ELEMENTS

OF THE

HY OF THE HUMAN MIND.

BY

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ADVERTISEMENT.

After an interval of more than twenty years, I venture to present to the public a Second Volume on the Philosophy of the Human Mind.

When the preceding Part was sent to the press, I expected that a few short chapters would comprehend all that I had further to offer concerning the Intellectual Powers; and that I should be able to employ the greater part of this Volume in examining those principles of our constitution, which are immediately connected with the Theory of Morals. On proceeding, however, to attempt an analysis of Reason, in the more

strict acceptation of that term, I found so many doubts crowding on me with respect to the logical doctrines then generally received, that I was forced to abandon the comparatively limited plan according to which I had originally intended to treat of the Understanding, and, in the mean time, to suspend the continuation of my work, till a more unbroken leisure should allow me to resume it with a less divided attention.

Of the accidents which have since occurred to retard my progress, it is unnecessary to take any notice here. I allude to them, merely as an apology for those defects of method, which are the natural, and perhaps the unavoidable consequences of the frequent interruptions by which the train of my thoughts has been diverted to other pursuits. Such of my readers as are able to judge how very large a proportion of my materials has been the fruit of my own meditations; and who are aware of the fugitive nature of our reasonings concerning phenomena so far removed from the perceptions of Sense, will easily conceive the difficulty I must occasionally have experienced, in decyphering the

short and slight hints on these topics, which I had committed to writing at remote periods of my life; and still more, in recovering the thread which had at first connected them together in the order of my researches.

I have repeatedly had occasion to regret the tendency of this intermitted and irregular mode of composition, to deprive my speculations of those advantages, in point of continuity, which, to the utmost of my power, I have endeavoured to give them. But I would willingly indulge the hope, that this is a blemish more likely to meet the eye of the author than of the reader; and I am confident, that the critic who shall honour me with a sufficient degree of attention, to detect it where it may occur, will not be inclined to treat it with an undue severity.

A Third Volume (of which the chief materials are already prepared) will comprehend all that I mean to publish under the title of the Philosophy of the Human Mind. The principal subjects allotted for it are Language; Imitation; the Varieties of Intel-

lectual Character; and the Faculties by which Man is distinguished from the lower animals. The two first of these articles belong, in strict propriety, to this second part of my work; but the size of the volume has prevented me from entering on the consideration of them at present.

The circumstances which have so long delayed the publication of these volumes on the Intellectual Powers, have not operated, in an equal degree, to prevent the prosecution of my inquiries into those principles of Human Nature, to which my attention was, for many years, statedly and forcibly called by my official duty. Much, indeed, still remains to be done in maturing, digesting, and arranging many of the doctrines which I was accustomed to introduce into my lectures; but if I shall be blessed, for a few years longer, with a moderate share of health and of mental vigour, I do not altogether despair of yet contributing something, *in the form of Essays*, to fill up the outline which the sanguine imagination of youth encouraged me to conceive, before I had duly measured the magnitude of my undertaking with the time or

with the abilities which I could devote to the execution.

The volume which I now publish is more particularly intended for the use of Academical Students; and is offered to them as a guide or assistant, at that important stage of their progress when, the usual course of discipline being completed, an inquisitive mind is naturally led to review its past attainments, and to form plans for its future improvement. In the prosecution of this design, I have not aimed at the establishment of new theories; far less have I aspired to the invention of any new *organ* for the discovery of truth. My principal object is to aid my readers in unlearning the scholastic errors which, in a greater or less degree, still maintain their ground in our most celebrated seats of learning; and by subjecting to *free*, but I trust, not *sceptical* discussion, the more enlightened though discordant systems of modern Logicians, to accustom the understanding to the unfettered exercise of its native capacities. That several of the views opened in the following pages appear to myself original, and of some importance, I

will not deny; but the reception these may meet with, I shall regard as a matter of comparative indifference, if my labours be found useful in training the mind to those habits of reflection on its own operations, which may enable it to superadd to the instructions of the schools, that higher education which no schools can bestow.

Kinneil-House, 22d November 1813.

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PRELIMINARY OBSERVATIONS ON THE VAGUENESS AND AMBIGUITY OF THE COMMON PHILOSOPHICAL LANGUAGE RELATIVE TO THIS PART OF OUR CONSTITUTION.—REASON AND REASONING,—UNDERSTANDING,—INTELLECT,—JUDGMENT, &c.

THE power of Reason, of which I am now to treat, is unquestionably the most important by far, of those which are comprehended under the general title of Intellectual. It is on the right use of this power, that our success in the pursuit both of knowledge and of happiness depends; and it is by the exclusive possession of it that Man is distinguished, in the most essential respects, from the lower animals. It is, indeed, from

their subserviency to its operations, that the other faculties, which have been hitherto under our consideration, derive their chief value.

In proportion to the peculiar importance of this subject are its extent and its difficulty;—both of them such as to lay me under a necessity, now that I am to enter on the discussion, to contract, in various instances, those designs in which I was accustomed to indulge myself, when I looked forward to it from a distance. The execution of them at present, even if I were more competent to the task, appears to me, on a closer examination, to be altogether incompatible with the comprehensiveness of the general plan which was sketched out in the advertisement prefixed to the former volume; and to the accomplishment of which I am anxious, in the first instance, to direct my efforts. If that undertaking should ever be completed, I may perhaps be able afterwards to offer additional illustrations of certain articles, which the limits of this part of my work prevent me from considering with the attention which they deserve. I should wish, in particular, to contribute something more than I can here introduce, towards a rational and practical system of Logic, adapted to the present state of human knowledge, and to the real business of human life.

