thus found, the motion of each body after the ſtroke is determined, as before.

10. IN the next place, ſuppoſe the bodies A and B were both moving the ſame way, but A with a ſwifter motion, ſo as to overtake B, and ſtrike againſt it. The effect of the percuſſion or ſtroke, when the bodies are not elaſtic, is diſcovered by finding the common motion, which the two bodies would have after the ſtroke, if B were at reſt, and A were to advance againſt it with a velocity equal to the exceſs of the preſent velocity of A above the velocity of B; and by adding to this common velocity thus found the velocity of B.

11. IF the bodies are elaſtic, the effect of the elaſticity is to be united with this other, as in the former caſes.

12. WHEN the bodies are perfectly elaſtic, the rule of HUYGENS [a] in this caſe is to prolong CD (fig. 7.) and to take in it thus prolonged CE in the ſame proportion to ED, as the greater velocity of A bears to the leſſer velocity of B; after which FG being taken equal to FE, the velocities of the two bodies after the ſtroke will be determined, as in the two preceding caſes.

13. THUS I have given the ſum of what has been written concerning the effects of percuſſion, when two bodies freely in motion ſtrike directly againſt each other; and the reſults here ſet down, as the conſequence of our reaſoning

[a] In the place above-cited.

from the laws of motion, anſwer moſt exactly to experience. A particular ſet of experiments has been invented to make trial of theſe effects of percuſſion with the greateſt exactneſs. But I muſt defer theſe experiments, till I have explained the nature of pendulums [a]. I ſhall therefore now proceed to deſcribe ſome of the appearances, which are cauſed in bodies from the influence of the power of gravity united with the general laws of motion; among which the motion of the pendulum will be included.

14. THE moſt ſimple of theſe appearances is, when bodies fall down merely by their weight. In this caſe the body increaſes continually its velocity, during the whole time of its fall, and that in the very ſame proportion as the time increaſes. For the power of gravity acts conſtantly on the body with the ſame degree of ſtrength: and it has been obſerved above in the firſt law of motion, that a body being once in motion will perpetually preſerve that motion without the continuance of any external influence upon it: therefore, after a body has been once put in motion by the force of gravity, the body would continue that motion, though the power of gravity ſhould ceaſe to act any farther upon it; but, if the power of gravity continues ſtill to draw the body down, freſh degrees of motion muſt continually be added to the body; and the power of gravity acting at all times with the ſame ſtrength, equal degrees of motion will conſtantly be added in equal portions of time.

[a] Theſe experiments are deſcribed in § 73.

15. This conclusion is not indeed absolutely true: for we shall find hereafter [a], that the power of gravity is not of the same strength at all distances from the center of the earth. But nothing of this is in the least sensible in any distance, to which we can convey bodies. The weight of bodies is the very same to sense upon the highest towers or mountains, as upon the level ground; so that in all the observations we can make, the forementioned proportion between the velocity of a falling body and the time, in which it has been descending, obtains without any the least perceptible difference.

16. From hence it follows, that the space, through which a body falls, is not proportional to the time of the fall; for since the body increases its velocity, a greater space will be passed over in the same portion of time at the latter part of the fall, than at the beginning. Suppose a body let fall from the point A (in fig. 8.) were to descend from A to B in any portion of time; then if in an equal portion of time it were to proceed from B to C; I say, the space BC is greater than AB; so that the time of the fall from A to C being double the time of the fall from A to B, AC shall be more than double of AB.

17. The geometers have proved, that the spaces, through which bodies fall thus by their weight, are just in a duplicate or two-fold proportion of the times, in which the body has been falling. That is, if we were to take the line DE in the same proportion to AB, as the time, which the body has imployed in falling from A to C, bears to the time of the fall

[a] Book II. Chap. 5.

from

from A to B; then AC will be to DE in the ſame proportion. In particular, if the time of the fall through AC be twice the time of the fall through AB; then DE will be twice AB, and AC twice DE; or AC four times AB. But if the time of the fall through AC had been thrice the time of the fall through AB; DE would have been treble of AB, and AC treble of DE; that is, AC would have been equal to nine times AB.

18. IF a body fall obliquely, it will approach the ground by ſlower degrees, than when it falls perpendicularly. Suppoſe two lines AB, AC (in fig. 9.) were drawn, one perpendicular, and the other oblique to the ground DE: then if a body were to deſcend in the ſlanting line AC; becauſe the power of gravity draws the body directly downwards, if the line AC ſupports the body from falling in that manner, it muſt take off part of the effect of the power of gravity; ſo that in the time, which would have been ſufficient for the body to have fallen through the whole perpendicular line AB, the body ſhall not have paſſed in the line AC a length equal to AB; conſequently the line AC being longer than AB, the body ſhall moſt certainly take up more time in paſſing through AC, than it would have done in falling perpendicularly down through AB.

19. THE geometers demonſtrate, that the time, in which the body will deſcend through the oblique ſtraight line AC, bears the ſame proportion to the time of its deſcent through the perpendicular AB, as the line it ſelf AC bears to AB. And in reſpect to the velocity, which the body will have ac-

quired in the point C, they likewiſe prove, that the length of the time imployed in the deſcent through AC ſo compenſates the diminution of the influence of gravity from the obliquity of this line, that though the force of the power of gravity on the body is oppoſed by the obliquity of the line AC, yet the time of the body's deſcent ſhall be ſo much prolonged, that the body ſhall acquire the very ſame velocity in the point C, as it would have got at the point B by falling perpendicularly down.

20. If a body were to deſcend in a crooked line, the time of its deſcent cannot be determined in ſo ſimple a manner; but the ſame property, in relation to the velocity, is demonſtrated to take place in all caſes: that is, in whatever line the body deſcends, the velocity will always be anſwerable to the perpendicular height, from which the body has fell. For inſtance, ſuppoſe the body A (in fig. 10.) were hung by a ſtring to the pin B. If this body were let fall, till it came to the point C perpendicularly under B, it will have moved from A to C in the arch of a circle. Then the horizontal line AD being drawn, the velocity of the body in C will be the ſame, as if it had fallen from the point D directly down to C.

21. If a body be thrown perpendicularly upward with any force, the velocity, wherewith the body aſcends, ſhall continually diminiſh, till at length it be wholly taken away; and from that time the body will begin to fall down again, and paſs over a ſecond time in its deſcent the line, wherein it aſcended; falling through this line with an increaſing velocity in ſuch a manner, that in every point thereof, through which

which it falls, it ſhall have the very ſame velocity, as it had in the ſame place, when it aſcended; and conſequently ſhall come down into the place, whence it firſt aſcended, with the velocity which was at firſt given to it. Thus if a body were thrown perpendicularly up in the line AB (in fig. 11.) with ſuch a force, as that it ſhould ſtop at the point B, and there begin to fall again; when it ſhall have arrived in its deſcent to any point as C in this line, it ſhall there have the ſame velocity, as that wherewith it paſſed by this point C in its aſcent; and at the point A it ſhall have gained as great a velocity, as that wherewith it was firſt thrown upwards. As this is demonſtrated by ~~the~~ geometrical writers; ſo, I think, it will appear evident, by conſidering only, that while the body deſcends, the power of gravity muſt act over again, in an inverted order, all the influence it had on the body in its aſcent; ſo as to give again to the body the ſame degrees of velocity, which it had taken away before.

22. AFTER the ſame manner, if the body were thrown upwards in the oblique ſtraight line CA (in fig. 9.) from the point C, with ſuch a degree of velocity as juſt to reach the point A; it ſhall by its own weight return again through the line AC by the ſame degrees, as it aſcended.

23. AND laſtly, if a body were thrown with any velocity in a line continually incurvated upwards, the like effect will be produced upon its return to the point, whence it was thrown. Suppoſe for inſtance, the body A (in fig. 12.) were hung by a ſtring AB. Then if this body be impelled any

way, it muſt move in the arch of a circle. Let it receive ſuch an impulſe, as ſhall cauſe it to move in the arch A C; and let this impulſe be of ſuch ſtrength, that the body may be carried from A as far as D, before its motion is overcome by its weight: I ſay here, that the body forthwith returning from D, ſhall come again into the point A with the ſame velocity, as that wherewith it began to move.

24. IT will be proper in this place to obſerve concerning the power of gravity, that its force upon any body does not at all depend upon the ſhape of the body; but that it continues conſtantly the ſame without any variation in the ſame body, whatever change be made in the figure of the body: and if the body be divided into any number of pieces, all thoſe pieces ſhall weigh juſt the ſame, as they did, when united together in one body: and if the body be of a uniform contexture, the weight of each piece will be proportional to its bulk. This has given reaſon to conclude, that the power of gravity acts upon bodies in proportion to the quantity of matter in them. Whence it ſhould follow, that all bodies muſt fall from equal heights in the ſame ſpace of time. And as we evidently ſee the contrary in feathers and ſuch like ſubſtances, which fall very ſlowly in compariſon of more ſolid bodies; it is reaſonable to ſuppoſe, that ſome other cauſe concurs to make ſo manifeſt a difference. This cauſe has been found by particular experiments to be the air. The experiments for this purpoſe are made thus. They ſet up a very tall hollow glaſs; within which near the top they lodge a feather and ſome very ponderous body, uſually a piece of gold,

this

this metal being the moſt weighty of any body known to us. This glaſs they empty of the air contained within it, and by moving a wire, which paſſes through the top of the glaſs, they let the feather and the heavy body fall together; and it is always found, that as the two bodies begin to deſcend at the ſame time, ſo they accompany each other in the fall, and come to the bottom at the very ſame inſtant, as near as the eye can judge. Thus, as far as this experiment can be depended on, it is certain, that the effect of the power of gravity upon each body is proportional to the quantity of ſolid matter, or to the power of inactivity in each body. For in the limited ſenſe, which we have given above to the word motion, it has been ſhewn, that the ſame force gives to all bodies the ſame degree of motion, and different forces communicate different degrees of motion proportional to the reſpective powers [a]. In this caſe, if the power of gravity were to act equally upon the feather, and upon the more ſolid body, the ſolid body would deſcend ſo much ſlower than the feather, as to have no greater degree of motion than the feather: but as both bodies deſcend with equal ſwiftneſs, the degree of motion in the ſolid body is greater than in the feather, bearing the ſame proportion to it, as the quantity of matter in the ſolid body to the quantity of matter in the feather. Therefore the effect of gravity on the ſolid body is greater than on the feather, in proportion to the greater degree of motion communicated; that is, the effect of the power of gravity on the ſolid body bears the ſame proportion to its effect on the feather, as the quanti-

[a] Chap. I. § 25, 26, 27, compared with § 15. &c.

