Divide twice the excefs (reduced to chains) by the perpendicular from C to AE; and the quotient fet from the laft angle E to I, gives the point of divifion.

Examp.

The field ABCDE contains 42 acres, the part CBAI to be cut off is 27 acres. The triangle CBA is 10.80; the triangle CAE is 24.46. The perpendicular 22.7.

$$
\begin{array}{r}
10.80 \\
24.46 \\
\hline
35.26 \\
27.00 \\
\hline
\text{over } 8.26 = 82.6 \text{ chains}
\end{array}
$$

$$
\begin{array}{r}
82.6 \\
2 \\
\hline
\end{array}
$$

$$
\begin{array}{l}
22.7)165.2(7.28 = \text{EI.} \\
\quad\quad\; 1589 \\
\quad\quad\; \overline{\;\;630} \\
\quad\quad\;\;\; 454 \\
\quad\quad\; \overline{\;1760}
\end{array}
$$

Therefore if EI be made 7.28. chains, and CI drawn; the part CBAI will contain 27 acres.

Cor. *By the fame method, a poligon may be divided into any number of equal parts; and even into any number of parts, whofe proportions are expreffed by any numbers.*

This is done either by dividing the bafe of the triangle FG in the ratio of the feveral parts. Or elfe cutting off the feveral parts of the figure one after

after another. And when there are many parts, it Fig. comes to no more, than dividing a triangle into several parts, by lines from the vertex, as explained in Prob. XX. 49.

This problem is of very great ufe in practice; and by thefe methods any pieces of land may be divided more expeditioufly, and as truly as by any method whatfoever. And for meafuring any poligon, when it is reduced to a triangle, it is done at once; which otherwife would require as many calculations, as there are triangles.

PROB. XXXIII.

To divide a poligon into any number of parts, equal or unequal; by lines drawn from a point within it.

To do this, you muft have the content of every triangle in the field, and of the whole field. And then beginning at any angle, draw a line to that angle, and take fo many of the firft triangles as will make the firft part required; and what they fall fhort of that part, cut off the next triangle. Then take the remainder of that triangle, and fo many of the next as will make up the fecond part. And fo proceed round the figure, till all the parts be laid out.

Examp.

Let ABCDEF be a field containing 69 *acres, which* 50. *is to be divided among three men, from a pond in the middle at G, fo that every man may have the benefit of the water. And the firft man to have* 25 *acres, the fecond* 20, *and the third* 24. *To fet out every man's fhare from the angle* C.

Draw the line GC for the beginning of the divifion; and alfo lines to all the reft of the angles, as GB, GA, GF, GE, and GD. Then meafure all the

For the triangles; and their contents will be as follows.

CGB	11.81
BGA	15.27
AGF	6.07
FGE	6.65
EGD	18.83
DGC	2.37
	61.00

Then since the first man's share is 25 acres, the first triangle CGB 11.81 is too little, and the first and second triangles together, CGB and BGA, comes to 27.08, which is more than 25; therefore subtract 25 therefrom, and there remains 2.08. And therefore we must cut off 2.08 acres from the triangle BGA towards A. To do which, measure the perpendicular from G to BA, 13.28 chains, and reduce 2.08 acres to chains, which will be 20.8; divide 41.6 or double these chains, by 13.28; and you have for the quotient 3.13 chains; which set off by measure, from A towards B, as to H, for the dividing point. And draw GH, so CGHB is the first man's share.

Then for the next man's share. Add this 2.08 acres cut off, to the next triangle AGF 6.07, the sum is 8.15 which is too little, add the next triangle FGE 6.65, makes 14.80 acres, which is still too little; and adding the next triangle EGD, it will be far too much. Therefore we shall cut off an additional part of the triangle EGD at E. Subtract 14.80 from 20 the second man's share, and there remains 5.2 acres, or 52 chains. The perpendicular from G upon ED is 13.9 chains. Therefore divide 104 (the double of 52) by 13.9, and the quotient is 7.48 chains; and this must be measured

ured off from E towards D, as to I. Then draw Fig. 50.
GI, and HGIEFA will be the second man's ſhare.

Laſtly, the remainder of the triangle GDE, is 11.63 equal to the part IGD; to this add the triangle DGC 12.37; and the ſum is 24 acres for the area GIDC, which is the third man's ſhare. So the field is divided as was required.

If the field was to be divided into any number of equal parts; divide the whole content of the field by the number of parts, the quotient ſhews the quantity of one part or ſhare. As if there were 12 equal ſhares in it, then one ſhare will be 7.5 acres. Therefore cut from the triangle CGB, 7.5 acres; and from that and the next 7.5 acres more, and ſo on; which may be done as before directed; or by Cor. Prob. XVIII.

If you deſire to cut off any number of acres given, without knowing the content of the whole field. You muſt meaſure triangle after triangle, till you get as much as you want, or very near it. And for the defect cut off a part of the next.

PROB. XXXIV.

To transfer a plan or map truly, from one paper to another.

Lay your plan upon the blank paper, and fix them both together upon ſome flat board or table, that they may not move from one another or part aſunder, till the operation is done. Then with a fine needle or pricker, make ſmall pricks thro' all the principal angles, or places on your plan, which will make ſo many marks in your paper. Then take off the plan, and draw lines from one point or mark to another, as you find them in the plan; till all the lines be laid down. If you be in any doubt about ſome point or prick; takes its diſtance from one

Fig. one or two points, you are certain of, and apply it to the plan, and that will ſet all to rights.

Or thus.

Take the map or plan, and rub the backſide all over with black lead; then lay your paper upon a flat table, and upon that lay the leaded ſide of the plan. Faſten them at the four corners, that they may keep together. Then with a ſharp pointed bodkin run over all the lines in your plan, laying pretty hard on; and ſuch other objects as you would expreſs. Then take off the map, and all the lines, that you traced with the bodkin, will appear upon the paper; which you may draw with ink, or may rub out at pleaſure, with crums of bread.

Otherwiſe thus.

Take a paper well oiled with linſeed oil; and laying the map upon a table, lay the oiled paper upon it, and faſten them together; then you may ſee all the lines and furniture of the map, thro' the oiled paper, as thro' a clear glaſs. Then with pen and ink, or with a bodkin, draw or trace all the lines in the map. So will you have a map in the oiled paper, which you may place upon another paper, if you pleaſe. Thus by an oiled paper, may the figure of any flower, fruit, or other curious thing, be taken from any draught of it.

Or thus.

The drawings in a map may be transferred to another paper, by taking every line from the one, and ſetting it upon the other, with a pair of compaſſes: but this is very tedious, and requires a deal of time; but to facilitate the work, there are three legged compaſſes made, to take off three points at once, and ſet them down on the new paper.

PROB.

Fig. 51.

PROB. XXXV.

To reduce any field to a greater or lesser size, in any proportion of the sides.

Take a point in a side or an angle, as at A. From that point, draw lines to all the other angles, as AE, AD, AC. Then on any side or diagonal, set off the length of that side or diagonal, according as you would have it reduced; suppose along AF, from A to *f*. From *f* draw *fe* parallel to FE, to intersect the next diagonal AE in *e*. From *e* draw *ed* parallel to ED, to intersect AD in *d*. From *d* draw *dc* parallel to DC to intersect AC in *c*. From *c* draw *cb* parallel to CB. Then *afedcba*, wil be the reduced figure.

Or you may take a point within the figure, as at P; from which draw lines to all the angles, PA, PB, PC, PD, PE. Then make P*a* the reduced length of any line as PA; and from *a* draw *ab* parallel to AB, to intersect the next line PB in *b*. In like manner draw *bc*, *cd*, *de*, *ea*, parallel to BC, CD, DE, EA; each from the intersection of the correspondent line drawn from P. And *abcde* will be the figure reduced, as was proposed. And by the same rules, they may be reduced to a bigger size, as well as a lesser. 52.

Cor. 1. *Hence the reduced figure is similar to the original figure.* 52.

For the angles are all equal; and the sides parallel and proportional, to those in the given figure.

Cor. 2. *Hence, a figure* ACE *may be divided in a given ratio, by lines parallel to the sides of the figure; by the same method.*

Making Af^2 to AF^2, or Pa^2 to PA^2, in the given ratio.

PROB. XXXVI.

To reduce a map or plan, to a less or greater size, in any proportion of the sides.

If the draught or plan is not large, lay it down again by a smaller scale; to do which you must take every line from the plan, and apply it to the scale it was laid down by; and then take the same numbers, off the lesser scale, to be laid down in the new plan. Thus you must perform the whole work geometrically, by scale and compass; and it is best to use a diagonal scale, by reason of their minute divisions. But when the plan is large, this method is tedious.

Otherwise.

Make a reducing scale with a small hole, in a line with the edge of it, and nearer one end than the other, in proportion of the size it is to be reduced to. These are both graduated with equal parts, in the same proportion. Then glue or fix your map to a table, and close by it edge by edge a paper, on which it is to be reduced into a lesser or greater form. Put a strong needle thro' the hole in the center of the reducing scale, and fix it in the table about the middle of the line where their edges join. Then, if you would reduce it to a lesser size; turn the edge of the longer end of the scale, to any object on the plan; and see what number of equal parts it cuts; and find the same number on the other end of the scale, and by its edge make a mark on the paper, for the place of that object. And thus you must proceed till all things on the map be laid down.

But if you would make a larger draught, you must turn the shorter end of the scale to the object on the map, and the longer end to the paper.

The

The rule must be a thin plate, that the numbers may be pricked off by the edge of it. Fig.

But plots or maps are best reduced by an instrument contrived on purpose. This is made with two drawing pens; so that tracing one of them over all the lines in the plan, the other pen will describe it anew on a clean paper, either in a lesser or a greater size, as one pleases.

Or thus.

Inclose the map in a large square, and divide that 53.
into several small squares. Also make a square upon 54.
your new plot, so much less than the other, as you would have it diminished; and divide it also into the same number of small squares. Then observe in what square, and what part of the square, any object, on the map, is placed; then place it in the like square, and in the same part of it, on your new plan. And thus transfer every remarkable point in the map, into your new plot; putting it in its proper square, and into the same situation. Thus in fig. 53. you have several closes to the west, and a hill near them; likewise a river running thro' the ground, two trees near it, and a house a little below, with a road leading to it. And in the reduced plan, fig. 54. you have also the closes to the west, and hill near them; and the river running thro' the ground, and two trees beside it, and a house below, and the road going to it; and all placed in the corresponding squares, and the same parts thereof. And thus all other things therein, ought to be placed. And the same way, a plan is reduced from a lesser to a greater size; but this seldom comes in practice.

This method is very proper for a large plan, because it contains a great variety of things.

PROB. XXXVII.

To reduce one sort of measure into another.

In some places, 'thro' custom, they reckon more feet to the perch than the statute allows; and therefore an acre with them will be greater than a statute acre. But to know how many statute acres there are in any field, surveyed by a different perch; and the (contrary); by having that perch, and the correspondent number of acres given. Multiply the number of acres by the square of the length of the perch it was surveyed and planned by; and divide the product by the square of the other perches length; and the quotient gives the number of acres, according to that perch.