"What subject (says Burke) does not branch out to infinity! "It is the nature of our particular scheme, and the single point "of view in which we consider it, which ought to put a stop "to our researches*." How forcibly does the remark apply to

* Conclusion of the Inquiry into the Sublime and the Beautiful.

all those speculations which relate to the principles of the Human Mind!

I have frequently had occasion, in the course of the foregoing disquisitions, to regret the obscurity in which this department of philosophy is involved, by the vagueness and ambiguity of words; and I have mentioned, at the same time, my unwillingness to attempt verbal innovations, wherever I could possibly avoid them, without essential injury to my argument. The rule which I have adopted in my own practice is, to give to every faculty and operation of the mind its own appropriate name; following, in the selection of this name, the *prevalent* use of our best writers; and endeavouring afterwards, as far as I have been able, to employ each word *exclusively*, in that acceptation in which it has hitherto been used *most generally*. In the judgments which I have formed on points of this sort, it is more than probable that I may sometimes have been mistaken; but the mistake is of little consequence, if I myself have invariably annexed the same meaning to the same phrase; —an accuracy which I am not so presumptuous as to imagine that I have uniformly attained, but which I am conscious of having, at least, uniformly attempted. How far I have succeeded, they alone who have followed my reasonings with a very critical attention are qualified to determine; for it is not by the statement of formal definitions, but by the habitual use of precise and appropriate language, that I have endeavoured to fix in my reader's mind the exact import of my expressions.

In appropriating, however, particular words to particular

ideas, I do not mean to censure the practice of those who may have understood them in a sense different from that which I annex to them; but I found that, without such an appropriation, I could not explain my notions respecting the human mind, with any tolerable degree of distinctness. This scrupulous appropriation of terms, if it can be called an *innovation*, is the only one which I have attempted to introduce; for in no instance have I presumed to annex a philosophical meaning to a technical word belonging to this branch of science, without having previously shewn, that it has been used in the same sense by good writers, in some passages of their works. After doing this, I hope I shall not be accused of affectation, when I decline to use it in any of the other acceptations in which, from carelessness or from want of precision, they may have been led occasionally to employ it.

Some remarkable instances of vagueness and ambiguity in the employment of words, occur in that branch of my subject of which I am now to treat. The word *Reason* itself is far from being precise in its meaning. In common and popular discourse, it denotes that power by which we distinguish truth from falsehood, and right from wrong; and by which we are enabled to combine means for the attainment of particular ends. Whether these different capacities are, with strict logical propriety, referred to the same power, is a question which I shall examine in another part of my work; but that they are all included in the idea which is generally annexed to the word *reason*, there can be no doubt; and the case, so far as I know, is the same with the corresponding term in all languages whatever. The fact probably is, that this word was first employed

to comprehend the principles, whatever they are, by which man is distinguished from the brutes; and afterwards came to be somewhat limited in its meaning, by the more obvious conclusions concerning the nature of that distinction, which present themselves to the common sense of mankind. It is in this enlarged meaning that it is opposed to *instinct* by Pope:

> "And Reason raise o'er Instinct as you can;
> "In this 'tis God directs, in that 'tis Man."

It was thus, too, that Milton plainly understood the term, when he remarked, that smiles imply the exercise of reason;

> "——— Smiles from Reason flow,
> "To brutes denied:"———

And still more explicitly in these noble lines:

> "There wanted yet the master-work, the end
> "Of all yet done; a creature who, not prone
> "And brute as other creatures, but endued
> "With sanctity of REASON, might erect
> "His stature, and upright with front serene
> "Govern the rest, self-knowing; and from thence,
> "Magnanimous, to correspond with heaven;
> "But, grateful to acknowledge whence his good
> "Descends, thither with heart, and voice, and eyes
> "Directed in devotion, to adore
> "And worship God Supreme, who made him chief
> "Of all his works."

Among the various characteristics of humanity, the power of devising means to accomplish ends, together with the power of distinguishing truth from falsehood, and right from wrong,

are obviously the most conspicuous and important; and accordingly it is to these that the word *reason*, even in its most comprehensive acceptation, is now exclusively restricted *.

By some philosophers, the meaning of the word has been, of late, restricted still farther; to the power by which we distin-

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* This, I think, is the meaning which most naturally presents itself to common readers, when the word *reason* occurs in authors not affecting to aim at any nice logical distinctions; and it is certainly the meaning which must be annexed to it, in some of the most serious and important arguments in which it has ever been employed. In the following passage, for example, where Mr Locke contrasts the light of Reason with that of Revelation, he plainly proceeds on the supposition, that it is competent to appeal to the former, as affording a standard of right and wrong, not less than of speculative truth and falsehood; nor can there be a doubt that, when he speaks of *truth* as the object of natural reason, it was principally, if not wholly, *moral* truth, which he had in his view: "Reason is natural revelation, whereby the eternal Father of Light, and fountain of all "knowledge, communicates to mankind that portion of truth which he has laid within "the reach of their natural faculties. Revelation is natural reason, enlarged by a new "set of discoveries, communicated by God immediately, which reason vouches the "truth of, by the testimony and proofs it gives that they come from God. So that "he who takes away reason to make way for revelation, puts out the light of both, "and does much the same, as if he would persuade a man to put out his eyes, the "better to receive the remote light of an invisible star by a telescope."—*Locke's Essay*, B. iv. c. 19.