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ty of matter in the ſolid body bears to the quantity of matter in the feather. Thus it is the proper deduction from this experiment, that the power of gravity acts not on the ſurface of bodies only, but penetrates the bodies themſelves moſt intimately, and operates alike on every particle of matter in them. But as the great quickneſs, with which the bodies fall, leaves it ſomething uncertain, whether they do deſcend abſolutely in the ſame time, or only ſo nearly together, that the difference in their ſwift motion is not diſcernable to the eye; this property of the power of gravity, which has here been deduced from this experiment, is farther confirmed by pendulums, whoſe motion is ſuch, that a very minute difference would become ſufficiently ſenſible. This will be farther diſcourſed on in another place [a]; but here I ſhall make uſe of the principle now laid down to explain the nature of what is called the center of gravity in bodies.

25. The center of gravity is that point, by which if a body be ſuſpended, it ſhall hang at reſt in any ſituation. In a globe of a uniform texture the center of gravity is the ſame with the center of the globe; for as the parts of the globe on every ſide of its center are ſimilarly diſpoſed, and the power of gravity acts alike on every part; it is evident, that the parts of the globe on each ſide of the center are drawn with equal force, and therefore neither ſide can yield to the other; but the globe, if ſupported at its center, muſt of neceſſity hang at reſt. In like manner, if two equal bodies A and B (in

[a] Book II. Chap. 5. § 3.

fig. 13.) be hung at the extremities of an inflexible rod CD, which ſhould have no weight; theſe bodies, if the rod be ſupported at its middle E, ſhall equiponderate; and the rod remain without motion. For the bodies being equal and at the ſame diſtance from the point of ſupport E, the power of gravity will act upon each with equal ſtrength, and in all reſpects under the ſame circumſtances; therefore the weight of one cannot overcome the weight of the other. The weight of A can no more ſurmount the weight of B, than the weight of B can ſurmount the weight of A. Again, ſuppoſe a body as AB (in fig. 14.) of a uniform texture in the form of a roller, or as it is more uſually called a cylinder, lying horizontally. If a ſtraight line be drawn between C and D, the centers of the extreme circles of this cylinder; and if this ſtraight line, commonly called the axis of the cylinder, be divided into two equal parts in E: this point E will be the center of gravity of the cylinder. The cylinder being a uniform figure, the parts on each ſide the point E are equal, and ſituated in a perfectly ſimilar manner; therefore this cylinder, if ſupported at the point E, muſt hang at reſt, for the ſame reaſon as the inflexible rod above-mentioned will remain without motion, when ſuſpended at its middle point. And it is evident, that the force applied to the point E, which would uphold the cylinder, muſt be equal to the cylinder's weight. Now ſuppoſe two cylinders of equal thickneſs AB and CD to be joined together at CB, ſo that the two axis's EF, and FG lie in one ſtraight line. Let the axis EF be divided into two equal parts at H, and the axis FG into two equal

equal parts at I. Then becauſe the cylinder AB would be upheld at reſt by a power applied in H equal to the weight of this cylinder, and the cylinder CD would likewiſe be upheld by a power applied in I equal to the weight of this cylinder; the whole cylinder AD will be ſupported by theſe two powers: but the whole cylinder may likewiſe be ſupported by a power applied to K, the middle point of the whole axis EG, provided that power be equal to the weight of the whole cylinder. It is evident therefore, that this power applied in K will produce the ſame effect, as the two other powers applied in H and I. It is farther to be obſerved, that HK is equal to half FG, and KI equal to half EF; for EK being equal to half EG, and EH equal to half EF, the remainder HK muſt be equal to half the remainder FG; ſo likewiſe GK being equal to half GE, and GI equal to half GF, the remainder IK muſt be equal to half the remainder EF. It follows therefore, that HK bears the ſame proportion to KI, as FG bears to EF. Beſides, I believe, my readers will perceive, and it is demonſtrated in form by the geometers, that the whole body of the cylinder CD bears the ſame proportion to the whole body of the cylinder AB, as the axis FG bears to the axis EF [a]. But hence it follows, that in the two powers applied at H and I, the power applied at H bears the ſame proportion to the power applied at I, as KI bears to KH. Now ſuppoſe two ſtrings HL and IM extended upwards, one from the point H and the other from I, and to be laid hold on by two powers, one ſtrong enough to hold up the cylinder AB, and the other of

[a] See Euclid's Elements, Book XII. prop. 13.

ſtrength

ſtrength ſufficient to ſupport the cylinder CD. Here as theſe two powers uphold the whole cylinder, and therefore produce an effect, equal to what would have been produced by a power applied to the point K of ſufficient force to ſuſtain the whole cylinder: it is manifeſt, that if the cylinder be taken away, the axis only being left, and from the point K a ſtring, as KN, be extended, which ſhall be drawn down by a power equivalent to the weight of the cylinder, this power ſhall act againſt the other two powers, as much as the cylinder acted againſt them; and conſequently theſe three powers ſhall be upon a balance, and hold the axis HI fixed between them. But if theſe three powers preſerve a mutual balance, the two powers applied to the ſtrings HL and IM are a balance to each other; the power applied to the ſtring HL bearing the ſame proportion to the power applied to the ſtring IM, as the diſtance IK bears to the diſtance KH. Hence it farther appears, that if an inflexible rod AB (in fig. 15.) be ſuſpended by any point C not in the middle thereof; and if at A the end of the ſhorter arm be hung a weight, and at B the end of the longer arm be alſo hung a weight leſs than the other, and that the greater of theſe weights bears to the leſſer the ſame proportion, as the longer arm of the rod bears to the ſhorter; then theſe two weights will equiponderate: for a power applied at C equal to both theſe weights will ſupport without motion the rod thus charged; ſince here nothing is changed from the preceding caſe but the ſituation of the powers, which are now placed on the contrary ſides of the line, to which they are fixed. Alſo for the same reason, if two weights A and B (in fig. 16.) were connected together by an inflexible rod CD, drawn from C the center of gravity of A to D the center of gravity of B; and if the rod CD were to be so divided in E, that the part DE bear the same proportion to the other part CE, as the weight A bears to the weight B: then this rod being supported at E will uphold the weights, and keep them at rest without motion. This point E, by which the two bodies A and B will be supported, is called their common center of gravity. And if a greater number of bodies were joined together, the point, by which they could all be supported, is called the common center of gravity of them all. Suppose (in fig. 17.) there were three bodies A, B, C, whose respective centers of gravity were joined by the three lines DE, DF, EF: the line DE being so divided in G, that DG bear the same proportion to GE, as B bears to A; G is the center of gravity common to the two bodies A and B; that is, a power equal to the weight of both the bodies applied to G would support them, and the point G is pressed as much by the two weights A and B, as it would be, if they were both hung together at that point. Therefore, if a line be drawn from G to F, and divided in H, so that GH bear the same proportion to HF, as the weight C bears to both the weights A and B, the point H will be the common center of gravity of all the three weights; for H would be their common center of gravity, if both the weights A and B were hung together at G, and the point G is pressed as much by them in their present situation, as it would be in that case. In the same manner from the common center of these three

weights,

weights, you might proceed to find the common center, if a fourth weight were added, and by a gradual progreſs might find the common center of gravity belonging to any number of weights whatever.

26. As all this is the obvious conſequence of the propoſition laid down for aſſigning the common center of gravity of any two weights, by the ſame propoſition the center of gravity of all figures is found. In a triangle, as A B C (in fig. 18.) the center of gravity lies in the line drawn from the middle point of any one of the ſides to the oppoſite angle, as the line B D is drawn from D the middle of the line A C to the oppoſite angle B [a]; ſo that if from the middle of either of the other ſides, as from the point E in the ſide A B, a line be drawn, as E C, to the oppoſite angle; the point F, where this line croſſes the other line B D, will be the center of gravity of the triangle [b]. Likewiſe D F is equal to half F B, and E F equal to half F C [c]. In a hemiſphere, as A B C (fig. 19.) if from D the center of the baſe the line D B be erected perpendicular to that baſe, and this line be ſo divided in E, that D E be equal to three fifths of B E, the point E is the center of gravity of the hemiſphere [d]

27. It will be of uſe to obſerve concerning the center of gravity of bodies; that ſince a power applied to this center alone can ſupport a body againſt the power of gravity, and

[a] Archimed. de æquipond. prop. 11.
[b] Ibid. prop. 12.
[c] Lucas Valerius De centr. gravit. ſolid. L. I. prop. 2.
[d] Idem L. II. prop. 2.

hold it fixed at reſt; the effect of the power of gravity on a body is the ſame, as if that whole power were to exert itſelf on the center of gravity only. Whence it follows, that, when the power of gravity acts on a body ſuſpended by any point, if the body is ſo ſuſpended, that the center of gravity of the body can deſcend; the power of gravity will give motion to that body, otherwiſe not: or if a number of bodies are ſo connected together, that, when any one is put into motion, the reſt ſhall, by the manner of their being joined, receive ſuch motion, as ſhall keep their common center of gravity at reſt; then the power of gravity ſhall not be able to produce any motion in theſe bodies, but in all other caſes it will. Thus, if the body AB (in fig. 20, 21.) whoſe center of gravity is C, be hung on the point A, and the center C be perpendicularly under A (as in fig. 20.) the weight of the body will hold it ſtill without motion, becauſe the center C cannot deſcend any lower. But if the body be removed into any other ſituation, where the center C is not perpendicularly under A (as in fig. 21.) the body by its weight will be put into motion towards the perpendicular ſituation of its center of gravity. Alſo if two bodies A, B (in fig. 22.) be joined together by the rod CD lying in an horizontal ſituation, and be ſupported at the point E; if this point be the center of gravity common to the two bodies, their weight will not put them into motion; but if this point E is not their common center of gravity, the bodies will move; that part of the rod CD deſcending, in which the common center of gravity is found. So in like manner, if theſe two bodies were connected together by any more complex contrivance; yet if if one of the bodies cannot move without ſo moving the other, that their common center of gravity ſhall reſt, the weight of the bodies will not put them in motion, otherwiſe it will.

