miſplace them in the draught.

5. An eſtate may be ſo ſituated, that the whole cannot be ſurveyed together; becauſe one part of the eſtate cannot be ſeen from another. In this caſe, you may divide it into three or four parts, and ſurvey the parts ſeparately, as if they were lands belonging to different perſons; and at laſt join them together.

6. As it is neceſſary to protact or lay down your work as you proceed in it, you muſt have a ſcale of a due length to do it by. To get ſuch a ſcale, you muſt meaſure the whole length of the eſtate in chains; then you muſt conſider how many inches long the map is to be; and from theſe you will know how many chains you muſt have in an inch, and make your ſcale, or chuſe one already made, accordingly.

7. The trees in every hedge row muſt be placed in their proper ſituation, which is ſoon done by the plane table; but may be done by the eye without an inſtrument; and being thus taken by gueſs, in a rough draught, they will be exact enough, being only to look at; except it be ſuch as are at any remarkable places, as at the ends of hedges, at ſtiles, gates, &c. and theſe muſt be meaſured. But all this need not be done till the draught be finiſhed. And obſerve in all the hedges, what ſide the gutter is on, and to whom the fences belong.

8. When you have long ſtations, you ought to have a good inſtrument to take angles with, which ſhould be exact to a quarter of a degree at leaſt; and hardly any common ſurveying inſtrument will come nearer. And tho' the plane table is not at all a proper inſtrument to ſurvey a whole lordſhip with; yet it may very properly be made uſe of, to take the ſeveral ſmall internal parts; and ſuch as cannot

be

be taken from the main ſtations; and is a very quick and ready inſtrument. Fig.

Example.

Walking over the lordſhip, I pitch upon the four ſtations A, B, C, D, from which I can command the greateſt part of it, there I ſet up marks. Then I meaſure along AB, which is a right line, and the boundary on one ſide of the land. In meaſuring along, I ſet down the diſtances meaſured, when I come at the corners of the fields *a*, *a*, *a*, *a*, where the hedges come in, and likewiſe where I croſs the brook *bb*. Then having got to B, I ſet down the whole length of AB. 30.

Next I meaſure from B to C, and in my way, I ſet down how far I have meaſured when I croſs the hedges at *c*, *c*, *c*, *c*; and likewiſe where I croſs the brook *bb* again. Thus I meaſure forward till I come at C, and then I ſet down the length of the ſtationary line BC.

After the ſame manner I meaſure along the ſtationary line CA, obſerving to ſet down the interſections with the hedges, as before; till I come at A, where I ſet down the length of CA. Then the three points A, B and C, are determined; and may be laid down in the plan; and all the foreſaid points.

Being come to A again, I go from A towards B, and in my way I ſurvey every ſingle field adjoining to the ſtationary line AB. To do which the ſhorteſt way, I take the angles at every interſection *a*, that the ſides of each field makes with the ſtationary line AB; and then I meaſure their lengths; by which every field is eaſily laid down. In the ſame manner I proceed from B to C, and meaſure every field adjoining to BC. And then I go to A, and meaſure every field in my way thither.

Next

Fig. 30. Next I go from A towards D, and ſet down as before, all my croſſing of the hedges; and the length AD, when I come at D. And in like manner, I meaſure along DC, ſetting down all the croſſings of the hedges as before, with whatever elſe is remarkable, as where a highway croſſes at *d*.

Having finiſhed all the main ſtations, we muſt begin to make inner ſtations. Therefore I take F and G for two ſtations, being in the lines AB and BC, the hedges from F to G running almoſt ſtreight; then I meaſure from F towards G, and at *f*, I find a hedge going to the left, and going on to *g*, I find another hedge going to the right; and at *h* I croſs the burn. At *i* there is an angle, to which I make an offset. Going on further I come at a croſs hedge *l* going to the right; and then meaſure on to G the end of the ſtation. Now in going from F to G, we can take all the angles that the ſides of the fields make with the ſtationary line FG, and then meaſure their lengths; by which theſe fields may be laid down on paper.

Then I take another inner ſtation at I, and meaſuring from A to *o*, I come to the oppoſite corner of the field; then meaſuring on to *p*, I croſs a hedge; then I proceed to my ſtation I. Then I meaſure from I to F, and take an offset to *n* where the hedge croſſes the brook. Then I come to the corner of the laſt field at *m*; and then meaſure to the oppoſite corner at F, the other ſtation. In your going from A to I, you may take the angles that the hedges make with your ſtationary line AI, and meaſure theſe hedges, and then they may be laid down. And the like, in going from I to F.

All this being done, take a new ſtation H, and meaſuring from B towards H, all the hedges lie almoſt in a right line. So going along we come at a croſs hedge, and going further we come at a tree, in the hedge we meaſure along; going further we come

come at two other croſs hedges; and a piece fur- Fig.
ther we croſs the brook, going on we come at a 30.
croſs hedge; going on ſtill we come at another croſs hedge; all theſe hedges are to the left. Then going on ſtill further, we have a windmill to the right; and afterwards a croſs hedge to the left, and then we meaſure on to the ſtation H. Then meaſuring from H towards C, we have a houſe on the left; and then go on to C. And the fields may be all ſurveyed as you go along BH and HC, and then laid down. And after this manner you muſt proceed thro' the whole, taking new ſtations, till all be done.

PROB. XXII.

To ſurvey a whole county, or large tract of land.

1. Chuſe two, three or four eminent places for ſtations, ſuch as the tops of high hills or mountains, towers, or church ſteeples, which may be ſeen from one another; and from which moſt of the towns, and other places of note, may alſo be ſeen. And let them be as far diſtant from one another as poſſible. Upon theſe places raiſe beacons, or long poles, with flags of different colours flying at them; ſo as to be viſible from all the other ſtations.

2. At all the places, which you would put down in the map, ſet up long poles with flags at them of ſeveral colours, to diſtinguiſh the places from one another; fixing them upon the tops of church ſteeples, or the tops of houſes, or in the centers of leſſer towns. But you need not have theſe marks at many places at once, as ſuppoſe half a ſcore at a time. For when the angles have been taken, at the two ſtations, to all theſe places, the marks may be moved to new places; and ſo ſucceſſively to all the places you want. Theſe marks then being ſet up at a convenient number of places, and ſuch as may

be

Fig. be ſeen from both ſtations; go to one of theſe ſtations; and with an inſtrument to take angles, ſtanding at that ſtation, take all the angles between the other ſtation, and each of theſe marks, obſerving which is blue, which red, &c. and which hand they lie on; and ſet all down with their colours. Then go to the other ſtation, and take all the angles between the firſt ſtation, and each of the former marks, and ſet them down with the others; each againſt his fellow with the ſame colour. You may if you can, alſo take the angles at ſome third ſtation, which may ſerve to prove the work, if the three lines interſect in that point, where any mark ſtands. The marks muſt ſtand till the obſervations are finiſhed at both ſtations; and then they muſt be taken down, and ſet up at freſh places. And the ſame operations muſt be performed, at both ſtations, for theſe freſh places; and the like for others. Your inſtrument for taking angles, muſt be an exceeding good one, made on purpoſe with teleſcopic ſights; and of three, four or five feet radius. A circumferentor is reckoned a good inſtrument for this purpoſe.

3. And tho' it is not abſolutely neceſſary to meaſure any diſtance, becauſe any ſtationary line being laid down from any ſcale, all the other lines will be proportional thereto. Yet ſome of the lines had better be meaſured, to aſcertain the diſtances of places in miles; and to know how many geometrical miles there are in any length; and from thence to make a ſcale to meaſure any diſtance in miles. In meaſuring any diſtance, it will not be exact enough to go along the high roads, by reaſon of their turnings and windings; and hardly ever lying in a right line between the ſtations, which muſt cauſe infinite reductions, and create endleſs trouble to make it a right line; for which reaſon it can never be exact. But a better way is to meaſure in a right line with a chain, between ſtation and ſtation,

over

over hills and dales or level fields, and all obſta- Fig.
cles. Only in caſe of water, woods, towns, crags, rocks, banks, &c. where one cannot paſs; ſuch parts of the line muſt be meaſured by the methods of inacceſſible diſtances; and beſides, allowing for aſcents and deſcents, when we meet with them. And a good compaſs that ſhews the bearing of the two ſtations, will always direct you to go ſtreight, when you do not ſee the two ſtations; and in your progreſs, if you can go ſtreight, you may take offſets to any remarkable places. Likewiſe note the interſection of your ſtationary line with all roads, rivers, &c.

4 And from all your ſtations, and in your whole progreſs, be very particular in obſerving ſea coaſts, rivers mouths, towns, caſtles, houſes, churches, wind-mills, water-mills, trees, rocks, ſands, roads, bridges, fords, ferries, woods, hills, mountains, rills, brooks, parks, beacons, ſluices, floodgates, locks, &c. and in general all remarks or rarities.

5. After you have done with your firſt and main ſtationary lines, which command the whole county; then you muſt take inner ſtations, at ſome places already determined, which will divide the whole into ſeveral partitions: and from theſe ſtations you muſt determine the places of as many of the remaining towns as you can. And if any remain in that part, you muſt take more ſtations, at ſome places already determined; from which you may determine the reſt. And thus we muſt go thro' all the parts of the county; taking ſtation after ſtation, till we have determined all we want. And in general, the ſtationary diſtances muſt always paſs thro' ſuch remarkable points as have been determined before, by the former ſtations.

6. Laſtly, the poſition of the ſtationary line you meaſure, or the point of the compaſs it lies on, muſt be determined by aſtronomical obſervation.

Hang

Fig. Hang up a thread and plummet in the ſun, and obſerve when the ſhadow runs along that ſtationary line; and at that moment take the ſun's altitude; then having his declination, and the latitude; the azimuth will be found, by caſe 11th of oblique ſpherical triangles. And the azimuth is the angle the ſtationary line makes with the meridian; and therefore a meridian may eaſily be drawn thro' the map.

Or a meridian may be drawn thro' it by hanging up two threads in a line with the pole ſtar, when he is juſt north, which may be known from the aſtronomical tables. Or thus, obſerve the ſtar *Allioth*, or that in the rump of the great bear (being that next the ſquare); or elſe *Caſſiopeia*'s *hip*; I ſay, obſerve by a line and plummet when either of theſe ſtars and the pole ſtar come into a perpendicular; and at that time they are full north. Therefore two perpendicular lines being fixt at that moment, towards theſe two ſtars, will give the poſition of the meridian.

County ſurveying is by far the moſt difficult part of ſurveying; for the ſtations are ſo long and ſo many, that if the inſtrument to take the angles, be not very accurate, great errors will be committed; eſpecially after ſo many repetitions of taking angles, at ſo many different ſtations. And to meaſure all the roads and diſtances of places, would be an endleſs labour, and beſides they would not be exact; for no road leading from one place to another, goes in a right line; ſo that any one of them, muſt meaſure to more conſiderably than what it is. Therefore I cannot find that any better method can be practiſed, than what is here laid down. For if a ſtationary diſtance, ſufficiently long be ſet out and meaſured for a baſe; and the angles taken from theſe two ſtations, to all the places that can be ſeen; the ſituation of theſe places will be had more-

exactly

exactly than by any other method. But when we go to theſe ſecond ſtations to take angles, more care is required; and more ſtill, if we go to third, or fourth ſtations, or farther. So that we had need to have good inſtruments to perform this. All this is a work of time and expence. Fig.

PROB. XXIII.

To ſurvey a city or great town.

Suppoſe ABCDEFG be ſeveral ſtreets in any town or city, which you want to ſurvey. Begin at the meeting of two or more of ſome of the principal ſtreets, along which you can have the longeſt proſpect, for getting the longeſt ſtationary diſtances. Let B be the firſt ſtation; then along theſe ſtreets BC and BF, as far as you can ſee, cauſe two men to ſtand for marks; or rather ſet up ſtation ſtaves in wood pedeſtals. Then with your inſtrument placed at B, take the angle CBF, which note down. Then let the mark at F be carried to B, and the mark at C to D. Then you muſt obſerve at B a ſtreet running off to the right; and going along BC, meaſuring with a chain of 50 feet long, you come to a ſtreet at *b* on the left hand, ſet that down, and how far you have meaſured; and proceeding on to C, ſet down the length of BC in your book of obſervations. 31.

Then planting the inſtrument at C, take the angle BCD, and ſet it down. Then let the mark at B be brought to C, and that at D to E, and obſerve at C a ſtreet running off to the right; then meaſure along from C to D; and at *m* you come to two ſtreets going off, one to the left, the other to the right; then meaſure along to D, and ſet all down.

Plant

Fig. 31. Plant the inſtrument at D, and take the angle CDE, and meaſure along from D to E, and at 2 you have a ſtreet going to the right, and ſet all down. Bring the marks from C to D, and from E to F.

Place the inſtrument at E, and take the angle DEF, which ſet down. Then carry the marks from D to E, and from F to B; and meaſure along EF, and you come to two ſtreets at *n*, one on the right, the other on the left, which ſet down; then meaſure to F, where you have a ſtreet going to the right; ſet all down.

Laſtly, place the inſtrument at F, and take the angle EFB, which ſet down; and at F you have a ſtreet going to the right. Meaſure from F to B, and in your way you have two ſtreets at *p*, one on the right, and the other on the left; ſet them down and proceed to B; and ſet down the length FB, and then you arrive at B where you begun; ſo that part of the ſurvey is finiſhed.

After the ſame manner the part ABFG muſt be ſurveyed, taking the angles at A and B, at F and G; and meaſuring the diſtances AB, BF, FG and GA. And ſo you muſt proceed piece by piece, by this method of circulation, till the whole be finiſhed. And all the parts muſt at laſt be joined together, in their true ſituations.