Examp.

Suppose a field of 40 acres was planned by a perch of 18 feet; how many statute acres does it contain?

18	$16\frac{1}{2}$	272.2|5)12960)47.60
18	$16\frac{1}{2}$	10890
144	96	20700
18	16	19054
	$16\frac{1}{4}$	
324		16460
40	$272\frac{1}{4}$	16332
12960		1280

Ans. 47.6 acres.

Cor. *The length of the perch, a field is measured by; is reciprocally as the square root of the number of acres, it measures to, by that perch.*

PROB.

Fig.

PROB. XXXVIII.

Having given the content of a field; to find what scale it was laid down by; or how many parts to an inch.

Take any number of parts to an inch for your scale, by which measure the content of the field; then multiply the content given by the square of your scale (or number of parts) and divide the product by the content found by that scale; the square root of the quotient, is the true scale, or number of parts in an inch.

Examp.

There is a field contains twelve acres, but measured by a scale of 10 parts in an inch, its content is 16½ acres. To find the scale.

```
   10    12
   10   100
  ---   ---
  100
  ---
16.5)2400(145.5        1̇45̇.5(12.07
     165.              1
     ---              ---
      750           22)45
      660              44
      ---              --
       90              1.50
       82
      ---
        8
      ---
```

So the scale contains 12.07 parts in an inch.

Cor. The scale or number of parts in an inch, is as the square root of the content found by it, in a given

SECT. III.

Of Measuring inaccessible Heights, and Distances.

PROB. I.

To measure the height of any perpendicular object AB, *where you can come to the base* A.

55. LET ABD be the object, as suppose a tower, whose height AB is required; AC a horizontal line; for here C is supposed to be level with the foot of the object at A.

Take your station at C, about as far from the object as the height of it; and where you can see both the top and foot of the object; and there place your instrument, with which take the angle BCA; that done, measure the distance CA, from the instrument to the foot of the tower. Then by plain trigonometry say,

As radius:
to the distance CA::
So the tangent of the angle BCA:
to the height of the tower AB.

To the height thus found, add the height of the instrument above the ground.

Examp.

Fig. 55.

Examp.

Suppose the angle BCA 51 52, and distance CA, 64 feet.

Rad	—	10
CA 64	—	1.80618
Tan. 51 52	—	10.10510
AB $81\frac{1}{2}$	—	1.91128

To which add 4 feet the height of the eye, the sum is $85\frac{1}{2}$ feet.

This may also be resolved by projection; draw CA, on which set 64 feet from any scale, to A. Erect the perpendicular AB. Then make the angle ACB 51 52; and draw CB intersecting the perpendicular in B, the top of the tower. Then AB measured on the same scale, gives $81\frac{1}{2}$.

Otherwise, by the shadow.

Upon plain ground, set up the staff *ab* perpendicular to the horizon; so that the sun shining, it may cast the shadow *ac*, and let AC be the shadow of the tower AB. Then measure the shadows *ac* and AC, and the length of the staff *ab*. Then say,

As the length of the staff's shadow ac:
is to the height of the staff ab : :
So is the length of the object's shadow AC :
to the height of the object AB.

Or thus by the shadow without the staff. Take, with an instrument, the altitude of the sun, when the shadow of the top of the tower B, falls at C. Then measure CA, and you have the angle C, and the base CA; to find the perpendicular BA; which is done by the first method.

SCHOLIUM.

In taking the angle ACB, if you move backwards or forwards, till that angle be juſt 45 degrees; then the diſtance CA, being meaſured, will be equal to the height AB.

If you be on the top of the tower, you may take the angle ABC, to any mark at C; then ſubtracting the angle ABC from 90, gives the angle ACB; then meaſuring AC; from theſe, AB will be found as before. And this comes to the ſame thing as meaſuring a depth.

Hence alſo if the height AB be given, the diſtance of any point C, as AC, may be found; having the angle ACB. It is only varying the proportion, given in the firſt method.

We may obſerve, that the altitude AB, may be found otherwiſe than by calculation, if we can get to the top of it. And that is, by help of a line and plummet, let down from the top B, to the bottom at A. For the length of the line, reaching from B to A, being meaſured, gives the height BA.

PROB. II.

To meaſure an inacceſſible height DB.

56. Suppoſe DB to be a tower and a ſpire, where we cannot come at the bottom of it; or in caſe of a mote or other hindrance, between us and the object; that we can get no nearer than A; we muſt proceed thus.

Suppoſe the line of ſtation BAC, drawn from the foot of the perpendicular DB, to be horizontal. In this line we muſt take two ſtations A and C, at a good diſtance from one another. Then place the inſtrument at A, and take the angle DAB, ſuppoſe 58°; then go to the other ſtation C, and take the angle

angle DCB 38 deg. Then measure the distance between the stations CA, 26 yards. Fig. 56.

Then to project it, draw a line CB, and at one end A, make the angle DCB 38 deg. and draw CD. Then set off the stationary distance 26 from C to A; and at A make the angle BAD 58 deg. and draw AD, to intersect CD in D. Then the distance of D from the line CB, applied to the scale, gives the height 40 yards.

Or thus, by Trigonometry.

Subtract BCD 38°, from BAD 58°; there remains 20°, for the angle ADC. Then,

As	Rad —	10.
	× S. ADC, 20	9.53405
To	CA, 26	1.41497
So is	S.C, 38	9.78934
	× S.A, 58	9.92842
		21.13273
To the height BD, 39.7		1.59868

To this height, add the height of the instrument above the ground for the whole height.

For by trigonometry, S.<D : CA : : S.<C : $AD = \frac{S.C \times CA}{S.D}$ and rad : AD or $\frac{S.C \times CA}{S.D}$: : S.A : BD, that is,

Rad × S.ADC : CA × S.ACD : : S.BAD : height BD.

or Rad × S.ADC : S.ACD × S.BAD : CA : BD.

Or thus, by the tangents.

Find the difference of the cotangents of the angles BAC, and BAD. Then,

As the dif. cotangents :
Radius : :
So the stationary distance CA :
to the height DB.

Cot.

Fig. Cot. 38 1.27994
Cot. 58 .62487

diff. .65507

then .655 : 1 : : 26 : DB.

1

.655)26.00(39.7 = DB.
19 65

6350
5895

455

For if DB be radius, CA will be the difference of the tangents of ADB and CDB, which are the complements of A and C. And therefore rad : that difference : : DB : CA.

Otherwise thus.

If you take two ſtations at A and C ſo that the angle DAB ſhall be double to DCB, Then CA will be equal to AD.

Then *Radius* :
the ſtationary diſtance AC : :
S.angle DAB :
to the height DB.

Or thus.

Take the ſtation at A, ſo that the angle DAB may be juſt 45 degrees ; and the angle DCB 26° 34′. Then the height DB will be equal to the ſtationary diſtance CA.

For when DAB is 45, DB = BA = CA, and CB = 2DB. But it is, as CB (or 2DB) : DB, or as 2 : 1 : : rad : tan. 26° 34′.

Fig. 56.

Cor. 1. *Hence one may find the height of one object, DEF, standing upon another GEF.*

First find the whole altitude DB, and then the height of the tower EG, by this Prob. And their difference, is the height of the spire EDF alone.

Cor. 2. *If the height of the tower* EG *be known; any distance* CA, *in the line* GC, *may be measured from* E.

For that is only the reverse of this Prob. and therefore CA may be found by working the proportion backwards, by the natural tangents; or by the sines; as laid down before.

PROB. III.

To measure an inaccessible height BD, *by means of any two stations; so that one of them* A, *may be level with the base* B *of the object.*

Draw the horizontal line BA, and taking one station at A in that line, take another any where as at C, no matter whether it be higher or lower. Place the instrument at A, and take the angles BAD and DAC; and go to C, measuring the stationary line AC; and at C plant the instrument, and take the angle ACD. Then subtract the sum of the angles DAC and DCA from 180, to get the angle ADC. 57.

Then say, as $\left\{ \begin{array}{l} \textit{Radius} \\ \times \text{ S.ADC} \end{array} \right.$

$\hphantom{\text{Then say, }}$ to $\left\{ \begin{array}{l} \text{S.ACD} \\ \times \text{ S.DAB} \end{array} \right.$

So the stationary distance AC:

to the height BD.

Examp.

There is a tree DE *standing upon a high hill* BE; *to find the height of its top* D, *above the level of* A.

First

Fig. 67. First I place the instrument at A, and take the angle BAD 58 deg. and the angle DAC 72° 10′. And then I measure the stationary distance AC 26 yards; and at C I fix my instrument, and take the angle ACD 64° 30′. Therefore, ADC is 43 20. Whence,

{ Rad.	10.
{ S.ADC, 43 20,	9.83647
{ S.ACD, 64 30,	9.95548
{ S.DAB, 58 0,	9.92842
AC, 26	1.41497
	21.29887
height BD, 29	1.46240

Then to this height, add the height of the instrument. The demonstration is the same here, as in the last Prob.

Or thus.

Take the station A, so that the angle BAD may be 30 deg. Then will DA be equal to 2DB; and it will be,

As S.ADC,
to $\frac{1}{2}$CA;
So S.ACD,
to the height DB.

Otherwise thus.

This supposes both stations A and C to be level with the base B of the object.

Here are given the angles of altitude BAD, BCD, and the horizontal angle at one of the stations, as ACB, and the stationary distance CA.

Make as tan. BCD : tan. BAD : : S.ACB : S.BAC, observe whether acute or obtuse.