28. I SHALL proceed in the next place to ſpeak of the mechanical powers. Theſe are certain inſtruments or machines, contrived for the moving great weights with ſmall force; and their effects are all deducible from the obſervation we have juſt been making. They are uſually reckoned in number five; the lever, the wheel and axis, the pulley, the wedge, and the ſcrew; to which ſome add the inclined plane. As theſe inſtruments have been of very ancient uſe, ſo the celebrated ARCHIMEDES ſeems to have been the firſt, who diſcovered the true reaſon of their effects. This, I think, may be collected from what is related of him, that ſome expreſſions, which he uſed to denote the unlimited force of theſe inſtruments, were received as very extraordinary paradoxes: whereas to thoſe, who had underſtood the cauſe of their great force, no expreſſions of that kind could have appeared ſurprizing.

29. ALL the effects of theſe powers may be judged of by this one rule, that, when two weights are applied to any of theſe inſtruments, the weights will equiponderate, if, when put into motion, their velocities will be reciprocally proportional to their reſpective weights. And what is ſaid of weights, muſt of neceſſity be equally underſtood of any other forces

equi-

equivalent to weights, ſuch as the force of a man's arm, a ſtream of water, or the like.

30. But to comprehend the meaning of this rule; the reader muſt know, what is to be underſtood by reciprocal proportion; which I ſhall now endeavour to explain, as diſtinctly as I can; for I ſhall be obliged very frequently to make uſe of this term. When any two things are ſo related, that one increaſes in the ſame proportion as the other, they are directly proportional. So if any number of men can perform in a determined ſpace of time a certain quantity of any work, ſuppoſe drain a fiſh-pond, or the like; and twice the number of men can perform twice the quantity of the ſame work, in the ſame time; and three times the number of men can perform as ſoon thrice the work; here the number of men and the quantity of the work are directly proportional. On the other hand, when two things are ſo related, that one decreaſes in the ſame proportion, as the other increaſes, they are ſaid to be reciprocally proportional. Thus if twice the number of men can perform the ſame work in half the time, and three times the number of men can finiſh the ſame in a third part of the time; then the number of men and the time are reciprocally proportional. We ſhewed above [a] how to find the common center of gravity of two bodies, there the diſtances of that common center from the centers of gravity of the two bodies are reciprocally proportional to the reſpective bodies. For CE in fig. 16. being in the ſame pro-

portion

portion to ED, as B bears to A; CE is ſo much greater in proportion than ED, as A is leſs in proportion than B.

31. Now this being underſtood, the reaſon of the rule here ſtated will eaſily appear. For if theſe two bodies were put in motion, while the point E reſted, the velocity, wherewith A would move, would bear the ſame proportion to the velocity, wherewith B would move, as EC bears to ED. The velocity therefore of each body, when the common center of gravity reſts, is reciprocally proportional to the body. But we have ſhewn above[a], that if two bodies are ſo connected together, that the putting them in motion will not move their common center of gravity; the weight of thoſe bodies will not produce in them any motion. Therefore in any of theſe mechanical engines, if, when the bodies are put into motion, their velocities are reciprocally proportional to their reſpective weights, whereby the common center of gravity would remain at reſt; the bodies will not receive any motion from their weight, that is, they will equiponderate. But this perhaps will be yet more clearly conceived by the particular deſcription of each mechanical power.

32. THE lever was firſt named above. This is a bar made uſe of to ſuſtain and move great weights. The bar is applied in one part to ſome ſtrong ſupport; as the bar AB (in fig. 23, 24.) is applied at the point C to the ſupport D. In ſome other part of the bar, as E, is applied the weight to be ſuſtained or moved; and in a third place, as F, is applied another weight or equivalent force, which is to ſuſtain or move

the weight at E. Now here, if, when the lever ſhould be put in motion, and turned upon the point C, the velocity, wherewith the point F would move, bears the ſame proportion to the velocity, wherewith the point E would move, as the weight at E bears to the weight or force at F; then the lever thus charged will have no propenſity to move either way. If the weight or other force at F be not ſo great as to bear this proportion, the weight at E will not be ſuſtained; but if the force at F be greater than this, the weight at E will be ſurmounted. This is evident from what has been ſaid above[a], when the forces at E and F are placed (as in fig. 23.) on different ſides of the ſupport D. It will appear alſo equally manifeſt in the other caſe, by continuing the bar BC in fig. 24. on the other ſide of the ſupport D, till CG be equal to CF, and by hanging at G a weight equivalent to the power at F; for then, if the power at F were removed, the two weights at G and E would counterpoize each other, as in the former caſe: and it is evident, that the point F will be lifted up by the weight at G with the ſame degree of force, as by the other power applied to F; ſince, if the weight at E were removed, a weight hung at F equal to that at G would balance the lever, the diſtances CG and CF being equal.

33. IF the two weights, or other powers, applied to the lever do not counterbalance each other; a third power may be applied in any place propoſed of the lever, which ſhall

[a] Pag. 65, 68.

hold

hold the whole in a juſt counterpoize. Suppoſe (in fig. 25.) the two powers at E and F did not equiponderate, and it were required to apply a third power to the point G, that might be ſufficient to balance the lever. Find what power in F would juſt counterbalance the power in E; then if the difference between this power and that, which is actually applied at F, bear the ſame proportion to the third power to be applied at G, as the diſtance CG bears to CF; the lever will be counterpoized by the help of this third power, if it be ſo applied as to act the ſame way with the power in F, when that power is too ſmall to counterbalance the power in E; but otherwiſe the power in G muſt be ſo applied, as to act againſt the power in F. In like manner, if a lever were charged with three, or any greater number of weights or other powers, which did not counterpoize each other, another power might be applied in any place propoſed, which ſhould bring the whole to a juſt balance. And what is here ſaid concerning a plurality of powers, may be equally applied to all the following caſes.

34. IF the lever ſhould conſiſt of two arms making an angle at the point C (as in fig. 26.) yet if the forces are applied perpendicularly to each arm, the ſame proportion will hold between the forces applied, and the diſtances of the center, whereon the lever reſts, from the points to which they are applied. That is, the weight at E will be to the force in F in the ſame proportion, as CF bears to CE.

35. BUT whenever the forces applied to the lever act obliquely to the arm, to which they are applied (as in fig. 27.)

then the ſtrength of the forces is to be eſtimated by lines let fall from the center of the lever to the directions, wherein the forces act. To balance the levers in fig. 27, the weight or other force at F will bear the ſame proportion to the weight at E, as the diſtance CE bears to CG the perpendicular let fall from the point C upon the line, which denotes the direction wherein the force applied to F acts: for here, if the lever be put into motion, the power applied to F will begin to move in the direction of the line FG; and therefore its firſt motion will be the ſame, as the motion of the point G.

36. When two weights hang upon a lever, and the point, by which the lever is ſupported, is placed in the middle between the two weights, that the arms of the lever are both of equal length; then this lever is particularly called a balance; and equal weights equiponderate as in common ſcales. When the point of ſupport is not equally diſtant from both weights, it conſtitutes that inſtrument for weighing, which is called a ſteelyard. Though both in common ſcales, and the ſteelyard, the point, on which the beam is hung, is not uſually placed juſt in the ſame ſtraight line with the points, that hold the weights, but rather a little above (as in fig. 28.) where the lines drawn from the point C, whereon the beam is ſuſpended, to the points E and F, on which the weights are hung, do not make abſolutely one continued line. If the three points E, C, and F were in one ſtraight line, thoſe weights, which equiponderated, when the beam hung horizontally, would alſo equiponderate in any other ſituation. But we ſee in theſe inſtruments, when they are charged with weights,