Obſerve in every part to take in as much of the town as you can at a time; and to cloſe (that is, to come round to the ſame point again) with the feweſt ſides or ſtations you can, not exceeding 5 or 6, if you can help it; for the more ſides the leſs exact. Your chain muſt be 50 feet long, a link to a foot.

At every ſtation, and at other places where it is neceſſary, meaſure the breadth of the ſtreets, taking them as offsets; and as you go along, take offſets to all remarkable places, on the right and left, and to all ends of ſtreets, and to all turnings and

corners;

corners; and this muſt be done with a rod 10 feet long divided into feet; for all theſe meaſures muſt be taken, and laid down in feet; and ſet all down regularly in your ſurvey book. And to help your memory, it will be convenient to have an eye draught of each part of the town, as you ſurvey it; and always take notice what way you go about. Fig. 37.

Having thus taken your high ſtreets, you may in the ſame manner proceed with the other ſmall ſtreets and lanes, noting down in your book, or eye draught, the offsets, as you find them to the right or left.

Your high ſtreets and croſs ſtreets being taken, you muſt take the meaſures of the houſes to the front, and the breadth of them, with your rod; and the dimenſions of courts and alleys, ſetting them down in your draught. You muſt be very exact and curious in taking the dimenſions of churches, the ſteeple, the buttreſſes, &c. And likewiſe halls, inns of court, colleges, eminent houſes, &c.

SECT. II.

Of Planning or Plotting of Ground; and of Caſting up, Dividing, Reducing, &c.

PROB. I.

To make a ſcale to lay down any plan with.

32. FIRST conſider the bigneſs of your ſcale, or how many equal parts you will have in an inch. Then draw the indefinite line AB, and two more parallel thereto, the one very near. Then take what length you deſign for 10 in your compaſſes; run over the line AB, with the compaſſes, as many of theſe lengths as you pleaſe. Divide the firſt part at A into 10 equal parts; and at all the other points of diviſion, write 0, 10, 20, 30, 40, &c. continued as far as you have occaſion for. And at the ends of the ſcale, and at all the points of diviſion, draw perpendiculars to AB, thro' all the three parallels, and the ſcale is finiſhed.

The uſe is this; take the length of any line you want, to lay down, off the ſcale in your compaſſes, and lay it upon any line in your plan, which you want it ſet off upon, by ſetting the compaſſes upon it; and it will reach to the point required.

And if you want to meaſure any line upon your plan; take the length of that line in your compaſſes, and apply it to the ſcale, and it will ſhew you the length.

There are other forms of scales, as diagonal scales, Fig.
where an inch or half an inch, is divided into 100 32.
parts; but these are so well known, that I need say no more about them.

If you have taken a survey, and want to put it into a map; you cannot lay it down, in the bounds intended, without a proper scale; and it will be difficult to light of one of a proper length already made; and therefore you must be obliged to make one yourself.

PROB. II.

To project or draw the plot of a triangular field.

If the field has been measured with the chain; 1.
then you have all the sides of the triangle ABC in chains, in your field book. Therefore draw any line AC for the base, and take the length of it with your compasses from the scale, and set it from A to C. Then take the length of AB from the scale, and setting one foot in A, describe an arch towards B. Then take the third side, BC from the scale in your compasses, and setting one foot in C, cross the former arch in B. Then from the point of intersection, draw the lines BA, BC; and BAC is the triangle required.

If the perpendicular and base be known, and either segment AP. You must set off AP equal to that segment, and raise the perpendicular PB of the length given, and draw BA and BC.

Otherwise, from an angle or two.

If you have two sides AC, CB; and the angle ACB. Draw AC, and with the compasses set off the length of it in chains from the scale. Then by the contract or line of chords, make the angle ACB of the given degrees; and draw CB, on which set the length (from the scale) to B; and draw AB.

Or if the ſide AC, and the angles A and C be
11. given. Draw AC of its due length taken off the ſcale. Then make two angles at A and C, equal to theſe given; and draw AB, CB to interſect in B.

Laſtly, if you have given AC, AD, DB, and angle ADB. Make the angle ADB equal to that given. And DA, DB of the lengths given; and AC equal the given baſe. And draw BC, BA.

Cor. *This is the foundation of deſcribing any poligon; for they may always be divided into triangles. And all the triangles may be deſcribed one after another, as above.*

PROB. III.

To lay down the plan of any field upon paper; from the meaſures taken in the field; and plotting offsets.

15. If the field has been divided into triangles, and all the ſides have been meaſured with the chain; as ABCDE; where the lengths of the ſides are ſet down in the field book, or elſe written upon the ſides, in a foul draught. Draw any line DE, and from the ſcale ſet off its length in chains, from D to E. Then with the length of EA and one foot in E, deſcribe an arch; and with the length of DA and one foot in D, croſs that arch in A; then A is another angle of the field. In like manner having the three ſides AD, DB, AB, lay down the triangle ADB, gives the angle B. And from the three ſides DC, DB, BC, lay down the triangle DBC, and we have all the ſides and angles of the figure.

16. But if all the lines are drawn from a point within as F. Then with the three ſides DF, EF, DF, lay down the triangle DEF; and in like manner the triangle EAF; then ABF; then BCF; and laſtly CDF. For in each triangle, there is given all the three ſides.

If the sides and included angles are given, as in the figures, ABD. Draw the line AS of the length given; and make the angle ASB equal to the given degrees; drawing SB, making it of its due length, and draw AB: then you have the triangle SAB. Then from the sides SB, SC and the included angle BSC, or else ASC, you will project the triangle BSC, in the same manner; as also CSD; and so on till the whole figure is laid down. Fig. 20. 21.

When all the sides round the figure are given, and an angle opposite to each of them, as in the figure ACE. First draw the side EA from the given angle, and set off its length to A. Then make the angle AEB equal to that given, drawing EB. Then take AB in your compasses, and with one foot in A, cross the line EB in B, and draw AB. Then make the angle AEC or BEC equal to the angle given; and with BC in your compasses and one foot in B, cross the line EC in C; and draw BC. Then make the angle AED or CED equal to that given; and take the length of CD in your compasses, and with one foot in C, cross the line ED in D, and draw CD; and you have laid down the whole figure. But *note*, you ought to know whether the angles ABE, EBC, ECD be acute or obtuse. 22.

When all the angles at one point or station, opposite to the sides or the field are given; and all the alternate angles round the field; and only one line FA measured. Draw the line FA of its due length, and make the angles AFB and BAB as given in your book, or foul draught; and the intersection of FB, AB, gives the point B. Again, make the angles BFC, FBC equal to their given angles, and BC, FC will intersect in C. Proceed thus round the figure, to find the points of intersection D and E. But if the station be taken within the field as S, and FS measured. Then SF, and 23. 24.

Fig. 24. the two angles at S and F, will determine the point A. Then SA, and the angles at S and A, will determine the point B. And the like for the points C, D, E. And at laſt you will have the whole figure.

25. When all the ſides and angles, in going round it, are meaſured, as in the figure ABCDE; where *a*, *b*, *c*, *d*, *e*, are the ſeveral ſtations. Make the angle *eab* equal to that given, and make *ab* equal to its given length. Then make the angle *abc* equal to its degrees, and *bc* equal to its number of chains. In like manner make the angles *bcd*, *cde*, *dea*, of their reſpective number of degrees; and the ſides *cd*, *de*, *ea*, of their reſpective lengths in chains; and the figure is deſcribed, all but the offsets.

And to deſcribe and plan all the offsets. At *a* ſet off the diſtances to the hedges at A. Alſo ſet off, in the line *ab*, the diſtances *af*, *ag*, *ah*, *ai*; and at *f*, *g*, *h*, *i*, raiſe perpendiculars, to *ab*, of their proper lengths, all taken from the field book; and at *b* make the offset to the hedge at B. Then thro' A and B, and the ends of all theſe perpendiculars, draw the hedge AB. Then at *b* and *c*, make offsets to B and C, and ſet off *bk* to *k* where it touches the hedge; and draw the hedge B*k*C. Then make the proper offsets at *c* and *d*, to C and D; and draw the hedge CD. Again, make an offset at *d* to D, and ſet off *dl*, where it touches the hedge; and *dm*, and at *m*, the offset; and at *e* an offset to E. Then draw D*l* and *l*E thro' the end of the perpendicular at *m*, for another hedge. Laſtly, ſet off two offsets from *e* to E, and from *a* to A, and draw the hedge EA. Then you have the true figure of the field laid down.

3. Again, let AB be any ſtationary line. Set off from A, the diſtances A*c*, A*d*, A*f*, AB, from the book. And on the points *c*, *d*, *f*, B, raiſe the perpendiculars,

...rticulars *co*, *do*, *fg*, *Bb*, of their due lengths, Fig.
... from the book; and thro' the ends *a*, *g*, *b*,
draw the hedge or boundary *Aacgb*. And the like
for any others.

General Observations.

All the lines belonging to the same figure must be taken from the same scale, with a pair of compasses; and it is best to use a diagonal scale, because the links can be expressed by hundredth parts; and the scale ought to be as large as the paper will permit, that it may contain the whole draught, and the bigger it is, the more exactly you will work. All offsets belonging to each line, must be set in their proper places, and of their due lengths, perpendicular to the station lines, and to be taken off the same scale. And every line that is inclined to the horizon (as in a hill side), must be first reduced to a horizontal line, before it be laid down.

And all angles must be laid down with a protractor or line of chords of a good radius. And those things must be laid down in their proper positions, places, and magnitudes, whether they be lines, angles, or offsets, or any remarkable objects that are to be determined; and all according to the directions of your field book, or foul draught, you plan it from. And all this according to the known rules of practical Geometry. And at last, the lines drawn thro' all the points thus determined, will give the true figure of the field.

PROB. IV.

To plot a field, as surveyed by the infallible.

17. 18. ...PO be the table with the paper of observations; there is here but one station, taken in the middle of the field. Unfold the paper of observation,

ſervation, and upon a vacant place of the paper, take a point S for your ſtation. Then
16. ſide of a parallel ruler upon the line 1, ſet the other ſide to the point S; and by its edge, draw the line SA, always the ſame way from S; that the line 1 lies from the center of the table; and ſet from S to A its length 16.20 chains. Then ſet one ſide of the rule to the line 2, and the other ſide to S, and draw the line SB equal to 10.12 chains. Then ſet to the line 3, and to S, and draw SC 14.68. Then ſet to the line 4 and to S, and draw SD 12.90. Then ſet to the line 5 and to S, and draw SE 7.25. Then ſet to the line 6 and to S, and draw SF 8.8[illegible]. Then ſet to the line 7, and to S, and draw SG 14.65. Then draw the lines ABCDEFGA for the field.

18. But if the ſurvey was taken by going round the
19. field. Let RT be the table. Unfold the paper, and chuſe any point A for the beginning. Lay one ſide of a parallel ruler upon the line 1, and ſet the other to the point A, and draw the line AB by its edge, to lye the ſame way from A, that the line 1 lies from the center. This muſt be obſerved in drawing all the lines; and make AB 18.10 chains. Set one ſide of the rule to the line 2, and the other to B, and draw the line BC, 6.18. Set one ſide to the line 3, and the other to C, and draw CD 17.75. Set one ſide to the line 4, and the other to D, and draw DE 10.32. Set to the line 5, and to E, and draw EF 10.45. Set to the line 6 and to F, and draw FG 9.00. Set to the line 7, and to G, and draw GA, which ought to be equal to the meaſured length 8.15, if the work is right. So your field is planned.

PROP. V.

To lay down the plat of a field, as surveyed by the magnetic needle.

Let ACF be a field surveyed by the needle, going round about, taking the ſtations at *a*, *b*, *c*, *d*, *e*. Thro' all the ſtations draw the meridians *am*, *bm*, *cm*, *dm*, *em*, as you come at them. Then beginning at *a*, make the angle *mab* equal to the bearing of the ſide *ab*, north-eaſterly, and thereon ſet its length in chains to *b*. The meridian *bm* being ſuppoſed to be drawn, make the angle *mbc* the bearing of *bc* ſouth-eaſterly, and make *bc* its length. Make the angle *mcd* the bearing of *d* ſouth-weſterly, and make *cd* its diſtance. Make the angle *mde* the bearing of *e*, ſouth-weſterly, and *de* its diſtance. Make the angle *mea* the bearing of *a* north-weſterly, and and *ea* its diſtance. Then the plan is finiſhed, excepting the offſets, which are laid down as directed in Prop. III. Thus, in going from *a* to *b*, you have no offſets at *s* and *t*, at B you touch the hedge. From *b* to *c*, you have four offſets at *k*, *f*, *h*, *i*, and you touch the hedge at C. From *c* to *d*, one offſet at *k*, at E touch the hedge. From *d* to *e*, five offſets at *m*, *n*, F, G, H; at *p* touch the hedge. From *e* to *a*, two offſets at H and A. The places of all theſe, muſt be marked upon the ſtationary lines; and their lengths ſet off perpendicular to.

Fig. 26.

PROB.

Fig.

PROB. VI.

To lay down the plan of a field, surveyed from two ſtations, by taking the angles round about, and meaſuring only the line between the ſtations.

27. Chuſe a convenient point upon your paper, for your firſt ſtation, as S. Thro' S draw a line, and thereon the ſtationary diſtance ST; then T is your ſecond ſtation. Then with a protractor or line of chords, make the angles TSA, TSB, TSC, TSD, TSE and TSF, each of the number of degrees, as you find them in your field book; and from S draw the lines 1, 2, 3, 4, 5, 6, out at length, numbering them as in the book. In like manner, make the angles at T your ſecond ſtation, STA, STB, STC, STD, STE, STF; and from T draw the lines 1, 2, 3, 4, 5, 6, as in the book. Then obſerve where the correſpondent lines interſect; as 1 and 1 interſect at A; 2 and 2 interſect at B; 3 and 3 at C; 4 and 4 at D; 5 and 5 at E; and 6 and 6 at F. Then A, B, C, D, E, F, are the corners of the field. Therefore drawing the lines, AB, BC, CD, &c. from angle to angle, ABCDEF will be the plot of the field. Here you muſt be ſure to take the interſections of the correſpondent lines, or thoſe numbered with the ſame figures; otherwiſe you'll get a falſe plan.

