LÁVATÍ.

COLEBROOKE'S TRANSLATION

OF THE

LÍLÁVATÍ.

WITH NOTES

BY

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PREFACE.

This little book is an edition of Colebrooke's translation of the *Lílávatí*, a standard work on Hindu mathematics, written by Bháskaráchàryya, a celebrated mathematician and astronomer who lived in the twelfth century of the Christian era.[1] The work forms the first part of a larger work of the author called the *Siddhánta-siromani*. This part is called by the author, *Páti-ganita* or Arithmetic; but this name has not been properly given. For, the work, besides dealing with subjects which lie within the province of Arithmetic, also treats of subjects which properly belong to Algebra and Geometry. It comprises the four simple rules, extraction of the square root and the cube root, vulgar fractions, Rule of Three, interest, alligation, problems producing simple and quadratic equations, arithmetical and geometrical progressions, permutations and combinations, indeterminate equations of the first degree, several properties of triangles and quadrilaterals, areas of circles, volumes of spheres, cones and pyramids, solid content of excavations, and several other matters. Some of the problems solved evince a great deal of progress in algebraical investigations. The author does not state the reasons for the various rules given by him. I have tried to supply the reasons as simply and shortly as they occurred to me; but still in some cases neater and shorter demonstrations may possibly be given. The explanations given have been printed in small type and enclosed within square brackets. It is thus hoped that the present edition will prove useful and interesting not

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[1] This date is ascertained from the fact that Bháskara himself informs us in a passage of his *Siddhánta-siromani*, that he was born in the year 1036 of the *Saka* era, and that he completed his great work when he was 36 years old. This gives 1150 A.D., as the date of the completion of the *Siddhánta-siromani*. See the *Goládhyáya* of the *Siddhánta-siromani*, Wilkinson's translation, XIII, 58.

only to the scholar and the antiquarian, but also to the student of modern algebra.

In his foot-notes, Colebrooke has given translations of extracts from the leading commentaries on the *Lílávatí*. These are:—(1) The commentary of Gangádhara, written about 1420 A.D.: (2) that of Súryadása, called *Ganitámrita*, written in 1538 A.D., containing a clear interpretation of the text, with concise explanations of the rules: (3) that of Ganesa, called *Buddhivilásiní*, the best of all the commentaries, written in 1545 A.D., comprising a copious exposition of the text, with demonstrations of the rules: (4) the gloss of Ranganátha on the *Vásaná*, or Bháskara's demonstratory annotations of the *Siddhánta-siromani*, written towards the beginning of the seventeenth century A.D.: (5) the *Manoranjana*, written by Rám Krishna Deva, of uncertain date: and (6) the *Ganitakaumudí*, which has not been recovered, but is known from the quotations cited from it by Súryadása and Ranganátha. Some of the translated extracts contain expositions of the rules and of technical terms, and some contain demonstrations of the rules in a few cases. Of these demonstrations which are given chiefly by Ganesa and Súryadása, those which are satisfactory and instructive, have been retained in the present edition; whilst others which are obscure and unsatisfactory, have been omitted. For convenience of reference, the *Lílávatí* in Sanskrit is printed at the end, with divisions into chapters and sections corresponding to those made in the translation. No such divisions were made by Bháskara.

H. C. B.

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LÍLÁVATÍ.

CHAPTER I.

INTRODUCTION.

1. Having bowed to the deity, whose head is like an elephant's[1]; whose feet are adored by gods; who, when called to mind, relieves his votaries from embarrassment; and bestows happiness on his worshippers; I propound this easy process of computation,[2] delightful by its elegance,[3] perspicuous with words concise, soft and correct, and pleasing to the learned.

DEFINITIONS OF TECHNICAL TERMS.

(Money by tale.)

2. Twice ten cowry shells[4] are a *kákiní;* four of these are a *pana;* sixteen of which must be here considered as a *dramma;* and in like manner, a *nishka,* as consisting of sixteen of these.

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[1] Ganesa, represented with an elephant's head and human body.
[2] *Pátíganita; pátí, paripátí,* or *vyaktaganita,* arithmetic.
[3] *Líláratí,* delightful: an allusion to the title of the book. See notes on §§ 18 and 277.
[4] Cypræa moneta. Sans., *Varátaka, kapardí.* Hindi, *Kauri.*

(Weights.)

3. A *gunja*[1] (or seed of *Abrus*) is reckoned equal t two barley-corns ; a *valla*, to three *gunjas;* and eight c these are a *dharana;* two of which make a *gadyánaka* In like manner one *dhataka* is composed of fourtee *vallas.*

4. Half ten *gunjas* are called a *másha*, by such a are conversant with the use of the balance ; a *karsha* contains sixteen of what are termed *máshas;* a *pala* four *karshas.* A *karsha* of gold is named *suvarna.*

(Measures.)

5—6. Eight breadths of a barley-corn[2] are here a finger; four times six fingers, a cubit[3]; four cubits, a staff[4]; and a *krosa* contains two thousand of these; and a *yojana*, four *krosas.*

So a bambu pole consists of ten cubits; and a field (or plane figure) bounded by four sides, measuring twenty bambu poles, is a *nivartana.*[5]

7. A cube,[6] which in length, breadth and thickness measures a cubit, is termed a solid cubit : and, in the meting of corn and the like, a measure, which contains

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[1] A seed of Abrus precatorius, black or red ; the one called *Krishnala*, the other *raktiká*, *ratti*, or *rattiká*.

[2] Eight barley-corns (*yava*) by breadth, or three grains of rice by length are equal to one finger (*angula*). —Gan.

[3] According to Ganesa, the cubit (*hasta*) means the practical cubit employed by artisans and called *gaj*. It is longer than the ordinary cubit of 18 inches.

[4] *Danda*, a staff : directed to be cut nearly of man's height. (Manu, II. 46.)

[5] A superficial measure containing 400 square poles.—Súr.

[6] *Dwádasásra*, lit. dodecagon, but meaning a parallelopiped; the term *asra* corner or angle, being here applied to the edge or line of incidence of two planes.

c

solid cubit, is a *khárí* of *Magadha*[1] as it is denomi-ated in science.

8. A *drona* is the sixteenth part of a *khárí;* an dhaka is a quarter of a *drona;* a *prastha* is a fourth art of an *ádhaka;* and a *kudaba* is by the ancients ermed a quarter of a *prastha*.[2]

The rest of the axioms, relative to time[3] and so forth, re familiarly known.

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[1] The country situated on the *Sonebhadrá* river.—Gan. It is South Behar.

[2] Another stanza occurs here in one copy of the text. It is explained in e *Manoranjana*, and by Gangádhara, but not by Ganesa and Súryadása. is therefore to be rejected as spurious and interpolated. It is as follows:— sera is here reckoned at twice seven *tankas*, each equal to three-fourths a *yadyánaka:* and a *mana*, at forty *seras*. The name is in use among the *urushkas*, for a weight of corn and like articles." See notes on §§ 97 and 233.

[3] The author has himself explained the measures of time in his *Siddhánta-iromani*. [See the *Goládhyáya*, Wilkinson's translation, IV. 5–12.—Ed.]

CHAPTER II.

SECTION I.

INVOCATION.[1]

9. Salutation to Ganesa, resplendent as a blue an spotless lotus; and delighting in the tremulous motio of the dark serpent, which is perpetually twining withi his throat.

NUMERATION.

10—11. Names of the places of figures have beer assigned for practical use by ancient writers,[2] increasing regularly[3] in decuple proportion: namely, unit, ten, hundred, thousand, myriad, hundred thousands, million, ten millions, hundred millions, thousand millions, ten thousand millions, hundred thousand millions, billion,

---

[1] A reason of this second introductory stanza is, that the foregoing definitions of terms are not properly a part of the treatise itself; none such having been premised by Árya Bhatta and other ancient authors in their treatises on arithmetic.—Gan. and *Mano.*

[2] According to the Hindus, numeration is of divine origin; the invention of nine figures (*anka*), with the device of places to make them suffice for all numbers, being ascribed to the beneficent Creator of the Universe, in Bháskara's *Vásaná* and its gloss; and in Krishna's Commentary on the *Víja-ganita.* Here nine figures are specified; the place, when none belongs to it, being shown by a blank (*súnya*), which, to obviate mistake, is denoted by a dot or small circle.

[3] From the right, where the first and lowest number is placed, toward the left hand.—Gan.

ten billions, hundred billions, thousand billions, ten thousand billions, hundred thousand billions.[1]

SECTION II.

EIGHT OPERATIONS[2] OF ARITHMETIC.

12. Rule of addition and subtraction[3]: half a stanza.

The sum of the figures according to their places is to be taken in the direct or inverse order[4]: or (in the case of subtraction) their difference.

[The rule as exemplified in the *Manoranjana* is more cumbrous than the ordinary rule.]

13. Example. Dear intelligent Lílávatí,[5] if thou be skilled in addition and subtraction, tell me the sum of two, five, thirty-two, a hundred and ninety-three, eighteen, ten, and a hundred, added together; and the remainder, when their sum is subtracted from ten thousand.

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[1] Sans. *eka, dasa, sata, sahasra, ayuta, laksha, prayuta, koti, arbuda, abja* or *padma, kharva, nikharva, mahápadma, sanku, jaladhi* or *samudra, antya, madhya, parárdha.*

A passage of the *Veda*, which is cited by Súryadása, contains the places of figures :—"Be these the milch kine before me, one, ten, a hundred, a thousand, ten thousand, a hundred thousand, a million, ... Be these milch kine my guides in this world."

[2] *Parikarmáshtaka*, eight operations, or modes of process; logistics or algorism.

[3] *Sankalana, sankalita, misrana, yuti, yoga, &c.*, summation, addition. *Vyavakalana, vyavakalita, sodhana, patana, &c.*, subtraction. *Antara*, difference, remainder.

[4] From the first on the right, towards the left; or from the last on the left, towards the right.—Gang.

[5] Seemingly the name of a female to whom instruction is addressed. But the term is interpreted in some of the commentaries, consistently with its etymology, 'charming.'—See §§ 1 and 277.

Statement : 2, 5, 32, 193, 18, 10, 100.

Result of the addition[1] : 360.

Statement for subtraction : 10000, 360.

Result of the subtraction : 9640.

14—15. Rule of multiplication[2] : two and a half stanzas.

Multiply the last[3] figure of the multiplicand by the multiplicator, and next the penult, and then the rest, by the same repeated. Or let the multiplicand be repeated under the several parts of the multiplicator, and be multiplied by those parts : and the products be added together. Or the multiplier being divided by any number which is an aliquot part of it, let the multiplicand be multiplied by that number, and then by the quotient, the result is the product. These are two methods of subdivision by form. Or multiply separately by the places of figures, and add the products together. Or multiply by the multiplicator diminished or increased by a quantity arbitrarily assumed ; adding or subtracting the product of the multiplicand taken into the assumed quantity.

[The author gives here six methods. The first method is the ordinary one, and includes the *tatstha* of the older authors, which is worked by repeating or moving the multiplier over or under every digit of the multiplicand, and which, according to Ganesa's

[1] Mode of working addition as shown in the *Manoranjana* :

Sum of the units,	2,5,2,3,8,0,0,	... ...	2 0
Sum of the tens,	3,9,1,1,0,	... ...	1 4
Sum of the hundreds,	1,0,0,1,	... ...	2
Sum of the sums		... ...	3 6 0

[2] *Gunana*, *abhyása* ; also *hanana* and any term implying a tendency to destroy. It is denominated *pratyutpanna* by Brahmagupta and by Srídhara. *Gunya*, multiplicand. *Gunaka*, multiplicator. *Ghâta*, product.