A passage still more explicit for my present purpose occurs in the pleasing and philosophical conjectures of Huyghens, concerning the planetary worlds. "Positis vero "ejusmodi planetarum incolis ratione utentibus, quæri adhuc potest, anne idem illic, "atque apud nos, sit hoc quod *rationem* vocamus. Quod quidem ita esse omnino di- "cendum videtur, neque aliter fieri posse: sive usum rationis in his consideremus quæ "ad mores et æquitatem pertinent, sive in iis quæ spectant ad principia et fundamenta "scientiarum. Etenim ratio apud nos est, quæ sensum justitiæ, honesti, laudis, cle- "mentiæ, gratitudinis ingenerat, mala ac bona in universum discernere docet: quæque "ad hæc animum disciplinæ, multorumque inventorum capacem reddit," &c: &c.—*Hugenii Opera Varia*, Vol. II. p. 663. Lugd. Batav. 1724.

guish truth from falsehood, and combine means for the accomplishment of our purposes;—the capacity of distinguishing right from wrong, being referred to a separate principle or faculty, to which different names have been assigned in different ethical theories. The following passage from Mr Hume contains one of the most explicit statements of this limitation which I can recollect: "Thus, the distinct boundaries and offices of reason "and of taste are easily ascertained. The former conveys the "knowledge of truth and falsehood; the latter gives the senti- "ment of beauty and deformity,—vice and virtue. Reason, "being cool and disengaged, is no motive to action, and di- "rects only the impulse received from appetite or inclination, "by shewing us the means of attaining happiness or avoiding "misery. Taste, as it gives pleasure or pain, and thereby con- "stitutes happiness or misery, becomes a motive to action, and "is the first spring or impulse to desire and volition *."

On the justness of this statement of Mr Hume, I have no remarks to offer here; as my sole object in quoting it was to illustrate the different meanings annexed to the word *reason* by different writers. It will appear afterwards, that, in consequence of this circumstance, some controversies, which have been keenly agitated about the principles of morals, resolve entirely into verbal disputes; or at most, into questions of arrangement and classification, of little comparative moment to the points at issue †.

* Essays and Treatises, &c. Appendix, concerning Moral Sentiment.

† In confirmation of this remark, I shall only quote at present a few sentences from an excellent discourse, by Dr Adams of Oxford, on the nature and obligations of virtue.

Another ambiguity in the word *reason*, it is of still greater consequence to point out at present; an ambiguity which leads us to confound our rational powers in general, with that particular branch of them, known among logicians by the name of the *Discursive faculty*. The affinity between the words *reason* and *reasoning* sufficiently accounts for this inaccuracy in common and popular language; although it cannot fail to appear obvious, on the slightest reflection, that, in strict propriety, *reasoning* only expresses *one* of the various functions or operations of *reason;* and that an extraordinary capacity for the former by no means affords a test by which the other constituent elements of the latter may be measured *. Nor is it to common and popular language that this inaccuracy is confined. It has extended itself to the systems of some of our most acute philosophers, and has, in various instances, produced an apparent diversity of opinion where there was little or none in reality.

" Nothing can bring us under an obligation to do what appears to our moral judgment " *wrong*. It may be supposed our interest to do this; but it cannot be supposed our " duty.——Power may compel, interest may bribe, pleasure may persuade; but " REASON only can oblige. This is the only authority which rational beings can own, " and to which they owe obedience."

It must appear perfectly obvious to every reader, that the apparent difference of opinion between this writer and Mr Hume, turns chiefly on the different degrees of latitude with which they have used the word *reason*. Of the two, there cannot be a doubt that Dr Adams has adhered by far the most faithfully, not only to its acceptation in the works of our best English authors, but to the acceptation of the corresponding term in the ancient languages. " Est quidem vera lex, recta ratio——quæ vocet ad " officium, jubendo; vetando, a fraude deterreat," &c. &c.

* " The two most different things in the world (says Locke) are, a logical chicaner, " and a man of reason."—*Conduct of the Understanding*, § 3.

"No hypothesis (says Dr Campbell) hitherto invented, hath "shewn that, by means of the discursive faculty, without the "aid of any other mental power, we could ever obtain a notion "of either the beautiful or the good *." The remark is undoubtedly true, and may be applied to all those systems which ascribe to reason the origin of our moral ideas, if the expressions *reason* and *discursive faculty* be used as synonymous. But it was assuredly not in this restricted acceptation, that the word *reason* was understood by those ethical writers at whose doctrines this criticism seems to have been pointed by the ingenious author. That the discursive faculty alone is sufficient to account for the origin of our moral ideas, I do not know that any theorist, ancient or modern, has yet ventured to assert.

Various other philosophical disputes might be mentioned, which would be at once brought to a conclusion, if this distinction between reason and the power of reasoning were steadily kept in view †.

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* Philosophy of Rhetoric, Vol. I. p. 204.

† It is curious, that Dr Johnson has assigned to this very limited, and (according to present usage) very doubtful interpretation of the word *reason*, the *first* place in his enumeration of its various meanings, as if he had thought it the sense in which it is most properly and correctly employed. "Reason (he tells us) is the power by which man "deduces one proposition from another, or proceeds from premises to consequences." The authority which he has quoted for this definition is still more curious, being manifestly altogether inapplicable to his purpose. "Reason is the director of man's will, "discovering in action what is good; for the laws of well-doing are the dictates of right "reason."—*Hooker.*

In the *sixth* article of the same enumeration, he states, as a distinct meaning of the same word, *ratiocination, discursive power.* What possible difference could he conceive

In the use which I make of the word *reason*, in the title of the following disquisitions, I employ it in a manner to which no philosopher can object—to denote merely the power by which we distinguish truth from falsehood, and combine means for the attainment of our ends: omitting for the present all consideration of that function which many have ascribed to it, of distinguishing right from wrong; without, however, presuming to call in question the accuracy of those by whom the term has been thus explained. Under the title of *Reason*, I shall consider also whatever faculties and operations appear to be more immediately and essentially connected with the discovery of *truth*, or the attainment of the objects of our pursuit,—more particularly the Power of Reasoning or Deduction; but distinguishing, as carefully as I can, our capacity of carrying

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between this signification and that above quoted? The authority, however, which he produces for this last explanation is worth transcribing. It is a passage from Sir John Davis, where that fanciful writer states a distinction between reason and understanding; to which he seems to have been led by a conceit founded on their respective etymologies.