28. If the ſtationary diſtance be taken in a ſide of the field, as EF, or even out of the field as *st*, the work would have been the very ſame. For the correſpondent lines 11, 22, 33, &c. that form the angles which you muſt make about F and E, or *s* and *t*, will always interſect in the points A, B, C, &c. which determine the angles of the field.

An

where all been more ſtations, proceed Fig.
the ſame manner, to lay down the angles taken
and to draw lines, to interſect in the points
required.

PROB. VII.

To lay down the plan of any part of a country, with ſuch objects as appear, by taking obſervations at ſeveral ſtations.

This is done much the ſame way as the laſt Prob. 29.
only there are more ſtations concerned, which require to be laid down, one after another; and the diſtances to the objects meaſured.

Upon a ſheet of paper, take a convenient point S for your firſt ſtation, from which draw the line ST, and ſet thereon the ſtationary diſtance ST as in your book; then T is your ſecond ſtation. With a protractor or line of chords, ſet off the angles TSK, TSL, TSA, TSB, and TSC, as in your book. And draw the lines SK, SL, SA, SB, SC; and ſet thereon ſet the meaſured diſtances of each, as you find them in the book. Then the places of the objects K, L, A, B, C are determined.

Then, in like manner at T, make the angles STD, STE, STF, STI, equal to the angles which the ſtationary line makes with TD, TE, &c. as you find them in the book; and drawing TD, TE, TF, TI, ſet thereon the meaſured diſtances of D, E, F, I; and then their places are determined.

Next make the angle STV equal to the obſerved angle between the ſtationary lines ST and TV; and drawing TV, ſet thereon the meaſured diſtance of the ſtations T and V, as ſet down; then V is a third ſtation. Therefore at V, make the angles with TV as you obſerved them, as TVG, TVH, &c. and ſet off VG, VH; and on VG, VH, ſet their meaſured

ed diſtances, all which you have in the n
and then the places of G and H are det
And if you have more ſtations, and mo objects,
you muſt proceed in the ſame manner, to lay down
the angles obſerved, and to ſet off the diſtances.
And thus may the plan of a ſquadron of ſhips be
taken, or the map of a river, or any remarkable
objects, tho' inacceſſible.

PROB. VIII.

To draw the plan of a manor or large eſtate; part of a river, or a great town.

1. Having provided ſome paper, vellum or parchment to draw it upon; conſider how large you deſign to have your map. Then collect from your field book, the dimenſions of the eſtate in chains; and from hence you may make an eſtimate, how many chains will be contained in an inch; and conſequently what ſize your ſcale muſt be of, to plot it with; and it will be beſt to uſe a diagonal ſcale.

30. 2. Then take a convenient point A upon yo
paper for the firſt ſtation, and from thence draw
your firſt ſtationary line AB, of its due length, from
the field book; then will B be your ſecond ſtation.
Then having the diſtances AC and BC, you will
find the place C of a third ſtation, upon the plan.
And in like manner, having CD and AD, the p
D will be found for a fourth ſtation. For
longeſt ſtationary lines, and the moſt general ſt
muſt be determined firſt; and then the ne
pending on them, and ſo on to the leaſt or moſt
minute. And ſuch lines as ſerve only to find the
boundaries, muſt be drawn with black lead, to be
rubbed out again.

3. Then for plotting your obſervations taken
upon any of theſe lines; you muſt take the ſeveral
diſtances

...om your diagonal ſcale, and ſet them upon Fig
...s, or with a thin ſcale graduated the ſame 30.
...ways, and applied to theſe lines, prick off theſe diſtances, by the edge. Likewiſe every angle muſt be laid down truly by a protractor or line of chords. And all theſe things muſt be done in that order you took them, as directed by your field book, firſt reducing inclined lines to horizontal ones, by Prob. 9, Sect. 1. Thus in going from A to B, when we come at *a* there is a hedge; therefore ſet the diſtance Aa from A to *a*, and at *a* lay down the angle that the hedge *aa* makes with A*a*, and ſet the length of it from *a* to *o*, for the corner of the firſt field. Then I ſet off the diſtance from A to the next hedge, and draw that hedge in like manner as the other. Then I ſet off the diſtance of the brook from A to *b*, and draw the brook, making ſuch an angle with AB as I find in my book. In this manner I proceed till all be laid down from A to B. After the ſame manner I lay down every thing from B to C, and from C to A; and likewiſe from A to D, and from D to C. Theſe general ſtations being finiſhed; the inner and leſſer lines muſt be plotted, as from F to G, from A to ..., from ... to E. Then from B towards H, where I project a tree, and the croſſing of the brook, and a windmill on the right; then from H to C, where I project a houſe on the left hand. Thus you muſt go thro' all the leſſer ſtations, planning every thing as you find in your book, in their due places; as all ... and all offsets to hedges, &c. till all houſes, ... trees, and other remarkable objects, be put into your plan; proceeding from one part to another till all the parts be laid down; and if all the parts cloſe or fill up the plan, it is a ſign the work is right. And the plan is beſt done upon paper, or elſe laid off upon your vellum, by laying the paper ... it. And make always the north ſide

Fig. ſide of the map the uppermoſt. To do
30. there is required no more than a little practic[al] geometry, attending duly to what you have given in your field book. Sometimes it may be convenient to plot different parts in different papers; and put them altogether at laſt.

4. All hills muſt be ſhadowed, to diſtinguiſh them [from the] draught; likewiſe ſhadow the buttreſſes of [the] valleys very deep; and alſo towers, caſtles, houſes, and all ſuch things.

5. Put the trees in each hedge, into their proper places in the map; as they have been taken on a foul draught; and make prick lines on the outſides of the hedges, for the gutters; and theſe will ſhew what hedges belong to each field. And where there are gates, the hedges muſt break off.

6. All the fields and cloſes being laid down in their true order and ſituation; write the name of each field ſomewhere in the middle of it; and if you think proper, the number of acres it contains: tho' ſome people chooſe to have their contents in a book on purpoſe.

7. Colour the hedges with different colours, and put into your map, the manor houſe, and all other out-houſes, barns, ſtables; alſo rivers, roads, bye-lanes; and any windmill, water-mill, bridge brook, pond, wood, coppice, foot-way, or horſe-way. Draw trees in woody grounds, ſhadow mountainous grounds with hills and valleys. Expreſs bogs, fens, groves, highways, rivers, gates, ſtiles, &c. Draw the meridian line, and put a mariners compaſs in ſome bye place; and a ſcale of equal parts or chains; and laſtly the title of the map, thus.

A true and perfect ſurvey of the manor
(Townſhip) of ——, in the county of ——
Being (part of) the eſtate of ——
Anno Domini ——.

8 In-

very great extent, there ought to be drawn Fig.
vertical and horizontal lines thro' the map, 30.
noted by letters at the top and bottom and sides,
for the ready finding any field therein, mentioned
in a table.

9. The same rules, by which any field is laid 31.
down in the method of circulation, will equally
serve for laying down the streets of a town. For
take the point B for the first station, and draw BF,
and make the angle FBC equal to the degrees ob-
served, as in the survey book; and drawing BC,
set its length upon it to C, taken from the book.
And in that line from B to C, make offsets on each
side, and set off such distances as you have noted
down. In the same manner, set off the angles
BCD, CDE, DEF, and EFB. And set off the
distances CD, DE, EF, FB; and make such off-
sets on each, as you found in going along them;
as the survey book will direct. Then draw the
inner streets, till all the part BCDEF be planned.
In like manner lay down the part BFGA, and so
one part after another, till the whole is laid down.
But it is best to lay down every part as soon as
it is surveyed, whilst you have it in your mind.

PROB. IX.

To draw a map or plan of a whole county.

The laying down the plan of a county is upon
the principle, as laying down a survey, taken
from stations, as delivered in the 6th Problem.
Only here we are obliged to have more stations,
and to take more angles. And as a county is of a
large extent, every thing must be laid down with
the utmost exactness. Wherefore, to do this, chuse
a convenient point upon your paper for your first
station. From that point draw your stationary line,

and

Fig. 31. and ſet upon it, from that point, the diſtance meaſured between your two main ſtations, taken from a proper ſcale; and this gives your ſecond ſtation upon the paper. And the diſtance between them is the only meaſured diſtance you have. Then take a good inſtrument for laying down angles, and place its center at the firſt ſtation; and make an angle there with the ſtationary line, equal to the angle you took (to any certain place) at the firſt obſervation, when at the firſt ſtation; which your book will ſhew. Then make an angle with the ſtationary line, at the ſecond ſtation, equal to that you took (to the ſame place) at your ſecond ſtation; where this line (terminating the angle) cuts the other, is the place at which that particular mark ſtands: thus one place is determined. Again, at the firſt ſtation, make an angle with the ſtationary line; equal to that you obſerved, to a ſecond place. And likewiſe an angle, at the ſecond ſtation, which was obſerved there to the ſame place; and where this line interſects the firſt, is the place of the ſecond mark; ſo a ſecond place is determined. And thus go on, to lay down two correſpondent angles, taken out of the book, one at one ſtation, and the other at the other; and the interſection of the lines of that angle, gives the correſpondent place. And this work is always to be done both at the main ſtations, and at all inner ſtations.

All the lines ought to be taken from a diagonal ſcale, of a mile or half a mile to an inch, according to the largeneſs of the county; which may be eaſily collected from the dimenſions of the deſigned map, compared with the dimenſions of the county, in miles.

2. All your work muſt be laid down on a paper ſpread upon a flat table, and paſted down at the ends, or otherways fixt, ſo as not to be altered, till the whole be laid down. Or one may take a fine copper

copper plate, the bigness of the map, to project upon, and one set of observations may be laid down after another, as they are taken at the two stations; till the whole be compleated; and the lines may be drawn with a fine steel point. But here such observations as are taken to one hand, must be laid to the contrary hand, as in all engraving. Fig.

3. And the instruments made use of in laying down the plan, must be exceeding good and large. Your protractor or line of chords, or sector, or whatever instrument you use to lay down angles, must be of as large a radius, as the instrument you took the observations with, and as accurately divided. For most of your operations will be to lay down angles; and if they be not as exactly laid down in your map, as they are taken by observation, the map will be erroneous. And you might as well not have taken them exactly, as not lay them down exactly. Here the distances must all be reduced to horizontal lines, before they be laid down. And all the lines and angles are best laid down in the order they were taken, as your book of observations will shew. And the principal and most eminent stations must be planned first; and then others that depend immediately upon them, and the next, those that depend upon these; and so on. And thus every thing must be laid down, according to what you find in your book.

4. When all the towns and remarkable places are thus determined and mapped. The hills and mountains must be shadowed, the sea coasts drawn; all towns, castles, houses, churches exactly drawn, the largest of them in perspective. All roads and rivers delineated, and all bridges drawn; and every thing properly shaded, to distinguish them and shew what they are. All the divisions of the country must be drawn in prickt lines. And the name of each part written therein. And the boundaries

Fig. will be more viſible and diſtinct, if they be colour-
ed with water colours. All this is a work of great nicety.

5. Write the names of each diviſion in great letters within it; and the names of all towns and other places, beſide them. The cities muſt be diſtinguiſhed from market towns, and theſe from leſſer towns, &c. by certain marks. Meridians and parallels muſt be drawn through the map; alſo a mariners compaſs in ſome bye place; and a ſcale of miles; the longitude and latitude of places at the ſides and ends, and the title of the map in large characters. And in all maps, take notice, that the north ſide muſt be uppermoſt.

PROB. X.

To reduce one triangle into another of equal area, to have a given baſe, or a given perpendicular.

33. Let ABC be the triangle to be reduced, and AD the new baſe. From the vertex C draw the line CD, and from B the end of the baſe, draw BE parallel to it, interſecting the oppoſite ſide in E; then draw ED, and the triangle AED is equal to the triangle ACB. For the part FBD is equal to the part EBC, being contained between the ſame parallels EB, CD; and the part AEB is common to both triangles.

And if the perpendicular be given; let it cut the ſide at the point E. Draw EB to the oppoſite angle B; and CD parallel to it, from the vertex C, to cut the baſe AB in D; draw ED, then AED is the triangle required; which is proved as before.

PRO

Fig.

PROB. XI.

To reduce one triangle to another equal triangle, from a given point in the ſide.

Let ABC be the triangle, and E the point given. 33.
From the given point E, to the oppoſite angle B, draw the line EB. And from the vertex C, the line CD parallel to it, to cut the baſe AB in D. Draw the line ED, and the triangle AED is equal to the triangle ACB.

For the triangle EDB is equal to the triangle ECB, being contained between the ſame parallels; therefore to each of them add the triangle AEB, and then AED is equal to ACB.

SCHOL.

This Problem will be uſeful, when the area is not to run beyond a given point E.

PROB. XII.

To reduce a poligon to an equal triangle, from any angle, or point given in a ſide.

This is done by taking away one angle after ano- 34.
ther, till the whole be reduced. Let the poligon be ABCDEF, and let the point be at the angle D. Produce any ſide, that does not join upon D, as AF. Draw the firſt diagonal AC, and BH parallel to it, to cut the baſe FA in H. Draw HD to the next angle, and CI parallel to it, cutting AF in I. And draw DI for one ſide. Here AC, BH, ID, &c. muſt be drawn lines.