[3] The digit standing last towards the left.

explanation, proceeds obliquely, joining products along compartments. The second is tedious, following from the formula, $a(b+c) = ab + ac$. The third is multiplication by factors. The fourth is practically the same as the first, and the fifth, the same as the second. The sixth follows from the formula, $a(b - c) = ab - ac$.]

16. Example. Beautiful and dear Lílávatí, whose eyes are like a fawn's! tell me the numbers resulting from one hundred and thirty-five, taken into twelve, if thou be skilled in multiplication by whole or by parts, whether by subdivision of form or separation of digits.[1] Tell me, auspicious woman, the quotient of the product divided by the same multiplier.

Statement: Multiplicand 135. Multiplicator 12.

Product (multiplying the digits of the multiplicand successively by the multiplicator) 1620.

Or, subdividing the multiplicator into parts, as 8 and 4; and severally multiplying the multiplicand by them; adding the products together, the result is the same, 1620.

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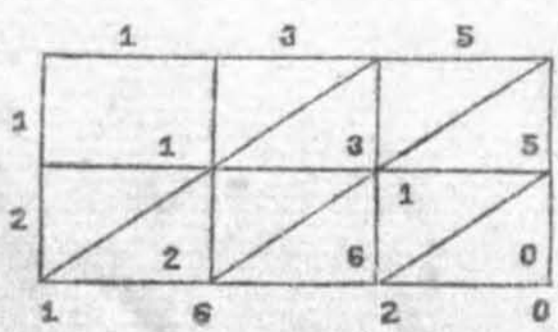
[1] The adjoined scheme of the process of multiplication is exhibited in Ganesa's commentary.



According to the *tatstha* method, the process will stand thus :—

12	12	12	or	135	135
1	3	5		1	2
12		60			270
	36				135
	1620				1620

Or, the multiplicator 12 being divided by 3, the quotient is 4 ; by which, and by 3, successively multiplying the multiplicand, the last product is the same, 1620.

Or, taking the digits as parts, *viz.*, 1 and 2 ; the multiplicand being multiplied by them severally, and the products added together, according to the places of figures, the result is the same, 1620.

Or, the multiplicand being multiplied by the multiplicator less 2, *viz.*, 10, and added to twice the multiplicand, the result is the same, 1620.

Or, the multiplicand being multiplied by the multiplicator increased by 8, *viz.*, 20, and eight times the multiplicand being subtracted, the result is the same, 1620.

17. Rule of division[1] : one stanza. That number, by which the divisor being multiplied balances the last digit of the dividend (and so on[2]), is the quotient in division : or, if practicable, first abridge[3] both the divisor and the dividend by an equal number, and proceed to division.

Example. Statement of the number produced by multiplication in the foregoing example, and of its multiplicator, for a dividend, 1620, and a divisor, 12.

Quotient 135 ; the same with the original multiplicand.[4]

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[1] *Bhága-hára, bhájana, harana, chhedana*, division. *Bhájya*, dividend. *Bhájaka, hara*, divisor. *Labdhi*, quotient.

[2] Repeating the divisor for every digit, like the multiplier in multiplication.—Gang.

[3] *Apavartya*, abridging. See note on § 249.

[4] The process of long division is exhibited in the *Manoranjana*, thus : The highest places of the proposed dividend, 16, being divided by 12, the quotient is 1 ; and 4 over. Then 42 becomes the highest remaining number, which divided by 12 gives the quotient 3, to be placed in a line with the

Or both the dividend and the divisor, being reduced to least terms by the common measure 3, are 540 and 4; or by the common measure 4, they become 405 and 3. Dividing by the respective reduced divisors, the quotient is the same, 135.

[The first part of the rule is vague and incomplete, although it is practically the same as the ordinary rule, as will be evident from the foot-note 4, p. 8. The second part follows from the identity $ab \div ac = b \div c$.]

18—19. Rule for the square[1] of a quantity: two stanzas.

The multiplication of two like numbers together is the square. The square of the last[2] digit is to be placed over it; and the rest of the digits, doubled and multiplied by that last, to be placed above them respectively; then repeating the number, except the last digit, again (perform the like operation). Or twice the product of two parts, added to the sum of the squares of the parts, is the square (of the whole number.)[3] Or the product of the sum and difference of the number and an assumed quantity, added to the square of the assumed quantity, is the square.[4]

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preceding quotient 1: thus 13. Remainder 60, which divided by 12 gives 5: and this being carried to the same line as before, the entire quotient is exhibited: *viz.*, 135.

[1] *Varga, kriti*, a square number.

[2] The process may begin with the first digit, as intimated by the author in § 24.

[3] The proposed quantity may be divided into three parts, instead of two; and the products of the first and second, first and third, and second and third, being added together and doubled, and added to the sum of the squares of the parts, the total is the square sought.—Gan.

[4] Another method is hinted in the author's note on this passage; consisting in adding together the product of the proposed quantity by any assumed one, and its product by the proposed less the assumed one.—Rang. [This follows from the identity, $ab + a(a-b) = a^2$.—Ed.]

["The square of the last digit, &c." The translation here is slightly incorrect. It should run thus :—"The square of the last digit, and the rest of the digits doubled and multiplied by that last, are to be placed one above the other (regard being had to the local values) ; then repeating, &c." It will then appear that the first two methods are really the same, and are based on the formula, $(a+b+c)^2=a^2+2a\ (b+c)+b^2+2bc+c^2$. The working of the first method is this :—Suppose we have to find the square of 297.

Then

7^2	=	49	or, 2^2	=	4
$7\times2\times29$	=	406	$2\times2\times97$	=	388
9^2	=	81	9^2	=	81
$9\times2\times2$	=	36	$9\times2\times7$	=	126
2^2	=	4	7^2	=	49
$\therefore(297)^2$	=	88209 ;	$\therefore(297)^2$	=	88209

The ciphers are omitted for simplicity. The third method follows from the identity, $(a+b)(a-b)+b^2=a^2$, a being the proposed, and b, the assumed quantity.]

20. Example. Tell me, dear woman, the squares of nine, of fourteen, of three hundred less three, and of ten thousand and five, if thou know the method of computing the square.

Statement : 9, 14, 297, 10005.

Proceeding as directed, the squares are found : 81, 196, 88209, 100100025. Or, put 4 and 5, parts of 9. Their product doubled 40, added to the sum of their squares 41, makes 81. So, taking 10 and 4, parts of 14, their product 40, being doubled, is 80 ; which, added to 116, the sum of the squares 100 and 16, makes the entire square, 196.

Or, putting 6 and 8, their product 48, doubled, is 96 ; which, added to the sum of the squares 36 and 64, *viz.*, 100, makes the same 196.

Again, 297, diminished by 3, is 294; and, in another place, increased by the same, is 300. The product of these is 88200; to which adding the square of 3, *viz.*, 9, the sum is as before the square, 88209.

21. Rule for the square root[1]: one stanza.

Having deducted from the last of the odd digits[2] the square number, double its root; and by that dividing the subsequent even digit, and subtracting the square of the quotient from the next uneven place, note in a line (with the preceding double number) the double of the quotient. Divide by the (number as noted in a) line the next even place, and deduct the square of the quotient from the following uneven one, and note the double of the quotient in the line. Repeat the process (until the digits be exhausted). Half the (number noted in the) line is the root.

[The rule is practically the same as the ordinary one for the extraction of the square root. Only it is a little more cumbrous, as will appear from the two processes placed side by side :—

```
           1__1__1
           8 8 2 0 9 ( 4
           4          18
           -----      ----
           482         58
   4×9 =   36          14
           ----       ----
           122         594
   9² =     81        ½ of 594
           -----      = 297,
           4109       the root required.
  7×58 =   406
           -----
             49
   7² =      49
           -----

                 8 8 2 0 9 ( 297
                 4
           49 |  482
              |  441
              |  ----
          587 |  4109
              |  4109
              |  ----
```

[1] *Varga múla*, root of the square; *múla*, *pada*, are synonyms of root.

[2] Every uneven place is to be marked by a vertical line, and the intermediate even digits by a horizontal line. But, if the last place be even, it is joined with the contiguous odd digit. Example, $\overset{1}{8}\overset{-}{8}\overset{1}{2}\overset{-}{0}\overset{1}{9}$

Thus we see that instead of directing the subtraction of 9×49 at once from 482, the rule directs first the subtraction of 9×40, and then from the remainder the subtraction of 9×9 or 9^2. And similarly for the next step. The process shown on the left-hand side is the same as that explained in the *Manoranjana.*]

22. Example. Tell me, dear woman, the root of four, and of nine, and those of the squares before found, if thy knowledge extend to this calculation.

Statement: 4, 9, 81, 196, 88209, 100100025. The roots are 2, 3, 9, 14, 297, 10005.

23—25. Rule for the cube[1]: three stanzas.

The continued multiplication of three like quantities is a cube. The cube of the last (digit) is to be set down; and next the square of the last multiplied by three times the first; and then the square of the first taken into the last and tripled; and lastly, the cube of the first: all these, added together according to their places, make the cube. The proposed quantity (consisting of more than two digits) is distributed into two portions, one of which is then taken for the last (and the other for the first); and in like manner repeatedly (if there be occasion.)[2] Or the same process may be begun from the first place of figures, either for finding the cube or the square. Or three times the proposed number, multiplied by its two parts, added to the sum of the cubes of those parts, give the cube. Or the

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[1] *Ghana*, a cube; lit., solid.

[2] The subdivision is continued until it comes to single digits. Ganesa confines it to the places of figures (*sthána-vibhága*), not allowing the portioning of the number (*rúpa-vibhága*); because the addition is to be made according to the places.

square root of the proposed number being cubed, that multiplied by itself, is the cube of the proposed square.[1]

[The different methods follow from the following formulæ :—

$$(a+b)^3=a^3+3a^2b+3ab^2+b^3.$$

$$(a+b)^3=a^3+b^3+3ab(a+b).$$

$$\left\{(\sqrt{a^2})^3\right\}^2=(a^2)^3 .]$$

26. Example. Tell me, dear woman, the cube of nine, the cube of the cube of three, and the cube of the cube of five, and the cube roots of these cubes, if thy knowledge be great in the computation of cubes.

Statement : 9, 27, 125.

The cubes in the same order are, 729, 19683, 1953125.[2]

The proposed number being 9, and its parts 4 and 5, 9 multiplied by them and by 3 is 540 ; which, added to the sum of the cubes 64 and 125, *viz.*, 189, makes

---

[1] This carries an allusion to the raising of quantities to higher powers than the cube. Ganesa specifies some of them. Thus the fourth power of a number is called *varga-varga ;* the sixth power, *varga-ghana* or *ghana-varga :* the eighth power, *varga-varga-varga ;* the ninth power, *ghana-ghana;* the fifth power, *varga-ghana-ghâta ;* and the seventh power, *varga-varga-ghana-ghâta.*

[2] The following process of finding the cube of 125 is given in the *Manoranjana.* The proposed number 125 is distributed into two parts 12 and 5 ; and the first of these again into two parts 1 and 2 :

Then, 1 cubed is	1
1, square of 1, tripled and multiplied by 2, is ...	6
4, square of 2, tripled and multiplied by 1, is ...	12
2 cubed is	8
	1728
Now, 12 cubed as above is	1728
144, square of 12, tripled and multiplied by 5, is ...	2160
25, square of 5, tripled and multiplied by 12, is ...	900
5 cubed is	125
Thus, 125 cubed is	1953125

the cube of 9, *viz.*, 729. The entire number being 27, its parts are 20 and 7; by which the number being successively multiplied, and then tripled, is 11340; and this added to the sum of the cubes of the parts, 8343, makes the cube 19683.