Then subtract the sum of the angles BCA and BAC from 180, gives the angle ABC. Then say,

As

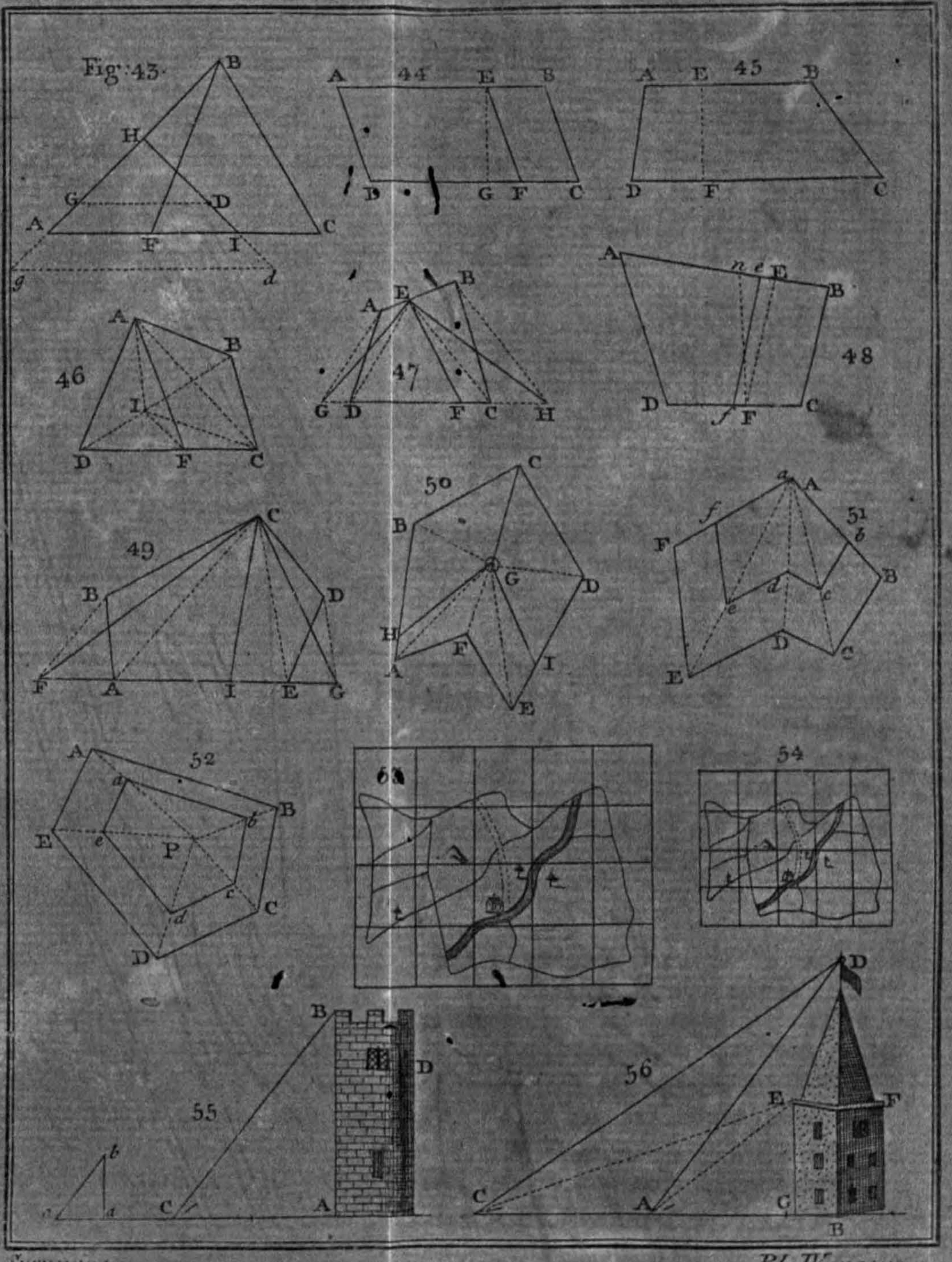


Fig: 43.
44
45
46
47
48
49
50
51
52
54
55
56

As S.ABC × radius : S.ACB × tan. BCD : : so Fig. stationary distance AC : to the height DB. 57.

This method is deduced from Prob. 125 of my Algebra.

In the former method, the point C may be either in or out of the plane DAB, as is evident. For DAB is perp. to the horizon; but ADC may be any way inclined to it. But in the latter method both ADB, and CDB are perp. to the horizon; and therefore the plane DCA must always be inclined to it.

Cor. 1. *A depth may be found the same way as an altitude; by assuming two stations, from both which, any spot in it may be seen, where you can set up a mark.*

This is easily understood, by only conceiving the point D to be below B.

Cor. 2. *If* CA *coincided with* AB, *this would come to the same case as the last Prob. And if* CA *was perpendicular to the horizon, this Prob. would be the same as finding one altitude* BD, *from having another altitude* AC *given; which therefore is easily resolved.*

After this manner the height of a hill or a mountain, or any object above the level of the place, is easily found.

Cor. 3. *Hence may be found the height* DE *of one object upon another; by first finding the height* DB, *and then* EB; *and the difference will be the height* DE.

PROB. IV.

To find the height of a very high mountain.

This might be resolved by what went before, if it was not for the refraction of the rays of light, 58. which do not come streight to the eye from such a height in the atmosphere, where the air must be greatly

Fig. greatly rarified. For this reaſon, a ray coming fr
58. the top of a mountain to the earth, deſcribes a cu
line, concave to the earth. Therefore an obſer
at the earth, taking the angle of altitude of
object upon the top of it, directs his eye along t
tangent to that curve; for in that direction t
rays come to the eye. But as that tangent run
above the mountain, he makes the angle of alt
tude higher than it ſhould be; and of conſequenc
the mountain itſelf higher than it really is. Now
to rectify this, let marks be ſet up at the ſtations A
and C; and one at the top of the mountain at D
And let the obſervers go, one to A, and the othe
to D; and let them both at the ſame time, mak
their obſervations thus; let the obſerver at A tak
the altitude of D above the horizon; where th
rays, inſtead of coming in the right line DA, wil
come in the direction FA, the tangent of the curve
at A: and therefore he really takes the altitude o
the point F. Alſo let the obſerver at D take th
depreſſion of A below the horizon; where th
rays, inſtead of coming to him in the line AD, wil
come in the direction FD, the tangent of the curv
at D. And therefore he has taken the depreſſion
of the point F, inſtead of D. Now by comparing
notes, the angle at A will be greater than that a
D, whereas they ought to be equal. Therefore
take half the difference of the obſerved angles, and
ſubtract it from the altitude taken at A, and that
gives the true altitude. For the altitude at A was
taken too great by the quantity DAF, and that a
D too little by ADF equal to the former. The
ſame thing is to be done at the ſecond ſtation C
Theſe angles being rectified, you may then procee
by either method laid down in the laſt Prob. (eſpe
cially the latter) to find the altitude; and for thi
purpoſe make your ſtationary diſtance AC as long
as you well can.

I

It may happen that two stations may be taken upon a mountain, more conveniently than below; and then you will only want one station below. But this creates no material difference in the work; for it is only finding a depth instead of a height. Fig. 58.

PROB. V.

To find the height of a cloud by its shadow.

Observe some small cloud which is coming directly towards you, or going directly from you; as C. Then observe when the middle of its shadow is at some remarkable point upon the ground, as at A; at that instant take the altitude ABC of the middle of the cloud. That done, take the sun's altitude at the place of your station B, and that will be equal to the angle BAC; also measure the distance BA between your station and the shadow. Then you have a triangle ABC, where all the angles and the side BA are given; which is easily laid down by rule and compass, and the height of C, above BA, found. Or by trigonometry. Subtract the sum of the angles at B and A from 180, to get the angle C. Then say, as rad × S.ACB : S.BAC × S.ABC :: to the stationary distance AB : to the height of the cloud. 59.

For S.C : AB :: S.B : AC $= \frac{S.B}{S.C} \times AB$. And rad : AC or $\frac{S.B}{S.C} \times AB$ :: S.A : perpendicular; or rad × S.C : AB :: S.A × S.B : height.

If the cloud be directly over your head at the time of observation, then CBA will be a right angle. Then rad : AB :: tan. sun's altitude CAB : height CB.

You must have a small cloud to observe by, because the observation must be made to a point. But

Fig. 59. But if you would observe a large cloud, take the altitude of the very edge, and observe the very edge of the shadow. And you must be upon some large plain or open ground, where you can see about you.

PROB. VI.

To find the altitude of an object whose top appears just in the horizon; having the distance in miles given.

This is a matter which frequently happens at sea in sailing to or from the land.

Square the distance in miles, which multiply by the decimal .2223, and the product is the height of the object in yards.

60. For let AD be an arch of the earth's surface, DB the height of an object, AB a tangent at A. Then in the right angled triangle CAB, $CB^2 = CD^2 + 2CD \times DB + DB^2 = CA^2 + AB^2$. But because $CA = CD$, $2CD \times DB + DB^2 = AB^2$, or (because BD is very small) $2CD \times DB = AB^2 = AD^2$, and $DB = \frac{AD^2}{2CD}$. But 2CD, or the earth's diameter, is 7916 miles; therefore $\frac{AD^2}{7916} =$ DB in miles, and $\frac{AD^2 \times 1760}{7916} =$ DB in yards; that is, $.2223 \times AD^2 =$ the height DB in yards. And the point B just appears or disappears to a spectator in the tangent at A.

PROB. VII.

To measure the length of a horizontal line AB accessible at both ends, A and B.

61. A line may be accessible at both ends, and yet cannot be measured, by reason of some impediment which happens to be in the way, as wood, water

ter, houſes, &c. In ſuch a caſe, ſet up a ſtake Fig.
ſome diſtance from the line, as at F; and thro' 61.
produce the lines AF, BF, beyond F, in which take FD equal to one of the lines as FA, FC, equal to the other FB; and leave two marks at D and C. Then meaſure the length of the line CD, which will be equal to that required AB.

If there is not room to meaſure out the lengths AF, BF; take half of them and meaſure from F to D and C; then meaſure CD and double it. Or take $\frac{1}{3}$ of each, and then CD is $\frac{1}{3}$ of AB.

Or thus.

Take a ſtation at ſome place where you can ſee A and B, as at F; and with an inſtrument take the angle AFB, then meaſure the two lines AF, BF. And then the triangle AFB is eaſily laid down with ſcale and compaſs.

Or take the angles at A and B, or any two angles; and meaſure one ſide; and then the triangle may be laid down, or computed by plain trigonometry.

PROB. VIII.

To meaſure a diſtance AB, acceſſible only at one end B.

Let the end B be one ſtation, and chuſe another 62.
at a convenient place as C; and having an object at A, produce the lines AB and AC to D and E, of any length. Then meaſure the ſides of the triangle BCD, and alſo the ſides of the triangle BCE. And by theſe the diſtance BA may be determined; and all this with the chain only.

Upon a paper draw the line BC, of its due length, and on that conſtruct the triangle BCD with the ſides you meaſured. Alſo upon the ſame line BC, make the triangle BCE, with the ſides, as you meaſured

Fig. 62. ſured them. Then produce DB and EC till th[ey] interſect in A; then BA meaſured, on the ſa[me] ſcale, will ſhew the diſtance of A from B, or [the] length of the line BA.

It may be alſo computed by plain trigonomet[ry]. In the triangle CBD there are given all the ſid[es], to find the angle DBC, and its ſupplement AB[C]. And in the triangle BCE, all the ſides are giv[en], to find the angle BCE, and the angle BCA its ſup-plement. And laſtly, in the triangle ABC all [the] angles will be known, and the ſide BC, whence [AB] will be found.

Or thus.

Take one ſtation at B, and another at a con[ve]nient diſtance as C; and ſet up marks. Then plac[e] the inſtrument at B, take the angle ABC, betw[een] ſome mark or object at A, and the mark at C. Th[en] meaſure the ſtationary line BC; and place the [in]ſtrument at C, and take the angle ACB, betw[een] the ſame object at A and the mark at B. Th[en] ſubtract the ſum of theſe angles B and C from 1[80°] to get the angle A. Then ſay,

As ſine of the angle A :
to the ſtationary diſtance BC : :
So ſine of the angle C :
to the diſtance AB.