which

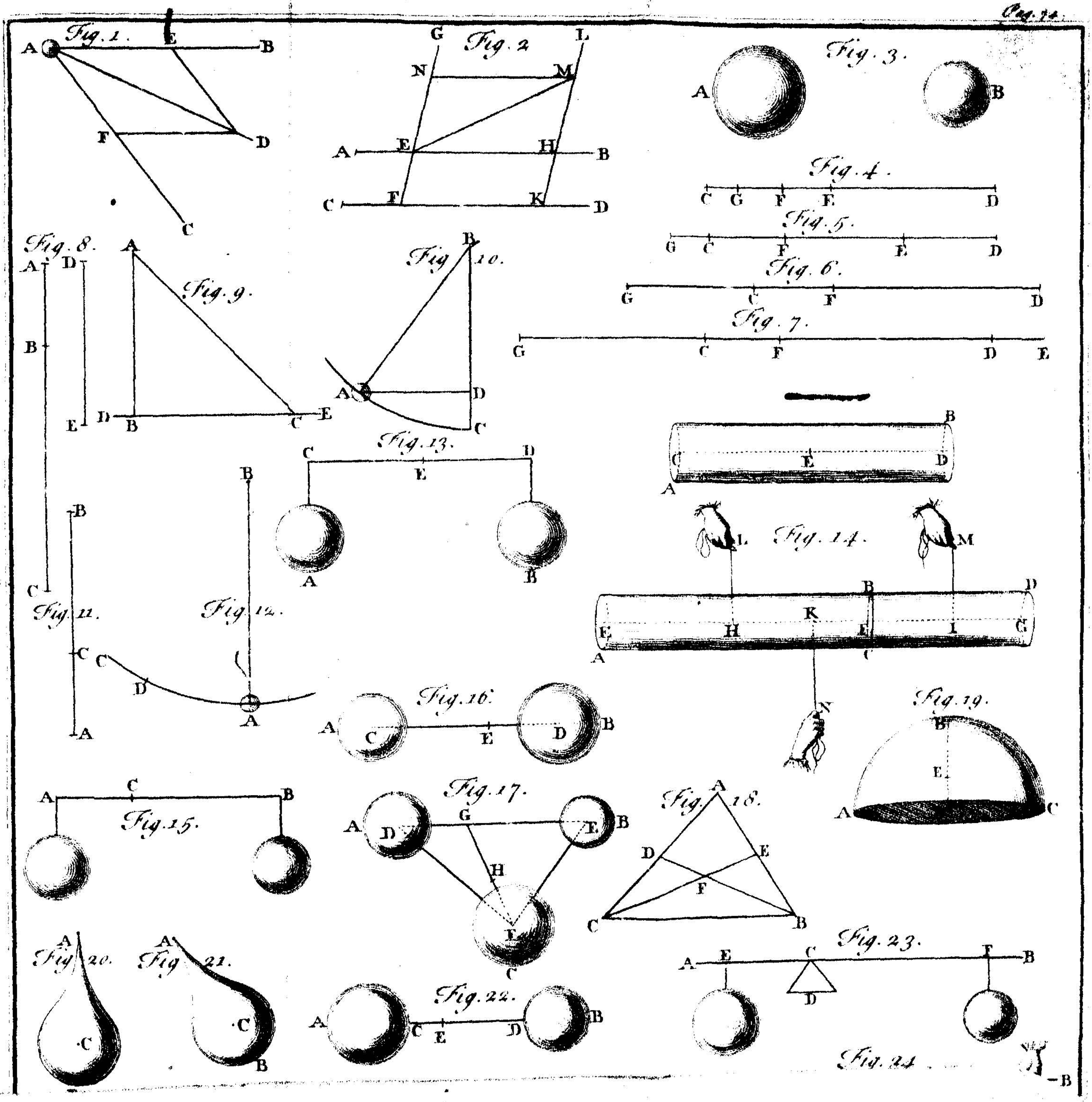
Fig. 1.
Fig. 2
Fig. 3.
Fig. 4.
Fig. 5.
Fig. 6.
Fig. 7.
Fig. 8.
Fig. 9.
Fig. 10.
Fig. 11.
Fig. 12.
Fig. 13.
Fig. 14.
Fig. 15.
Fig. 16.
Fig. 17.
Fig. 18.
Fig. 19.
Fig. 20.
Fig. 21.
Fig. 22.
Fig. 23.
Fig. 24.

which equiponderate with the beam hanging horizontally; that, if the beam be inclined either way, the weight moſt elevated ſurmounts the other, and deſcends, cauſing the beam to ſwing, till by degrees it recovers its horizontal poſition. This effect ariſes from the forementioned ſtructure: for by this ſtructure theſe inſtruments are levers compoſed of two arms, which make an angle at the point of ſupport (as in fig. 29, 30.) the firſt of which repreſents the caſe of the common balance, the ſecond the caſe of the ſteelyard. In the firſt, where CE and CF are equal, equal weights hung at E and F will equiponderate, when the points E and F are in an horizontal ſituation. Suppoſe the lines EG and FH to be perpendicular to the horizon, then they will denote the directions, wherein the forces applied to E and F act. Therefore the proportion between the weights at E and F, which ſhall equiponderate, are to be judged of by perpendiculars, as CI, CK, let fall from C upon EG and FH: ſo that the weights being equal, the lines CI, CK, muſt be equal alſo, when the weights equiponderate. But I believe my readers will eaſily ſee, that ſince CE and CF are equal, the lines CI and CK will be equal, when the points E and F are horizontally ſituated.

37. IF this lever be ſet into any other poſition (as in fig. 31.) then the weight, which is raiſed higheſt, will outweigh the other. Here, if the point F be raiſed higher than E, the perpendicular CK will be longer than CI: and therefore the weights would equiponderate, if the weight at F were

were leſs than the weight at E. But the weight at F is equal to that at E; therefore is greater, than is neceſſary to counterbalance the weight at E, and conſequently will outweigh it, and draw the beam of the lever down.

38. In like manner in the caſe of the ſteelyard (fig. 32.) if the weights at E and F are ſo proportioned, as to equiponderate, when the points E and F are horizontally ſituated; then in any other ſituation of this lever the weight, which is raiſed higheſt, will preponderate. That is, if in the horizontal ſituation of the points E and F the weight at F bears the ſame proportion to the weight at E, as CI bears to CK; then, if the point F be raiſed higher than E (as in fig. 32.) the weight at F ſhall bear a greater proportion to the weight at E, than CI bears to CK.

39. Farther a lever may be hung upon an axis, and then the two arms of the lever need not be continuous, but fixed to different parts of this axis; as in fig. 33, where the axis AB is ſupported by its two extremities A and B. To this axis one arm of the lever is fixed at the point C, the other at the point D. Now here, if a weight be hung at E, the extremity of that arm, which is fixed to the axis at the point C; and another weight be hung at F, the extremity of the arm, which is fixed on the axis at D; then theſe weights will equiponderate, when the weight at E bears the ſame proportion to the weight at F, as the arm DF bears to CE.

40. This

40. THIS is the caſe, if both the arms are perpendicular to the axis, and lie (as the geometers expreſs themſelves) in the ſame plane; or, in other words, if the arms are ſo fixed perpendicularly upon the axis, that, when one of them lies horizontally, the other ſhall alſo be horizontal. If either arm ſtand not perpendicular to the axis; then, in determining the proportion between the weights, inſtead of the length of that arm, you muſt uſe the perpendicular let fall upon the axis from the extremity of that arm. If the arms are not ſo fixed as to become horizontal, at the ſame time; the method of aſſigning the proportion between the weights is analogous to that made uſe of above in levers, which make an angle at the point, whereon they are ſupported.

41. FROM this caſe of the lever hung on an axis, it is eaſy to make a tranſition to another mechanical power, the wheel and axis.

42. THIS inſtrument is a wheel fixed on a roller, the roller being ſupported at each extremity ſo as to turn round freely with the wheel, in the manner repreſented in fig. 34, where AB is the wheel, CD the roller, and EF its two ſupports. Now ſuppoſe a weight G hung by a cord wound round the roller, and another weight H hung by a cord wound about the wheel the contrary way: that theſe weights may ſupport each other, the weight H muſt bear the ſame proportion to the weight G, as the thickneſs of the roller bears to the diameter of the wheel.

43. SUPPOSE

43. Suppose the line kl to be drawn through the middle of the roller; and from the place of the roller, where the cord, on which the weight G hangs, begins to leave the roller, as at m, let the line mn be drawn perpendicularly to kl; and from the point, where the cord holding the weight H begins to leave the wheel, as at o, let the line op be drawn perpendicular to kl. This being done, the two lines op and mn repreſent two arms of a lever fixed on the axis kl; conſequently the weight H will bear to the weight ~~G the~~ ſame proportion, as mn bears to op. But mn bears the ſame proportion to op, as the thickneſs of the roller bears to the diameter of the wheel; for mn is half the thickneſs of the roller, and op half the diameter of the wheel.

44. If the wheel be put into motion, and turned once round, that the cord, on which the weight G hangs, be wound once more round the axis; then at the ſame time the cord, whereon the weight H hangs, will be wound off from the wheel one circuit. Therefore the velocity of the weight G will bear the ſame proportion to the velocity of the weight H, as the circumference of the roller to the circumference of the wheel. But the circumference of the roller bears the ſame proportion to the circumference of the wheel, as the thickneſs of the roller bears to the diameter of the wheel, conſequently the velocity of the weight G bears to the velocity of the weight H the ſame proportion, as the thickneſs of the roller bears to the diameter of the wheel, which is the proportion that the weight H bears to the weight G. Therefore as before in the lever, ſo here alſo the general rule laid down

down above is verified, that the weights equiponderate, when their velocities would be reciprocally proportional to their refpective weights.

45. IN like manner, if on the fame axis two wheels of different fizes are fixed (as in fig. 35.) and a weight hung on each; the weights will equiponderate, if the weight hung on the greater wheel bear the fame proportion to the weight hung on the leffer, as the diameter of the leffer wheel bears to the diameter of the greater.

46. IT is ufual to join many wheels together in the fame frame, which by the means of certain teeth, formed in the circumference of each wheel, fhall communicate motion to each other. A machine of this nature is reprefented in fig. 36. Here ABC is a winch, upon which is fixed a fmall wheel D indented with teeth, which move in the like teeth of a larger wheel EF fixed on the axis GH. Let this axis carry another wheel I, which fhall move in like manner a greater wheel KL fixed on the axis MN. Let this axis carry another fmall wheel O, which after the fame manner fhall turn about a larger wheel PQ fixed on the roller RS, on which a cord fhall be wound, that holds a weight, as T. Now the proportion required between the weight T and a power applied to the winch at A fufficient to fupport the weight, will moft eafily be eftimated, by computing the proportion, which the velocity of the point A would bear to the velocity of the weight. If the winch be turned round, the point A will defcribe a circle as AV. Suppofe the wheel EF to have ten times the number of teeth, as the

the wheel D; then the winch muſt turn round ten times to carry the wheel EF once round. If the wheel KL has alſo ten times the number of teeth, as I, the wheel I muſt turn round ten times to carry the wheel KL once round; and conſequently the winch ABC muſt turn round an hundred times to turn the wheel KL once round. Laſtly, if the wheel PQ has ten times the number of teeth, as the wheel O, the winch muſt turn about one thouſand times in order to turn the wheel PQ, or the roller RS once round. Therefore here the point A muſt have gone over the circle AV a thouſand times, in order to lift the weight T through a ſpace equal to the circumference of the roller RS: whence it follows, that the power applied at A will balance the weight T, if it bear the ſame proportion to it, as the circumference of the roller to one thouſand times the circle AV; or the ſame proportion as half the thickneſs of the roller bears to one thouſand times AB.