"When she rates things, and moves from ground to ground,
"The name of Reason she obtains by this;
"But when by Reason she the truth hath found,
"And standeth fixt, she Understanding is."

The adjective *reasonable*, as employed in our language, is not liable to the same ambiguity with the substantive from which it is derived. It denotes a character in which *reason* (taking that word in its largest acceptation) possesses a decided ascendant over the temper and the passions; and implies no particular propensity to a display of the discursive power, if indeed it does not exclude the idea of such a propensity. In the following stanza, Pope certainly had no view to the logical talents of the lady whom he celebrates:

"I know a thing that's most uncommon,
"(Envy be silent and attend)
"I know a *reasonable* woman,
"Handsome and witty, yet a friend."

on this logical process, from those more comprehensive powers which Reason is understood to imply.

The latitude with which this word has been so universally used, seemed to recommend it as a convenient one for a *general title*, of which the object is rather comprehension than precision. In the discussion of particular questions, I shall avoid the employment of it as far as I am able; and shall endeavour to select other modes of speaking, more exclusively significant of the ideas which I wish to convey *.

Of this *reasonable woman*, we may venture to conjecture, with some confidence, that she did not belong to the class of those *femmes raisonneuses*, so happily described by Moliere:

" Raisonner est l'emploi de toute ma maison
" Et le raisonnement en bannit la raison."

* Mr Locke too has prefixed the same title, Of *Reason*, to the 17th chapter of his Fourth Book, using the word in a sense nearly coinciding with that very extensive one which I wish my readers to annex to it here.

After observing, that by reason he means " that faculty whereby man is supposed to be " distinguished from brutes, and wherein it is evident he much surpasses them;" he adds, that " we may in reason consider these four degrees;—the first and highest is the discover- " ing and finding out of proofs; the second, the regular and methodical disposition of " them, and laying them in a clear and fit order, to make their connection and force be " plainly and easily perceived; the third is the perceiving their connection; and the " fourth is making a right conclusion."

Dr Reid's authority for this use of the word is equally explicit: " The power of rea- " soning is very nearly allied to that of judging. We include both under the name of " *reason*."—*Intellect. Powers*, p. 671. 4to edit.

Another authority to the same purpose is furnished by Milton:

——————" Whence the soul
" Reason receives; and Reason is HER BEING—
" —*Discursive* or intuitive." *Par. Lost*, B. v. l. 486.

[I presume that Milton, who was a logician as well as a poet, means by the words *her being*, her *essential* or characteristical endowment.]

Another instance of the vagueness and indistinctness of the common language of logicians, in treating of this part of the Philosophy of the Human Mind, occurs in the word *Understanding*. In its popular sense, it seems to be very nearly synonymous with *reason*, when that word is used most comprehensively; and is seldom or never applied to any of our faculties, but such as are immediately subservient to the investigation of truth, or to the regulation of our conduct. In this sense, it is so far from being understood to comprehend the powers of Imagination, Fancy, and Wit, that it is often stated in direct opposition to them; as in the common maxim, that a sound understanding and a warm imagination are seldom united in the same person. But philosophers, without rejecting this use of the word, very generally employ it, with far greater latitude, to comprehend all the powers which I have enumerated under the title of intellectual; referring to it Imagination, Memory, and Perception, as well as the faculties to which it is appropriated in popular discourse, and which it seems indeed most properly to denote. It is in this manner that it is used by Mr Locke in his celebrated Essay; and by all the logicians who follow the common division of our mental powers into those of the Understanding and those of the Will.

In mentioning this ambiguity, I do not mean to cavil at the phraseology of the writers from whom it has derived its origin,

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To these quotations I shall only add a sentence from a very judicious French writer; which I am tempted to introduce here, less on account of the sanction which it gives to my own phraseology, than of the importance of the truth which it conveys.

"Reason is commonly employed as an instrument to acquire the sciences; whereas, "on the contrary, the sciences ought to be made use of as an instrument to give reason "its perfection."—*L'Art de Penser*, translated by Ozell, p. 2. London, 1717.

but only to point it out as a circumstance which may deserve attention in some of our future disquisitions. The division of our powers which has led to so extraordinary an extension of the usual meaning of language, has an obvious foundation in the constitution of our nature, and furnishes an arrangement which seems indispensable for an accurate examination of the subject: nor was it unnatural to bestow on those faculties, which are all subservient in one way or another to the right exercise of the Understanding, the name of that power, from their relation to which their chief value arises.

As the word *understanding*, however, is one of those which occur very frequently in philosophical arguments, it may be of some use to disengage it from the ambiguity just remarked; and it is on this account that I have followed the example of some late writers, in distinguishing the two classes of powers which were formerly referred to the Understanding and to the Will, by calling the former *intellectual*, and the latter *active*. The terms *cognitive* and *motive* were long ago proposed for the same purpose by Hobbes; but they never appear to have come into general use, and are indeed liable to obvious objections.