The diſtance BA may alſo be found here by r[ule] and compaſs, by laying down the triangle ABC.

Or thus.

Make the angle ABC of 60 degrees, or if [you] cannot at B, take ſome other point in the line [BA]. Then chooſe ſome other point C, in the line [BC], that the angle BCA may alſo be 60. Then m[ea]ſure BC, and that is the length of BA. F[or] ABC be 60°, and BCA 60, then CAB will be [60], and the triangle ABC will be equilateral.

Or elſe make the angle ABC a right angle, [and] ſomewhere in the line BC, take the point C, the angle BCA may be 45 degrees. Then measure with the chain the length of BC, and that is the length of BA. For the angle at A is also 45 degrees; and therefore the sides opposite to the equal angles, AB and BC, will be equal. Fig. 62.

Or you may make the angle ABC a right angle, and the angle ACB 60 degrees. Then measure the length of BC; then BA will be equal to the square root of $3BC^2$, or equal to $BC\sqrt{3}$. For in that case $AC = 2BC$, and $AB^2 = AC^2 - BC^2 = 4BC^2 - BC^2 = 3BC^2$.

Or make the angle ABC what you will, and the angle BCA half the supplement of it; then measuring the length of BC, you will have the length of BA. For then the angle BAC will also be half the supplement of ABC, since they are both together the whole supplement. Therefore their opposite sides are equal.

PROB. IX.

To measure an inaccessible distance AB; or one that cannot be come to at any end.

Set up a stake somewhere in the line BA produced, as at C; then chuse another station as at D. 63.
Then with an instrument at C, take the angle ACD. Then measure the distance CD; and placing the instrument at D, take the angles CDA, ADB. Subtract the sum of the angles ACD and ADC from 180, and there remains the angle CAD, or rather add the two angles ACD and ADC together, and you have the angle DAB. Likewise subtract the sum of the angles DAB and ADB from 180, and you have the angle ABD. Then make, as S.ABD × S.ABD : S.ACD × S.ADB :: so is the stationary distance CD : to the distance sought

For

Fig. 63. For $S.A : CD :: S.C : AD = \frac{S.C}{S.A} \times CD.$

$$\text{And } S.B : S.ADB :: AD \text{ or } \frac{S.C}{S.A} \times CD : AB.$$

$$\text{Or } S.A \times S.B : S.C \times S.ADB :: CD : AB.$$

It may alſo be laid down by projection. Dr the line CB, and make the angle BCD equal to t obſerved; ſet off CD, the diſtance meaſured; and at D, make the obſerved angles CDA, ADB; and the part AB intercepted between DA and DB, is the diſtance required.

Or thus.

64. When you cannot get a ſtation in the right line AB; take a proper ſtation at F, where you can ſee A and B; and produce the lines AF and BF, to D and C, and make FD and FC, of any convenient lengths, by meaſuring them. Then placing the inſtrument at F, take the angle AFB. Then go to C and take the angle ACB; and likewiſe at D, the angle ADB. Then the work may be laid down thus.

Draw any line AD and at any point as F make the angle AFB equal to that obſerved, and draw the line BFC; and ſet off the diſtances FD and FC. Then at the point C, make the angle BCA, equal to that you meaſured, drawing CA to interſect DA in A, one end of the line. Again, mak the angle ADB equal to its meaſured quantity, an draw DB, interſecting CB in B, the other end o the line; then AB meaſured on the ſcale is the diſtance required.

To reſolve it by trigonometry. Subtract the ang ACB from the angle AFB, and you have the an gle CAF; then in the triangle CAF, all the angle and the ſide CF are given, therefore to find AF, will be,

As sine of the angle CAF Fig. 64.
to the side CF ::
So the sine of the angle ACF :
to the side AF.

Again, subtract the angle ADB from AFB, and there remains the angle FBD. Therefore to find FB in the triangle DFB, say,

As sine of the angle DBF :
to the side FD ::
So the sine of the angle FDB :
to the side FB.

Then in the triangle AFB, there are given the two sides AF, FB, and the included angle AFB; by which the side AB will be found.

If the points C, D, be taken in the lines BF, AF produced, so that the angles ACB and ADB may be each of them half the angle AFB; then the line AB will be equal to CD. For then the angle ACF is equal to CAF, and FDB equal to FBD; therefore CF is equal to AF, and DF equal to BF; whence AB is equal to CD.

If there is not room to go back to C and D, you may take a station somewhere about G; and find the length of AF (by Prob. VIII.) by the method of measuring distances, accessible only at one end. And the same way, find the length of BF. And then the line AB will be found as before.

Otherwise thus.

Having chosen two points of station C and D at a proper distance, in the same plane with AB, whose distance may be measured, and from which, A and B may be seen. There set up two marks; and placing the instrument at C, take the angles ACB, BCD. Then measure the stationary distance CD; and at D, placing the instrument, take the angles CDA, ADB. Then you have the angles ACD, and CDB. Take the sum of the angles ACD and CDA from 180, and there remains

 the

Fig. 64. the angle CAD. And in the same manner the angle CBD is found.

Now we have in the triangle ACD all the angles and the side CD, from which the side AC will be found. Then in the triangle CBD, we have all the angles and the side CD, whence we can find the side CB. Lastly in the triangle ACB, we have the two sides AC, BC, and the included angle ACB; from whence the required side AB is found.

It may also be laid down by rule and compass, to save the trouble of so many calculations. Draw any line as CD, on which set off the distance of the stations at C and D. Then at C, make the angles BCD, BCA, drawing the lines CB, CA. Again make at D, the angles CDA and ADB, all equal to these measured; and draw DA, which intersects CA in A. And draw DB, which intersects CB in B, and AB drawn and measured is the distance sought. And thus you may measure any of the lines CA, CB, or DA, DB, by applying them to your scale.

It is no matter whether the line AB be horizontal, perpendicular, or inclined, provided CD be in the same plane with AB. For then the length of AB may always be determined by the two stations C and D. And therefore if AB be an inaccessible altitude, and C, D, two stations either in a perpendicular CD, or upon any hill side; the height AB may be found as above; as will be evident by supposing the plane CADB perpendicular to the horizon.

Otherwise thus.

If we cannot get two stations C, D, in the same plane with AB, the line to be measured; then we must take at C, the three angles ACB, BCD, ACD; and at D the three angles CDA, ADB, and CDB; and measure CD; and then proceed in the very same manner, to find AB, as before.

this may be done two ways, 1. finding AC and CB, and then AB; or 2. finding AD and DB, and then AB. And if it be done both ways it will be a proof of the work. And this method is universal, however the station C, D, be taken; and whatever position the line AB is in, whether parallel to the horizon, or perpendicular, or any way inclined; provided that A and B can but be seen from both the stations C and D. Fig. 64.

PROB. X.

To take an inaccessible distance AB, when both ends of it, A and B, cannot be seen from any two stations.

Assume some station D, from which both A and B are visible. Take two other side stations, C and E, so that A may be seen from C, and B from E; and both C and E from D. Then by Prob. VIII. find the inaccessible distance DB from the two stations D, E; and the inaccessible distance DA, from the stations D and C, which is to be done thus. Measure the distance DE, and the angles BDE, BED, from which find the side BD. Also measure CD, and the angles ADC and ACD; from which the side AD will be found. Then measure the angle ADB; and in the triangle ADB, there are given the two sides AD, BD, and the included angle ADB, to find AB as required. 65.

The construction of this, by rule and compass, will be easy. From any point D, draw DE of a proper length, and at D make the angles EDB, BDA, and ADC; on DC set its measured distance to C. Then at E, make the angle DEB; and at C, the angle DCA. Then the lines EB and CA will cut DB and DA in B and A; therefore AB drawn and measured, will be the distance sought.

Fig. 66.

Or thus.

When no ſtation can be found, from which both A and B can be ſeen; take two ſtations C and D, ſo that A may be ſeen from C, and B from D. Then take two ſide ſtations E and F; by help of which, find the inacceſſible diſtances DB and CA (by Prob. VIII.) by meaſuring the diſtance DE, and angles EDB and DEB; and alſo CF and the angles FCA, CFA. Then meaſure the angles ACD, and CDB, and the ſtationary diſtance CD. Then in the triangle ACD, the ſides AC, CD, and included angle ACD being given; the ſide AD and the angle ADC will be found; and ADC ſubtracted from BDC, gives ADB. Then in the triangle ADB, the two ſides AD, DB, and the included angle ADB are given; to find AB the diſtance required.

The conſtruction is eaſy enough. Draw CD, and make the angles FCA, ACD, CDB and BDE, and ſet off the diſtances CF, DE; then make the angles CFA, DEB; and FA, EB, will interſect CA, DB, in A and B; and AB is the diſtance ſought.

Other Problems concerning heights and diſtances might be deviſed, from the various ſituations that objects may be ſuppoſed to have. But the foundation of all is contained in what has been here laid down; and to add more, would only confound the memory, and ſwell the book to little purpoſe.

PROB. XI.

To find the level of two places, or to find the aſcent or deſcent from one place to the other.

This is beſt done by a ſpirit level with teleſcopic ſights; which may be ſet level by ſcrews, which

raiſe or lower one end of it. You muſt alſo have Fig. 66.
two poles, four yards long, with white papers or marks upon them to ſlide up and down, and croſs the middle of the mark a black line is drawn; and you muſt have two aſſiſtants to carry theſe two poles.

To take a level from A to I. Let one of the 67.
men ſtand at A and the other at C, and at a convenient place as P, between A and C, place the level, and ſet it horizontal by help of the ſcrews. Let the man at A hold the pole upright in his hand at A, whilſt from P, you look through the ſights towards A, and cauſe the man at A to ſlip the mark up or down, till the black ſtroke on the mark at B, be level with the ſights; then meaſure BA. Then cauſe the other man to ſtand at C, and hold the pole upright, then turning the inſtrument round at P, and looking towards C, cauſe him to raiſe or lower his mark, till thro' the ſights, you can ſee the black ſtroke at D; then meaſure the height above the ground CD. And the difference of the heights AB and CD, is the aſcent or deſcent, if there be no more ſtations. But if there are many ſtations make a table with two columns, one for the back ſtations, and the other for the fore ſtations. Now to proceed, ſet down the two heights AB, CD; and let the man at A go to F with his pole, and remove the inſtrument to Q, and level it; then direct the ſights towards C, and let the man at C ſlip his mark up or down till the black line appears thro' the ſights, as at E; and meaſure CE. Then turning the inſtrument about, look towards F, till thro' the ſights you ſee the black mark at G, and meaſure GF; in the mean time the man at C removes to K, and placing the inſtrument at R, direct it towards F backward, and to K forwards. And ſo proceed from ſtation to ſtation to the end. And ſet down the meaſures you took of the back ſtations,

Fig. 67. ſtations, AB, CE, Fo and the fore ſtations CD, FG, IK, in their proper columns. And at laſt ſum up both the columns, and take the difference of them; and if the fore ſtations exceed, you have a deſcent; but if the back ſtations exceed, it is an aſcent. If they be equal, you have a level.