47. I SHALL now explain the effect of the pulley. Let a weight hang by a pulley, as in fig. 37. Here it is evident, that the power A, by which the weight B is ſupported, muſt be equal to the weight; for the cord CD is equally ſtrained between them; and if the weight B move, the power A muſt move with equal velocity. The pulley E has no other effect, than to permit the power A to act in another direction, than it muſt have done, if it had been directly applied to ſupport the weight without the intervention of any ſuch inſtrument.

48. AGAIN, let a weight be ſupported, as in fig. 38; where the weight A is fixed to the pulley B, and the cord, by which

which the weight is upheld, is annexed by one extremity to a hook C, and at the other end is held by the power D. Here the weight is ſupported by a cord doubled; inſomuch that although the cord were not ſtrong enough to hold the weight ſingle, yet being thus doubled it might ſupport it. If the end of the cord held by the power D were hung on the hook C, as well as the other end; then, when both ends of the cord were tied to the hook, it is evident, that the hook would bear the whole weight; and each end of the ſtring would bear againſt the hook with the force of half the weight only, ſeeing both ends together bear with the force of the whole. Hence it is evident, that, when the power D holds one end of the weight, the force, which it muſt exert to ſupport the weight, muſt be equal to juſt half the weight. And the ſame proportion between the weight and power might be collect-ed from comparing the reſpective velocities, with which they would move; for it is evident, that the power muſt move through a ſpace equal to twice the diſtance of the pulley from the hook, in order to lift the pulley up to the hook.

49. IT is equally eaſy to eſtimate the effect, when many pulleys are combined together, as in fig. 39, 40; in the firſt of which the under ſet of pulleys, and conſequently the weight is held by ſix ſtrings; and in the latter figure by five: therefore in the firſt of theſe figures the power to ſupport the weight, muſt be one ſixth part only of the weight, and in the latter figure the power muſt be one fifth part.

50. There are two other ways of ſupporting a weight by pulleys, which I ſhall particularly conſider.

51. One of theſe ways is repreſented in fig. 41. Here the weight being connected to the pulley B, a power equal to half the weight A would ſupport the pulley C, if applied immediately to it. Therefore the pulley C is drawn down with a force equal to half the weight A. But if the pulley D were to be immediately ſupported by half the force, with which the pulley C is drawn down, this pulley D will uphold the pulley C; ſo that if the pulley D be upheld with a force equal to one fourth part of the weight A, that force will ſupport the weight. But, for the ſame reaſon as before, if the power in E be equal to half the force neceſſary to uphold the pulley D; this pulley, and conſequently the weight A, will be upheld: therefore, if the power in E be one eighth part of the weight A, it will ſupport the weight.

52. Another way of applying pulleys to a weight is repreſented in fig. 42. To explain the effect of pulleys thus applied, it will be proper to conſider different weights hanging, as in fig. 43. Here, if the power and weights balance each other, the power A is equal to the weight B; the weight C is equal to twice the power A, or the weight B; and for the ſame reaſon the weight D is equal to twice the weight C, or equal to four times the power A. It is evident therefore, that all the three weights B, C, D together are equal to ſeven times the power A. But if theſe three weights were joined in one, they would produce the caſe of fig. 40: ſo that in that figure the weight

weight A, where there are three pulleys, is seven times the power B. If there had been but two pulleys, the weight would have been three times the power; and if there had ben four pulleys, the weight would have been fifteen times the power.

53. THE wedge is next to be confidered. The form of this inftrument is fufficiently known. When it is put under any weight (as in fig. 44.) the force, with which the wedge will lift the weight, when drove under it by a blow upon the end AB, will bear the fame proportion to the force, where-with the blow would act on the weight, if directly applied to it; as the velocity, which the wedge receives from the blow, bears to the velocity, wherewith the weight is lifted by the wedge.

54. THE fcrew is the fifth mechanical power. There are two ways of applying this inftrument. Sometimes it is fcrewed into a hole, as in fig. 45, where the fcrew AB is fcrewed through the plank CD. Sometimes the fcrew is applied to the teeth of a wheel, as in fig. 46, where the thred of the fcrew AB turns in the teeth of a wheel CD. In both thefe cafes, if a bar, as AE, be fixed to the end A of the fcrew; the force, wherewith the end B of the fcrew in fig. 45 is forced down, and the force, wherewith the teeth of the wheel CD in fig. 44 are held, bears the fame proportion to the power applied to the end E of the bar; as the velocity, wherewith the end E will move, when the fcrew is turned, bears to the velocity, wherewith the end B of the fcrew in fig. 43, or the teeth of the wheel CD in fig. 46, will be moved.

 55. THE

55. The inclined plane affords alſo a means of raiſing a weight with leſs force, than what is equal to the weight it ſelf. Suppoſe it were required to raiſe the globe A (in fig. 47.) from the ground BC up to the point, whoſe perpendicular height from the ground is ED. If this globe be drawn along the ſlant DF, leſs force will be required to raiſe it, than if it were lifted directly up. Here if the force applied to the globe bear the ſame proportion only to its weight, as ED bears to FD, it will be ſufficient to hold up the globe; and therefore any addition to that force will put it in motion, and draw it up; unleſs the globe, by preſſing againſt the plane, whereon it lies, adhere in ſome degree to the plane. This indeed it muſt always do more or leſs, ſince no plane can be made ſo abſolutely ſmooth as to have no inequalities at all; nor yet ſo infinitely hard, as not to yield in the leaſt to the preſſure of the weight. Therefore the globe cannot be laid on ſuch a plane, whereon it will ſlide with perfect freedom, but they muſt in ſome meaſure rub againſt each other; and this friction will make it neceſſary to imploy a certain degree of force more, than what is neceſſary to ſupport the globe, in order to give it any motion. But as all the mechanical powers are ſubject in ſome degree or other to the like impediment from friction; I ſhall here only ſhew what force would be neceſſary to ſuſtain the globe, if it could lie upon the plane without cauſing any friction at all. And I ſay, that if the globe were drawn by the cord GH, lying parallel to the plane DF; and the force, wherewith the cord is pulled, bear the ſame proportion to the weight of the globe, as ED bears to DF; this

this force will ſuſtain the globe. In order to the making proof of this, let the cord GH be continued on, and turned over the pulley I, and let the weight K be hung to it. Now I ſay, if this weight bears the ſame proportion to the globe A, as DE bears to DF, the weight will ſupport the globe. I think it is very manifeſt, that the center of the globe A will lie in one continued line with the cord HG. Let L be the center of the globe, and M the center of gravity of the weight K. In the firſt place let the weight hang ſo, that a line drawn from L to M ſhall lie horizontally; and I ſay, if the globe be moved either up or down the plane DF, the weight will ſo move along with it, that the center of gravity common to both the weights ſhall continue in this line LM, and therefore ſhall in no caſe deſcend. To prove this more fully, I ſhall depart a little from the method of this treatiſe, and make uſe of a mathematical propoſition or two: but they are ſuch, as any perſon, who has read EUCLID's ELEMENTS, will fully comprehend; and are in themſelves ſo evident, that, I believe, my readers, who are wholly ſtrangers to geometrical writings, will make no difficulty of admitting them. This being premiſed, let the globe be moved up, till its center be at G, then will M the center of gravity of the weight K be ſunk to N; ſo that MN ſhall be equal to GL. Draw NG croſſing the line ML in O; then I ſay, that O is the common center of gravity of the two weights in this their new ſituation. Let GP be drawn perpendicular to ML; then GL will bear the ſame proportion to GP, as DF bears to DE; and MN being equal to GL, MN will bear the ſame proportion

to

to GP, as DF bears to DE. But NO bears the ſame proportion to OG, as MN bears to GP; conſequently NO will bear the ſame proportion to OG, as DF bears to DE. In the laſt place, the weight of the globe A bears the ſame proportion to the other weight K, as DF bears to DE; therefore NO bears the ſame proportion to OG, as the weight of the globe A bears to the weight K. Whence it follows, that, when the center of the globe A is in G, and the center of gravity of the weight K is in N, O will be the center of gravity common to both the weights. After the ſame manner, if the globe had been cauſed to deſcend, the common center of gravity would have been found in this line ML. Since therefore no motion of the globe either way will make the common center of gravity deſcend, it is manifeſt, from what has been ſaid above, that the weights A and K counterpoize each other.

56. I shall now conſider the caſe of pendulums. A pendulum is made by hanging a weight to a line, ſo that it may ſwing backwards and forwards. This motion the geometers have very carefully conſidered, becauſe it is the moſt commodious inſtrument of any for the exact meaſurement of time.

57. I have obſerved already [a], that if a body hanging perpendicularly by a ſtring, as the body A (in fig. 48.) hangs by the ſtring AB, be put ſo into motion, as to be made to aſcend up the circular arch AC; then as ſoon as it has arrived

[a] § 25.

at

at the highest point, to which the motion, that the body has received, will carry it; it will immediately begin to descend, and at A will receive again as great a degree of motion, as it had at first. This motion therefore will carry the body up the arch AD, as high as it ascended before in the arch AC. Consequently in its return through the arch DA it will acquire again at A its original velocity, and advance a second time up the arch AC as high as at first; by this means continuing without end its reciprocal motion. It is true indeed, that in fact every pendulum, which we can put in motion, will gradually lessen its swing, and at length stop, unless there be some power constantly applied to it, whereby its motion shall be renewed; but this arises from the resistance, which the body meets with both from the air, and the string by which it is hung: for as the air will give some obstruction to the progress of the body moving through it; so also the string, whereon the body hangs, will be a farther impediment; for this string must either slide on the pin, whereon it hangs, or it must bend to the motion of the weight; in the first there must be some degree of friction, and in the latter the string will make some resistance to its inflection. However, if all resistance could be removed, the motion of a pendulum would be perpetual.