Again, draw IH, and EG parallel to it, to cut the baſe AF in G, then draw DG for the other

Fig. 34.

In ſhort thus.

From AC open your parallel ruler to B, to cut AF at H. From HD open to C, to cut at I; and draw ID.

From DF open to E to cut at G, and draw DG. Then IDG is equal to ABCDEF.

For by reaſon of the parallels CA, BH, the triangle CBA is equal to the triangle CHA. Therefore inſtead of the two ſides CB, BA; the ſingle ſide CH may be taken. Again, by reaſon of the parallels DH, CI; the triangle DHC is equal to the triangle DHI; and therefore inſtead of the two ſides DC, CH; the ſingle ſide DI may be taken. In like manner the triangle DEF is equal to DGF; whence the triangle IDG is equal to the figure ABCDEF.

Cor. 1. *Hence any poligon may be reduced to fewer ſides, by taking away one or more ſides of it.*

Thus ABCDEF is reduced to HCDEF, and that to IDEF, and that to IDG.

Cor. 2. *Hence a triangle may be reduced back to a polygon, or a polygon to another of more ſides. Which is uſeful in reduction, when the area is to terminate at a certain point, as* F.

Thus to reduce GD to the point F. Draw from F, to the next angle D, the line FD; and from G, the parallel GE, to cut the other ſide DE in E, if there be ſuch a ſide; otherwiſe take any point E in the parallel GE, and draw the two lines DE, EF.

Likewiſe to reduce DI to the point H; draw HD to the next angle D, and IC parallel to it, in which take the point C, and draw CD, CH.

Again, to reduce CH to the point A: draw AC to the firſt angle C, and HB parallel to it, and take B therein, and draw CB, BA.

SCHOL

Fig. 34.

SCHOL.

It will be beſt to take ſuch a ſide for the baſe, (and ſuch a point for the vertex, of the triangle (which may be in a ſide, as well as an angle), that when the operation is finiſhed, the height and baſe of the triangle may not be very unequal; obſerving that the higher the vertex is, the ſhorter the baſe will be. If the point for the vertex be taken in a ſide, it is the ſame thing, as if the poligon had one ſide more.

PROB. XIII.

To caſt up the content of a triangular field.

Let ABC be the triangle propoſed. Having got 11.
the baſe AC, and perpendicular BP, either by meaſuring them in the field; or elſe meaſuring them by the ſcale, upon the draught, laid down upon paper. Then

Multiply the baſe by the perpendicular, in chains and decimals. And take half the product for the ſquare chains, which divide by 10, gives the acres, and decimal parts; or, which is the ſame thing, from half the product, cut off five figures for a decimal. If you have a mind to find the roods and perches, multiply the decimal, firſt by 4, and then by 40.

Note, you may multiply half the baſe by the perpendicular, or the baſe by half the perpendicular, for the content in chains, &c.

Fig. 11.

Examp.

Baſe AC	8.27
Perpen. EP	4.23
	4135
	1654
	3308
Prod. —	35.1475
	10) 17.5737
Content	1.757 acres.

Or thus.

When two ſides AC, CB, and the included angle C, are given; or when AC, DB, and angle ADB are given.

Add together the logarithms of AC, CB, and ſine of the angle C; or elſe of AC, DB, and S.ADB; and from the ſum abate 10. Find the number anſwering to that logarithm. Then half that number divided by 10, is the content in acres.

Examp.

AC	8.27	—	0.91750
CB	6.88	—	0.83758
S.∠C	38 20	—	9.79255
	35.29	—	1.54763
Content	1.764 acres.		

Or thus.

When a ſide AC, and the adjoining angles A C, are given. To twice the log. baſe AC, add th log. ſines of the angles A and C; from the ſum take the log. ſine of the ſum of theſe angles (or

the third angle B), and throw away 10. Find the number of this logarithm; and divide half of it by 10, for the content. Fig.

Examp.

AC	8.27	0.91750
		0.91750
S. ∠C	38 20	9.79255
S. ∠A	55 30	9.91599
		21.54354
S. ∠B	86 10	9.99902
	35.04	1.54452

Content 1.752 acres.

Or thus, by the laſt data.

Divide the ſquare of the baſe AC, by the ſum of the natural cotangents of the angles A, C. Half the quotient divided by 10, gives the area.

8.27 1.2647
8.27 .6873

5789 1.9520)68.393 (35.04
1654 58 560 1.752 content
6616 in acres.

8330
7600

73000

68.3929

Otherwiſe thus.

When the three ſides are given. Find half the ſum of the three ſides, and ſubtract each ſide from it. Then multiply that half ſum and the three re-

Fig. 11. mainders continually; and the ſquare root of the laſt product, divided by 10, gives the area.

Rather add their four logarithms together, and take half of the ſum; and the number being found, the tenth part of it is the area in acres.

Examp.

AC	8.27,	
CB	6.88	
AB	5.14	
	20.29	Log.
½ ſum	10.14	1.00604
1 diff.	1.87	0.27184
2 diff.	3.27	0.51454
3 diff.	5.00	0.69897
		2.49139
	17.61	1.24569

Content 1.761 acres.

PROB. XIV.

To caſt up the content of a ſquare, a rectangle, or of any parallelogram.

Multiply the length, by the breadth (which is the neareſt diſtance from one ſide to the other); and the product, divided by 10, is the area.

Examp.

Fig. 11.

Examp.

Length 16.27
Breadth 10.60

97620
16270

172.4620

Content 17.246 acres, which may be reduced to roods and perches, by multiplying by 4 and 40.

PROB. XV.

To caſt up the content of a trapezium, ABCD.

12. Having found the diagonal BD, and perpendiculars AF, CG, either by meaſuring in the field, or upon paper, after it is laid down. Then

Multiply the diagonal, by the ſum of the perpendiculars; and half the product, divided by 10, is the content.

Examp.

Suppoſe the diagonal and perpendiculars, the ſame as were meaſured in Prob. XII. Sect. I.

1 perp.	5.18	Diag.	8.37
2 perp.	4.86		10 04
	10.04		33 48
			83700
			84.0348

Content 4.202 acres.

Or

Fig.

Or thus.

13. Where the lengths of the diagonals, and the angle they make, are given. Multiply the diagonals and the nat. ſine of the angle, continually; and the product divided by 10, gives the content.

Or rather, add the logarithms of the diagonals, and the log. ſine of the angle together, rejecting 10; find the number belonging; and divide half of it by 10 (or the whole by 20), gives the content.

Examp.

Suppoſe the angle and diagonals, as meaſured in Prob. XII. Sect. I.

AC,	7.54,	0.87737
DB,	6.17,	0.79028
S.AEB,	66 20,	9.96184
	42.61,	1.62949

Content 2.130 acres.

PROB. XVI.

To caſt up the content of any field, in form of any polygon.

14. Since we do not always want to make a draught of a field, but only to find its content; in ſuch caſes, we only meaſure ſuch lines as are neceſſary for finding the content; and then fewer meaſures will do, than for plotting.

The ſhorteſt way then to find the content, is to divide the field into trapeziums, as few as you can, or into trapeziums and triangles; and to meaſure the diagonals and perpendiculars in the field, and ſet them down; and likewiſe the baſes and perpendiculars of the triangles, if there be any. Then find

find the contents of each of the trapeziums, by the Fig. 3d Prob. and of the triangles, by Prob. XIII. and 14. the sum of all is the content of the field.

Examp.

Let the measures be taken as in Prob. XIII. Sect. I.

1. *Trapezium* CDEF.

1 perp.	3.54	Diag. 11.88
2 perp.	2.52	6.06
	6.06	7128
		71280
		71.9928

2. *Trapezium* BCFG.

1 perp.	4.47	8.19
2 perp.	3.72	Diag. 12.00
	8.19	1638
		819
		98.28

The triangle ABG.

base	10.56	71.9928
perp.	3.32	98.28
	2112	35.0592
	3168	205.3320
	3168	Content 10.2666 acres.
	35.0592	

Otherwise

Fig. 14.

Otherwiſe thus.

When we have not got the diagonals and perpendiculars, requiſite for caſting up. The field muſt be laid down on paper, by Prob. III. And then, by dividing it into trapezia and triangles, we muſt meaſure theſe as before. Therefore what we have to do, is, to meaſure the lengths of theſe diagonals, and baſes (with the ſame ſcale the field was drawn by) in chains and decimal parts; and their correſpondent perpendiculars, by taking the neareſt diſtance from the vertex or angle, to the baſe or diagonal.

Then multiply the diagonal of each trapezium by the ſum of its perpendiculars; and the baſe of every triangle by its perpendicular; and take half the ſum of all theſe products, which divide by 10, gives the acres and decimals. And the decimal may be reduced to roods and perches, if you pleaſe, by multiplying by 4 and then by 40.

And *note*, if any part of the area caſt up is without the field, ſuch part muſt be ſubtracted from the whole.

As for the offsets, and parts of irregular curves, if there be any; they muſt be computed ſeparately, and added to the former contents, if they lie without the ſtationary lines; or ſubtracted, if within. To find their contents will be ſhewn in the following Problem.

Otherwiſe thus.

4. Reduce the field into a triangle by Prob. XII. And if any ſide in it be very irregular, running in and out, a right line may be drawn by the eye, that takes in, as much as it leaves out. Then the baſe of this triangle multiplied by its height, and half the product divided by 10, will give the area in acres, as uſual.

This

This reducing a field to a triangle, is as exact a way as caſting up the contents of all the trapeziums and triangles ſeverally; and far more eaſily and ſpeedily done. Fig. 34.

Examp.

Let ABCDEF *be any field; which is reduced to the triangle* IDG. *And let the baſe* IG *be* 9.00, *and the perp. or the neareſt diſtance from* D *to* AF 6.46.

6.46
9.00

58.14

Content 2.907 acres in the triangle.

Therefore the content of the polygon is 2.907 acres.

SCHOL.

In uneven hilly ground it has been diſputed, whether the horizontal baſe, or the true ſuperficies, ought to be computed; and the greateſt part of ſurveyors ſeem to be agreed, that the ſuperficies ſhould be meaſured. And indeed, ſince ſurveying is but a mechanical practice, we may lay down this general rule, that ſince we meaſure by the chain, we muſt compute by ſuch dimenſions as we find by the chain. But if we compute from a map or draught, we muſt needs take our meaſures from the map, to compute by. And in theſe caſes there will always be a difference between the area meaſured and the area planned; more or leſs, as the ground is more or leſs hilly and irregular; and ſubject to aſcents and deſcents, and to hills and dales. But tho' the ſuperficies ought to be meaſured, only the horizontal baſe muſt be planned; for otherwiſe a map cannot be made too cloſe, *i. e.* for the adjoining parts to come together.

It

Fig. 34. It may be obſerved, that ſince all ground is more or leſs irregular, as well as the fences that incloſe it; therefore if ſeveral perſons ſurvey the ſame piece of ground; no two of them will give exactly the ſame content. Nay, if even the ſame perſon ſurveys it twice, he will differ from himſelf more or leſs; eſpecially if he ſurveys it by different methods.

If any geometrical curve is given to be ſurveyed; you muſt meaſure it by the rules of menſuration, for that purpoſe; for I cannot here lay down rules for all ſorts of figures, but ſuch as are common, and ſuch as come in practice every day. But for geometrical curves of all ſorts, I muſt refer to that book. But theſe things rarely come in queſtion.

PROB. XVII.

To caſt up the contents of offsets and parts of irregular curves.

1. The parts of a field contained by offsets are trapezoids and triangles; and therefore they muſt be meaſured as ſuch. And to find their contents, multiply each part of the ſtationary line by the ſum of the offsets on each end of it, and this being repeated as often as there are offsets; add them all together, and take half the ſum, and divide it by 10, gives the whole area contained by theſe offsets; which muſt be added or ſubtracted, according as they lay within or without the field.

If the offsets are at equal diſtances, find a mean offset, by dividing the ſum of them all, by their number. Then multiply this mean offset by the whole baſe, for the content in chains, which divide by 10.

Fig. 3.

Examp.

Suppoſe AB *be the ſtationary line, and* Aoegh *the bulge; and let the offsets be as in* Prob. III. Sect. I. *To find the area* ABhgeoA.

Offset at A	0.	Offset at *c*,	0.87
at *c*,	0.87	at *d*,	0.23
	0.87		1.10
Ac	1.94	*cd*,	1.48
	348		880
	783		440
	87		110
	1.6878		1.6280
Offset at *d*,	0,23	Offset at *f*,	0.81
at *f*,	0,81	at B,	0.47
	1.04		1.28
df,	.75	*f*B,	.28
	520		1024
	728		256
	.7800		.3584
			1.6878
			1.6280
			.7800
			4.4542

Content .2227 in acres, for the area of ABhgeo.

And after the ſame manner muſt all the offset areas be found in the field ADF, Fig. 26, and ſubtracted from the whole content, as they lay without the field.

2. If

Fig. 3. 2. If any part of the area be bounded by a regular curve, and the height be ſmall, multiply the baſe by the height, and take $\frac{2}{3}$ the product for the content in chains.

35. 3. When ſome part of the ground is bounded by an irregular curve, as *abc*, meaſure the baſe AC. Then for finding the content of AC*ca*; erect the perpendicular ordinates or offsets A*a*, B*b*, C*c*, at equal diſtances, and meaſure them. Then $\frac{Aa + Cc + 4Bb}{6} \times AC$, will be the area of AC*ca*, in chains.

Or more exactly, if *ad* be a curve; divide the baſe AD into 3 equal parts at B, C, and meaſure it, and the perpendicular ordinates A*a*, B*b*, C*c*, D*d*; then $\frac{Aa + Dd + 3Bb + 3Cc}{8} \times AD$, will be the area of AD*da*, in chains.