It has probably been owing to the very comprehensive meaning annexed in philosophical treatises to the word *Understanding*, that the use of it has so frequently been supplied of late by *Intellect*. The two words, as they are commonly employed, seem to be very nearly, if not exactly, synonymous; and the latter possesses the advantage of being quite unequivocal, having never acquired that latitude of application of which the former admits. The adjective *intellectual*, indeed, has had its

meaning extended as far as the substantive *understanding*; but, as it can be easily dispensed with in our particular arguments, it may, without inconvenience, be adopted as a distinctive epithet, where nothing is aimed at but to mark, in simple and concise language, a very general and obvious classification. The word *intellect* can be of no essential use whatever, if the ambiguity in the signification of the good old English word *understanding* be avoided; and as to *intellection*, which a late very acute writer * has attempted to introduce, I can see no advantage attending it, which at all compensates for the addition of a new and uncouth term to a phraseology which, even in its most simple and unaffected form, is so apt to revolt the generality of readers.

The only other indefinite word which I shall take notice of in these introductory remarks is *judgment*; and, in doing so, I shall confine myself to such of its ambiguities as are more peculiarly connected with our present subject. In some cases, its meaning seems to approach to that of *understanding*; as in the nearly synonymous phrases, *a sound understanding*, and *a sound judgment*. If there be any difference between these two modes of expression, it appears to me to consist chiefly in this, that the former implies a greater degree of positive ability than the latter; which indicates rather an exemption from those biasses which lead the mind astray, than the possession of any uncommon reach of capacity. To *understanding* we apply the epithets strong, vigorous, comprehensive, profound: To *judgment*, those of correct, cool, unprejudiced, impartial, solid. It was

* Dr Campbell. See his Philosophy of Rhetoric, Vol. I. p. 103. 1st edit.

in this sense that the word seems to have been understood by Pope, in the following couplet:

> "'Tis with our judgments as our watches; none
> "Go just alike, yet each believes his own."

For this meaning of the word, its primitive and literal application to the judicial decision of a tribunal accounts sufficiently.

Agreeably to the same fundamental idea, the name of *judgment* is given, with peculiar propriety, to those acquired powers of discernment which characterize a skilful critic in the fine arts; powers which depend, in a very great degree, on a temper of mind free from the undue influence of authority and of casual associations. The power of Taste itself is frequently denoted by the appellation of *judgment*; and a person who possesses a more than ordinary share of it is said to be *a judge* in those matters which fall under its cognizance.

The meaning annexed to the word by logical writers is considerably different from this; denoting one of the simplest acts or operations of which we are conscious, in the exercise of our rational powers. In this acceptation, it does not admit of definition, any more than *sensation*, *will*, or *belief*. All that can be done, in such cases, is to describe the occasions on which the operation takes place, so as to direct the attention of others to their own thoughts. With this view, it may be observed, in the present instance, that when we give our assent to a mathematical axiom; or when, after perusing the demonstration of a theorem, we assent to the conclusion; or, in general, when

we pronounce concerning the truth or falsity of any proposition, or the probability or improbability of any event, the power by which we are enabled to perceive what is true or false, probable or improbable, is called by logicians the faculty of *judgment*. The same word, too, is frequently used to express the particular acts of this power, as when the decision of the understanding on any question is called a judgment of the mind.

In treatises of logic, *judgment* is commonly defined to be an act of the mind, by which one thing is affirmed or denied of another; a definition which, though not unexceptionable, is perhaps less so than most that have been given on similar occasions. Its defect (as Dr Reid has remarked) consists in this,—that although it be by affirmation or denial that we express our judgments to others, yet judgment is a solitary act of the mind, to which this affirmation or denial is not essential; and therefore, if the definition be admitted, it must be understood of mental affirmation or denial only; in which case, we do no more than substitute, instead of the thing defined, another mode of speaking perfectly synonymous. The definition has, however, notwithstanding this imperfection, the merit of a conciseness and perspicuity, not often to be found in the attempts of logicians to explain our intellectual operations.

Mr Locke seems disposed to restrict the word *judgment* to that faculty which pronounces concerning the verisimilitude of doubtful propositions; employing the word *knowledge* to express the faculty which perceives the truth of propositions, either intuitively or demonstratively certain. "The faculty which "God has given man to supply the want of clear and certain

"*knowledge* in cases where that cannot be had, is *judgment*; "whereby the mind takes its ideas to agree or disagree; or, "which is the same thing, any proposition to be true or false, "without perceiving a demonstrative evidence in the proofs.

"Thus, the mind has two faculties, conversant about truth "and falsehood.

"First, *knowledge*, whereby it certainly perceives, and is un- "doubtedly satisfied of the agreement or disagreement of any "ideas.

"Secondly, *judgment*, which is the putting ideas together, "or separating them from one another in the mind, when their "agreement or disagreement is not perceived, but presumed "to be so; which is, as the word imports, taken to be so, be- "fore it certainly appears. And if it so unites, or separates "them, as in reality things are, it is *right judgment**."

For this limitation in the definition of *judgment*, some pretence is afforded by the literal signification of the word, when applied to the decision of a tribunal; and also, by its metaphorical application to the decisions of the mind, on those critical questions which fall under the province of Taste. But, considered as a technical or scientific term of logic, the practice of our purest and most correct writers sufficiently sanctions the more enlarged sense in which I have explained it; and, if I do not much deceive myself, this use of it will be

* Essay on the Human Understanding, Book iv. Chap. 14.

found more favourable to philosophical distinctness than Mr Locke's language, which leads to an unnecessary multiplication of our intellectual powers. What good reason can be given for assigning one name to the faculty which perceives truths that are *certain*, and another name to the faculty which perceives truths that are *probable*? Would it not be equally proper to distinguish, by different names, the power by which we perceive one proposition to be *true*, and another to be *false*?