58. BUT to proceed, the first property, I shall take notice of in this motion, is, that the greater arch the pendulous body moves through, the greater time it takes up: though the length of time does not increase in so great a proportion as the arch. Thus if CD be a greater arch, and EF a lesser, where CA is equal to AD, and EA equal to AF; the body,

when

when it swings through the greater arch C D, shall take up in its swing from C to D a longer time than in swinging from E to F, when it moves only in that lesser arch; or the time in which the body let fall from C will descend through the arch C A is greater than the time, in which it will descend through the arch E A, when let fall from E. But the first of these times will not hold the same proportion to the latter, as the first arch C A bears to the other arch E A, which will appear thus. Let C G and E H be two horizontal lines. It has been remarked above [a], that the body in falling through the arch C A will acquire as great a velocity at the point A, as it would have gained by falling directly down through G A; and in falling through the arch E A it will acquire in the point A only that velocity, which it would have got in falling through H A. Therefore, when the body descends through the greater arch C A, it shall gain a greater velocity, than when it passes only through the lesser; so that this greater velocity will in some degree compensate the greater length of the arch.

59. The increase of velocity, which the body acquires in falling from a greater height, has such an effect, that, if straight lines be drawn from A to C and E, the body would fall through the longer straight line C A just in the same time, as through the shorter straight line E A. This is demonstrated by the geometers, who prove, that if any circle, as ABCD (fig. 49.) be placed in a perpendicular situation; a body shall fall obliquely through every line, as A B drawn from the lowest point A in the circle to any other point in the circum-

[a] § 20.

ference

ference just in the same time, as would be imployed by the body in falling perpendicularly down through the diameter CA. But the time in which the body will descend through the arch, is different from the time, which it would take up in falling through the line AB.

60. IT has been thought by some, that because in very small arches this correspondent straight line differs but little from the arch itself; therefore the descent through this straight line would be performed in such small arches nearly in the same time as through the arches themselves: so that if a pendulum were to swing in small arches, half the time of a single swing would be nearly equal to the time, in which a body would fall perpendicularly through twice the length of the pendulum. That is, the whole time of the swing, according to this opinion, will be four fold the time required for the body to fall through half the length of the pendulum; because the time of the body's falling down twice the length of the pendulum is half the time required for the fall through one quarter of this space, that is through half the pendulum's length. However there is here a mistake; for the whole time of the swing, when the pendulum moves through small arches, bears to the time required for a body to fall down through half the length of the pendulum very nearly the same proportion, as the circumference of a circle bears to its diameter; that is very nearly the proportion of 355 to 113, or little more than the proportion of 3 to 1. If the pendulum takes so great a swing, as to pass over an arch equal to one sixth part of the whole circumference of the

N circle,

circle, it will ſwing 115 times, while it ought according to this proportion to have ſwung 117 times; ſo that, when it ſwings in ſo large an arch, it loſes ſomething leſs than two ſwings in an hundred. If it ſwing through $\frac{1}{10}$ only of the circle, it ſhall not loſe above one vibration in 160. If it ſwing in $\frac{1}{20}$ of the circle, it ſhall loſe about one vibration in 690. If its ſwing be confined to $\frac{1}{40}$ of the whole circle, it ſhall loſe very little more than one ſwing in 2600. And if it take no greater a ſwing than through $\frac{1}{60}$ of the whole circle, it ſhall not loſe one ſwing in 5800.

61. Now it follows from hence, that, when pendulums ſwing in ſmall arches, there is very nearly a conſtant proportion obſerved between the time of their ſwing, and the time, in which a body would fall perpendicularly down through half their length. And we have declared above, that the ſpaces, through which bodies fall, are in a two fold proportion of the times, which they take up in falling [a]. Therefore in pendulums of different lengths, ſwinging throug hſmall arches, the lengths of the pendulums are in a two fold or duplicate proportion of the times, they take in ſwinging; ſo that a pendulum of four times the length of another ſhall take up twice the time in each ſwing, one of nine times the length will make one ſwing only for three ſwings of the ſhorter, and ſo on.

62. This proportion in the ſwings of different pendulums not only holds in ſmall arches; but in large ones alſo,

[a] § 17.

provided

provided they be ſuch, as the geometers call ſimilar; that is, if the arches bear the ſame proportion to the whole circumferences of their reſpective circles. Suppoſe (in fig. 48.) A B, C D to be two pendulums. Let the arch E F be deſcribed by the motion of the pendulum A B, and the arch G H be deſcribed by the pendulum C D; and let the arch E F bear the ſame proportion to the whole circumference, which would be formed by turning the pendulum A B quite round about the point A, as the arch G H bears to the whole circumference, that would be formed by turning the pendulum C D quite round the point C. Then I ſay, the proportion, which the length of the pendulum A B bears to the length of the pendulum C D, will be two fold of the proportion, which the time taken up in the deſcription of the arch E F bears to the time employed in the deſcription of the arch G H.

63. THUS pendulums, which ſwing in very ſmall arches, are nearly an equal meaſure of time. But as they are not ſuch an equal meaſure to geometrical exactneſs; the mathematicians have found out a method of cauſing a pendulum ſo to ſwing, that, if its motion were not obſtructed by any reſiſtance, it would always perform each ſwing in the ſame time, whether it moved through a greater, or a leſſer ſpace. This was firſt diſcovered by the great HUYGENS, and is as follows. Upon the ſtraight line A B (in fig. 49.) let the circle C D E be ſo placed, as to touch the ſtraight line in the point C. Then let this circle roll along upon the ſtraight line A B, as a coach-wheel rolls along upon the ground. It is evident, that, as

 ſoon

ſoon as ever the circle begins to move, the point C in the circle will be lifted off from the ſtraight line AB; and in the motion of the circle will deſcribe a crooked courſe, which is repreſented by the line CFGH. Here the part CH of the ſtraight line included between the two extremities C and H of the line CFGH will be equal to the whole circumference of the circle CDE; and if CH be divided into two equal parts at the point I, and the ſtraight line IK be drawn perpendicular to CH, this line IK will be equal to the diameter of the circle CDE. Now in this line if a body were to be let fall from the point H, and were to be carried by its weight down the line HGK, as far as the point K, which is the loweſt point of the line CFGH; and if from any other point G a body were to be let fall in the ſame manner; this body, which falls from G, will take juſt the ſame time in coming to K, as the body takes up, which falls from H. Therefore if a pendulum can be ſo hung, that the ball ſhall move in the line ~~AGFE~~, all its ſwings, whether long or ſhort, will be performed in the ſame time; for the time, in which the ball will deſcend to the point K, is always half the time of the whole ſwing. But the ball of a pendulum will be made to ſwing in this line by the following means. Let KI (in fig. 52.) be prolonged upwards to L, till IL is equal to IK. Then let the line LMH equal and like to KH be applied, as in the figure between the points L and H, ſo that the point which in this line LMH anſwers to the point H in the line KH ſhall be applied to the point L, and the point anſwering to the point K ſhall be applied to the point H. Alſo let ſuch another line LNC be applied between L and C in the ſame manner.

manner. This preparation being made; if a pendulum be hung at the point L of ſuch a length, that the ball thereof ſhall reach to K; and if the ſtring ſhall continually bend againſt the lines HML and LNC, as the pendulum ſwings to and fro; by this means the ball ſhall conſtantly keep in the line CKH.

64. NOW in this pendulum, as all the ſwings, whether long or ſhort, will be performed in the ſame time; ſo the time of each will exactly bear the ſame proportion to the time required for a body to fall perpendicularly down, through half the length of the pendulum, that is from I to K, as the circumference of a circle bears to its diameter.

65. IT may from hence be underſtood in ſome meaſure, why, when pendulums ſwing in circular arches, the times of their ſwings are nearly equal, if the arches are ſmall, though thoſe arches be of very unequal lengths; for if with the ſemidiameter LK the circular arch OKP be deſcribed, this arch in the lower part of it will differ very little from the line CKH.

66. IT may not be amiſs here to remark, that a body will fall in this line CKH (fig. 53.) from C to any other point, as Q or R in a ſhorter ſpace of time, than if it moved through the ſtraight line drawn from C to the other point; or through any other line whatever, that can be drawn between theſe two points.

67. BUT as I have obſerved, that the time, which a pendulum takes in ſwinging, depends upon its length; I ſhall now ſay ſomething concerning the way, in which this length of the pendulum is to be eſtimated. If the whole ball of the pendulum could be crouded into one point, this length, by which the motion of the pendulum is to be computed, would be the length of the ſtring or rod. But the ball of the pendulum muſt have a ſenſible magnitude, and the ſeveral parts of this ball will not move with the ſame degree of ſwiftneſs; for thoſe parts, which are fartheſt from the point, whereon the pendulum is ſuſpended, muſt move with the greateſt velocity. Therefore to know the time in which the pendulum ſwings, it is neceſſary to find that point of the ball, which moves with the ſame degree of velocity, as if the whole ball were to be contracted into that point.

68. THIS point is not the center of gravity, as I ſhall now endeavour to ſhew. Suppoſe the pendulum AB (in fig. 54.) compoſed of an inflexible rod AC and ball CB, to be fixed on the point A, and lifted up into an horizontal ſituation. Here if the rod were not fixed to the point A, the body CB would deſcend directly with the whole force of its weight; and each part of the body would move down with the ſame degree of ſwiftneſs. But when the rod is fixed at the point A, the body muſt fall after another manner; for the parts of the body muſt move with different degrees of velocity, the parts more remote from A deſcending with a ſwifter motion, than the parts nearer to A; ſo that the body will receive a kind of rolling motion while it deſcends. But it has been

been obſerved above, that the effect of gravity upon any body is the ſame, as if the whole force were exerted on the body's center of gravity [a]. Since therefore the power of gravity in drawing down the body muſt alſo communicate to it the rolling motion juſt deſcribed; it ſeems evident, that the center of gravity of the body cannot be drawn down as ſwiftly, as when the power of gravity has no other effect to produce on the body, than merely to draw it downward. If therefore the whole matter of the body C B could be crouded into its center of gravity, ſo that being united into one point, this rolling motion here mentioned might give no hindrance to its deſcent; this center would deſcend faſter, than it can now do. And the point, which now deſcends as faſt, as if the whole matter of the body C B were crouded into it, will be farther removed from the point A, than the center of gravity of the body C B.