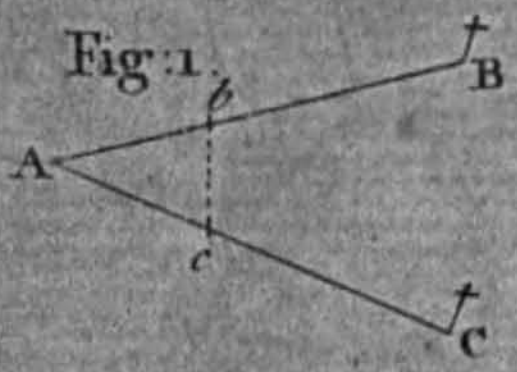
Or more exactly ſtill. Divide the baſe AE into four equal parts, at B, C, D; and meaſure the offsets or ordinates, A*a*, B*b*, C*c*, D*d*, E*e*, (as in Cor. Prob. III. Sect. I.) then will $\frac{7 \times \overline{Aa + Ee} + 32 \times \overline{Bb + Dd} + 12Cc}{90} \times AE$ be the area of AE*ea*, in ſquare chains.

SCHOL.

If a lordſhip is to be ſurveyed; after all the fields have been meaſured and laid down in the plan, the contents of them all are to be caſt up, one by one, by ſome or other of the rules and directions beforegoing; and their contents written in the field, or elſe put into a table, to have recourſe to upon occaſion. And ſuch a table or book is beſt made, to have the names of the cloſes go on alphabetically; eſpecially if the lordſhip be large, and contains a great number of cloſes.

PROB.

Fig: 1.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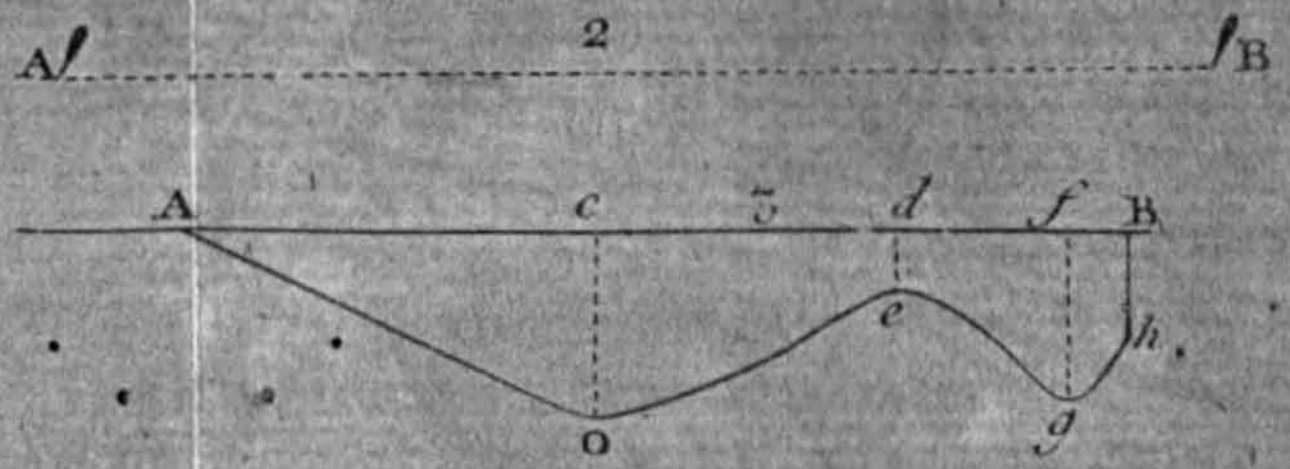
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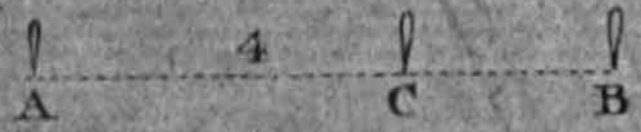
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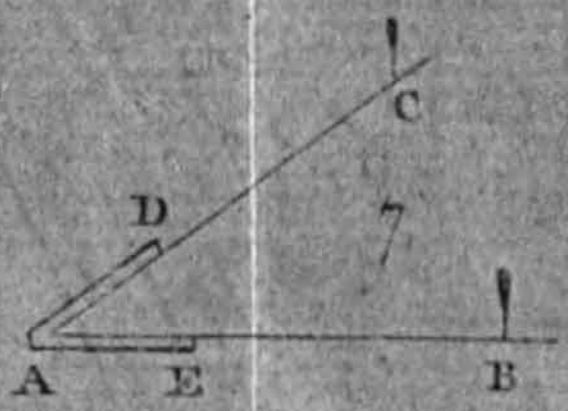
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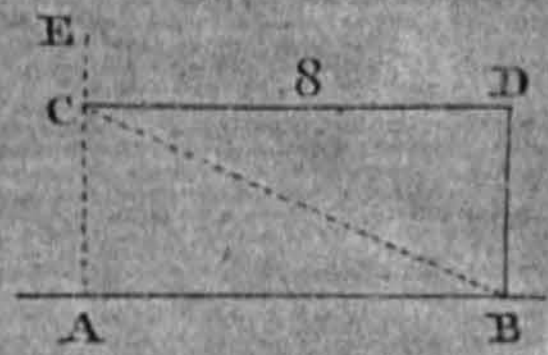
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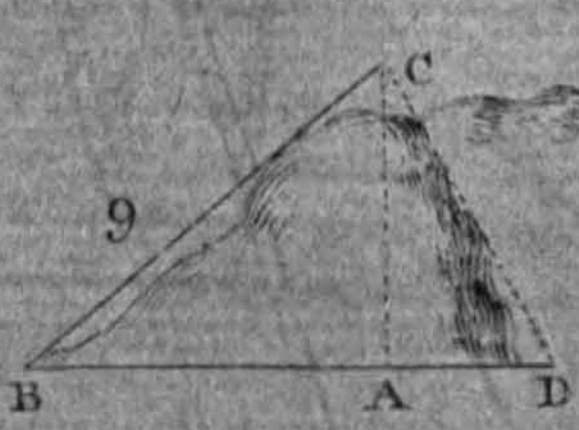
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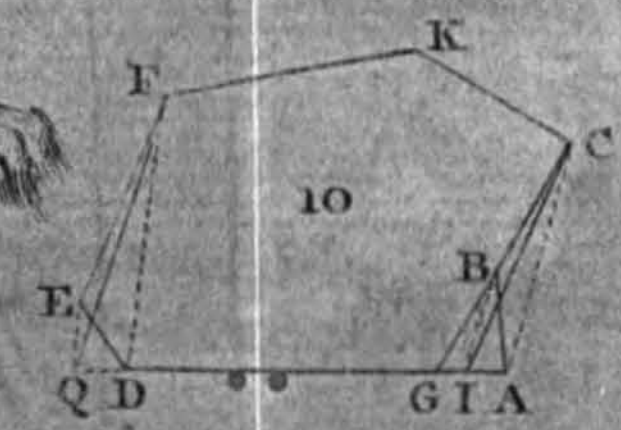
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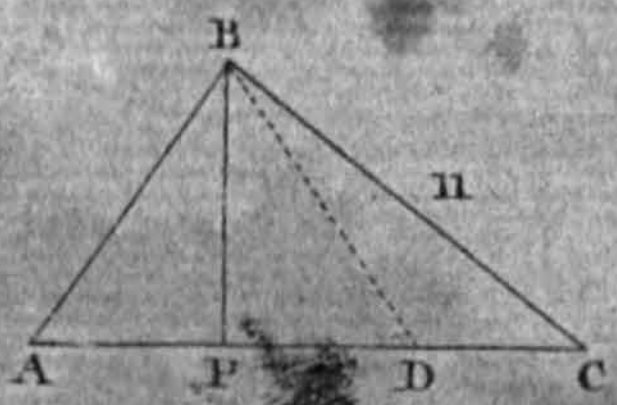
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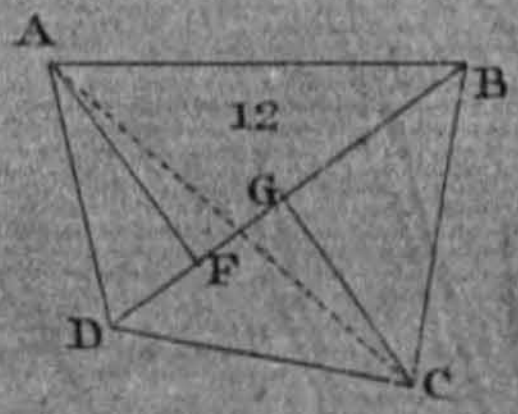
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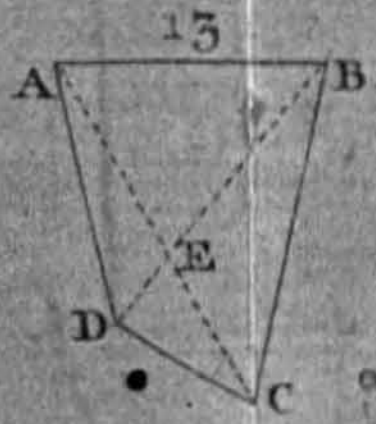
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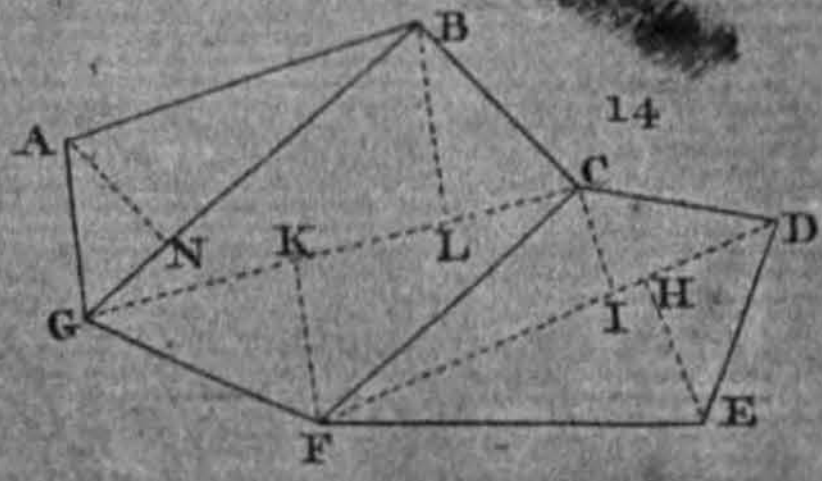
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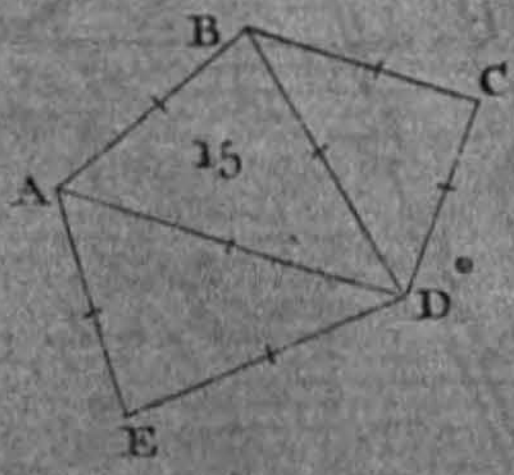
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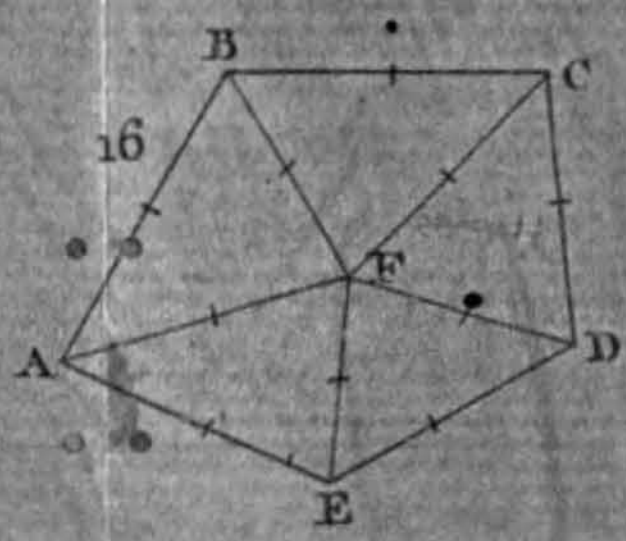
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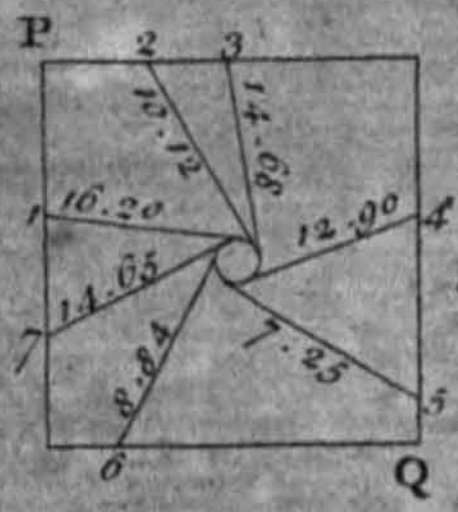
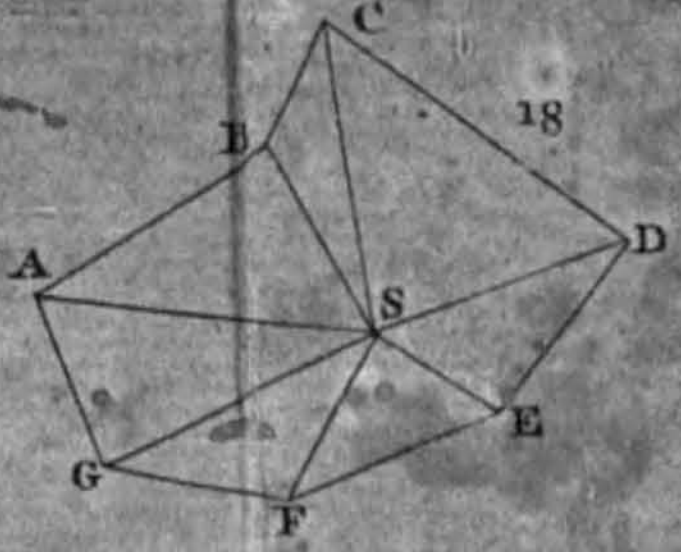
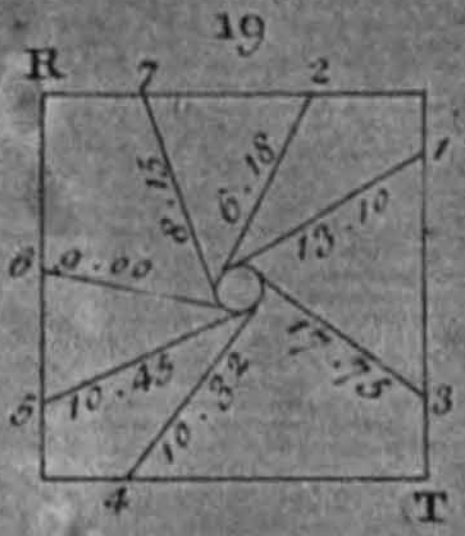
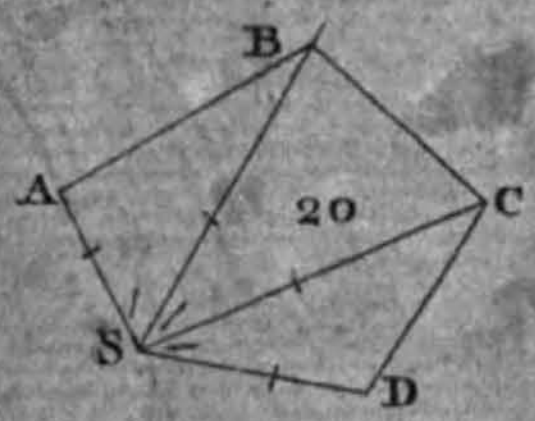
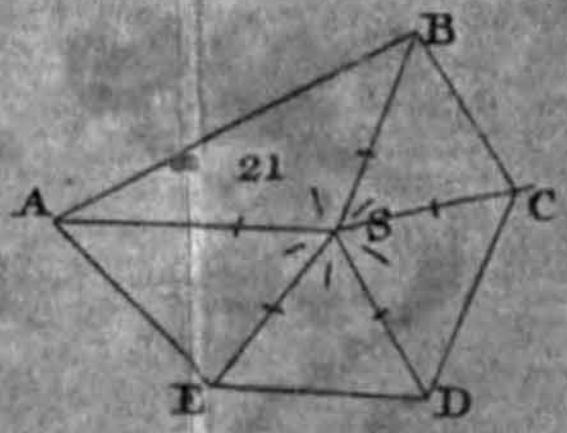
Fig: 17.