The proposed number being a square as 4, its root 2 cubed is 8. This taken into itself gives 64, the cube of 4. So 9 being proposed, its square root 3, cubed, is 27; the square of which, 729, is the cube of 9. In short, the square of the cube is the same as the cube of the square.

["In short, the square, &c." The translation should be, "in short, the cube of a square number is the same as the square of the cube of the square root of the number." This follows from the third formula given in the preceding article.]

27—28. Rule for the cube root[1]: two stanzas.

The first (digit) is a cube's place; and the two next, uncubic; and again, the rest in like manner. From the last cubic place take the (nearest) cube, and set down its root apart. By thrice the square of that root divide the next (or uncubic) place of figures, and note the quotient in a line (with the quantity before found). Deduct its square taken into thrice the last (term), from the next (digit); and its cube from the succeeding one. Thus the line (in which the result is reserved) is the root of the cube. The operation is repeated (as necessary).

Example. Statement of the foregoing cubes for extraction of the root: 729, 19683, 1953125.

The cube roots respectively are 9, 27, 125.

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[1] *Ghana-múla*, root of the cube.

[The rule is more cumbrous than the ordinary one, as will appear below :—

$$
\begin{array}{rr|l}
 & \overset{1}{1}9\overset{1}{5}3\overset{1}{1}25 & 1 \\
1^3 = & 1 & 2 \\
\hline
 & 953125 & 12 \\
3 \times 1^2 = 3 & & 5 \\
3 \times 2 = & 6 & 125 \\
\hline
 & 353125 & \\
3 \times 2^2 \times 1 = & 12 & \\
\hline
 & 233125 & \\
2^3 = & 8 & \\
\hline
 & 225125 & \\
3 \times 12^2 = 432 & & \\
5 \times 432 = & 2160 & \\
\hline
 & 9125 & \\
5^2 \times 12 \times 3 = & 900 & \\
\hline
 & 125 & \\
5^3 = & 125 & \\
\hline
\end{array}
$$

$$
\begin{array}{rl|l}
 & \dot{1}95\dot{3}12\dot{5} & 125 \\
 & 1 & \\
\hline
3 \times 10^2 = & 300 & 953 \\
3 \times 10 \times 2 = & 60 & \\
2^2 = & 4 & \\
\hline
 & 364 & 728 \\
\hline
 & & 225125 \\
3 \times 120^2 = & 43200 & \\
3 \times 120 \times 5 = & 1800 & \\
5^2 = & 25 & \\
\hline
 & 45025 & 225125 \\
\hline
\end{array}
$$

[The process shown on the left-hand side is the same as that explained in the *Manoranjana*. The ciphers are omitted for simplicity.]

SECTION III.

FRACTIONS.[1]

FOUR RULES FOR THE ASSIMILATION OR REDUCTION OF FRACTIONS TO A COMMON DENOMINATOR.[2]

29. Rule for the simple reduction of fractions[3]: one

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[1] *Bhinna*, a fraction; lit., a divided quantity, or one obtained by division. —Gan. An incomplete quantity or non-integer (*apúrna*).—Gang.

[2] *Bhága-játi-chatushtaya*, *játi-chatushtaya*, or four modes of assimilation or process for reducing to a common denominator, fractions having dis-similar denominators, preliminary to addition and subtraction of fractions.

[3] *Bhága-játi* or *ansa-savarnana*, assimilation of fractions, reducing them to uniformity.

stanza. The numerator and denominator[1] being multiplied reciprocally by the denominators of the two quantities,[2] they are thus reduced to the same denominator. Or both numerator and denominator may be multiplied by the intelligent calculator into the reciprocal denominators abridged by a common measure.

[This is the ordinary rule for reducing fractions to their least common denominator. The first part of the rule is meant for fractions whose denominators are prime to each other.]

30. Example. Tell me the fractions reduced to a common denominator which answer to three and a fifth, and one-third, proposed for addition ; and those which correspond to a sixty-third and a fourteenth offered for subtraction.

Statement[3] : $\frac{3}{1}$, $\frac{1}{5}$, $\frac{1}{3}$.

Reduced to a common denominator, $\frac{45}{15}$, $\frac{3}{15}$, $\frac{5}{15}$. Sum $\frac{53}{15}$.

Statement of the second example : $\frac{1}{63}$, $\frac{1}{14}$.

The denominators being abridged, or reduced to least terms, by the common measure 7, the fractions become $\frac{1}{9}$, $\frac{1}{2}$. Numerator and denominator, multiplied by the abridged denominators, give respectively $\frac{2}{126}$ and $\frac{9}{126}$. Subtraction being made, the difference is $\frac{7}{126}$. This abridged by 7 is $\frac{1}{18}$.

---

[1] *Bhága, ansa, vibhága, lava, &c.*, the numerator of a fraction. *Hara, hára, chheda, &c.*, the denominator of a fraction. That which is to be divided is the part (*ansa*) ; and that by which it is to be divided is *hara*, the divisor.—Gan. and Súr.

[2] *Rási*, a quantity. § 36.

[3] Among astronomers and other arithmeticians, oral instruction has taught to place the numerator above and the denominator beneath.—Gan.

No line is interposed in the original ; but it has been introduced in the translation to conform to the modern practice. Bháskara subsequently directs (§ 36) an integer to be written as a fraction by placing under it unity for its denominator. The same is done by him in this place in the text.

31. **Rule for the reduction of subdivided fractions**[1]: half a stanza.

The numerators being multiplied by the numerators, and the denominators by the denominators, the result is a reduction to homogeneous form in subdivision of fractions.

[This is the ordinary rule for reducing a compound fraction to a simple fraction.]

32. Example. The quarter of a sixteenth of the fifth of three-quarters of two-thirds of a moiety of a *dramma* was given to a beggar by a person, from whom he asked alms: tell me how many cowry shells the miser gave, if thou be conversant, in arithmetic, with the reduction termed subdivision of fractions.

Statement: $\frac{1}{1}\ \frac{1}{2}\ \frac{2}{3}\ \frac{3}{4}\ \frac{1}{5}\ \frac{1}{16}\ \frac{1}{4}$.

Reduced to homogeneousness, $\frac{6}{7680}$, or in least terms. $\frac{1}{1280}$.

Answer: A single cowry shell was given.[2]

33. Rule for the reduction of quantities increased or decreased by a fraction:[3] a stanza and a half.

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[1] *Prabhága-játi*, assimilation of sub-fractions, or making uniform the fraction of a fraction.—Gan.

Prabhága, a divided fraction or fraction of a fraction: as a part of a moiety, and so forth.—Gang.

[2] For a cowry shell is in the tale of money the 1280th part of a *dramma*, § 2.

[3] *Bhágánubandhajáti*, assimilation of fractional increase, reduction to uniformity of an increase by a fraction, or the addition of a part; from *anubandha*, junction.—Gan. *Bhágápavádhajáti*, assimilation of fractional decrease; from *apaváha*, deduction.—Gan.

These, as remarked by Ganesa, are merely particular cases of addition and subtraction. The fractions may be parts of an integer, or parts of the proposed quantity itself. Hence we get two sorts of each, named by Gangádhara and Súryadása, *rúpa-bhágánubandha*, addition of the fraction of a unit; *rúpa-bhágápaváha*, subtraction of the fraction of a unit; *rási-bhágánubandha*, addition of a fraction of the quantity; *rási-bhágápaváha*, subtraction of a fraction of the quantity.

The integer being multiplied by the denominator, the numerator is made positive or negative,[1] provided parts of a unit be added or be subtractive. But if indeed the quantity be increased or diminished by a part of itself, then, in the addition and subtraction of fractions, multiply the denominator by the denominator standing underneath[2], and the numerator by the same augmented or lessened by its own numerator.

[The first part of the rule follows from the identity, $a \pm \frac{b}{c} = \frac{ac \pm b}{c}$, and the second part, from the identities, $\frac{a}{b} \pm \frac{c}{d} \times \frac{a}{b} = \frac{ad \pm ac}{bd} = \frac{a(d \pm c)}{bd}$.

Hence, if we write $\frac{c}{d}$ underneath $\frac{a}{b}$, we get the second part of the rule.

The process may be repeated, if necessary.

Thus, $\frac{a}{b} + \frac{c}{d} \times \frac{a}{b} + \frac{e}{f}\left(\frac{a}{b} + \frac{c}{d} \times \frac{a}{b}\right) = \frac{a(d+c)(f+e)}{bdf}$.

An application of this last formula occurs in the examples given in § 35.]

34\. Example. Say how much two and a quarter, and three less a quarter, are, when reduced to uniformity, if thou be acquainted with fractional increase or decrease.

Statement: 2 3
$\frac{1}{4}$ $\cdot\frac{1}{4}$

Reduced to homogeneousness, they become $\frac{9}{4}$ and $\frac{11}{4}$.

[In the original a dot (·) is used instead of the sign *minus* (−).]

---

[1] *Dhana*, positive; *rina*, negative.

[2] Indian arithmeticians write fractions under the quantities to which they are additive, or from which they are subtractive. Accordingly, the numerators and denominators are put in their order, one under the other.

35. Example. How much is a quarter added to its third part, with a half of the sum, and how much are two-thirds lessened by one-eighth of them, and then diminished by three-sevenths of the residue? Tell me likewise, how much half less its eighth part, added to nine-sevenths of the residue is, if thou be skilled, dear woman, in fractional increase and decrease.

Statement: $\frac{1}{4}$, $\frac{2}{3}$, $\frac{1}{2}$
$\frac{1}{3}$, $\cdot\frac{1}{8}$, $\cdot\frac{1}{8}$
$\frac{1}{2}$, $\cdot\frac{3}{7}$, $\frac{9}{7}$

Reduced to uniformity, the results are $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{1}$.

[In the above examples we may apply the last formula given in § 33. Thus in the first example, we have $a=1$, $b=4$, $c=1$, $d=3$, $e=1$, $f=2$, and all the signs are *plus*. Hence the result is $\frac{1\times(3+1)\times(2+1)}{4\times3\times2}=\frac{12}{24}=\frac{1}{2}$; and similarly for the other two. The same process is exhibited in the *Manoranjana.*]

The Eight Rules of Arithmetic applied to Fractions.[1]

36. Rule for addition and subtraction of fractions[2]: half a stanza. The sum or (in the case of subtraction) the difference of fractions having a common denominator, is (taken). Unity[3] is put denominator of a quantity[4] which has no divisor.[5]

[This rule and the others which follow are all ordinary rules.]

---

[1] *Bhinna-parikarmáshtaka*, the eight modes of process, as applicable to fractions: the preceding Section relating to those arithmetical processes as applicable to integers (*abhinna-parikarmáshtaka.*)

[2] *Bhinna-sankalita*, addition of fractions; *bhinna-vyavakalita*, subtraction of fractions.

[3] *Rúpa*, the species or form; anything having bounds.—Gang. In the singular, the arithmetical unit; in the plural, any integer.

[4] *Rási*, a congeries; a heap of things, of which unity is the scale of numeration; a quantity or number.

[5] That is, it is put denominator of an integer.

37. Example. Tell me, dear woman, quickly, how much a fifth, a quarter, a third, a half, and a sixth, make when added together. Say instantly what the residue of three is, subtracting those fractions.