As to *knowledge*, I do not think that it can, with propriety, be contrasted with *judgment*; nor do I apprehend that it is at all agreeable, either to common use or to philosophical accuracy, to speak of knowledge as a *faculty*. To me it seems rather to denote the *possession* of those truths about which our faculties have been previously employed, than any separate power of the understanding, by which truth is perceived *.

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* In attempting thus to fix the logical import of various words in our language which are apt to be confounded, in popular speech, with *reason*, and also with *reasoning*, some of my readers may be surprised, that I have said nothing about the word *wisdom*. The truth is, that the notion expressed by this term, as it is employed by our best writers, seems to presuppose the influence of some principles, the consideration of which belongs to a different part of my work. In confirmation of this, it may be remarked, that whereas the province of our reasoning powers (in their application to the business of life), is limited to the choice of *means*, *wisdom* denotes a power of a more comprehensive nature, and of a higher order; a power which implies a judicious selection both of *means* and of *ends*. It is very precisely defined by Sir William Temple, to be "that which makes men judge what are the best ends, and what the best means "to attain them."

Of these two modifications of wisdom, the one denotes a power of the mind which obviously falls under the view of the logician; the examination of the other, as obviously, belongs to ethics.

Before concluding these preliminary remarks, I cannot help expressing my regret, that the subject on which I am about to enter will so frequently lay me under the necessity of criticising the language, and of disputing the opinions of my predecessors. In doing so, I am not conscious of being at all influenced by a wish to indulge myself in the captiousness of controversy; nor am I much afraid of this imputation from any of my readers who shall honour these speculations with an attentive perusal. My real aim is, in the *first* place, to explain the grounds of my own deviations from the track which has been commonly pursued; and, *secondly*, to facilitate the progress of such as may follow me in the same path, by directing their attention to those points of divergency in the way, which may suggest matter for doubt or hesitation. I know, at the same time, that, in

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A distinction similar to this was plainly in the mind of Cudworth when he wrote the following passage, which, although drawn from the purest sources of ancient philosophy, will, I doubt not, from the uncouthness of the phraseology, have the appearance of extravagance to many in the present times. To myself it appears to point at *a fact* of the highest importance in the moral constitution of man.

"We have all of us by nature μαντευμα τι (as both Plato and Aristotle call it) a "certain *divination*, *presage*, and *parturient vaticination* in our minds, of some higher "good and perfection, than either *power* or *knowledge*. Knowledge is plainly to be pre- "ferred before power, as being that which guides and directs its blind force and impe- "tus; but Aristotle himself declares, that there is λογυ τι κρειττον, which is λογυ αρχη; "*something better than reason and knowledge, which is the principle and original of it.* "For (saith he) λογυ αρχη υ λογος, αλλα τι κρειττον. *The principle of reason is not rea-* "*son, but something better.*"—*Intellectual System*, p. 203.

Lord Shaftesbury has expressed the same truth more simply and perspicuously in that beautiful sentence which occurs more than once in his writings: "True wisdom "comes more from the heart than from the head."——Numberless illustrations of this profound maxim must immediately crowd on the memory of all who are conversant with the most enlightened works on the theory of legislation; more particularly, with those which appeared, during the eighteenth century, on the science of political economy.

the opinion of many, the best mode of unfolding the principles of a science is to state them systematically and concisely, without any historical retrospects whatever; and I believe the opinion is well-founded, in those departments of knowledge, where the difficulty arises less from vague ideas and indefinite terms, than from the length of the logical chain which the student has to trace. But, in such disquisitions as we are now engaged in, it is chiefly from the gradual correction of verbal ambiguities, and the gradual detection of unsuspected prejudices, that a progressive, though slow approximation to truth is to be expected. It is indeed a slow approximation, at best, that we can hope to accomplish at present, in the examination of a subject where so many powerful causes (particularly those connected with the imperfections of language) conspire to lead us astray. But the study of the human mind is not, on that account, to be abandoned. Whoever compares its actual state with that in which Bacon, Des Cartes, and Locke found it, must be sensible how amply their efforts for its improvement have been repaid, both by their own attainments, and by those of others who have since profited by their example. I am willing to hope, that some useful hints for its farther advancement, may be derived even from my own researches; and, distant as the prospect may be of raising it to a level with the physical science of the Newtonian school, by uniting the opinions of speculative men about fundamental principles, my ambition as an author will be fully gratified, if, by the few who are competent to judge, I shall be allowed to have contributed my share, however small, towards the attainment of so great an object.

In the discussions which immediately follow, no argument will, I trust, occur beyond the reach of those who shall read them with the attention which every inquiry into the human mind indispensably requires. I have certainly endeavoured, to the utmost of my abilities, to render every sentence which I have written, not only intelligible but perspicuous; and, where I have failed in the attempt, the obscurity will, I hope, be imputed, not to an affectation of mystery, but to some error of judgment. I can, without much vanity, say, that, with less expence of thought, I could have rivalled the obscurity of Kant; and that the invention of a new technical language, such as that which he has introduced, would have been an easier task, than the communication of clear and precise notions (if I have been so fortunate as to succeed in this communication), without departing from the established modes of expression.