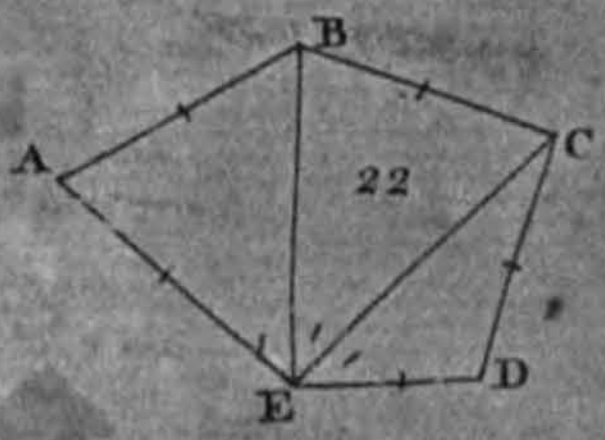


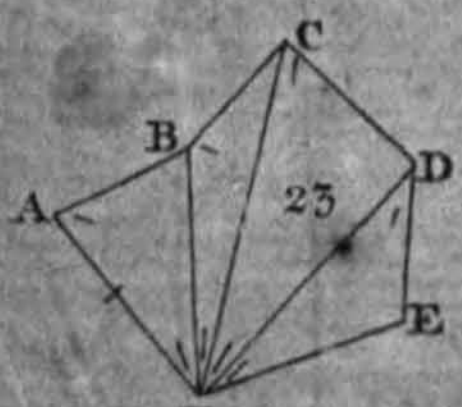


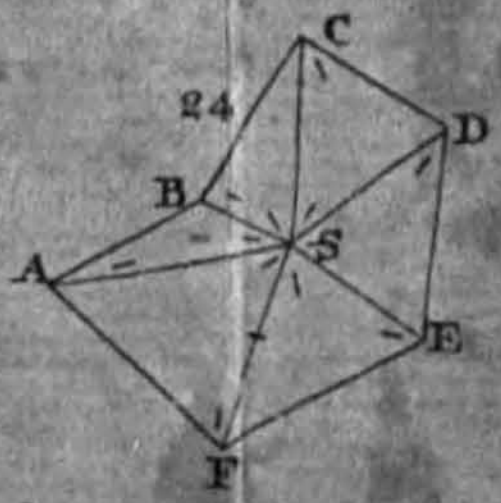


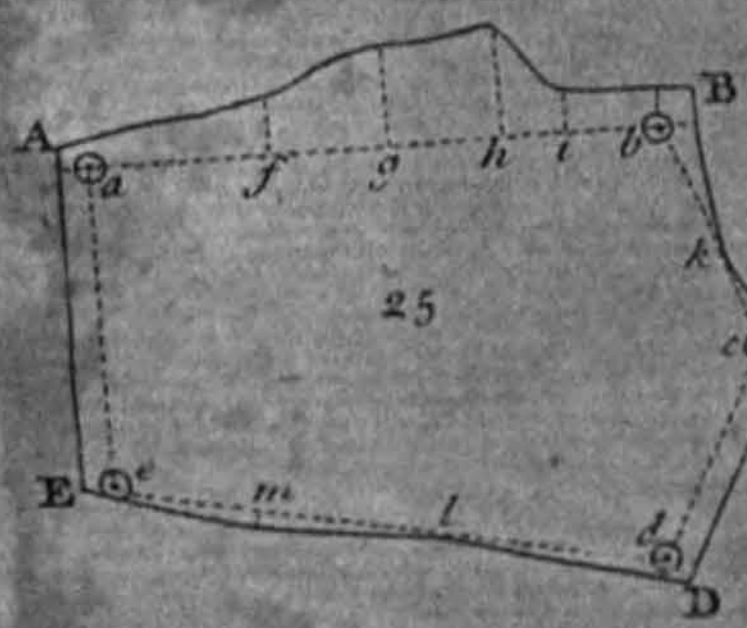


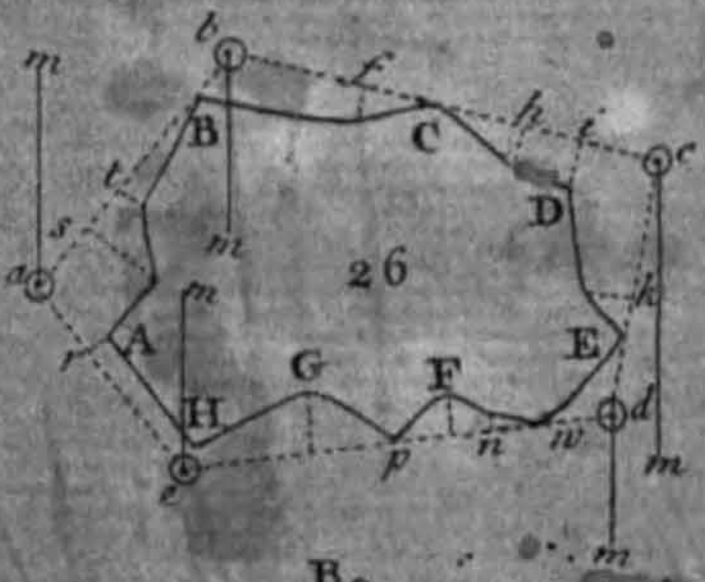


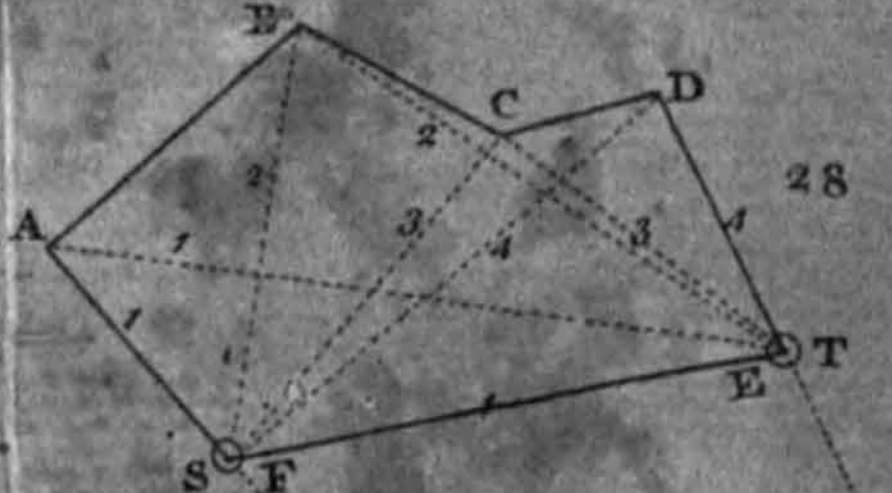


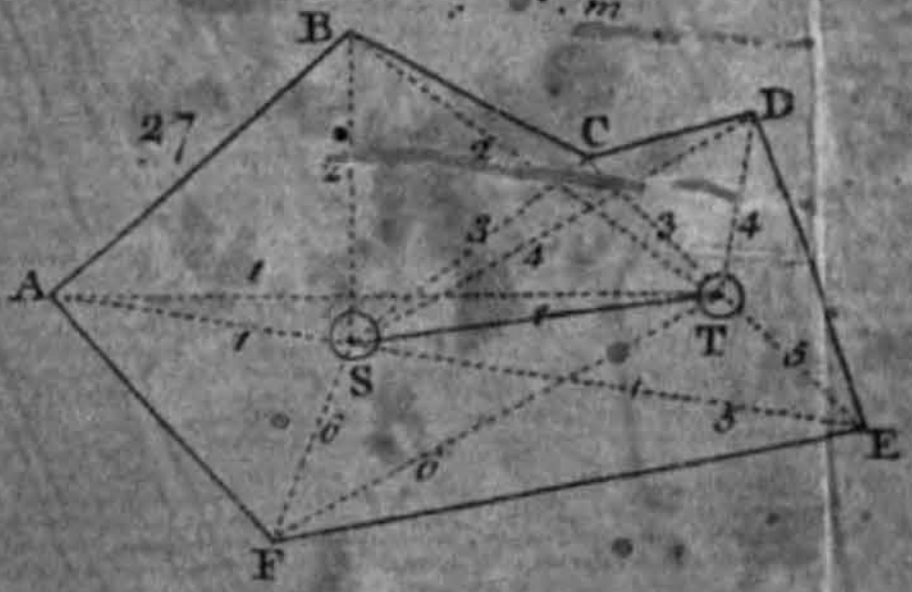












Surveying. Pl. II. page 90.

Fig.

PROB. XVIII.

To lay out any number of acres in a triangle, whose base and containing angle is given.

Multiply the base AC by the nat. sine of the angle C, for a divisor, by which divide 20 times the acres given; and the quotient is the side BC. 36.

Or thus by logarithms. To the log. of the number of acres, add the constant log. 11.30103, and reserve the sum. To the log. base AC, add the log. sine of the angle C; and subtract this sum from the former sum. The number of the remaining log. is the side CB.

For if A be the number of acres, then $\frac{AC \times CB \times S.C}{2 \times 10} = \text{area} = A$, therefore $CB = \frac{20A}{AC \times S.C.}$

Examp.

Suppose 17.64 *acres to be laid out in a triangle, whose base* AC *is* 24.81 *chains; and angle* C, 38 20. *To find the other side* CB.

	17.64,	1.24649
	const. log.	11.30103
		12.54752
	AC 24.81,	1.39463
	S.C 38 20,	9.79255
		11.18718
	CB 29.92,	1.36034 chains.

Cor. If you would lay out any number of acres in a triangle, of a given height.

Fig. 36. Divide twenty times the number of acres, by that height or perpendicular; the quotient is the length of the baſe, for any triangle.

PROB. XIX.

To lay out any number of acres in a rectangle, whoſe baſe is given.

Divide ten times the given number of acres, by the baſe given; and the quotient is the height of the rectangled parallelogram.

Examp.

Suppoſe 64.25 *acres laid down in a rectangle, one ſide of which is* 18.27 *chains; to find the other ſide.*

18.27)642.5(35.16 the height.
5481

9440
9135

3050
1827

1223

Cor. *If the acres are to lie in a ſquare, extract the ſquare root of ten times the number of acres given for the ſide of the ſquare.*

PROB

Fig.

PROB. XX. 37.

To divide a triangle in any ratio, from an angle; or to cut off any part of it, by a line drawn from an angle B.

Divide the oppoſite ſide AC in the point G, in the ſame ratio that the triangle is to be divided; and draw the line BG.

Or if the area be known; ſay as the whole area ABC, to the part to be cut off ABG; ſo is the whole baſe AC, to the part AG to be cut off, then draw the line BG.