Statement: $\frac{1}{5}$, $\frac{1}{4}$, $\frac{1}{3}$, $\frac{1}{2}$, $\frac{1}{6}$.

Added together the sum is $\frac{29}{20}$.

Subtracting those fractions from three, the remainder is $\frac{31}{20}$.

38. Rule for multiplication of fractions[1]: half a stanza.

The product of the numerators, divided by the product of the denominators, (gives a quotient, which) is the result of multiplication of fractions.

39. Example. What is the product of two and a seventh, multiplied by two and a third, and of a moiety multiplied by a third? Tell, if thou be skilled in the method of multiplication of fractions.

Statement: 2 2
$\frac{1}{3}$ $\frac{1}{7}$.

Reducing to uniformity we get $\frac{7}{3}$, $\frac{15}{7}$. The product is $\frac{5}{1}$.

Statement: $\frac{1}{2}$ $\frac{1}{3}$.

The product is $\frac{1}{6}$.

40. Rule for division of fractions[2]: half a stanza.

After reversing the numerator and denominator of the divisor, the remaining process for division of fractions is that of multiplication.

41. Example. Tell me the result of dividing five by two and a third; and a sixth by a third; if thy

---

[1] *Bhinna-guṇana*, multiplication of fractions.

[2] *Bhinna-bhágahára*, division of fractions.

understanding, sharpened into confidence, be competent for the division of fractions.[1]

Statement: $\begin{matrix} 2 \\ \frac{1}{3} \end{matrix}$ $\left(\frac{7}{3}\right)$ $\frac{5}{1}$. $\frac{1}{3}$, $\frac{1}{6}$.

Proceeding as directed, the quotients are $\frac{15}{7}$ and $\frac{1}{2}$.

42. Rule for involution and evolution of fractions[2]: half a stanza.

If the square be sought, find both squares; if the cube be required, both cubes: or, to discover the root (of cube or square), extract the roots of both (numerator and denominator).

43. Example. Tell me quickly the square of three and a half; and the square root of the square; and the cube of the same; and the cube root of that cube; if thou be conversant with fractional squares and roots.

Statement: $\begin{matrix} 3 \\ \frac{1}{2} \end{matrix}$ or reduced $\frac{7}{2}$.

Its square is $\frac{49}{4}$; of which the square root is $\frac{7}{2}$. The cube of it is $\frac{343}{8}$; of which again the cube root is $\frac{7}{2}$.

SECTION IV.

CIPHER.[3]

44—45. Rule for arithmetical process relative to cipher: two couplets.

In addition, cipher makes the sum equal to the additive.[4] In involution and (evolution) the result is cipher. A definite quantity,[5] divided by cipher, is the

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[1] Ganesa omits the latter half of the stanza. Gangádhara gives it entire.

[2] *Bhinna-varga*, square of a fraction; *bhinna-ghana*, cube of a fraction.

[3] *Súnya*, *kha*, and other synonyms of vacuum or etherial space; nought or cipher; a blank or the privation of specific quantity.—Krishna on *Vija-ganita*.

[4] *Kshepa*, that which is cast or thrown in; additive.—Gang.

[5] Rási. . See § 36.

submultiple of nought.[1] The product of cipher is nought : but it must be retained as a multiple of cipher,[2] if any further operation impend. Cipher having become a multiplier, should nought afterwards become a divisor, the definite quantity must be understood to be unchanged. So likewise any quantity, to which cipher is added, or from which it is subtracted, (is unaltered).

[The first four rules are clear. The rule, *viz.*, "cipher having become a multiplier, &c.," is not accurate. For $\frac{a\times 0}{0}=\frac{0}{0}=$ indeterminate, and *not*$=a$, as the rule says. The idea of *infinity* is not introduced here by the author. It is, however, introduced by him in the *Víja-ganita*, and also by Ganesa in his commentary on the above couplets.]

46. Example. Tell me how much cipher added to five is, and the square of cipher, and its square root, its cube, and cube root ; and five multiplied by cipher ; and how much ten is subtracting cipher ; and what number it is, which multiplied by cipher, and added to half itself, and multiplied by three, and divided by cipher, amounts to a given number sixty-three.

Statement: 0. Cipher added to 5 makes 5. Square of cipher, 0. Square root, 0. Cube of cipher, 0. Cube root, 0.

---

[1] *Kha-hara*, a fraction with cipher for its denominator. According to the remark of Ganesa, it is an infinite quantity: since it cannot be determined how great it is. It remains unaltered by the addition or subtraction of finite quantities: since, in the preliminary operation of reducing both fractional expressions to a common denominator, preparatory to taking their sum or difference, both numerator and denominator of the finite quantity vanish. Ranganátha affirms that it is infinite, because the smaller the divisor is, the greater is the quotient; now cipher, being in the utmost degree small, gives a quotient infinitely great.

[2] *Khaguna*, a quantity which has cipher for its multiplier. Cipher is set down by the side of the multiplicand, to denote it.—Gan.

Statement : 5. This multiplied by cipher makes 0.

Statement : 10. This divided by cipher gives $\frac{10}{0}$.

Statement : an unknown quantity; its multiplier, 0; additive, $\frac{1}{2}$; multiplicator, 3; divisor, 0; given number, 63; assumption, 1.

Then, either by inversion or position, as subsequently explained (§47 and §50), the number is found, 14. This mode of computation is of frequent use in astronomical calculation.

[The last example as translated by Colebrooke appears to be meaningless and absurd. If we put x for the required number, we get the equation,

$$\frac{3\,(x\times 0+\frac{1}{2}x)}{0}=63,$$

which is manifestly absurd. The correct translation, however, would lead to the equation,

$$\frac{0\times(x+\frac{1}{2}x)\times 3}{0}=63,$$

of which $x=14$ is a solution.]

CHAPTER III.
MISCELLANEOUS RULES.[1]

SECTION I.
INVERSION.

47—48. Rule of inversion[2]: two stanzas. To investigate a quantity, one being given,[3] make the divisor a multiplicator; and the multiplier a divisor; the square, a root; and the root, a square[4]; turn the negative into positive, and the positive into negative. If a quantity is to be increased or diminished by its own proportionate part, let the (lower[5]) denominator, being increased or diminished by its numerator, become the (corrected[6]) denominator, and the numerator remain unchanged; and then proceed with the other operations of inversion, as before directed.

[The reason for the rule is clear from the example given in § 49. If we want an arithmetical solution of such a problem, we must begin from the end, and invert every operation indicated in the problem. If a quantity is to be increased or diminished by its own proportionate part, *i.e.*, if we have an equation

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[1] *Prakírna*, miscellaneous. The rules contained in the first five sections of this chapter have none answering to them in the Arithmetic of Brahmagupta and Sridhara.

[2] *Viloma-vidhi*, *Viloma-kriyá*, *Vyasta-vidhi*, inversion.

[3] *Drisya*, the quantity or number, which is visible; the given quantity.

[4] And the cube, a cube root; and the cube root, a cube.—Gan.

[5] Gangádhara.

[6] Gangádhara.

of the form $x\left(1\pm\frac{a}{b}\right)=c$, then evidently $x=\frac{bc}{b\pm a}=c\mp\frac{ac}{b\pm a}$. This explains the latter part of the rule.]

49. Example. Pretty girl, with tremulous eyes, if thou know the correct method of inversion, tell me the number, which multiplied by three, and added to three-quarters of the product, and divided by seven, and reduced by subtraction of a third part of the quotient, and then multiplied into itself, and having fifty-two subtracted from the product, and the square root of the remainder extracted, and eight added, and the sum divided by ten, yields two.[1]

Statement: Multiplier 3. Additive $\frac{3}{4}$. Divisor 7. Decrease $\frac{1}{3}$. Square —. Subtractive 52. Square root —. Additive 8. Divisor 10. Given number 2.

Proceeding as directed, the result is 28, the number sought.

Section II.

Supposition.

50. Rule of supposition[2]: one stanza. Any number assumed at pleasure is treated as specified in the particular question, being multiplied and divided, raised or diminished by fractions; then the given quantity, being multiplied by the assumed number and

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[1] All the operations are inverted. The known number 2, multiplied by the divisor 10 converted into a multiplicator, makes 20; from which the additive 8, being subtracted, leaves 12; the square whereof (extraction of the root being directed) is 144; and adding the subtractive 52, it becomes 196; the root of this (square being directed) is 14; added to its half, 7, it amounts to 21, which multiplied by 7, is 147. This again divided by 7 and multiplied by 3 makes 63, which, subtracted from 147, leaves 84; and this divided by 3, gives 28.—*Mano.*

[2] *Ishta-karman*, operation with an assumed number. It is the rule of false position, supposition, and trial and error.

divided by that (which has been found), yields the number sought. This is called the process of supposition.

51. Example. What is that number, which multiplied by five, and having the third part of the product subtracted, and the remainder divided by ten, and one-third, a half and a quarter of the original quantity added, gives two less than seventy ?

Statement: Multiplier 5. Subtractive $\frac{1}{3}$ of itself. Divisor 10. Additive $\frac{1}{3}$ $\frac{1}{2}$ $\frac{1}{4}$ of the quantity. Given 68.

Putting 3 ; this multiplied by 5 is 15 ; less its third part, is 10 ; divided by 10, yields 1. Added to the third, half and quarter of the assumed number 3, *viz.*, $\frac{3}{3}$ $\frac{3}{2}$ $\frac{3}{4}$, the sum is $\frac{17}{4}$. By this divide the given number 68 taken into the assumed one 3 ; the quotient is 48.

The answer is the same with any other assumed number, as 1, &c.

Thus, by whatever number the quantity is multiplied or divided in any example, or by whatever fraction of the quantity it is increased or diminished, by the same should the like operations be performed on a number arbitrarily assumed ; and by that, which results, divide the given number taken into the assumed one ; the quotient is the quantity sought.

[The rule in § 50 is a clumsy way of solving a simple equation. The reason for it will appear below.

Let x denote the number sought in § 51.

Then, $5x \times \frac{2}{3} \times \frac{1}{10} + (\frac{1}{3} + \frac{1}{2} + \frac{1}{4})x = 68$.

Multiply both sides by any assumed integer k, and we get

$$x = \{68 \times k\} \div \{k \times 5 \times \tfrac{2}{3} \times \tfrac{1}{10} + (\tfrac{1}{3} + \tfrac{1}{2} + \tfrac{1}{4})k\}.$$

Thus we see that there is no need at all of assuming an integer k, as the rule directs.]

52. Example of reduction of a given quantity.[1] Out of a heap of pure lotus flowers, a third part, a fifth and a sixth were offered respectively to the gods Siva, Vishnu and the Sun; and a quarter was presented to Bhaváni. The remaining six lotuses were given to the venerable preceptor. Tell quickly the whole number of lotuses.

Statement: $\frac{1}{3}$ $\frac{1}{5}$ $\frac{1}{6}$ $\frac{1}{4}$; known 6. Putting one for the assumed number, and proceeding as above, the quantity is found 120.

[In Pandit Jívánanda Vidyáságara's edition, there is an example before this, which is omitted by Colebrooke. It is as follows:—

Out of a herd of elephants, half together with a third part of itself was roaming in a forest; a sixth part together with a seventh of itself was drinking water in a river; and an eighth part together with a ninth of itself was playing with lotuses. The leader of the herd was seen accompanied by three females. What was the number of elephants in the herd?

This may be solved either by an arithmetical or an algebraical method, both being practically the same. Adopting the latter, and putting x for the required number, we get the equation $\{\frac{1}{2}(1+\frac{1}{3})+\frac{1}{6}(1+\frac{1}{7})+\frac{1}{8}(1+\frac{1}{9})\}x+4=x$, whence $x=1008$.