To the following observations of D'Alembert (with some trifling verbal exceptions) I give my most cordial assent; and, mortifying as they may appear to the pretensions of bolder theorists, I should be happy to see them generally recognized as canons of philosophical criticism: "Truth in metaphysics "resembles truth in matters of taste. In both cases, the seeds "of it exist in every mind; though few think of attending to "this latent treasure, till it be pointed out to them by more cu-"rious inquirers. It should seem that every thing we learn "from a good metaphysical book is only a sort of reminiscence "of what the mind previously knew. The obscurity, of which "we are apt to complain in this science, may be always justly

" ascribed to the author; because the information which he " professes to communicate requires no technical language ap- " propriated to itself. Accordingly, we may apply to good " metaphysical authors what has been said of those who excel " in the art of writing, that, in reading them, every body is " apt to imagine, that he himself could have written in the same " manner.

" But, in this sort of speculation, if all are qualified to under- " stand, all are not fitted to teach. The merit of accommo- " dating easily to the apprehension of others, notions which " are at once simple and just, appears, from its extreme rarity, " to be much greater than is commonly imagined. Sound me- " taphysical principles are truths which every one is ready to " seize, but which few men have the talent of unfolding; so " difficult is it in this, as well as in other instances, to appro- " priate to one's self what seems to be the common inheritance " of the human race *."

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* " Le vrai en métaphysique ressemble au vrai en matiere de goût; c'est un vrai dont " tous les esprits ont le germe en eux-mêmes, auquel la plupart ne font point d'atten- " tion, mais qu'ils reconnoissent dès qu'on le leur montre. Il semble que tout ce qu'on " apprend dans un bon livre de métaphysique, ne soit qu'une espèce de réminiscence de " ce que notre ame a déja su; l'obscurité, quand il y en a, vient toujours de la faute de " l'auteur, parce que la science qu'il se propose d'enseigner n'a point d'autre langue que " la langue commune. Aussi peut-on appliquer aux bons auteurs de métaphysique " ce qu'on a dit des bons écrivains, qu'il n'y a personne qui en les lisant, ne croie pou- " voir en dire autant qu'eux.

" Mais si dans ce genre tous sont faits pour entendre, tous ne sont pas faits pour in- " struire. Le mérite de faire entrer avec facilité dans les esprits des notions vraies et " simples, est beaucoup plus grand qu'on ne pense, puisque l'expérience nous prouve

I am, at the same time, fully aware, that whoever, in treating of the human mind, aims to be understood, must lay his account with forfeiting, in the opinion of a very large proportion of readers, all pretensions to depth, to subtlety, or to invention. The acquisition of a new nomenclature is, in itself, no inconsiderable reward to the industry of those, who study only from motives of literary vanity; and, if D'Alembert's idea of this branch of science be just, the wider an author deviates from truth, the more likely are his conclusions to assume the appearance of discoveries. I may add, that it is chiefly in those discussions which possess the best claims to originality, where he may expect to be told by the multitude, that they have learned from him nothing but what they knew before.

The latitude with which the word *metaphysics* is frequently used, makes it necessary for me to remark, with respect to the foregoing passage from D'Alembert, that he limits the term entirely to an account of the origin of our ideas. "The generation "of our ideas (he tells us) belongs to metaphysics. It forms one "of the *principal* objects, and perhaps ought to form the *sole* "object of that science *."—If the meaning of the word be extended, as it too often is in our language, so as to comprehend

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"combien il est rare; les saines idées métaphysiques sont des vérités communes que "chacun saisit, mais que peu d'hommes ont le talent de développer; tant il est diffi- "cile, dans quelque sujet que ce puisse être, de se rendre propre ce qui appartient à "tout le monde."—*Elemens de Philosophie.*

* "La génération de nos idées appartient à la métaphysique; c'est un de ses objets "principaux, et peut-être devroit elle s'y borner."—Ibid.

all those inquiries which relate to the theory and to the improvement of our mental powers, some of his observations must be understood with very important restrictions. What he has stated, however, on the inseparable connection between perspicuity of style and soundness of investigation in metaphysical disquisitions, will be found to hold equally in every research to which that epithet can, with any colour of propriety, be applied.

CHAPTER FIRST.

OF THE FUNDAMENTAL LAWS OF HUMAN BELIEF; OR THE PRIMARY ELEMENTS OF HUMAN REASON.

THE propriety of the title prefixed to this Chapter will, I trust, be justified sufficiently by the speculations which are to follow. As these differ, in some essential points, from the conclusions of former writers, I found myself under the necessity of abandoning, in various instances, their phraseology; —but my reasons for the particular changes which I have made, cannot possibly be judged of, or even understood, till the inquiries by which I was led to adopt them be carefully examined.

I begin with a review of some of those primary truths, a conviction of which is necessarily implied in all our thoughts and in all our actions; and which seem, on that account, rather to form constituent and essential *elements* of reason, than *objects* with which reason is conversant. The import of this last remark will appear more clearly afterwards.

The primary truths to which I mean to confine my attention at present are, 1. Mathematical Axioms: 2. Truths (or more properly speaking, Laws of Belief,) inseparably connected with

the exercise of Consciousness, Perception, Memory, and Reasoning.—Of some additional laws of Belief, the truth of which is tacitly recognized in all our reasonings concerning contingent events, I shall have occasion to take notice under a different article.

Section I.

Of Mathematical Axioms.

I have placed this class of truths at the head of the enumeration, merely because they seem likely, from the place which they hold in the elements of geometry, to present to my readers a more interesting, and at the same time an easier subject of discussion, than some of the more abstract and latent elements of our knowledge, afterwards to be considered. In other respects, a different arrangement might perhaps have possessed some advantages, in point of strict logical method.

I.

On the evidence of mathematical axioms it is unnecessary to enlarge, as the controversies to which they have given occasion are entirely of a speculative, or rather scholastic description; and have no tendency to affect the certainty of that branch of science to which they are supposed to be subservient.