As if you want $\frac{1}{2}$, $\frac{1}{3}$ or $\frac{1}{4}$ of the triangle cut off, make AG to be $\frac{1}{2}$, $\frac{1}{3}$ or $\frac{1}{4}$ of AC, and draw BG.

For triangles on the ſame baſe, and of the ſame height, are as the baſes; therefore ABG : ABC :: AG : AC.

Cor. 1. *If you want to cut off a given quantity from the triangle, without knowing its content; proceed by* Prob. XVIII.

Cor. 2. *If you would divide the triangle into any number of equal parts; it is only dividing the ſide* AC *into the ſame number of equal parts; and drawing lines to theſe points, from the oppoſite angle.*

PROB. XXI.

To divide a triangle ABC *in any proportion, by a line* DE *drawn parallel to a given ſide* BC: *or to cut off any part of it.*

Divide either ſide AC, in the given proportion, 38.
i. e. And make AE a mean proportional between AF and AC. Thro' E draw ED parallel to BC, which divides the triangle as required.

38. *Or thus.*

Say as the whole triangle, to the part next the angle A to be cut off; ſo is the ſquare of either ſide as AB, to a fourth number; extract the ſquare root of it, which ſet from A to D, and draw DE parallel to AC, for the dividing line.

For the triangles ABC and ADE are ſimilar; therefore ABC : ADE : : AB^2 : AD^2 : : AC^2 : AE^2 :: AC : AF.

PROB. XXII.

To divide a triangle ABC *in a given ratio, by a line drawn from a given point* D *in a ſide; or to cut off any part of it.*

39. From the given point D, draw a line DB, to the oppoſite angle B. Then divide AC in F in the given ratio; and from F draw FG parallel to DB, to cut CB in G; then draw DG, which divides the triangle as required; the part DGC being at FC; and the part ADGB, as AF.

For drawing BF, the triangles GFB, GFD, contained between the ſame parallels, are equal; add to both CFG, then CFB is equal to CDG; and therefore either of them is to the whole CBA as CF to CA.

Cor. 1. *If you want to divide the triangle* ABC *into any number of equal parts; divide the baſe* AC *into the ſame number of equal parts; from all the points draw parallels to* BD; *and where they intersect the ſides* AB, CB, *draw lines from thence to* D.

Cor. 2. *If you want to cut off a given quantity from the triangle, without knowing its content; proceed according to* Prob. XVIII.

PROB

PROB. XXIII.

Fig. 40.

To divide a triangle ABC *in a given ratio, or cut off any part of it, by a line* GI *perpendicular to any ſide* AC.

Draw BD perpendicular to the ſame ſide AC. And let r repreſent the part to be cut off, and s the whole area. Then take $AG = \sqrt{\frac{r}{s} \times AD \times AC}$, and at G erect the perpendicular GI, then AGI is the part required. If G falls between D and C, take the other part $(s - r)$, and meaſure from C.

For, triangle AGI : triangle ACB : : AG × GI : AC × BD : : (by ſimilar triangles) $AG^2 : AC \times AD$: : $\frac{r}{s} \times AD \times AC : AC \times AD$: : $r : s$.

PROB. XXIV.

41. *To divide a triangle* ABC *into three equal parts, by three lines drawn from the three angles,* A, B, C.

Make AD a third part of AC; thro' D, draw DE parallel to the next ſide AB. Biſect DE in F, and from F, draw to the three angles A, B, C, the three lines FA, FB and FC, which will divide the triangle as propoſed.

For ſuppoſing a line drawn from B to D, the triangle ABD would be $\frac{1}{3}$ of ABC, but ABD and ABF are equal, being contained between the ſame parallels; therefore ABF is $\frac{1}{3}$ the triangle ABC. Therefore the two equal triangles AFC and BFC are equal to $\frac{2}{3}$ of ABC, and each is $\frac{1}{3}$ of it.

Cor. *Hence a triangle* ABC *may be divided into 3 parts, by 3 lines drawn from the three angles, which ſhall be as any three numbers a, b and c.*

For make AD to AC as a to $a + b + c$; and
41. draw DE parallel to AB. Then make DF to DE as b to $b + c$. And draw AF, BF and CF. Then it is plain, AFB is as the number a, and AFC as b, and BFC as c. For AFD : BFE : : DF (b) : FE (c) : : DFC : EFC. Or AFC : BFC : : b : c.

PROB. XXV.

To divide a triangle ABC *into* 3 *parts from a given point* D *within it; to be to one another, as the numbers* a, b, *and* c.

42. In any ſide AC, take AH to AC, as a to $a + b + c$, and draw DH, and from the oppoſite angle B, draw BF parallel to DH; and draw DB and DF, then ABDF is the part repreſented by a.

In like manner take CG to AC, as another of the numbers b, to $a + b + c$; and draw DC, and BI parallel to it; and draw DI. But as I falls without the figure, to reduce it (by Cor. 2. Prob. 12.) draw DC and IN parallel to it, to cut BC in N, and draw DN; then BDN is the part repreſented by b. Therefore FDNC is the third part repreſented by c.

For ſuppoſing BH drawn, then by conſtruction ABH will be equal to a = ABF + FBH = ABF + FBD = ABDF.

Again, if BG be ſuppoſed drawn; then by conſtruction GBC = b = GBI — CBI = BDI — BCI = (DOI being a right line) BDO — OCI = BDI because DON = COI, by reaſon of the paralls DC and NI.

Cor. *By the ſame way, one may divide a triangle into as many parts as he pleaſes, from a given point* D *within it; and in any ratio aſſigned.*

But ſuch Problems as theſe, are more for the exerciſe of Geometry, than for any great uſe they are

are of in ſurveying. I ſhall only add another, about Fig. 42.
triangles of the ſame ſort.

PROB XXVI.

To divide a triangle ABC *in a given ratio, by a line* IH, *drawn thro' a given point* D.

Make AF to AC as the part AHI to the whole ABC, and draw BF, and alſo DG parallel to AC. 43.

Then put $P = \frac{AB \times AF}{DG}$. And take $AH = \frac{1}{2}P + \sqrt{\frac{1}{4}P - AG \times P}$, and draw IDH. And when AH comes out greater than AB, work for the angle C, inſtead of A:

For, the triangles AHI and AFB will be equal, whence $AH \times AI = AF \times AB$, and $AI = \frac{AF \times AB}{AH}$. And by the ſimilar triangles HGD, HAI, $HG : GD :: HA : AI$ or $\frac{AF \times AB}{AH} :: AH^2 : AF \times AB$.

Whence $HA^2 \times GD = HG \times AF \times AB = \overline{HA - AG} \times AF \times AB = HA \times AF \times AB - AG \times AF \times AB$, and $HA^2 = \frac{AB \times AF}{GD} \times HA - \frac{AB \times AF}{GD} \times AG = P \times HA - P \times AG$. Which reduced, and the root extracted, gives $HA = \frac{1}{2}P + \sqrt{\frac{1}{4}PP - AG \times P}$.

Cor. *If the point given, is without the field, at* g, *then will* $AH = \frac{1}{2}P + \sqrt{\frac{1}{4}P + Ag \times P}$. For then g being on the other ſide of A, Ag will be affirmative.

Fig. 44.

PROB. XXVII.

To divide a parallelogram in any ratio, or to cut off any part of it; by a line parallel to one of the ſides.

Divide the other ſide AB at E, in the given ratio, and make CF equal to BE, and draw the line EF, which will divide the parallelogram as was required.

If you want a certain part of it cut off; take BE the ſame part of BA, and CF of CD; and draw EF.

If you deſire to cut off ſo many acres, without knowing the content of the whole; divide your number of acres by the breadth of the parallelogram, EG; and the quotient will ſhew how far you are to meaſure along the ſide BA or CD; where you muſt draw a line, parallel to the ſide BC or AD, as EF.

Cor. *Hence, you may divide a parallelogram into any number of equal parts; by dividing the oppoſite ſides,* AB *and* DC *into the ſame number of equal parts, and drawing lines thro' the correſpondent points.*

PROB. XXVIII.

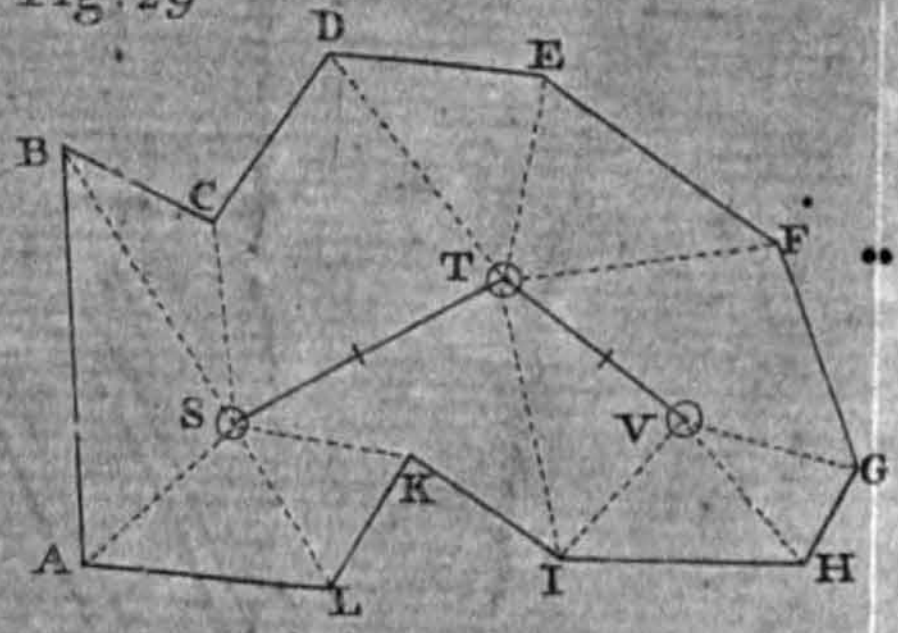
To divide a trapezoid in a given ratio, or to cut any part of it; by lines croſſing the parallel ſides.

45. Divide the parallel ſides AB, DC in the ſame ratio, and to the ſame hand, at E and F; and draw the line EF, which will divide the trapezoid in the ſame given ratio.

Otherwiſe, to cut any part off it; make AE the ſame part of AB, and DF the ſame part of DC; and draw EF.

Cor.