It may be observed here that all the examples in this Section are problems producing simple equations, which are solved not by the ordinary method of solving simple equations, but by the author's method stated in § 50. They may also be worked out by a purely arithmetical method.

The algebraical solution of the problem in § 52 is as follows:—

Let x denote the whole number of lotuses.

Then $(\frac{1}{3}+\frac{1}{5}+\frac{1}{6}+\frac{1}{4})x+6=x$, whence $x=120$].

---

[1] *Drisya-játi*, reduction of the visible or given quantity with fractions affirmative or negative; here, with negative; in the preceding example, with affirmative.

53. Example of reduction of residues.[1] A traveller, engaged in a pilgrimage, gave half his money at *Prayága;* two-ninths of the remainder at *Kásí;* a quarter of the residue in payment of taxes on the road; six-tenths of what was left at *Gayá;* there remained sixty-three *nishkas*, with which he returned home. Tell me the amount of his original stock of money, if you have learned the method of reduction of fractions of residues.

Statement : $\frac{1}{2}$ $\frac{2}{9}$ $\frac{1}{4}$ $\frac{6}{10}$; known 63. Putting one for the assumed number, subtracting the numerator from its denominator, multiplying denominators together, and in other respects proceeding as directed, the remainder is found $\frac{7}{60}$. By this dividing the given number 63 taken into the assumed quantity, the original sum comes out 540.

Or it may be found by the method of reduction of fractional decrease (§ 33)

Statement: $\frac{1}{1}\cdot\frac{1}{2}\cdot\frac{2}{9}\cdot\frac{1}{4}\cdot\frac{6}{10}$. Being reduced to homogeneous form, the result is $\frac{7}{60}$: whence the sum is deduced 540.

Or this may also be found by the rule of inversion (§ 47).

[Let x denote the original stock in *nishkas*. Then, at *Prayága*, there remained $\frac{1}{2}x$; at *Kásí*, $\frac{2}{9}$ of this being spent, there remained $\frac{7}{9}$ of $\frac{1}{2}x$; similarly, on the road, there remained $\frac{3}{4}$ of $\frac{7}{9}$ of $\frac{1}{2}x$, and at *Gayá*, $\frac{4}{10}$ of $\frac{3}{4}$ of $\frac{7}{9}$ of $\frac{1}{2}x$ or $\frac{7}{60}x$; hence we get $\frac{7}{60}x=63$, whence $x=540$.]

54. Example of reduction of differences.[2] Out of a swarm of bees, one-fifth part settled on a blossom of

[1] *Sesha-játi*, assimilation of residue; reduction of fractions of residues or successive fractional remainders.

[2] *Vislesha-játi*, assimilation of difference; reduction of fractional differences.

Kadamba,[1] and one-third on a flower of *silíndhrí*[2]; three times the difference of those numbers flew to the bloom of a *Kutaja*.[3] One bee, which remained, hovered and flew about in the air, allured at the same moment by the pleasing fragrance of a jasmine and pandanus. Tell me, charming woman, the number of bees.

Statement : $\frac{1}{5}$ $\frac{1}{3}$ $\frac{2}{15}$; known quantity, 1 ; assumed, 30.

A fifth of the assumed number is 6; a third is 10; difference 4 ; multiplied by 3 gives 12 ; and the remainder is 2. Then the product of the known quantity by the assumed one, being divided by this remainder shows the number of bees 15.

Here also putting unit for the assumed quantity, the number of the swarm is found 15.

So in other instances likewise.[4]

[Let x denote the number of bees.

Then, $\frac{1}{5}x+\frac{1}{3}x+3(\frac{1}{3}x-\frac{1}{5}x)+1=x$, whence $x=15$.]

SECTION III.

55. Rule of concurrence : half a stanza.

The sum with the difference added and subtracted,

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[1] *Kadamba*, Nauclea orientalis or N. Kadamba.

[2] *Silíndhrí*, a plant resembling the *Kachora*.—Krishna or *Víja-ganita*.

[3] Echites antidysenterica.

[4] The *Manoranjana* introduces one more example, which is there placed after the second, and is here subjoined.—"The third part of a necklace of pearls, broken in an amorous struggle, fell to the ground ; its fifth part rested on the couch ; the sixth part was saved by the wench ; and the tenth part was taken up by her lover ; six pearls remained strung. Say of how many pearls the necklace was composed."

Statement : $\frac{1}{3}$ $\frac{1}{5}$ $\frac{1}{6}$ $\frac{1}{10}$. Rem. 6.

Answer : 30.

being halved, gives the two quantities. This is termed concurrence.[1]

[Let x and y denote the required numbers. Then,
$x+y=k$, $x-y=l$, where k and l are given quantities ;
whence $x=\frac{1}{2}(k+l)$, $y=\frac{1}{2}(k-l)$.]

56. Example. Tell me the numbers, the sum of which is a hundred and one, and the difference, twenty-five ; if thou know the rule of concurrence, dear child.

Statement : sum 101 ; difference 25.

The two numbers are 38 and 63.

57. Rule of dissimilar operation[2] ; half a stanza.

The difference of the squares, divided by the difference of the radical quantities, gives their sum[3] ; whence the quantities are found in the mode before directed.

[Let x and y be the numbers. Then,

$$x^2-y^2=m,\ x-y=n.$$

$$\therefore x+y=\frac{m}{n}, \text{ \&c.}]$$

58. Example. Tell me quickly, skilful calculator, what numbers they are, of which the difference is eight, and the difference of squares four hundred.

Statement : difference of the quantities 8 ; difference of the squares 400.

The numbers are 21 and 29.

---

[1] *Sankramana*, concurrence or mutual penetration in the shape of sum and difference.—Gang. Investigation of two quantities concurrent or grown together in the form of sum and difference.—Gan. Calculation of quantities latent within those exhibited.—Súr.

[2] *Vishama-karman*, the finding of the quantities, when the difference of their squares is given, and either the sum or the difference of the quantities.—Gan. A species of concurrence.—Gang. See below, §135.

[3] Or divided by their sum, gives their difference.—Gan.

Section IV.

Problem concerning Squares.[1]

A certain problem relative to squares is propounded in the next instance.

59. Rule. The square of an arbitrary number, multiplied by eight and lessened by one, then halved and divided by the assumed number, is one quantity; its square, halved and added to one, is the other. Or unity, divided by double an assumed number and added to that number, is a first quantity; and unity is the other. These give pairs of quantities, the sum and difference of whose squares, lessened by one, are squares.

[Let n be the assumed number. Then, by the first part of the rule, the two numbers are,

$$\frac{1}{2n}(8n^2-1) \text{ and } \tfrac{1}{2}\left\{\frac{1}{2n}(8n^2-1)\right\}^2+1.$$

The sum of the squares of these numbers lessened by 1 is

$$\tfrac{1}{4}\left(4n-\frac{1}{2n}\right)^4+2\left(4n-\frac{1}{2n}\right)^2$$

$$=\left(4n-\frac{1}{2n}\right)^2\left\{\tfrac{1}{4}\left(4n-\frac{1}{2n}\right)^2+2\right\}$$

$$=\left(4n-\frac{1}{2n}\right)^2\left(2n+\frac{1}{4n}\right)^2,$$ a perfect square. Similarly, the difference of the squares of the numbers lessened by 1 is a perfect square.

Again, by the second part of the rule, the numbers are $\frac{1}{2n}+n$ and 1; and $\left\{\left(\frac{1}{2n}+n\right)^2 \pm (1)^2\right\}-1=\left(\frac{1}{2n}\pm n\right)^2$, which are perfect squares.

Hence the reason for the rule is evident.]

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[1] *Varga-karman*, operation relative to squares; an indeterminate problem, admitting innumerable solutions.

60. Tell me, my friend, numbers, the sum and difference of whose squares, less one, afford square roots, which dull smatterers in algebra labour to excruciate, puzzling for it in the six-fold method of discovery there taught.[1]

To bring out an answer by the first rule, let the number put be $\frac{1}{2}$. Its square, $\frac{1}{4}$, multiplied by 8, is 2; which lessened by 1 is 1. This halved is $\frac{1}{2}$, and divided by the assumed number $\frac{1}{2}$ gives 1 for the first quantity. Its square halved is $\frac{1}{2}$, which, added to 1, makes $\frac{3}{2}$. Thus the two quantities are 1 and $\frac{3}{2}$.

So, putting 1 for the assumed number, the numbers obtained are $\frac{7}{2}$ and $\frac{57}{8}$. With the supposition of 2, they are $\frac{31}{4}$ and $\frac{993}{32}$.

By the second method, let the assumed number be 1. Unity divided by the double of it is $\frac{1}{2}$, which added to the assumed number makes $\frac{3}{2}$. The first quantity is thus found. The second is unity. With the supposition of 2, the quantities are $\frac{9}{4}$ and 1. Putting 3, they are $\frac{19}{6}$ and 1.

61. Another Rule.[2] The square of the square of an arbitrary number, and the cube of that number, respectively multiplied by eight, adding one to the first product, are such quantities, equally in arithmetic and in algebra.

Put $\frac{1}{2}$. The square of the square of the assumed number is $\frac{1}{16}$, which multiplied by 8 makes $\frac{1}{2}$. This

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[1] This question, found in some copies of the text, and interpreted by Gangádhara and the *Manoranjana*, is unnoticed by the other commentators. [We do not know what the author means by *the six-fold method of discovery*. Colebrooke does not say anything about it.—Ed.]

[2] To bring out answers in whole numbers, the two preceding solutions giving fractions.—Gan. and Súr.

added to 1 is $\frac{3}{2}$, which is the first quantity. Again put $\frac{1}{2}$. Its cube is $\frac{1}{8}$, which multiplied by 8 gives the second quantity 1. Next supposing 1, the two quantities are 9 and 8. Assuming 2, they are 129 and 64. Putting 3, they are 649 and 216. And so on, without end, by means of various suppositions, in the several foregoing methods.

It is said that algebraic solution similar to arithmetical rules appears obscure; but it is not so to the intelligent; nor is it six-fold, but manifold.

[Let n be the arbitrary number. Then, by the rule, the numbers are, $8n^4 + 1$ and $8n^3$;
and $\{(8n^4+1)^2 \pm (8n^3)^2\} - 1 = (4n^2)^2 \; (2n^2 \pm 1)^2$, which are perfect squares.

Hence the reason for the rule is obvious.]

Section v.

62—63. Rule for assimilation of the root's coefficient[1]: two stanzas.

The sum or difference of a quantity and of a multiple of its square root being given, the square of half the coefficient[2] is added to the given number, and the square root of their sum (is extracted; that root,) with half the coefficient added or subtracted, being squared, is the quantity sought by the interrogator. If the quantity have a fraction (of itself) added or subtracted, divide the number given and the multiplicator of the root, by unity increased or lessened by the fraction,

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[1] *Múla-játi*, *múla-gunaka-játi* or *ishta-múlánsa-játi*, assimilation and reduction of the root's coefficient with a fraction.

[2] *Guna*, multiplicator; *múla-guna*, root's multiplier, the coefficient of the root.

and the required quantity may be then discovered, proceeding with those quotients as above directed.

A quantity, increased or diminished by its square root multiplied by some number, being given, add the square of half the multiplier of the root to the given number; and extract the square root of the sum. Add half the multiplier, if the difference were given; or subtract it, if the sum were so. The square of the result will be the quantity sought.