Sta.	Back ſta.		Fore ſta.	
	f.	in.	f.	in.
1	4	0	2	6
2	5	4	7	3
3	0	0	3	0
	9	4	12	9
			3	5

So as the fore-ſtations are greater, there is a deſcent of 3 feet 5 inches, from A to I. But in the length of a mile, 8 inches deſcent muſt be allowed, upon account of the earth's curvature; that is, if you meaſure along the tangent, at the point A. But if the water is to run, a greater allowance muſt be made, in proportion to the velocity it is to run with. And thus it will be known whether or no, water can be conveyed from one place to another. But it is difficult to take a height, the length of a mile to a few inches. For many propoſitions may be firm and true in geometrical rigor; which reduced to practice amount to nothing at all.

Otherwiſe thus.

To know if water can be conveyed in an open channel; take two boards half a yard long; make two holes in the middle of them, in which ſet up two ſmall ſticks, perpendicular, and exactly of the ſame height; at the tops of which, fix white marks, or pieces of paper. Set the boards in the

w

water to float, at a competent distance from one Fig. 67.
another, the further the better. Then as far from the pond as you can, set up a third stick, with a mark to slip up or down, which must be in a line with the other two. Then looking along the tops of the two first sticks, slip the mark on the third, up or down, till it be exactly in the same horizontal line. Then measuring the distance of the mark from the ground, and comparing it with the length of the sticks, you will know, which place is higher, and how much. And thus you may proceed to further places, by setting up more sticks, or removing the boards thither.

After this manner setting your boards in a ditch, and a stick at a good distance, with a mark on it, in a line with the tops of the other two; you will find if the ground descend.

If water is to be conveyed to any assigned place, in an open channel; no part of the channel must be lower than the place it leads to; for if it is, the water can never rise again to the height of the last place. Therefore, that you may always cause it to go by an easy descent, carry the trench by any turnings and windings, by the sides of hills or otherwise, keeping near a level. Not being solicitous to have it go streight, tho' the nearer to a streight line the better, if the ground will permit. But if there happen to be any valleys or low grounds; the water must be carried over them in troughs, supported by bridges or pillars. And if a hill interferes, it must be cut through to the same level.

But if water be to be conveyed in close pipes, you may descend into valleys, because water in the pipes will rise to the same height. But where there is a hill it must be cut thro'. If the descent be great, the pipes must be laid one up, and another down; in and out, to check the violent motion of the current; and not in a right line.

If

Fig. 67. If a canal is to be cut thro' a country, for the benefit of carriage; be sure to take it from a place high enough to carry the water over the intermediate places; and where there is plenty of water. For you may descend when you will, but you cannot rise; by this means, by turnings and windings, you may conduct it over any rising grounds. Yet it will often be better to build bridges over hollow places, to carry the canal, than go too far about. And when there is plenty of water, the navigation will be quick; and the water, lost by opening the locks or flood gates, will soon be supplied. But if water be scarce, these defects and losses cannot be supplied readily enough, which will retard the navigation, and prolong the time; and therefore such a canal can never answer the purpose.

And tho' we must necessarily take the water from a height at least equal to the place, where the goods must be taken up or laid down; yet it will be a fault to take it higher; because then we shall have a greater descent to make; the consequence of which is, that we must have more flood gates. And the number of flood gates is known by dividing the whole descent by the descent of one. And the descent at one flood gate may be 4, 5, or 6 feet, according to the bigness of the vessels, and the largeness of the canal.

68. To illustrate the nature of this; let FA be the part of a canal, EC, DB, two flood gates, which may be opened and shut at pleasure; let FED be the surface of the water, when the gate DB is shut, and EC open. And CBA the surface of the water, when EC is shut, and DB open. Now a vessel coming up the part AB, has to make an ascent into the part EF. In this case DB must be open, and consequently the vessel comes into the part BC, upon the same level; and when she is got in there, the gate DB is shut, and EC opened, and then the part ECDB instantly fills from the part EF; and the vessel rises with

the

the water from the level CB to the level ED, and the Fig.
gate EC being open, she can go along the level sur- 68.
face of the water DEF, and thus she has ascended
from the height of AB to the height of EF. And
then she can go along EF till she meet with ano-
ther flood gate, to make another rise.

In like manner a vessel coming from F towards E, if EC be open, she goes into the part ED. Then shutting EC and opening DB, she goes out of EB, into the part BA, and so away.

It may be observed, that both gates must never be open at once; for then the water in FE would flow into BA, and would all be lost. And when the gate DB is shut, and EC opened, the part EB is filled instantly, without sensibly decreasing the height of EF. For EF is a long part of the canal, and EB a very short part, and soon filled. And as the loss of water, is the quantity ECDB; therefore the gates E and D ought to be set as near together as they can, to take in one or more vessels.

I shall only add this; when a canal is to be cut thro' a country, it is often attended with difficulties that make it impracticable. For in digging the canal, they often meet with rocks, thro' which they can hardly make any progress; and when that is done, the clefts of the rocks will probably take away all the water. Likewise when they meet with loose ouzy ground; by its porous nature it drinks up, and destroys a great part of the water. And even when they meet with solid ground, yet the expence of digging, levelling, &c. is so great, and requires so much time to execute it; especially when they are to go many miles in length; that considering all things, hardly any of them can afford a profit answerable to such a monstrous expence. When the ground is very much upon the descent, a great number of locks or flood gates are required, and therefore a great number of hands will be necessary, which

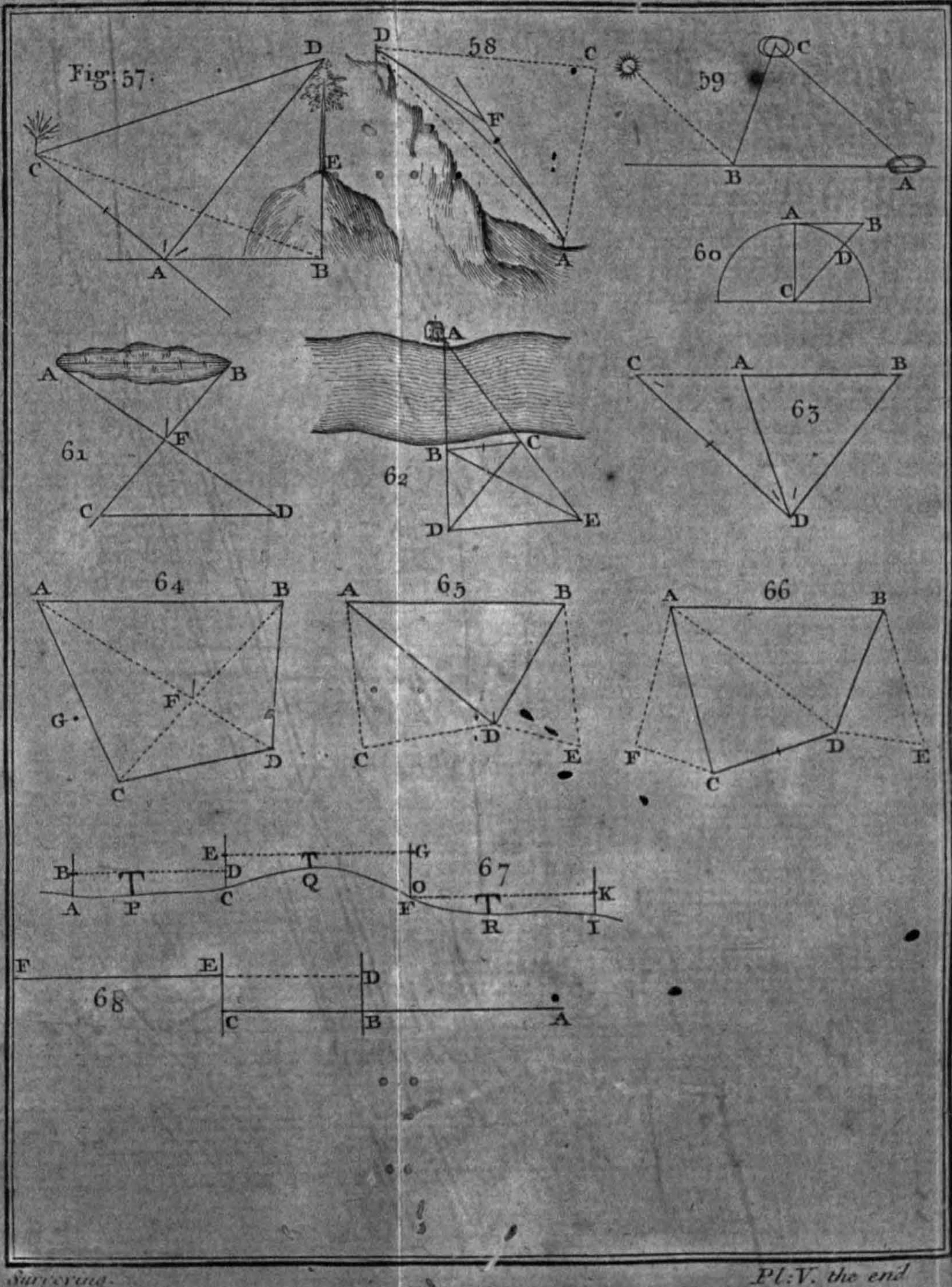
Fig. 68. which loses a deal of time; and such accidents make the thing extremely difficult, if not impossible.

And now having gone thro' all the principal branches of the Mathematics; I shall conclude with a few directions, to such as design to make these Sciences their chief study; which are by far the most difficult, as well as the most useful parts, of human learning. And first, the young student must take care to be well instructed in Arithmetick; for this is the basis of all manner of computations; not only in these sciences, but in all sorts of business. In the next place, he must get a complete knowledge of Geometry; for on this depends all computations relating to lines, which are infinite. Without a thorough knowledge of this, he will be obliged continually to turn back, or else stick fast in the road. These two branches lay the foundation of every thing that is to follow. Then the practical parts may take place, and Trigonometry and the doctrine of the sphere may be read, which make way for Astronomy. And it will be proper then, to study the Conic Sections, and the higher Geometry, which considers the nature of all sorts of curves. As Algebra is a universal science, and applicable to any thing; he may begin to read that, after he is acquainted with Arithmetic and Geometry; but then he can apply it to nothing but numerical and geometrical problems; till such time as he has learned the principles of any other science, that he would apply it to. And in this book of Algebra, there is a sufficient fund for exercising all the branches of the Mathematics. Fluxions being a science of the highest nature, and greatest extent, cannot be attained without the knowledge of all the rest; and particularly Algebra, by which all computations therein are performed. And this is the method that the young mathematician must proceed

proceed in, to make the beſt progreſs, and in the leaſt time. But he muſt frequently expect to meet with obſtacles and difficulties, thro' which his own natural ſagacity muſt carry him, joined with the delight he finds in the knowledge of theſe ſciences. For I doubt he will meet with no other reward for his induſtry, nor even for any diſcoveries he makes, but the pleaſing ſatisfaction of finding out truth. And it is this pleaſure that muſt be his main ſupport under the great fatigue of ſtudying theſe difficult ſciences. For when truth at laſt emerges out of a *chaos* of intricate calculations, and appears in its native brightneſs and luſtre, it poſſeſſes the whole mind; and the labour and fatigue of ſeeking after it inſtantly vaniſhes, and is no more thought on or remembered. Fig.