69. AGAIN, ſuppoſe the pendulum A B (in fig. 55.) to hang obliquely. Here the power of gravity will operate leſs upon the ball of the pendulum, than before: but the line D E being drawn ſo, as to ſtand perpendicular to the rod A C of the pendulum; the force of gravity upon the body C B, now it is in this ſituation, will produce the ſame effect, as if the body were to glide down an inclined plane in the poſition of D E. But here the motion of the body, when the rod is fixed to the point A, will not be equal to the uninterrupted deſcent of the body down this plane; for the body

[a] § 27.

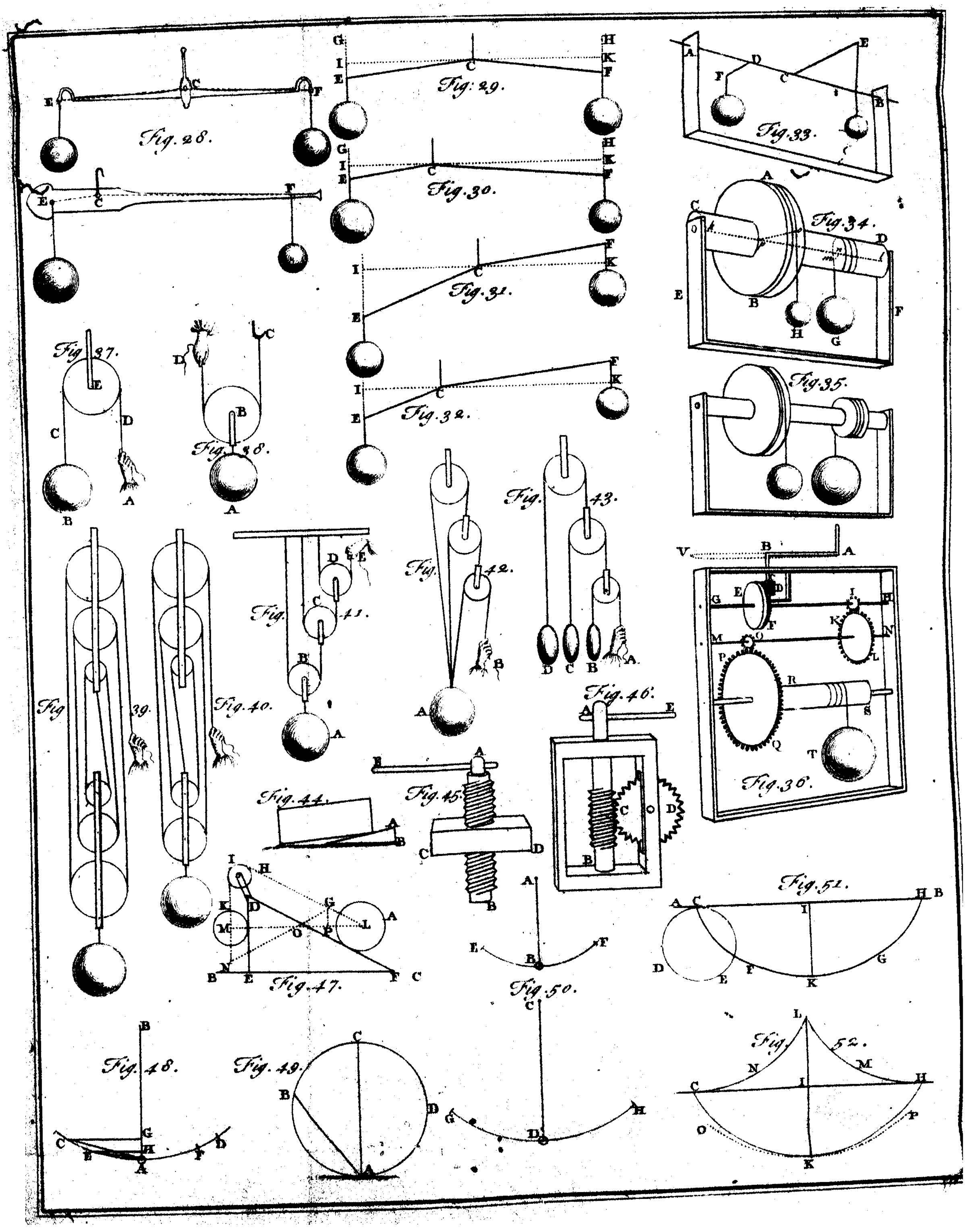
Fig. 28.
Fig. 29.
Fig. 30.
Fig. 31.
Fig. 32.
Fig. 33.
Fig. 34.
Fig. 35.
Fig. 36.
Fig. 37.
Fig. 38.
Fig. 39.
Fig. 40.
Fig. 41.
Fig. 42.
Fig. 43.
Fig. 44.
Fig. 45.
Fig. 46.
Fig. 47.
Fig. 48.
Fig. 49.
Fig. 50.
Fig. 51.
Fig. 52.

will here alſo receive the ſame kind of rotation in its motion, as before; ſo that the motion of the center of gravity will in like manner be retarded; and the point, which here deſcends with that degree of ſwiftneſs, which the body would have, if not hindered by being fixed to the point A; that is, the point, which deſcends as faſt, as if the whole body were crouded into it, will be as far removed from the point A, as before.

70. THIS point, by which the length of the pendulum is to be eſtimated, is called the center of oſcillation. And the mathematicians have laid down general directions, whereby to find this center in all bodies. If the globe AB (in fig. 56.) be hung by the ſtring CD, whoſe weight need not be regarded, the center of oſcillation is found thus. Let the ſtraight line drawn from C to D be continued through the globe to F. That it will paſs through the center of the globe is evident. Suppoſe E to be this center of the globe; and take the line G of ſuch a length, that it ſhall bear the ſame proportion to ED, as ED bears to EC. Then EH being made equal to $\frac{2}{5}$ of G, the point H ſhall be the center of oſcillation [a]. If the weight of the rod CD is too conſiderable to be neglected, divide CD (fig. 57) in I, that DI be equal to $\frac{1}{3}$ part of CD; and take K in the ſame proportion to CI, as the weight of the globe AB to the weight of the rod CD. Then having found H, the center of oſcillation of the globe, as before, divide IK in L, ſo that IL ſhall bear the ſame pro-

[a] Hugen. Horolog. oſcillat. pag. 141, 142.

portion

portion to LH, as the line CH bears to K; and L ſhall be the center of oſcillation of the whole pendulum.

71. THIS computation is made upon ſuppoſition, that the center of oſcillation of the rod CD, if that were to ſwing alone without any other weight annexed, would be the point I. And this point would be the true center of oſcillation, ſo far as the thickneſs of the rod is not to be regarded. If any one chuſes to take into conſideration the thickneſs of the rod, he muſt place the center of oſcillation thereof ſo much below the point I, that eight times the diſtance of the center from the point I ſhall bear the ſame proportion to the thickneſs of the rod, as the thickneſs of the rod bears to its length CD[a].

72. IT has been obſerved above, that when a pendulum ſwings in an arch of a circle, as here in fig. 58, the pendulum AB ſwings in the circular arch CD; if you draw an horizontal line, as EF, from the place whence the pendulum is let fall, to the line AG, which is perpendicular to the horizon: then the velocity, which the pendulum will acquire in coming to the point G, will be the ſame, as any body would acquire in falling directly down from F to G. Now this is to be underſtood of the circular arch, which is deſcribed by the center of oſcillation of the pendulum. I ſhall here farther obſerve, that if the ſtraight line EG be drawn from the point, whence the pendulum falls, to the loweſt point of the arch; in the ſame or in equal pendulums the velocity, which the

[a] See Hugen. Horolog. Oſcillat. p. 142.

pendulum acquires in G, is proportional to this line: that is, if the pendulum, after it has descended from E to G, be taken back to H, and let fall from thence, and the line HG be drawn; the velocity, which the pendulum shall acquire in G by its descent from H, shall bear the same proportion to the velocity, which it acquires in falling from E to G, as the straight line HG bears to the straight line EG.

73. We may now proceed to those experiments upon the percussion of bodies, which I observed above might be made with pendulums. This expedient for examining the effects of percussion was first proposed by our late great architect Sir Christopher Wren. And it is as follows. Two balls, as A and B (in fig. 59.) either equal or unequal, are hung by two strings from two points C and D, so that, when the balls hang down without motion, they shall just touch each other, and the strings be parallel. Here if one of these balls be removed to any distance from its perpendicular situation, and then let fall to descend and strike against the other; by the last preceding paragraph it will be known, with what velocity this ball shall return into its first perpendicular situation, and consequently with what force it shall strike against the other ball; and by the height to which this other ball ascends after the stroke, the velocity communicated to this ball will be discovered. For instance, let the ball A be taken up to E, and from thence be let fall to strike against B, passing over in its descent the circular arch EF. By this impulse let B fly up to G, moving through the circular arch HG. Then EI and GK being drawn horizontally, the

the ball A will ſtrike againſt B with the velocity, which it would acquire in falling directly down from I; and the ball B has received a velocity, wherewith, if it had been thrown directly upward, it would have aſcended up to K. Likewiſe if ſtraight lines be drawn from E to F and from H to G, the velocity of A, wherewith it ſtrikes, will bear the ſame proportion to the velocity, which B has received by the blow, as the ſtraight line EF bears to the ſtraight line HG. In the ſame manner by noting the place to which A aſcends after the ſtroke, its remaining velocity may be compared with that, wherewith it ſtruck againſt B. Thus may be experimented the effects of the body A ſtriking againſt B at reſt. If both the bodies are lifted up, and ſo let fall as to meet and impinge againſt each other juſt upon the coming of both into their perpendicular ſituation; by obſerving the places into which they move after the ſtroke, the effects of their percuſſion in all theſe caſes may be found in the ſame manner as before.