[The third paragraph is in prose in the original, and is added by the author by way of explanation of the two preceding metrical rules.

Suppose we have the equations,

$$x \pm a\sqrt{x} = b \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \quad (1).$$

Then, completing the square, we get

$$x \pm a\sqrt{x} + \left(\frac{a}{2}\right)^2 = b + \left(\frac{a}{2}\right)^2;$$

$$\therefore \sqrt{x} = \sqrt{b + \left(\frac{a}{2}\right)^2} \mp \frac{a}{2};$$

$$\therefore \quad x = \left\{\sqrt{b + \left(\frac{a}{2}\right)^2} \mp \frac{a}{2}\right\}^2.$$

Hence the reason for the first part of the rule is clear. It is the ordinary rule for solving an equation reducible to a quadratic by completing the square.

The second part of the rule is meant for equations of the form

$$x \pm \frac{c}{d}x \pm a\sqrt{x} = b \quad . \; . \; . \; . \; . \; . \quad (2),$$

whence we get

$$x \pm \frac{a\sqrt{x}}{1 \pm \frac{c}{d}} = \frac{b}{1 \pm \frac{c}{d}},$$

which is of the form (1), and may be solved as above. Thus we see the reason for the second part of the rule.]

64. Example: the root subtracted, and the difference given. One pair out of a flock of geese remained

sporting in the water, and saw seven times the half of the square root of the flock proceeding to the shore tired of the diversion. Tell me, dear girl, the number of the flôck.

Statement : coeff. $\frac{7}{2}$; given 2. Half the coefficient is $\frac{7}{4}$; its square $\frac{49}{16}$ added to the given number, makes $\frac{81}{16}$, the square root of which is $\frac{9}{4}$. Half the coefficient being added, the sum is $\frac{16}{4}$; or, reduced to least terms, 4. This squared is 16; the number of the flock, as required.

[Let x denote the number of the flock.

Then, $2+\frac{7}{2}\sqrt{x}=x$;

$$\therefore\ x-\frac{7}{2}\sqrt{x}+\left(\frac{7}{4}\right)^2=2+\left(\frac{7}{4}\right)^2=\left(\frac{9}{4}\right)^2;$$

$$\therefore\ \sqrt{x}-\frac{7}{4}=\frac{9}{4};$$

whence $x=\left(\frac{7}{4}+\frac{9}{4}\right)^2=16.$

This is an instance of the first part of the preceding rule.]

65. Example : the root added, and the sum given. Tell me what the number is, which, added to nine times its square root, amounts to twelve hundred and forty.

Statement : coeff. 9 ; given 1240. Proceeding by the rule, the required number is 961.

[Let x denote the number required. Then, $x+9\sqrt{x}=1240$, whence x.]

66. Example; the root and a fraction both subtracted. Of a flock of geese, ten times the square root of the number departed for the *Mánasa* lake,[1] on the

[1] Wild geese are observed to quit the plains of India, at the approach of the rainy season; and the lake called *Mánasasarovara* is covered with water-fowl, especially geese, during that season. The *Hindus* suppose the whole tribe of geese to retire to the holy lake at the approach of rain. The bird is sacred to Brahmá. [See *Raghuvansa*, XIII, 55.—Ed.]

approach of a cloud : an eighth part went to a forest of *Sthalapadminís*[1] : three couples were seen engaged in sport on the water abounding with delicate fibres of the lotus. Tell, dear girl, the whole number of the flock.

Statement : coeff. 10 ; fraction $\frac{1}{8}$; given 6.

Proceeding by the (second) rule (§63), unity, less the fraction, is $\frac{7}{8}$; and the coefficient and the given number, being both divided by that, become $\frac{80}{7}$ and $\frac{48}{7}$; and the half coefficient is $\frac{40}{7}$. With these, proceeding by the (first) rule (§62), the number of the flock is found 144.

[Let x denote the whole number of the flock.

$$\text{Then, } 10\sqrt{x} + \frac{1}{8}x + 6 = x;$$

$$\therefore \frac{7}{8}x - 10\sqrt{x} = 6 \text{ ; whence } x.$$

This is an instance of the second part of the rule in §62—63.]

67. Example. The son of Prithá,[2] irritated in fight, shot a quiver of arrows to slay Karna. With half his arrows, he parried those of his antagonist ; with four times the square root of the quiver-full, he killed his horses ; with six arrows, he slew Salya[3] ; with three he demolished the umbrella, standard and bow ; and with one, he cut off the head of the foe. How many were the arrows which Arjuna let fly ?

Statement : fraction $\frac{1}{2}$; coeff. 4 ; given 10.

The given number and coefficient being divided by unity less the fraction become 20 and 8 ; and proceeding by the rule (§62), the number of arrows comes out 100.

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[1] The plant intended is not ascertained. The context would seem to imply that it is arboreous, as the term signifies forest.

[2] Arjuna, surnamed Pártha ; his matronymic from Prithá or Kuntí.

[3] One of the *Kauravas*, and charioteer of Karna.

[Let x denote the number of arrows.

Then, $\frac{1}{2}x + 4\sqrt{x} + 6 + 3 + 1 = x$, whence x.]

68. Example. The square root of half the number of a swarm of bees is gone to a shrub of jasmin[1]; and so are eight-ninths of the whole swarm : a female is buzzing to one remaining male that is humming within a lotus in which he is confined, having been allured to it by its fragrance at night.[2] Say, lovely woman, the number of bees.

Here eight-ninths of the quantity and the root of its half are negative (and consequently subtractive) from the quantity : and the given number is two of the specific things. The negative quantity, and the given number halved, bring out half the quantity sought.[3] Thus :—

Statement : fraction $.\frac{8}{9}$; coeff. $.\frac{1}{2}$; given 1.

A fraction of half the quantity is the same as half the fraction of the quantity ; the fraction is therefore set down (unaltered).

Here proceeding as above directed, there comes out half the quantity, 36; which being doubled is the number of bees in the swarm, 72.

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[1] *Málati*, jasminum grandiflorum.

[2] The lotus being open at night and closed in the day, the bee might be caught in it.—Gan.

[3] In such questions, it is necessary to observe whether the coefficient of the root be so of the root of the whole number, or of that of its part ; for that quantity is found, of whose root the coefficient is used. But in the present case, the root of half the quantity is proposed ; and accordingly, the half of the quantity will be found by the rule. The number given, however, belongs to the entire quantity. Therefore, taking half the given number, half the required number is to be brought out by the process before directed.—*Mano.* and Súr.

[Let x denote the number of bees.

Then, $\sqrt{\frac{1}{2}x} + \frac{8}{9}x + 2 = x$.

Put $y = \frac{1}{2}x$, and we get

$$y - \frac{8}{9}y - \frac{1}{2}\sqrt{y} = 1,$$

whence by the rule in §§ 62-63, we obtain $y = 36$, and $\therefore$ $x = 72$.

Thus the reason for the process given in the text is clear. The reason given by the author and the commentators is not very clear.]

69. Example: the root and a fraction both added. Find quickly, if thou have skill in arithmetic, the quantity which added to its third part and eighteen times its square root, amounts to twelve hundred.

Statement: fraction $\frac{1}{3}$; coeff. 18; given 1200.

Here, dividing the coefficient and given number by unity added to the fraction (§63), and proceeding as before directed, the number is brought out, 576.

[Let x denote the number required.

Then, $x + \frac{1}{3}x + 18\sqrt{x} = 1200$; whence x.]

Section VI.

Rule of Proportion.[1]

70. Rule of three terms[2]: one stanza.

The first and last terms, which are the argument and requisition, must be of like denomination; the fruit, which is of a different species, stands between them: and that, being multiplied by the demand and divided

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[1] [A more literal translation would be 'Rule of Three,' the word in the original being *trairásika*.—Ed.]

[2] *Trairásika*, calculation belonging to a set of three terms.—Gang. Rule of Three. The first term is *pramána*, the measure or argument; the second is its fruit, *phala*, or produce of the argument; the third is *ichchhá*, the demand, requisition, desire or question.—Gan.

by the first term, gives the fruit of the demand.[1] In the inverse method, the operation is reversed.[2]

[The rule is the ordinary mechanical one for solving problems involving the Rule of Three, direct and inverse. It is not stated in the light of the principle of proportion, and is practically the same as that given in Mr. Barnard Smith's well-known work on Arithmetic, Art. 155.]

71. Example. If two and a half *palas* of saffron be obtained for three-sevenths of a *nishka*, say instantly, best of merchants, how much is got for nine *nishkas*.

Statement : $\frac{3}{7}$ $\frac{5}{2}$ $\frac{9}{1}$. Answer : 52 *palas* and 2 *karshas*.

[This is an example of the Rule of Three direct. Worked out by the principle of proportion, the process will stand thus :—

Let x denote the quantity sought in *palas*.

Then, we evidently have the proportion,

$$\tfrac{5}{2} : x :: \tfrac{3}{7} : 9,$$

$$\therefore x \times \tfrac{3}{7} = \tfrac{5}{2} \times 9,$$

$$\therefore \quad x = \frac{\frac{5}{2} \times 9}{\frac{3}{7}} = 52\tfrac{1}{2}.$$

Thus the answer is $52\frac{1}{2}$ *palas* = 52 *palas*, 2 *karshas*.

The reason for the rule in § 70 is obvious.]

72. Example. If one hundred and four *nishkas* are got for sixty-three *palas* of best camphor, consider and tell me, friend, what may be obtained for twelve and a quarter *palas*.

Statement: 63 104 $\frac{49}{4}$. Answer: 20 *nishkas*, 3 *drammas*, 8 *panas*, 3 *kákinís*, 11 cowryshells and $\frac{1}{9}$th part.

[This also is an instance of the Rule of Three direct, and may be worked out as above.]

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[1] *Ichchhá-phala*, produce of the requisition, or fruit of the question ; it is of the same denomination or species with the second term.

[2] See § 74.

73. Example. If a *khári* and one-eighth of rice may be procured for two *drammas*, say quickly what may be had for seventy *panas*.

Statement, reducing *drammas* to *panas* : 32 $\frac{9}{8}$ 70. Answer : 2 *kháris*, 7 *dronas*, 1 *ádhaka*, 2 *prasthas*.

[This is a third instance of the Rule of Three direct.]

74. Rule of Three inverse.[1]

If the fruit diminish as the requisition increases, or augment as that decreases, they who are skilled in accounts consider the Rule of Three terms to be inverted.[2]

When there is diminution of fruit, if there be increase of requisition, and increase of fruit if there be diminution of requisition, then the inverse Rule of Three is (employed.)

[This is the ordinary definition of inverse variation.]

75. For instance, when the value of living beings[3] is regulated by their age ; and in the case of gold, where the weight and touch are compared[4]; or when heaps[5] are subdivided ; let the inverted Rule of Three terms be (used).

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[1] *Vyasta-trairásika* or *Viloma-trairásika*, rule of three terms inverse.

[2] The method of performing the inverse rule has been already taught (§ 70), *viz.*, " in the inverse method, the operation is reversed;" *i.e.*, the fruit is to be multiplied by the argument and divided by the demand.—Súr.

When the fruit increases or decreases, as the demand is augmented or diminished, the direct rule (*Krama-trairásika*) is used ; else the inverse.—Gan.

[3] Slaves and cattle. The price of the older is less ; of the younger greater.—Gang. and Súr.

[4] Colour on the touchstone. See Alligation, § 101.

[5] *See* Chap. X. When heaps of grain, which had been meted with a small measure, are again meted with a larger one, the number decreases ; and when those, which had been meted with a large measure, are again meted with a smaller one, there is increase of number.—Gang. and Súr.