The little encouragement there is in this nation, for promoting theſe ſciences, is the reaſon, that few people go any great length this way. If they have gained ſo much knowledge, as to be able to teach a common ſchool, to get a living by, they think it ſufficient. Few make any farther progreſs; and if they do, they get the pleaſure and pains of it for their labour. Or when a man is eminent for his diſcoveries, perhaps he is dignified with the title of F. R. S. but then he is to pay a quarterly Ceſs for the honour of it. This is the way that ingenuity is rewarded in England.

As to this courſe of Mathematics, that I have here drawn up, it has been the work of my life; for in almoſt all the books I peruſed, I always found either a defect of matter or a defect of method; and ſometimes a redundance of things unneceſſary, and of little conſequence. This put me upon writing, firſt one branch, and then another, till I had got together a tolerable ſyſtem of mathematical learning; where I believe every branch is treated with as much perſpicuity, and contains as much matter, Fig. as any one can wiſh for, in that compaſs. For the whole is only deſigned as an introduction for young ſtudents, and to enable them to go thro' larger Treatiſes. Theſe things were all originally deſigned only for my own uſe, and have always proved extremely uſeful to me; and would have been much more ſo, if I had had them ſooner in life. And becauſe what has been ſerviceable to myſelf, I ſuppoſe will be of the ſame ſervice to others, under like circumſtances; therefore I determined to publiſh them. But before this could be done, I had them all to reviſe, correct, and methodize, and to add ſuch things as later improvements had found out; which has coſt me a great deal of time and labour. But then the reader is furniſhed with ſubjects, both eaſy and common, as well as difficult and ſublime; which I expect will fully anſwer his greateſt expectations. However, I have uſed my beſt endeavours. But, if not, I have taken a great deal of pains to no purpoſe; being ſenſible that without pains and patience nothing is to be done: *Sed tandem poſt ardua quies.*



Surveying. *Pl: V. the end*

APPENDIX.

Containing Corrections and Additions to all the foregoing Volumes.

ARITHMETIC.

PAGE 7, The last (*read*) of them, taken all together.

Page 31, line 3, &c. The operation being wrong, correct it thus,

```
   C.  ft.  lb.  C.  ft.  lb.
 6)72   6   11(12   1    1 the quotient.
   72
   --
    0    6)6(1
           6
          --
           0   6)11(1
                  6
                 --
                  5 remains.
                 --
```

Page 79, l. 8 (*read*) *Ex.* 5.
ib. l. 17 (*read*) *Ex.* 6.
ib. l. 27 (*read*) In *Ex.* 5.
Page 80, l. 18 (*read*) In the 6th *Ex.*
ib. l. 22 (*read*) *Ex.* 7.
Pag. 104, l. 16 (*read*) *the single rule of three.*
Pag. 114, l. 12 b (*read*) of 4 degrees of hardneſs, and 232.
ib. l. 10 b (*read*) dig a trench, of 7 degrees
ib. l. 6 b (*read*) * deg. —— 7 deg.
Pa. 146, l. 4 b (*read*) to the leaſt error; to be added, if the number was too little; but ſubtract-

K ed,

ed, if too great. Put out all the 2d article to the bottom of the page, as being needlefs.

Pa. 150, l. 1 (*read*) the correction . . . 5.86940

ib. after l. 16, *add*,

Note, this rule will anfwer all queftions that single rule of falfe will anfwer.

Page 156, l. 10 (*read*) 6 . . . . . 0.7781512

Pa. 157, l. 8 (*read*) fquare of 426

Pa. 170, l. 3 b (*read*) and 38 l. a year

Pa. 171, l. 3 (*read*) 36 . . . 684

ib. l. 4, &c. correct it thus,

$$
\begin{array}{r}
172)1774(10.32 \text{ months} \\
\underline{172} \\
54 \\
\underline{51} \\
\underline{3}
\end{array}
$$

P. 186, l. 12 (*read*) two the leaft whole num

ib. l. 13, &c. (*read*) 5 and 3 every w

for 365 and 219.

ib. l. 16, 17 (*read*) Therefore the ti

meeting will be $3 \times 24\frac{1}{3} = 73$ days.

Or thus,

$$
\begin{array}{rr}
 & C = 10 \\
 & \underline{B = 18} \\
\text{diff.} & \underline{2}
\end{array}
$$

Then $2^{m} : 1^{d} :: 73^{m} : 36\frac{1}{2}$ days, the time in w C will overtake B again; then find the leaft nu (by Prob. III. Ch IV. B. II.) which $24\frac{1}{3}$, 14 $36\frac{1}{2}$, meafure; and that is the anfwer, viz. 7

Pa. 223, l. 8 b (*read*), circulate, or come a again, according

Pa. 229, at the end of Prob. I. *add*,

Sc

Fig.

SCHOL.

If one or both numbers (in any operation) be fractional, reduce them to a common denominator. Then proceed with the numerators as before directed; and at last, subscribe the common denominator.

All the examples throughout the book should be in italic.

PROPORTIONS.

There wants Sect. I, II, III, all along in the running title.

GEOMETRY.

Page 26, 27, instead of Prop. XXV and XXVI. insert these.

PROP. XXV.

If an angle A of a triangle be bisected by a right line AD, which cuts the base; the segments of the base will be proportional to the adjoining sides of the triangle; BD : DC : : AB : AC. 1.

And if the external angle EAC *be bisected by the line* A*d*, *cutting the base in d*; *then the distances (of the intersection) from the angles, are as the adjoining sides*; B*d* : C*d* : : BA : CA.

Produce BA, and make AE = AC, and draw the line CE; because AE = AC, the <ACE = E (Prop. III.) = BAC (Prop. I.) = BAD (hyp.); therefore DA, CE are parallels (cor. 3. Prop. IV); and therefore BA : AE or AC : : BD : DC (Prop. XII.)

Again, make C*e* = CA; then the < A*e*C = *e*AC = EA*d* = FAB; therefore *e*C, AB are parallels, whence *d*B : *d*C : : AB : C*e* or CA.

Cor. 1. *If the sides be as the segments of the base; the line* AD, *bisects the angle* A.

For since BA : AC or AE : : BD : DC; DA and CE are parallels (cor. 1. Prop. XII); and BAD = <E, and DAC = ACE = E (Prop. III). Whence BAD = DAC, and A is bisected by AD.

Fig. Cor. 2. *If a line bisecting the vertical angle of*
1. *triangle, cuts the base; it will be*

As the sum of the sides, BA + AC :
To their difference, BA — AC : :
So the base, BC :
To the difference of the segments, BD — DC.

For BA : AC : : BD : DC (Prop. XXV), a
BA + AC : BA — AC : : BD + DC (BC) : BD —
DC or 2DO (Prop. XIII. Proportion), where O
the middle of the base BC.

Cor. 3. *Hence* DB : DC : : dB : dC.

2. Cor. 4. *If* AB, AC, AD, *be continually propo*
tional, and AE *equal to* AC; *then* CE *drawn, bise*
the angle BCD.

For the triangles ABC, ACD are similar; when
CB : CD : : AB : AC or AE : : AC or AE : AD
AE — AB (BE) : AD — AE (ED). Therefo
(Cor. 1.) angle BCE = ECD.

PROP. XXVI.

3. *If an angle* A *of a triangle* ABC, *be bisected by*
right line AD, *which cuts the base; the square of t*
bisecting line, together with the rectangle of the s
ments, is equal to the rectangle of the sides; AD^2 +
BDC = BAC.

Produce AD, and make the angle <DBP = DA
Then the 3 triangles CDA, PDB, and PBA are
milar. For CAD = PBD = PAB, and CDA =
PDB (2.I), whence <C = P, and ADC = A
(cor. 1. Prop. II). Therefore CD : DA : : PD :
(Prop. XIII), whence DA × PD = CD × BD (
Proportion). Again, CA : DA : : AP or AD
DP : AB (Prop. XIII); therefore CA × AB
AD^2 + DA × DP (12 Proportion) = AD^2 +
× BD (Ax 3).

Cor. *If the external angle* EAC *be bisected by*
line Ad, *cutting the base in* d; *then the rectangle* B
— the square of the bisecting line Ad, *is equal to*
rectangle BAC.

For produce dA to F, and make $\angle dBF = dAC$; Fig. then the triangles CdA, FdB, and FBA are similar. 3. For $CAd = dAE = FAB$; and $CdA = FdB$. Whence $ACd = AFB$, and $AdC = ABF$ (cor. 1. Prop. II.) Therefore, $Cd : dA :: Fd : dB$ (XIII); and $dA \times Fd = Cd \times Bd$ (12 Proportion). Again $CA : dA :: AF$ or $dF - dA : AB$ (13). Therefore $CA \times AB = dA \times dF - dA^2$ (12 Proportion) $= Cd \times Bd - dA^2$.

PROP. XXVII, &c.

Page 73, l. 20, (*read*) *between the sum and difference of the two radii*;
P. 107, l. 16, (*read*) DFKHCILG =
P. 121, l. 2, (*fig*. 191)
ib. l. 25, (*read*) DCH, be 3
P. 125, l. 18, fig. 195.
P. 126, l. 2, fig. 195.
ib. l. 8, fig. 196.
ib. l. 19, fig. 197.
ib. l. 4 b, fig. 198.
P. 127, l. 2, fig. 198.
P. 128, l. 2, fig. 198.
ib. l. 2[illegible], fig. 199, 200.
P. 129, l. 2, fig. 199, 200.
P. 137, l. 5, fig. 209.
P. 156, l. 2, (*read*) *Practical Geometry, or The Construction*, &c.
P. 157, 8, Prob. V and VI, (*read*) *To bisect, or divide a given*
ib. [illegible] Prob. VII, *add*,

Or thus,

If the angle be given in degrees; from the given point B, draw the line BD. Then upon a line of Chords extend your compasses from the beginning to 60 degrees (called the *Sweep of* 60), then setting one foot in B, with the other describe the arch DC. Then extend from the beginning, to the number

Fig. of degrees given, and set that extent from D to C, upon the arch DC. Thro' C draw the line BC, and DBC is the angle required.

Page 162, at the end of Prob. XII. *add,*

Cor. *If it be required to set any number of equal parts upon a right line, it is done thus.*

Take with your compasses, so many equal parts from a diagonal scale, or any convenient scale of equal parts; and set that extent upon your line.