74. Sir ISAAC NEWTON has deſcribed theſe experiments; and has ſhewn how to improve them to a greater exactneſs by making allowance for the reſiſtance, which the air gives to the motion of the balls [a]. But as this reſiſtance is exceeding ſmall, and the manner of allowing for it is delivered by himſelf in very plain terms, I need not enlarge upon it here. I ſhall rather ſpeak to a diſcovery, which he made by theſe experiments upon the elaſticity of bodies. It has been explained above [b], that when two bodies ſtrike, if they be not elaſtic,

[a] Princip. Philoſ. pag. 22. [b] Chap. 1. § 29.

they remain contiguous after the ſtroke; but that if they are elaſtic, they ſeparate, and that the degree of their elaſticity determines the proportion between the celerity wherewith they ſeparate, and the celerity wherewith they meet. Now our author found, that the degree of elaſticity appeared in the ſame bodies always the ſame, with whatever degree of force they ſtruck; that is, the celerity wherewith they ſeparated, always bore the ſame proportion to the celerity wherewith they met: ſo that the elaſtic power in all the bodies, he made trial upon, exerted it ſelf in one conſtant proportion to the compreſſing force. Our author made trial with balls of wool bound up very compact, and found the celerity with which they receded, to bear about the proportion of 5 to 9 to the celerity wherewith they met; and in ſteel he found nearly the ſame proportion; in cork the elaſticity was ſomething leſs; but in glaſs much greater; for the celerity, wherewith balls of that material ſeparated after percuſſion, he found to bear the proportion of 15 to 16 to the celerity wherewith they met [a]

75. I SHALL finiſh my diſcourſe on pendulums, with this farther obſervation only, that the center of oſcillation is alſo the center of another force. If a body be fixed to any point, and being put in motion turns round it; the body, if uninterrupted by the power of gravity or any other means, will continue perpetually to move about with the ſame equable motion. Now the force, with which ſuch a body

[a] Princip. Philoſ. pag. 25.

moves, is all united in the point, which in relation to the power of gravity is called the center of oscillation. Let the cylinder ABCD (in fig. 60.) whose axis is EF, be fixed to the point E. And supposing the point E to be that on which the cylinder is suspended, let the center of oscillation be found in the axis EF, as has been explained above [a]. Let G be that center: then I say, that the force, wherewith this cylinder turns round the point E, is so united in the point G, that a sufficient force applied in that point shall stop the motion of the cylinder, in such a manner, that the cylinder should immediately remain without motion, though it were to be loosened from the point E at the same instant, that the impediment was applied to G: whereas, if this impediment had been applied to any other point of the axis, the cylinder would turn upon the point, where the impediment was applied. If the impediment had been applied between E and G, the cylinder would so turn on the point, where the impediment was applied, that the end BC would continue to move on the same way it moved before along with the whole cylinder; but if the impediment were applied to the axis farther off from E than G, the end AD of the cylinder would start out of its present place that way in which the cylinder moved. From this property of the center of oscillation, it is also called the center of percussion. That excellent mathematician, Dr. BROOK TAYLOR, has farther improved this doctrine concerning the center of percussion, by shewing, that if through this point G a line, as GHI, be drawn perpendicular to EF, and lying

[a] § 71.

in

in the courſe of the body's motion; a ſufficient power applied to any point of this line will have the ſame effect, as the like power applied to G [a]: ſo that as we before ſhewed the center of percuſſion within the body on its axis; by this means we may find this center on the ſurface of the body alſo, for it will be where this line HI croſſes that ſurface.

76. I SHALL now proceed to the laſt kind of motion, to be treated on in this place, and ſhew what line the power of gravity will cauſe a body to deſcribe, when it is thrown forwards by any force. This was firſt diſcovered by the great GALILEO, and is the principle, upon which engineers ſhould direct the ſhot of great guns. But as in this caſe bodies deſcribe in their motion one of thoſe lines, which in geometry are called conic ſections; it is neceſſary here to premiſe a deſcription of thoſe lines. In which I ſhall be the more particular, becauſe the knowledge of them is not only neceſſary for the preſent purpoſe, but will be alſo required hereafter in ſome of the principal parts of this treatiſe.

77. THE firſt lines conſidered by the ancient geometers were the ſtraight line and the circle. Of theſe they compoſed various figures, of which they demonſtrated many properties, and reſolved divers problems concerning them. Theſe problems they attempted always to reſolve by the deſcribing ſtraight lines and circles. For inſtance, let a ſquare ABCD (fig. 61.) be propoſed, and let it be required to make ano-

[a] See Method. Increment. prop. 25.

ther

ther ſquare in any aſſigned proportion to this. Prolong one ſide, as D A, of this ſquare to E, till A E bear the ſame proportion to A D, as the new ſquare is to bear to the ſquare A C. If the oppoſite ſide B C of the ſquare A C be alſo prolonged to F, till B F be equal to A E, and E F be afterwards drawn, I ſuppoſe my readers will eaſily conceive, that the figure ABFE will bear to the ſquare A B C D the ſame proportion, as the line A E bears to the line A D. Therefore the figure A B F E will be equal to the new ſquare, which is to be found, but is not it ſelf a ſquare, becauſe the ſide A E is not of the ſame length with the ſide E F. But to find a ſquare equal to the figure A B F E you muſt proceed thus. Divide the line D E into two equal parts in the point G, and to the center G with the interval G D deſcribe the circle D H E I; then prolong the line A B, till it meets the circle in K; and make the ſquare AKLM, which ſquare will be equal to the figure ABFE, and bear to the ſquare A B C D the ſame proportion, as the line A E bears to A D.

78. I SHALL not proceed to the proof of this, having only here ſet it down as a ſpecimen of the method of reſolving geometrical problems by the deſcription of ſtraight lines and circles. But there are ſome problems, which cannot be reſolved by drawing ſtraight lines or circles upon a plane. For the management therefore of theſe they took into conſideration ſolid figures, and of the ſolid figures they found that, which is called a cone, to be the moſt uſeful.

79. A

79. A cone is thus defined by Euclide in his elements of geometry [a]. If to the ſtraight line A B (in fig. 62.) another ſtraight line, as A C, be drawn perpendicular, and the two extremities, B and C be joined by a third ſtraight line compoſing the triangle A C B (for ſo every figure is called, which is included under three ſtraight lines:) then the two points A and B being held fixed, as two centers, and the triangle A C B being turned round upon the line A B, as on an axis; the line A C will deſcribe a circle, and the figure A C B will deſcribe a cone, of the form repreſented by the figure BCDEF (fig. 63.) in which the circle CDEF is uſually called the baſe of the cone, and B the vertex.

80. Now by this figure may ſeveral problems be reſolved, which cannot by the ſimple deſcription of ſtraight lines and circles upon a plane. Suppoſe for inſtance, it were required to make a cube, which ſhould bear any aſſigned proportion to ſome other cube named. I need not here inform my readers, that a cube is the figure of a dye. This problem was much celebrated among the ancients, and was once inforced by the command of an oracle. This problem may be performed by a cone thus. Firſt make a cone from a triangle, whoſe ſide A C ſhall be half the length of the ſide B C. Then on the plane A B C D (fig. 64.) let the line E F be exhibited equal in length to the ſide of the cube propoſed; and let the line F G be drawn perpendicular to E F, and of ſuch a length, that it bear the ſame proportion to E F, as the

[a] Lib. XI. Def.

cube

[illegible] to be ſought is required to bear to the cube propoſed. Through the points E, F, and G let the circle F H I be deſcribed. Then let the line E F be prolonged beyond F to K, that F K be equal to F E, and let the triangle F K L, having all its ſides F K, K L, L F equal to each other, be hung down perpendicularly from the plane A B C D. After this, let another plane M N O P be extended through the point L, ſo as to be equidiſtant from the former plane A B C D, and in this plane let the line Q L R be drawn ſo, as to be equidiſtant from the line E F K. All this being thus prepared, let ſuch a cone, as was above directed to be made, be ſo applied to the plane M N O P, that it touch this plane upon the line Q R, and that the vertex of the cone be applied to the point L. This cone, by cutting through the firſt plane A B C D, will croſs the circle F H I before deſcribed. And if from the point S, where the ſurface of this cone interſects the circle, the line S T be drawn ſo, as to be equidiſtant from the line E F; the line F T will be equal to the ſide of the cube ſought: that is, if there be two cubes or dyes formed, the ſide of one being equal to E F, and the ſide of the other equal to F T; the former of theſe cubes ſhall bear the ſame proportion to the latter, as the line E F bears to F G.

81. INDEED this placing a cone to cut through a plane is not a practicable method of reſolving problems. But when the geometers had diſcovered this uſe of the cone, they applied themſelves to conſider the nature of the lines, which will be produced by the interſection of the ſurface of a cone

and a plane; whereby they might be enabled both to reduce these kinds of solutions to practice, and also to render their demonstrations concise and elegant.

82. Whenever the plane, which cuts the cone, is equidistant from another plane, that touches the cone on the side; (which is the case of the present figure;) the line, wherein the plane cuts the surface of the cone, is called a parabola. But if the plane, which cuts the cone, be so inclined to this other, that it will pass quite through the cone (as in fig. 65.) such a plane by cutting the cone produces the figure called an ellipsis, in which we shall hereafter shew the earth and other planets to move round the sun. If the plane, which cuts the cone, recline the other way (as in fig. 66.) so as not to be parallel to any plane, whereon the cone can lie, nor yet to cut quite through the cone; such a plane shall produce in the cone a third kind of line, which is called an hyperbola. But it is the first of these lines named the parabola, wherein bodies, that are thrown obliquely, will be carried by the force of gravity; as I shall here proceed to shew, after having first directed my readers how to describe this sort of line upon a plane, by which the form of it may be seen.

83. To any straight line AB (fig. 67.) let a straight ruler CD be so applied, as to stand against it perpendicularly. Upon the edge of this ruler let another ruler EF be so placed, as to move along upon the edge of the first ruler CD, and keep always perpendicular to it. This being so disposed, let any point, as G, be taken in the line AB, and let a string equal