[Some instances of inverse variation are here mentioned. The reason is clear from the foot-notes appended. The author does not mention the common instance of time and agency required for a given piece of work.]

76. Example of age and price of living beings. If a female slave, sixteen years of age, bring thirty-two (*nishkas*), what will one aged twenty cost? If an ox which has been worked two years sell for four *nishkas*, what will one, which has been worked six years, cost?

1st Statement: 16 32 20. Answer: $25\frac{3}{5}$ *nishkas*.

2nd Statement: 2 4 6. Answer: $1\frac{1}{3}$ *nishka*.

[Let x denote the cost in the first example. Then ∵ the greater is the age, the smaller is the cost, we have the proportion,

$$16 : 20 :: x : 32,$$

Whence $x = \frac{16 \times 32}{20} = 25\frac{3}{5}$.

Similarly the second example as well as those in the next two articles may be worked out.]

77. Example of touch and weight of gold. If a *gadyánaka* of gold of the touch of ten may be had for one *nishka* (of silver), what weight of gold of fifteen touch may be bought for the same price?

Statement: 10 1 15. Answer $\frac{2}{3}$.

78. Example of subdivision of heaps. A heap of grain having been meted with a measure containing seven *ádhakas*, if a hundred such measures were found, what would be the result with one containing five *ádhakas*?

Statement: 7 100 5. Answer 140.

79. Rule of compound proportion[1]: one stanza.

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[1] [A more literal translation would be, "Rule of Five and so forth," the word in the original being, *pancharásikádau*.—Ed.] This, which is the compound Rule of Three, comprises, according to Ganesa, two or more sets of three terms (*trairásika*); or two or more proportions (*anupáta*), as Súryadása

In the method of five, seven, nine or more[1] terms, transpose the fruit and divisors[2]; and the product of the larger set of terms, being divided by the product of the less set of terms[3], the quotient is the produce (sought)

[This is practically the same rule, rather incompletely stated, for solving problems involving the Double Rule of Three, as that given in Mr. Barnard Smith's work on Arithmetic, Art. 161.* It is a clumsy and a purely mechanical rule, having no connection whatever with the principle of proportion. The meaning of the phrase, "transpose the fruit and divisors," as explained by Ganesa, will appear from the foot-notes appended to the following articles. It should be observed here that "the product of the larger set" is not necessarily the numerically larger product; see the example in § 82. For

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observes. Thus the rule of five (*pancha-rásika*) comprises two proportions; that of seven (*sapta-rásika*) three; that of nine (*nava-rásika*) four; and that of eleven (*ekádasa-rásika*) five.

[1] Meaning eleven. *Mano.* and Súr.

[2] Ganesa and the commentator of the *Vásaná* understand this last word (*chhid*, divisor) as relating to denominators of fractions; and the transposing of them (if any there be) is indeed right: accordingly the author gives under this rule an example of working with fractions (§ 81). But the *Manoranjana* and Súryadása explain it otherwise; and the latter cites as, ancient commentary entitled *Ganita-Kaumudí* in support of his exposition. "There are two sets of terms; those which belong to the argument, and those which appertain to the requisition. The fruit in the first set is called produce of the argument; that in the second is named divisor of the set. They are to be transposed, or reciprocally brought from one set to the other; *i.e.*, put the fruit in the second set, and the divisor in the first. Would it not be enough to say, transpose the fruits of both sets? The author of the *Kaumudí* replies, 'the designation of divisor serves to indicate that after transposition, the fruit of the second set being included in the product of the less set of terms, the product of the greater set is to be divided by it'. Some, however, intepret it as relative to fractions. But that is wrong: for the word would be superfluous." [This explanation is not very clear.—Ed.]

[3] *Bahu-rási-paksha*, set of many terms. That to which the fruit is brought is the larger set.—Gang. Or, if there be fruit on both sides, that in which the fruit of the requisition is, is the larger set.—Gan. *Laghu-rási-paksha*, set of fewer terms.

the meaning of the term "larger set," see foot-note 3 p. 42. The phrase has been rather loosely used.]

80. Example. If the interest of a hundred for a month be five, say what the interest of sixteen is for a year. Find likewise the time from the principal and interest; and knowing the time and produce, tell the principal sum.

Statement:

1	12
100	16
5	

Answer[1]: the interest is $9\frac{3}{5}$.

To find the time; statement:

1	
100	16
5	$\frac{48}{5}$

Answer[2]: months 12.

To find the principal; statement:

1	12
100	
5	$\frac{48}{5}$

Answer[3]: principal 16.

[Worked out by the principle of proportion, the process will stand thus :—

Let x denote the interest required. Then, the int. of 16 for 1 year = int. of 16 × 12 for 1 month; and ∵ with a given time,

---

[1] Transposing the fruit,

1	12
100	16
	5

Product of the larger set, 960. Do. of the less set, 100. Quotient, $\frac{960}{100}$ or $\frac{48}{5}$.

[2] Transposing both fruits,

1	
100	16
$\frac{48}{5}$	5

and the denominator,

1	
100	16
48	5
	5

Product of the larger set, 4800. Do. of the less set, 400. Quotient, 12.

[3] Transposing both fruits,

1	12
100	
$\frac{48}{5}$	5

and the denominator,

1	12
100	
48	5
	5

Product of the larger set, 4800. Do. of the less set, 300. Quotient, 16.

the interest varies directly as the principal, we get the proportion,
$100 : 16 \times 12 :: 5 : x$,
whence $x = \frac{16 \times 12 \times 5}{100} = 9\frac{3}{5}$,
and the reason for the rule in § 79 is evident. Similarly for the other parts of the example.]

81. Example. If the interest of a hundred for a month and one-third be five and one-fifth, say what the interest is of sixty-two and a half for three months and one-fifth.

Statement: $\begin{array}{cc} \frac{4}{3} & \frac{16}{5} \\ 100 & \frac{125}{2} \\ \frac{26}{5} & \end{array}$ Answer[1]: interest $7\frac{4}{5}$.

[This may be worked out similarly as the preceding example.]

82. Example of the Rule of Seven. If eight best variegated silk scarfs, measuring three cubits in breadth and eight in length, cost a hundred (*nishkas*); say quickly, merchant, if thou understand trade, what a like scarf, three and a half cubits long and half a cubit wide, will cost.

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[1] Transposing the fruit, $\begin{array}{cc} \frac{4}{3} & \frac{16}{5} \\ 100 & \frac{125}{2} \\ & \frac{26}{5} \end{array}$ and the denominators, $\begin{array}{cc} 4 & 16 \\ 5 & 3 \\ 100 & 125 \\ 2 & \\ 5 & 26 \end{array}$

Abridging by correspondent reduction on both sides, $\begin{array}{cc} 1 & 4 \\ 5 & 3 \\ 4 & 5 \\ 2 & \\ 5 & 26 \end{array}$ and by further reduction, $\begin{array}{cc} 1 & 1 \\ 1 & 3 \\ 1 & 1 \\ 1 & \\ 5 & 13 \end{array}$

Product of the larger set, 39. Do. of the less set, 5. Quotient, $7\frac{4}{5}$.

The abridgment of the work by reduction of terms on both sides by their common divisors is taught by the *Manoranjana*.

Statement:

3	$\frac{1}{2}$
8	$\frac{7}{2}$
8	1
100	

Answer[1]: *Nishka* 0, *drammas* 14, *panas* 9, *kákiní* 1, cowry-shells $6\frac{2}{3}$.

[Let x denote the cost required.
Then the area of cloth in the first case
$= 8 \times 3 \times 8$ sq. cubits;
and in the second case
$= 1 \times \frac{7}{2} \times \frac{1}{2}$ sq. cubits.
Hence, the quality of the cloth remaining the same, we get the proportion,

$$8 \times 3 \times 8 : 1 \times \frac{7}{2} \times \frac{1}{2} :: 100 : x,$$

whence $x = \frac{1 \times 7 \times 1 \times 100}{8 \times 3 \times 8 \times 2 \times 2}$,
and the reason for the process given in the foot-note is evident.]

83. Example of the Rule of Nine. If thirty benches, twelve fingers thick, square of four wide, and fourteen cubits long, cost a hundred (*nishkas*); tell me, my friend, what price fourteen benches will fetch, which are four less in every dimension.

Statement:

12	8
16	12
14	10
30	14
100	

Answer[2]: *nishkas* $16\frac{2}{3}$.

---

[1] Transposing fruit and denominators,

3	1
2	
8	7
2	
8	1
	100

Product of the larger set, 700.
Do. of the less set, 768.
Quotient: 0, 14, 9, 1, $6\frac{2}{3}$.

[2] Transposing fruit,

12	8	Abridging by	1	1
16	12		2	1
14	10	correspondent	1	1
30	14	reduction on	3	1
	100	both sides,		100

Product of the larger set, 100.
Do. of the less set, 6.
Quotient, $16\frac{2}{3}$.

[Here, putting x for the price, and proceeding as above, we get the proportion,
$30 \times 12 \times 16 \times 14 \times 24 : 14 \times 8 \times 12 \times 10 \times 24 :: 100 : x$, whence x is known.]

84. Example of the Rule of Eleven. If the hire of carts to convey the benches of the dimensions first specified (in the preceding example), through a distance of one league (*gavyúti*[1]) be eight *drammas*; say what the cart-hire should be for bringing the benches last mentioned, four less in every dimension, through a distance of six leagues.

Statement:			
	12	8	
	16	12	Answer[2]: *drammas* 8.
	14	10	
	30	14	
	1	6	
	8		

[Let x denote the cart-hire required.

Then in the first case, solid content of benches $= 30 \times 12 \times 16 \times 14 \times 24$ cubic fingers, and they are carried through a distance of 1 league; and in the second case, solid content of benches $= 14 \times 8 \times 12 \times 10 \times 24$ cubic fingers, and they are carried through a distance of 6 leagues, which is the same as a solid content of $14 \times 8 \times 12 \times 10 \times 24 \times 6$ cubic fingers carried through 1 league;

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[1] *Gavyúti,* two *krosas* or half a *yojana*; it contains rather more than 8000 yards, and is more than 4½ English miles.

[2] Transposing the fruit,

12	8
16	12
14	10
30	14
1	6
	8

Abridging by correspondent reduction on both sides,

1	1
2	1
1	1
3	1
1	6
	8

and by further reduction,

1	1
1	1
1	1
1	1
1	2
	4

Product of the larger set, 8.
Do. of the less set, 1.
Quotient, 8.

hence, since for a given distance, the hire will vary directly as the solid content of benches carried, we get the proportion,
$30 \times 12 \times 16 \times 14 \times 24 \times 1 : 14 \times 8 \times 12 \times 10 \times 24 \times 6 :: 8 : x$,
whence $x = \frac{14 \times 8 \times 12 \times 10 \times 6 \times 8}{30 \times 12 \times 16 \times 14} = 8$.]

85. Rule of barter[1]; half a stanza. So in barter likewise, the same process is (followed); transposing both prices, as well as the divisors.[2]

[The reason for the rule will appear from the example which follows.]

86. Example. If three hundred mangoes be had in the market for one *dramma*, and thirty ripe pomegranates for a *pana*; say quickly, friend, how many should be had in exchange for ten mangoes.

Statement :			Answer[3] : 16 pomegranates.
	16	1	
	300	30	
	10		

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[1] *Bhánda-prati-bhándaka,* commodity for commodity; computation of the exchange of goods (*vastu-vinimaya-ganita,*—Gang.); barter.