TRIGONOMETRY.

Page 100, after Prop. II. *add,*

Cor. 1. *If two right angled triangles, have the same or equal hypothenuses, the perpendiculars, are as the sines of the angles at the base; and the bases are as the sines of the angles at the vertex.*

For they are both in the given ratio of the hypothenuse (AC) to the radius.

Cor 2. *If two right angled triangles have equal bases, the perpendiculars are as the tangents of the angles at the base; and the hypothenuses are reciprocally as the sines of the angles at the vertex.*

For the perpendiculars are tan. opposite angles are in the given ratio of the base to the radius.

And if B be the base; H, *b*, the hypothenuses; V, *v*, the sines of the angles or the vertex. Then by this Prop. as $V : B :: \text{rad} : H = \frac{B}{V} \times \text{rad}$; and $v : B :: \text{rad} : h = \frac{B}{v} \times \text{rad}$. therefore $H : h :: \frac{B}{V} \times \text{rad} : \frac{B}{v} \times \text{rad} :: \frac{1}{V} : \frac{1}{v} :: v : V$.

Cor. 3. *If two right angled triangles have equal perpendiculars; the bases are as the tangents of the angles at the vertex; and the hypothenuses are reciprocally as the sines of the angles at the base.*

This is plain by Cor. 2, making the base to become the perpendicular. Fig.

Page 104, l. 8 b, (*read*) greater side :

Page 154, after Prop. XX. *add*,

Corol. *In a right angled spherical triangle, if the hypothenuse be a quadrant, one of the sides will be a quadrant, and its opposite angle a right angle.* fig. 37.

Let BAC be the triangle, A the right angle, the hyp. BC a quadrant. AB (produced at least) passes thro' the pole of AC; and if the quadrant CB be supposed to revolve about C, it will cut AB in the pole of AC; but B is the point where they cut; therefore B is the pole of AC; and consequently BA, BC are both quadrants; and A being a right angle, C will also be a right angle (by Prop. XIV).

Page 158, put out the two Corols. at bottom. 4.

Page 159, line 1, (*read*) Cor. 1.

ib. Instead of the two Corols. at bottom, insert

Cor. 1.

In right angled spherical triangles ABC, ADF, *that have equal angles at the base; then,*

1. *The sines of the hypothenuses, are as the sines of the perpendiculars.*

2. *The sines of the bases are as the tangents of the perpendiculars.*

3. *The tangents of the hypothenuses, are as the tangents of the bases.*

4. *The cosines of the hypothenuses are as the cotangents of the angles at the vertex.*

5. *The cosines of the bases, are as the cosines of the vertical angles.*

6. *The cosines of the perpendiculars, are reciprocally as the sines of the angles at the vertex.*

All these are easily demonstrated by the help of the two last propositions; by extending the sides of the triangles to quadrants, after the manner of fig. 41, and finding the proportions, in the complemental triangles EDF, ADC, ECB; which will come out as follows.

Fig. 4. For rad : S.A : : S.AC : S.CB : : S.AF : S.FD.
and rad : tan. A : : S.AB : tan. BC : : S.AD : t. DF.
and rad : cos. A : : tan. AC : tan. AB : : t. AF : t. AD.
and rad : t. A : : cos. AC : : cot. C : : cos. AF : cot. F.
and rad : S.A : : cos. AB : : cos. C : : cos. AD : cos. F.
and rad $\times$ cos. A $=$ cos. CB $\times$ S.C $=$ cos. FD $\times$ S.F.

Cor. 2.

In right angled spherical triangles, AFB, AFD, *that have equal hypothenuses; then,*

1. *The sines of the bases are as the sines of the vertical angles.*

2. *The tangents of the bases, are as the cosines of the angles at the base.*

3. *The tangents of the angles at the base, are as the cotangents of the angles at the vertex.*

4. *The cosines of the bases, are reciprocally, as the cosines of the perpendiculars.*

For rad : S.AF : : S.AFB : S.AB : : S.AFD : S.AD.
and rad : t. AF : : cos. FAB : t. AB : : cos. FAD : t. AD.
and rad : cos. AF : : t. FAB : cos. AFB : : t. FAD : cot. AFD.
and rad $\times$ cos. AF $=$ cos. AB $\times$ cos. BF $=$ cos. AD $\times$ cos. DF.

Cor. 3.

In right angled spherical triangles ABC, ABF, *having equal bases; then,*

1. *The tangents of the perpendiculars, are as the tangents of the angles at the base.*

2. *The sines of the angles at the base, are as the cosines of the angles at the vertex.*

3. *The sines of the perpendiculars, are as the cotangents of the vertical angles.*

4. *The cosines of the perpendiculars, are as the cosines of the hypothenuses.*

5. *The cotangents of the hypothenuses, are as the cosines of the angles at the base.*

6. *The sines of the hypothenuses, are reciprocally as the sines of the vertical angles.*

For,

For, rad : S.AB :: { tan. CAB : tan. CB. / tan. FAB : tan. FB. } Fig. 4.

and rad : cos. AB :: { S.CAB : cos. ACB / S.FAB : Cos. F. }

and rad : tan. AB :: { cotan. ACB : S.CB. / cotan. F : S.FB. }

and rad : cos. AB :: { cos. CB : cos. AC. / cos. FB : cos. AF. }

and rad : cotan. AB :: { cos. CAB : cotan. AC. / cos. FAB : cotan. AF. }

And rad $\times$ S.AB $=$ S.ACB $\times$ S.AC, or $=$ S.F $\times$ S.AF.

Note, either side may be taken for the base.

Page 196, after line 7, *add*,

If the solution is neither found in ABF nor EDF; produce the sides from F, thro' A and B to quadrants as before, to find a 3d triangle; and then the solution will be found in this new complemental triangle.

P. 208, l. 6 b, (*read*) tan. $\frac{1}{2}$ AB.

p. 211, l. 7, (*read*) let fall FI a perp.

p. 213, l. 6 b, (*read*) if AF + FB < 180.

p. 239, l. 4 b, (*read*) proportionals between

Tab. natural sines.

8° 38′——sine .[illegible]01106

24°, at bottom right hand, (*read*) 114 deg.

41° 3′——[illegible] 35

43° 53′——sine .6931922

Tab. artificial sines.

15° 60′——sine 9.4403381

20° 60′——sine 5306

Tab. Logarithms.

Num. 354

Num. 987

1027——log. 8817

Num. [illegible]804

ALGE-

Fig.

ALGEBRA.

page	line	read
4	7 b	compound quantity; and the ſum taken all together.
47	1 b	$= \frac{\sqrt[n]{a}}{\sqrt[n]{x}}$.
119	9 b	*agc — bcf*.
148	4	wants a line underneath.
258	8	$a = \frac{d}{b+a} =$
282	6 b	$\frac{.26106}{10.598} =$
300	1	fig. 4.
328	3 b	$a = \frac{3s}{5} =$
336	8	$bfa - bdc$
342	13	$x^9 - x^4 = bx^6$
347	2	fig. 22.
353	9 b	$R^t - 1 = R^t$.
ib.	8 b	$\left(\frac{R^t}{rR^t}\, a = \right)$
358	6	2 miles, the ſecond 3, the third 4,
	9	$\overline{x - 1} + 2 =$ his laſt
364	4	*of the means, or their ſquares (c)*;
401	1 b	area, and AC $= y$.
408	1	fig. 54, put A for C, B for A, and C for B.
409	5 b	$\overline{+a - c}^2 = 2aa + 2cc =$
413	6	$\overline{s + b} \times \overline{s - b} =$
	7	$z \times \overline{z - a} \times \overline{z - b} \times \overline{z - c}$.
430	12	$\sqrt{tt \times \overline{c + y}^2} -$
441	1 b	GPV, VPM,
442	7	ſpherical triangle ABC,
465	5 b	(mechan. 56 and cor.)
	3 b	*leave out (ib. cor. 5.)*
466	3	$n \times \sqrt{\overline{\ \ }^2 - 1}$.

(mechan

page	line		Fig.
477	1	(mechan. 27. cor. 2)	
	2	dele fig. 132.	
497	10	last term will.	
524	7 b	(mec. 18. cor. 3)	
525	16 b	(mec. 23. cor. 1)	
526	10	(mec. 23. cor. 1)	
	6 b	(mec. 23. cor. 1)	
		Postscript.	
1	13	met with such	
2	7 b	but Dr. Halley and Fr. Robarts, esq; have	
	5 b	leave them	
		ARITHMETIC OF INFINITES, &c.	
3	1	$= \frac{1}{4} n \times \overline{na}$, or	
	3 b	$s = \overline{n+1.a}^{m}$	
7	13	all the x'es	
12	11	$c = -A + 2b + C =$	
17	21,	&c. Prop. V. should be italic	
29	4	and thus you may	
4	6 b	FINIS.	
		CONIC SECTIONS.	
9	17	$CA^2 : CN^2 :: BPA : PM^2$.	4
18	6 b	(XIII. Cor. 1) =	
20		at the end of Prop. 23. *add*, Cor. 1. *rectangle* $NMV = SMF - CE^2$. Cor. 2. $SC^2 : CE^2 ::$ *rectangle* $SMF - CE^2 : PM^2$. fig. 18. For (Prop. 19.) $CA^2 : CE^2 :: PN^2 : PM^2$, and $CA^2 - CE^2 : CE^2 :: PN^2 - PM^2 : PM^2$, that is, $SC^2 : CE^2 ::$ rect. $NMV : PM^2 ::$ (cor. 1.) $SMF - CE^2 . PM^2$.	
29	6	Cor. 1.	
		after line 7. *add*, Cor. 2. $FM : FT :: \sqrt{FMS} : CD$.	

Cor.

Fig. page	line	read
		Cor. 3. *Draw* MO *perp. to the tangent* MH. *Then* $CI \times MO = CD^2$. For (cor. 2. Prop. 35.) MO : CD :: CK : CA :: (cor. 1. of this) CD : CI.
34	5 b	$OD \times DN :: CF^2 : CK^2$.
56	7	*more lines el,*
60	2	(ib. Prop. III.)
131	4	multiplying by 4, $QZ^2 =$

PLATES.

Plate	iii	*f.* 28. draw MO perp. to MH.
plate	x	pa. 66, (at bottom).
pl xii	fig.	102, draw CD, PM.
pl xxi		pa. 148, (at bottom).

CURVE LINES.

89	7	whence y^2
90		after line 5, *add*, Cor. *In a very small part of any curve of finite curvature, as* AG; *the subtense of the angle of contact, is as the square of the tangent, cord, or arch; for these in their evanescent state, are all equal; that is,* $BG \propto AL^2$ or AG^2. fig. 86. *It is also observable, that the cords, sines, tangents, versed sines, &c. of a very small arch of a curve, have the same properties as those of the circular arch.*
94	12 b	line MF, are
	7 b	Since $Mv = ms$,
95		at the bottom, *add*,

SCHOLIUM.