Fig: 29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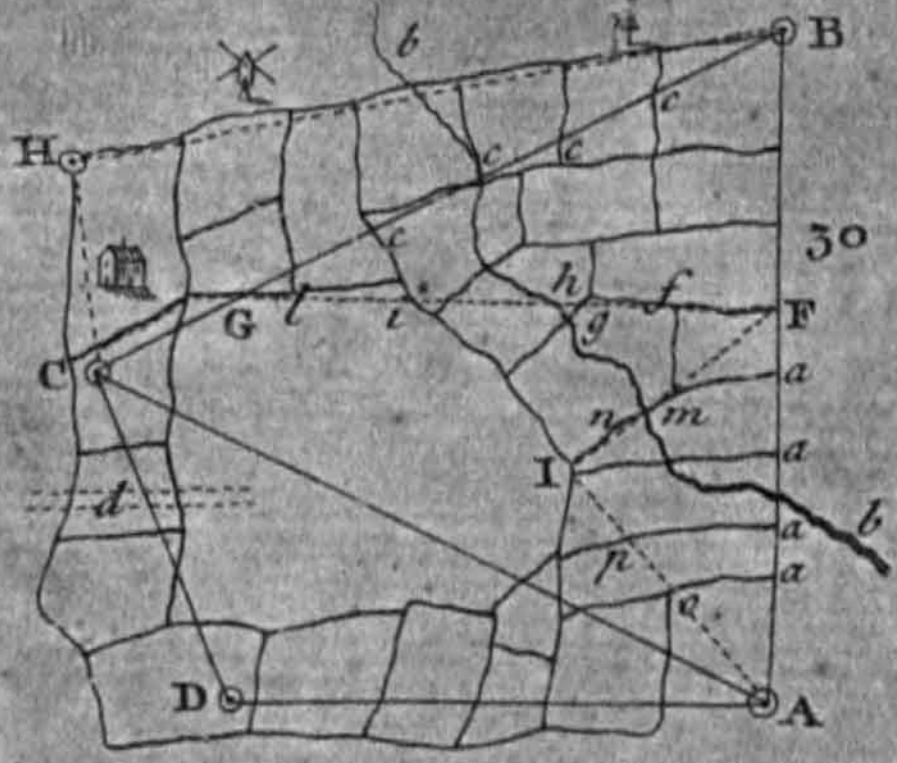
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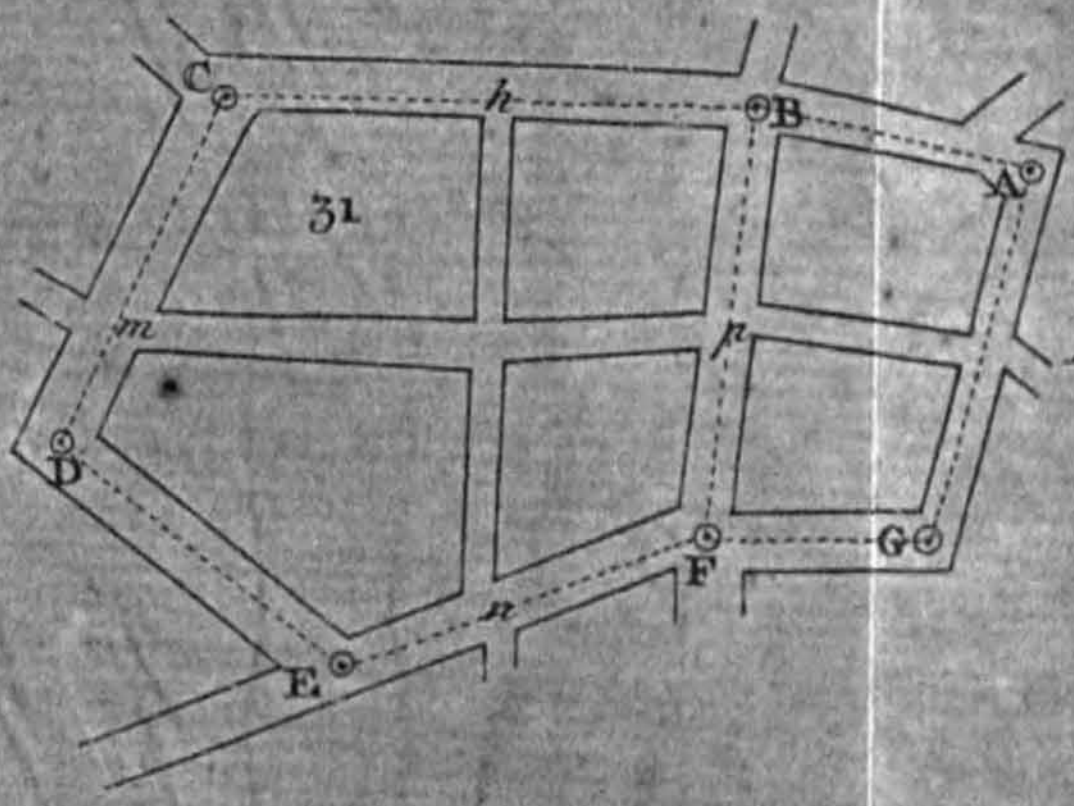
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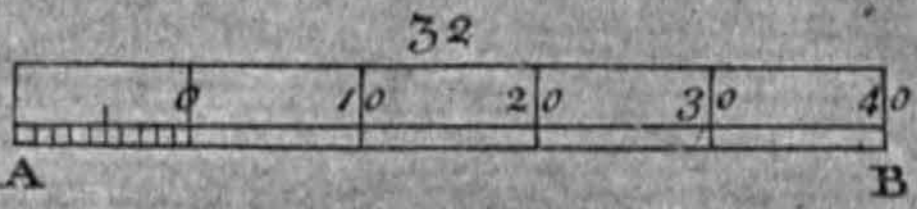
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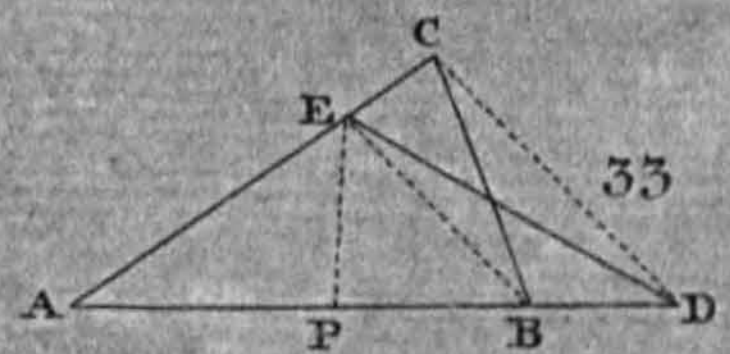
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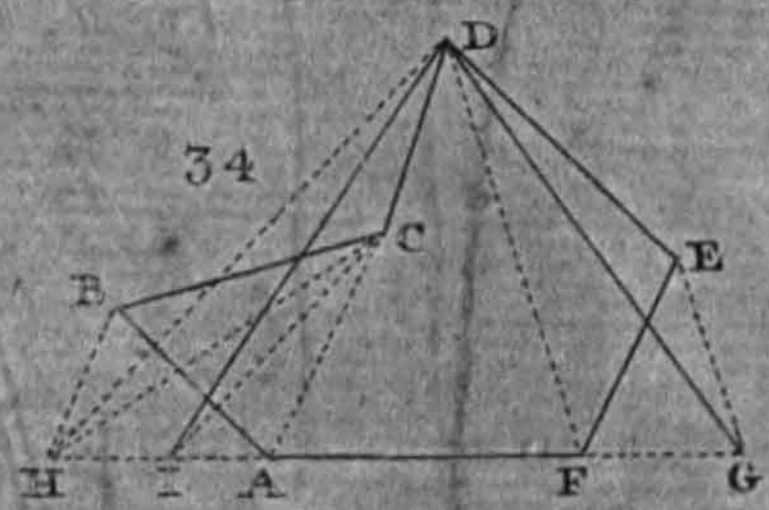
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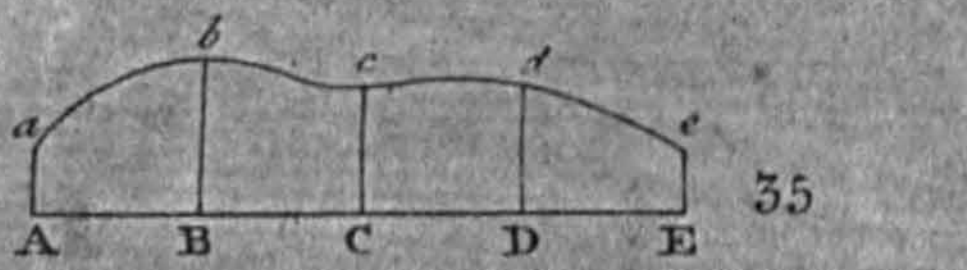
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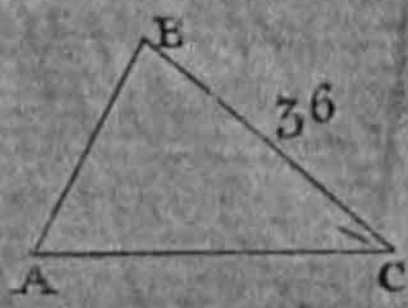
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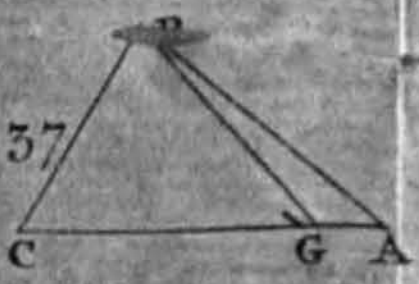
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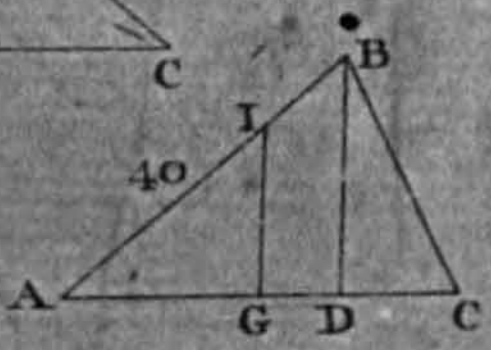
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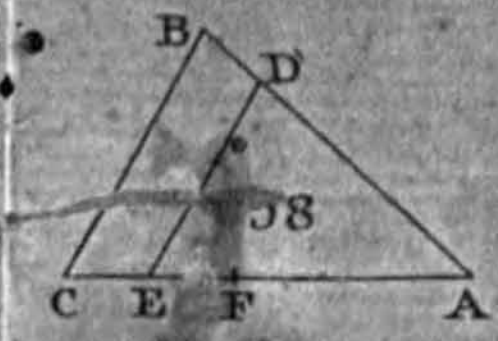
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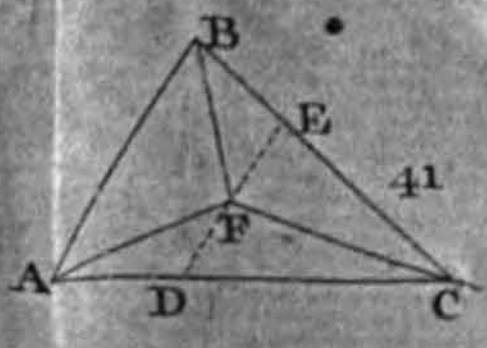
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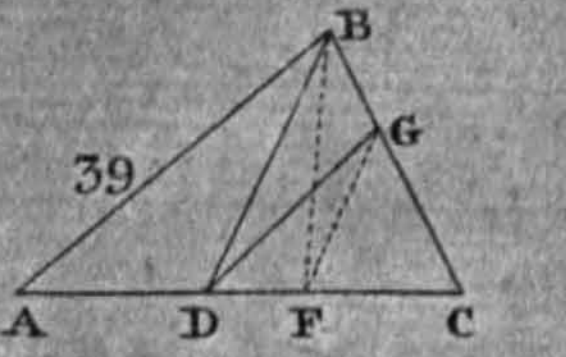
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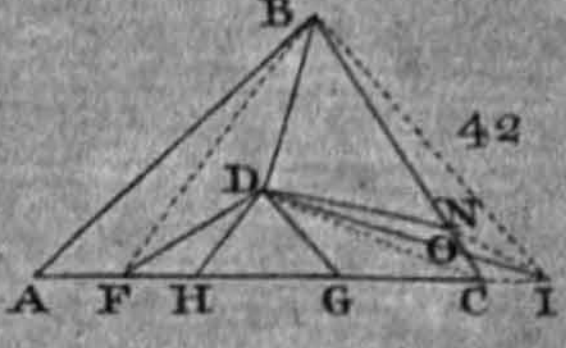
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Cor. *Hence a trapezoid may be divided into as many equal parts as we please; by dividing the two parallel sides, into the same number of equal parts; and drawing the correspondent lines.* Fig. 45.

PROB. XXIX.

To divide a trapezium in any given ratio of the parts, from an angle A.

Draw the diagonal BD oppoſite to the given angle A; and divide it at I, in the ratio given; and draw the lines AI and IC, which will divide the trapezium as required. 46.

But, if you would have one right line drawn from A to divide it; draw the other diagonal AC, and IF parallel to it, to cut DC in F, then draw AF, which will divide it in the given ratio.

For the triangle AID is to AIB, in the ratio of DI to IB, that is in the given ratio; and CID is to CIB in the ſame ratio; therefore the part ADFI is to the part ABCI in the given ratio. But by reaſon of the parallels AC, IF, the triangle AIC is equal to AFC; and therefore ABCI is equal to ABCF; and ADCI equal to ADF.

PROB. XXX.

To divide the trapezium ABCD, *in any given ratio, by a line drawn from* E, *a point in the ſide* AB.

Produce the oppoſite ſide DF; and reduce the trapezium ABCD into the triangle GEH, from the point E, by Prob XII. Then divide the baſe GH at F, in the given ratio, and draw EF, which will divide the trapezium as required. 47.

If the point F falls not between D and C, but between C and H; then BC muſt be produced for

the

the baſe, inſtead of DC. Or elſe reduce it back, Fig. 47. by Cor. 2. Prob. XII.

For the part DAE is equal to EGE, and ECI equal to ECB. Therefore EADF is equal to EGF, and EBCF equal to EHF, whence theſe parts are as GF and FH, that is, in the given ratio.

PROB. XXXI.

To cut off any quantity of land from a trapezium, by a line parallel to one ſide, as CB.

48. Divide the number of acres, given to be cut off, by the ſide CB, to which you are to go parallel. And the quotient ſet out from B to E, and from C to F; and draw EF. But if EF is greater than BC you muſt ſcarce go ſo far, from B or C; or if it be leſs you may go ſomething further. But when this is ſet out, as the field is irregular, you muſt meaſure the quantity cut off CBEF, and ſee what it is over or ſhort. Then meaſure FE and reduce that difference of the areas, to ſquare, chains, which divide by the length of FE, and the quotient muſt be ſet from E towards A, if you had cut off too little, as to *e*; and from F to *f*. But the contrary way if you had cut off too much. Then draw *ef* for the true line of diviſion.

Note, you may ſet it all to one end if you like; and then you muſt double the quotient, and ſet it from E to *n*, on the greater ſide, and draw *n*F. And indeed the courſe of the ridges in a field often confine us; ſo that when we expect they run parallel to the hedge, we find them otherwiſe; and therefore we may lay out more at one end than the other, in theſe caſes.

Cor. *Hence we may apply this method to any poligon; provided the ſides* AB, CD, *that we work upon be not far from parallel. Otherwiſe a proper allow-*

ance

ance muſt be made for the increaſe or decreaſe of the Fig.
line EF. 48.

PROB. XXXII.

Any poligon, regular or irregular, being given; to divide it in a given ratio, from a point in an angle or a ſide; or to cut off any quantity from it.

Let ABCDE be the poligon; reduce it into a 49.
triangle CFG, by Prob. XII. whoſe vertex is C, and baſe FG. Then divide FG in the point I, in the ratio given; and draw the line CI, which divides the triangle FCG, and alſo the poligon ABCDE, as was required.

Note, if CI do not cut AE, but ſome other ſide AB or ED, then that ſide it cuts muſt be produced for the baſe inſtead of AE, the point C being ſtill the vertex; or elſe it may be reduced by Cor. 2. Prop. XII.

If a given quantity of acres was to be cut off; reduce that quantity to chains, and take the neareſt diſtance from C to AE for the perpendicular; divide twice that number of chains by this perpendicular, and the quotient will be the length of the baſe, which ſet from F to I, then the line CI drawn, will cut off the quantity propoſed ABCI.

Otherwiſe thus.

If the ratio of the two parts be given, and the whole quantity; then either part is eaſily found.

Therefore to cut off any quantity towards CB, meaſure ſo many triangles next that ſide, as CBA, CAE, &c. till you have ſomething too much (or wants a ſmall matter); then cut off the overplus from the laſt triangle (or the defect from the next); and draw the line of diviſion.

The method to cut off the due part of the laſt triangle is this, ſe. CAE be the triangle, then di-

vide