[2] Gangádhara, Súryadása and the *Manoranjana,* so read this passage, *haránscha-múlye.* But Ganesa and Ranganátha have the affirmative adverb *sadáhi* (always) in place of the word *haránscha* (and the divisors). At all events, the transposition of denominators takes place, as usual; and so does that of the lowest term or fruit, as in the Rule of Five, to which, as Súryadása remarks, this is analogous. It comprises two proportions, thus stated by him from the example in the text:—"If for one *pana,* thirty pomegranates may be had, how many for sixteen? Answer, 480. Again, if for 300 mangoes, 480 pomegranates may be had, how many for ten? Answer, 16. Here thirty is first multiplied by sixteen and then divided by one; and then multiplied by ten and divided by three hundred. For brevity, the prices are transposed, and the result is the same."

[3] Transposing the prices,

1	16
300	30
10	

and transposing the fruit,

1	16
300	30
	10

Product of the larger set, 4800. Do. of the less set, 300. Quotient, 16.

[The example involves two proportions, as Súryadása observes. First find how many pomegranates can be had for one *dramma* or 16 *panas*.

Putting y for this number, we get

$$30 : y :: 1 : 16,$$

$$\therefore y = 30 \times 16.$$

Hence by the question, 300 mangoes are equivalent to 30×16 pomegranates.

Then, putting x for the number of pomegranates required, we get the proportion,

$$300 : 10 :: 30 \times 16 : x,$$

$$\text{whence } x = \frac{30 \times 16 \times 10}{300} = 16,$$

and the reason for the rule in § 85 is obvious.]

CHAPTER IV.

INVESTIGATION OF MIXTURE.[1]

SECTION I.

INTEREST.

87—88. Rule: a stanza and a half.[2]

The argument[3] multiplied by its time, and the fruit multiplied by the mixed quantity's time, being severally set down, and divided by their sum and multiplied by the mixed quantity, are the principal and interest (composing the quantity). Or the principal being found by the rule of supposition (§ 50), that, taken from the mixed quantity, leaves the amount of interest.

[The rule refers specially to the example given in § 89. By the word *argument* is meant 100, and by the word *fruit* is meant the interest on 100 for 1 month, or, as we would call it, the rate per cent. per mensem.

Let r = rate per cent. per mensem.
t = time in years.
P = principal.
I = interest.
A = amount.

[1] *Misra-vyavahára*, investigation of mixture, ascertainment of composition, as principal and interest, and so forth.—Gan. It is chiefly grounded on the rule of proportion.—*Ibid.* The rules in this chapter bear reference to the examples which follow them. Generally they are *quæstiones otiosæ*, problems for exercise.

[2] To investigate the principal and interest, the amount, time and rate being given.—Gan.

[3] *Pramána*, argument; *phala*, fruit (§ 70): principal and interest.

Then, $I = \frac{P \times r \times 12 \times t}{100}$.

$$\therefore A = P \left\{ 1 + \frac{r \times 12 \times t}{100} \right\} = P \times \frac{100 + r \times 12 \times t}{100}.$$

$$\therefore P = \frac{A \times 100}{100 + r \times 12 \times t}, \text{ and } I = \frac{A \times r \times 12 \times t}{100 + r \times 12 \times t}.$$

The last two formulæ stated in words give the first part of the rule. It is evident that these formulæ may also be derived by a simple proportion, as observed by Ganesa.

In the latter part of the rule, the principal is found by the author's method of supposition, which is practically the same as the solution of a simple equation.]

89. Example. If the principal sum, with interest at the rate of five on the hundred by the month, amount in a year to one thousand, tell the principal and interest respectively.

Statement: 1 12
100 1000.
5

Answer[1]: principal, 625 ; interest, 375.

Or, by the rule of position, put one ; and proceeding according to that rule (§ 50), the interest of unity is $\frac{3}{5}$, which, added to one, makes $\frac{8}{5}$. The given quantity 1000, multiplied by unity and divided by that ($\frac{8}{5}$), shows the principal 625. This, taken from the mixed amount, leaves the interest[2] 375.

[The second solution follows the latter part of the preceding rule.

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[1] 100 multiplied by 1 is 100 ; 5 by 12 is 60. Their sum 160 is the divisor. The first number 100, multiplied by 1000, and divided by 160, is 625. The second 60, multiplied by 1000, and divided by 160, gives 375.—Gang.

[2] Or the principal being known, the interest may be found by the Rule of Five.—Súr.

Let x denote the principal required. Then ∵ the rate per cent. per annum is 60, we get

$$x\left(1+\frac{60}{100}\right)=1000,$$

whence $x=\frac{1000}{1+\frac{3}{5}}$.]

90. Rule[1]: The arguments taken into their respective times are divided by the fruit taken into the elapsed times; the several quotients, divided by their sum and multiplied by the mixed quantity, are the parts as severally lent.

[The rule refers specially to the example in § 91.

Let x, y, z, be the portions lent at r_1, r_2, r_3 per cent. per mensem, and let I = common interest in t_1, t_2, t_3 months respectively.

Then, $x+y+z=a$, a given quantity;

and $$\frac{x\times r_1\times t_1}{100}=\frac{y\times r_2\times t_2}{100}=\frac{z\times r_3\times t_3}{100}=I;$$

$$\therefore x:y:z::\frac{100\times 1}{r_1\times t_1}:\frac{100\times 1}{r_2\times t_2}:\frac{100\times 1}{r_3\times t_3};$$

$$\therefore x=\frac{100\times 1}{r_1\times t_1}\times\frac{a}{\frac{100\times 1}{r_1\times t_1}+\frac{100\times 1}{r_2\times t_2}+\frac{100\times 1}{r_3\times t_3}},$$

with similar values for y and z. Hence the rule. Ganesa's explanation is practically the same as the above, but it is rather obscure: see footnote 2.]

91. Example. The sum of six less than a hundred *nishkas* being lent in three portions at interest of five, three and four per cent., an equal interest was obtained on all three portions, in seven, ten and five months respectively. Tell, mathematician, the amount of each portion.[2]

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[1] For determining parts of a compound sum.—Súr.

[2] Since the amount of interest on all the portions is the same, put unity for its arbitrarily assumed amount: whence corresponding principal sums

Statement :						
	1	7	1	10	1	5
	100		100		100	
	5		3		4	

Mixed amount 94.

Answer[1] : the portions are 24, 28 and 42. The equal amount of interest is $8\frac{2}{5}$.

92. Rule[2] : half a stanza.

The contributions[3] being multiplied by the mixed amount and divided by the sum of the contributions, are the respective fruits.[4]

[If we divide the mixed amount into parts proportional to the contributions, we shall get the mixed amount as due to each contribution. And this amount less the contribution is the gain. Hence the reason for the rule is obvious.]

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are found by the Rule of Five. For instance, if a hundred be the capital, of which five is the interest for a month, what is the capital, of which unity is the interest for seven months? and in like manner, the other principal sums are to be found. Thus, a compound proportion being wrought, the time is multiplied by the argument to which it appertains, and divided by the fruit taken into the elapsed time. Then, as the total of those principal sums is to them severally, so is the given total to the respective portions lent. They are thus severally found by the Rule of Three.—Gan.

[1] Multiplying the argument and fruit by the times, and dividing one product by the other, there result the fractions $\frac{100}{35}$, $\frac{100}{30}$, $\frac{100}{20}$, or $\frac{20}{7}$, $\frac{20}{6}$, $\frac{20}{4}$; which reduced to a common denominator and summed, make $\frac{1880}{168}$. Multiplied by the mixed amount 94, the fractions become $\frac{1880}{7}$, $\frac{1880}{6}$, $\frac{1880}{4}$; and then divided by the sum $\frac{1880}{168}$, they give $\frac{168}{7}$, $\frac{168}{6}$, $\frac{168}{4}$, or 24, 28, 42.—*Mano.*

To find the interest, employ the Rule of Five;

1	7
100	24.
5	

Answer, $8\frac{2}{5}$. By the same method, with all three portions, the interest comes out the same.—Súr.

[2] The capital sums, their aggregate amount, and the sum of the gains being given, to apportion the gains.—Gan.

[3] *Prakshepaka*, that which is thrown in or mixed.—Gan. Joined together.—Súr.

[4] The principle of the rule is obvious, being simply the Rule of Three.—Gan.

If by this sum of contributions, this contribution be had, then by the compound sum what will be? The numbers thus found, less the contributions, are the gains.—Ranganátha on the *Vásaná.*

93. Example. Say, mathematician, what the portioned shares are of three traders, whose original capitals were respectively fifty-one, sixty-eight and eighty-five, which have been raised by commerce conducted by them on joint stock, to the aggregate amount of three hundred.

Statement: 51, 68, 85; sum, 204. Mixed amount, 300.

Answer: 75, 100, 125. These, less the capital sums, are the gains: *viz.*, 24, 32, 40.

Or the mixed amount, less the sum of the capitals, is the profit on the whole: *viz.*, 96. This being multiplied by the contributions and divided by their sum, gives the respective gains: *viz.*, 24, 32, 40.

[The whole gain being divided into parts proportional to the contributions, gives the respective gains.]

Section II.

94. Rule[1]: half a stanza. Divide denominators by numerators; and then divide unity by those quotients added together. The result will be the time of filling (a cistern by several fountains).

[The rule refers to an example of the class given in § 95.

Let the times in which the fountains can severally fill the cistern be $\frac{a_1}{b_1}$, $\frac{a_2}{b_2}$, &c., of a day. Then in one day, the parts of the cistern filled up by the fountains severally are $\frac{b_1}{a_1}$, $\frac{b_2}{a_2}$, &c.; $\therefore$ if they work together, the part filled in one day will be $=\frac{b_1}{a_1}+\frac{b_2}{a_2}+$&c.;

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[1] To apportion the time for a mixture of springs to fill a well or cistern.—Gan. To solve an instance relative to fractions.—Súr.

the time in which the whole cistern will be filled $= \dfrac{1}{\dfrac{b_1}{a_1}+\dfrac{b_2}{a_2}+\&c.}$ of a day,
whence the reason for the rule is evident. The explanations of this rule given by the commentators are all vague and unsatisfactory.]

95. Example. Say quickly, friend, in what portion of a day (four) fountains, being let loose together, will fill a cistern, which, if severally opened, they would fill in one day, half a day, the third, and the sixth part, respectively.

Statement : $\frac{1}{1}$, $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{6}$.

Answer : $\frac{1}{12}$th part of a day.

Section III.

Purchase and Sale.

96. Rule.[1] By the (measure of the) commodities,[2] divide their prices taken into their respective portions (of the purchase); and by the sum of the quotients divide both them and those portions severally multiplied by the mixed sum : the prices and quantities are found in their order.

[The reason for the rule will appear from the solution of the example in § 97.]

97. Example. If three and a half *mánas*[3] of rice may be had for one *dramma*, and eight of kidney-beans[4]

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[1] For a case where a mixture of portions and composition of things are given.—Gan. Concerning measure of grain, &c.—Súr.

[2] *Panya*, the measure of the grain or other commodity procurable for the current price in the market.—Súr. and the *Manor.*

[3] *Mána* or *mánaka*, a measure; seemingly intending a particular one. According to Ganesa, the *mana* (apparently the same as the *mána*) is at most an eighth of a *khári*; being a cubic span. See note to § 236. A spurious couplet (see note on § 2) makes it the modern measure of weight containing forty *sers*.

[4] *Mudga*, phaseolus mungo ; a sort of kidney-bean.