It may be observed, that the circle of equal curvature with the curve at M, is that which passes thro' three points of the curve, supposed to approach infinitely near one another.

page	line	read	Fig.
		For two points do not determine it, because an infinite number of circles may pass thro' two points. But there is only one that can pass thro' three. And if a circle passes thro' two points that approach infinitely near, and at last coincide, it only denotes that this circle touches the curve in that point; which an infinite number of circles may do.	
97	3 b	$\dot{q} = \dot{p} + \dot{p}$;	
98	14 b	*axis*, $\dot{y} = 0$,	
99	2	$= y\dot{x} \times \frac{\dot{y}\ddot{y}}{\dot{z}}$	
114	6	$Mn = \dot{x}$, mn	
Pl. 8		fig. 100, wants o.	

OPTICS.

page	line	read	Fig.
85	5	$\dfrac{Cf \times x}{\frac{rr}{2Cf} - x}$	
9		after Cor. 3, *add*, Cor. 4. *In different mediums, it will be*, AB : BC : : AD : DE.	
150	2	*the focal distance. And the same for a water Lens*, (by Schol. Prop. II.)	
159	14 b	attends old people,	
169	14	$+ \overline{\frac{q-s}{q+s}}\Big\vert^{2} s$,	
170	4 b	in the same point P; and a stop with a hole in it must be placed in the middle between AB and CD, or nearer AB, to cut off straggling rays, that colour the object; and then the telescope is fit for use. The telescope —	
173	15	thro' the lens B,	

power

Fig. page	line	read
179	12	*power of a refracting telescope.*
181	3	*thro' a refracting telescope.*
184	2 b	*in different refracting te-*
185	15	*In similar refracting telescopes;*
193, 194, 195	1	fig. 116.
203	1	p. 203.
204	5	*leave out,* (by Prop. XVII. B. III.)
	9 b	to do; its magnifying
220	8 b	Microscope, which is shorter. Where
224	7 b	focus of AB, it
pl.viii		fig. 106, GLI should be streight.
pl.ix.		fig. 107, put Q near the end of P.
pl. x		fig. 116, the two axes at C, D shou[ld] cross one another.
pl.xiii		fig. 131, the square LMIK to be prick'[d]

PERSPECTIVE.

63	1	Distances of 13, &c.
pl. ii		fig. 16, the whole plane RGCQ shou[ld] be shaded.
		fig. 19, for *t* put *l*.
pl. ix		fig. 55, 1, 1 shou[l]d be at the angl[es] above.
pl. x.		fig. 57, the prick'd line[s] 41, 1S, wanting, and some others.
pl.xv		fig. 77, produce R*l*, a little beyond *l*.

MECHANICS.

32		After line 11, *add,* Cor. 5. *The space described by a fallin[g] body in the time of 1 vibration : is to half the length of the pendulum :: as the square of the circumference : to the square of the diameter.*

Fc

page	line	read	Fig.
		For the ſpaces being as the ſquares of the times, the ſpace fallen : $\frac{1}{2}$length of the pendulum : : as the ſquare of the time of vibration : to the ſquare of the time of falling thro' $\frac{1}{2}$length of the pendulum : : (by this Prop.) as the ſquare of the circumference ; to the ſquare of the diameter.	
35	10	Cor. 1. Hence,	
145	14	Cone ABC	
146		after line 11 b, *add*,	
		I lately contrived another ſort of barometer, which ſhews the aſcent and deſcent of the mercury at the bottom.	
		ABC is a recurve tube, cloſe at the top, where the bucket C is, and open at the end A. The length of CB is 32 or 33 inches, and of AB 6 or 7. The bucket C ſhould contain about as much as the end AB. And the bucket and end CB muſt be quite filled with mercury, as far as B, a little beyond the turn. The wider the bucket C is, the better. The ſcale ſet to the end AB, muſt be graduated downwards, for the mercury falls in this, when it riſes in the other ſort. This being placed againſt a wall, will ſhew the height of the mercury, as in the common ones. And this way is more commodious, as it ſaves the labour of clambering up, upon chairs, to ſee it, as one muſt do, in the common ſort, to ſee exactly.	7.
		A WATER BAROMETER	

PROJECTION.

5	1	fig. 2.	
8	3	fig. 5.	

off

Fig. page	line	read
9	3 b	off the ſines.
13	17	fig. 12.
14	2 b	the two laſt lines ſhould be roman and indented.
	21	$CpA + CAp$;
15	3	ſhould be indented.
		3, 4, 5 lines roman.
16	5	$ECI = CAD$
18	4	if p, q, be
	12	harmonically
20	7	projection C,
	9	equal arches EF
21	15	$\frac{OG + OD}{2}$:
25	9 b	points A, B, G,
26	8	draw EB to the point B, and BF perpendicular to it, cutting CH in F. To &c.
27		After *l* laſt *add*,
		But if the circle DLE was to lie on the other ſide of DGE, the arch HI muſt be taken on the other ſide of H.
29	6 b	interſection P,
30	2	gCt required.
	4	circle DEH, or
	11 b	pole p, draw
		After l. 8 b, *add*,
		For a right circle ACB.
		Thro' C (fig. 34) draw DE perpendicular to AB, then E is the pole of AB. Set the number of degrees from D to H, and draw EGH, then GC contains the degrees required. Or if GC be given, draw EGH, then HD in degrees, is the meaſure of GC.
		And in like manner AG contains the degrees in AH, or AH is the meaſure of AG.

be

Fig.

page	line	read
79	9	cast by *Spain*,
82	6	as the land of
1[illegible]	2 b	long night be
128	14	dif. lat. AR.
	4 b	velocity A*k*.
130	13	*ts* + *sr*, &c.
132	16	ser than Az;
137	10	whence z = 2.20
144	10	$HI = BE \times \overline{1 + \frac{1}{2}qss}$.
162	22	fig. 18.
169		after Cor. 2. *add*,
		Cor. 3. *And from hence will be known how much the watch is too fast or too slow; which now may be set to the true time.*

DIALLING.

page	line	read
9	15	*tag. his rays.*
16	27	*removed westward or eastward.*
17	10	*carried back thro'*
18	5	*situations on that meridian.*
58	11	fig. 20.
	5 b	put out fig. 20.
77	1	fig. 43.
131	1	put out fig. 12.
151		at bottom, *add*;
		See Cor. 3. Prop. V. Sect. III. Projection.
163	14 b	Chambau
		Pl. IX. fig. 34. 9 and 3 are wrong placed; they should stand where *qb*, *qd* cut AB.

COMBINATIONS

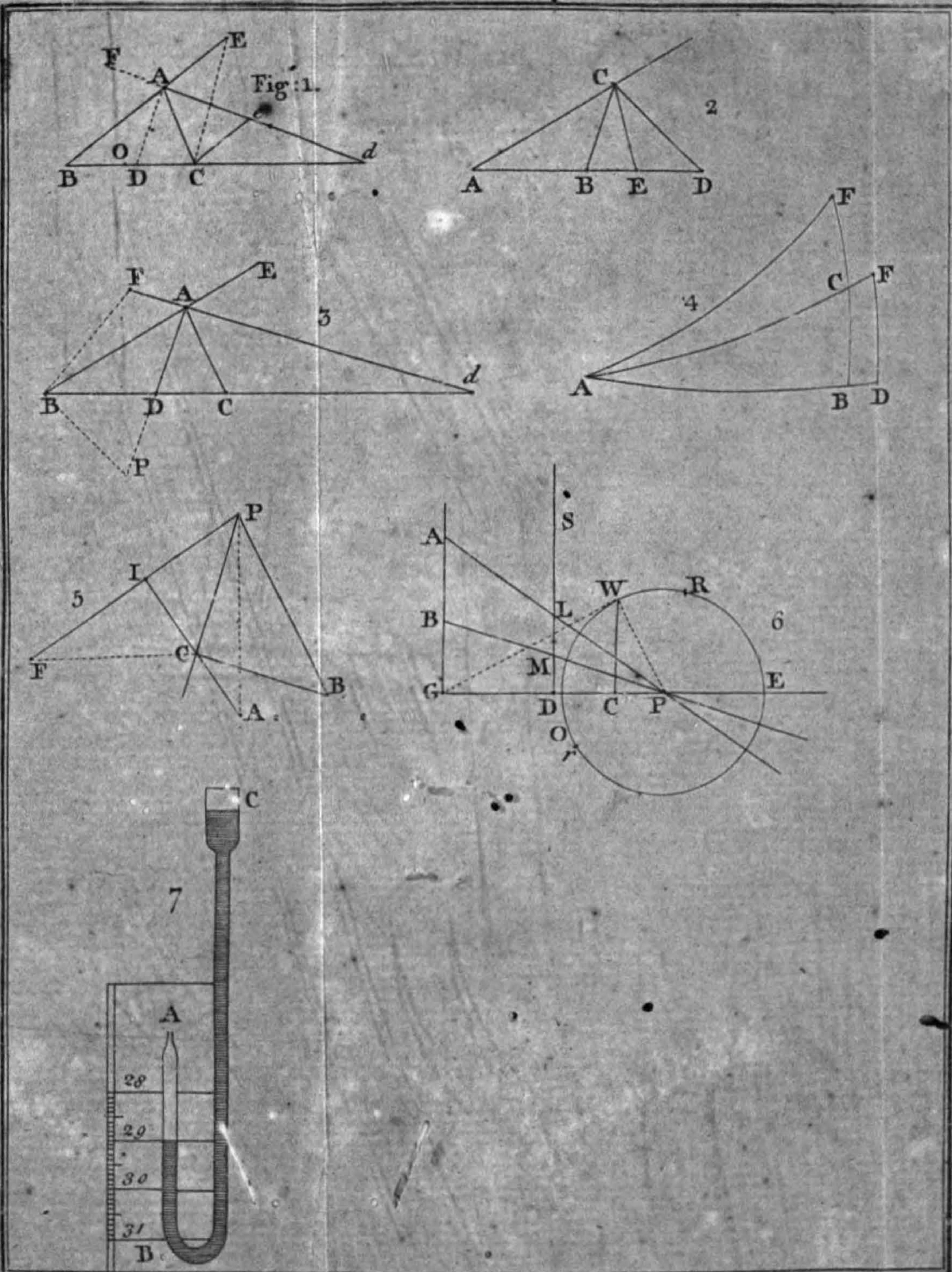
page	line	read
25	5 b	but there should

M

MEN-

Fig. page	line	read
		MENSURATION.
26	4	$37^{l}\ 12^{s}\ 6^{d}$
66	12	$3 : \frac{9}{2}$
69	1	fig. 8.
89	o· b	dele *Examp.*
118	12	— 1.895091
		9
125	12	176.16
		SURVEYING.
		angle *abc*, and
38	19	880
89	12	440
	&c.	110
		1.6280

FINIS.



Appendix *the end*