It must at the same time be confessed, with respect to this class of propositions (and the same remark may be extended to axioms in general), that some of the logical questions connected with them continue still to be involved in much obscu-

rity. In proportion to their extreme simplicity is the difficulty of illustrating or of describing their nature in unexceptionable language; or even of ascertaining a precise criterion by which they may be distinguished from other truths which approach to them nearly. It is chiefly owing to this, that, in geometry, there are no theorems of which it is so difficult to give a rigorous demonstration, as those, of which persons unacquainted with the nature of mathematical evidence are apt to say, that they require no proof whatever. But the inconveniences arising from these circumstances are of trifling moment; occasioning, at the worst, some embarrassment to those mathematical writers, who are studious of the most finished elegance in their exposition of elementary principles; or to metaphysicians, anxious to display their subtilty upon points which cannot possibly lead to any practical conclusion.

It was long ago remarked by Locke, of the axioms of geometry, as stated by Euclid, that although the proposition be at first enunciated in *general* terms, and afterwards appealed to, in its *particular* applications, as a principle *previously* examined and admitted, yet that the truth is not less evident in the latter case than in the former. He observes farther, that it is in some of its particular applications, that the truth of every axiom is originally perceived by the mind; and, therefore, that the general proposition, so far from being the *ground* of our assent to the truths which it comprehends, is only a verbal generalization of what, in particular instances, has been already acknowledged as true.

The same author remarks, that some of these axioms "are no

" more than bare verbal propositions, and teach us nothing but " the respect and import of names one to another. *The whole* " *is equal to all its parts*: what real truth, I beseech you, does " it teach us? What more is contained in that maxim, than " what the signification of the word *totum*, or the *whole*, does " of itself import? And he that knows that the word *whole* " stands for what is made up of all its parts, knows very little " less, than that 'the whole is equal to all its parts.' And up- " on the same ground, I think, that this proposition, *A hill is* " *higher than a valley*, and several the like, may also pass for " maxims."

Notwithstanding these considerations, Mr Locke does not object to the form which Euclid has given to his axioms, or to the place which he has assigned to them in his Elements. On the contrary, he is of opinion, that a collection of such maxims is *not without reason* prefixed to a mathematical system; in order that learners, " having in the beginning perfect- " ly acquainted their thoughts with these propositions made in " general terms, may have them ready to apply to all particu- " lar cases as formed rules and sayings. Not that, if they be " equally weighed, they are more clear and evident than the " instances they are brought to confirm; but that, being more " familiar to the mind, the very naming of them is enough to " satisfy the understanding." In farther illustration of this, he adds very justly and ingeniously, that " although our know- " ledge begins in particulars, and so spreads itself by degrees " to generals; yet, afterwards, the mind takes quite the con- " trary course, and having drawn its knowledge into as gene- " ral propositions as it can, makes them familiar to its thoughts,

"and accustoms itself to have recourse to them, as to the stan-
"dards of truth and falsehood."

But although, in mathematics, some advantage may be gained, without the risk of any possible inconvenience, from this arrangement of axioms, it is a very dangerous example to be followed in other branches of knowledge, where our notions are not equally clear and precise; and where the force of our pretended axioms (to use Mr Locke's words), "reaching only "to the sound, and not to the signification of the words, serves "only to lead us into confusion, mistakes, and error." For the illustration of this remark, I must refer to Locke.

Another observation of this profound writer deserves our attention, while examining the nature of axioms;—"that "they are not the foundations on which any of the sci-"ences is built; nor at all useful in helping men forward to "the discovery of unknown truths *." This observation I intend to illustrate afterwards, in treating of the futility of the syllogistic art. At present I shall only add, to what Mr Locke has so well stated, that, even in *mathematics*, it cannot with any propriety be said, that the axioms are the foundation on which the science rests; or the first principles from which its more recondite truths are deduced. Of this I have little doubt that Locke was perfectly aware; but the mistakes which some of the most acute and enlightened of his disciples have committed in treating of the same subject, convinces me, that a further elucidation of it is not altogether super-

* Book iv. chap. 7, § 11.—2. 8.

fluous. With this view, I shall here introduce a few remarks on a passage in Dr Campbell's Philosophy of Rhetoric, in which he has betrayed some misapprehensions on this very point, which a little more attention to the hints already quoted from the Essay on Human Understanding might have prevented. These remarks will, I hope, contribute to place the nature of axioms, more particularly of mathematical axioms, in a different and clearer light than that in which they have been commonly considered.

"Of intuitive evidence (says Dr Campbell) that of the following propositions may serve as an illustration: One and four make five. Things equal to the same thing are equal to one another. The whole is greater than a part; and, in brief, all axioms in arithmetic and geometry. These are, in effect, but so many expositions of our own general notions, taken in different views. Some of them are no more than definitions, or equivalent to definitions. To say, one and four make *five*, is precisely the same thing as to say, we give the name of *five* to one added to four. In fact, they are all in some respects reducible to this axiom, whatever is, is. I do not say they are deduced from it, for they have in like manner that original and intrinsic evidence, which makes them, as soon as the terms are understood, to be perceived intuitively. And, if they are not thus perceived, no deduction of reason will ever confer on them any additional evidence. Nay, in point of time, the discovery of the less general truths has the priority, not from their superior evidence, but solely from this consideration, that the less general are sooner objects of perception to us. But I affirm, that though

" not deduced from that axiom, they may be considered as " particular exemplifications of it, and coincident with it, inas- " much as they are all implied in this, that the properties of " our clear and adequate ideas can be no other than what the " mind clearly perceives them to be.

" But, in order to prevent mistakes, it will be necessary fur- " ther to illustrate this subject. It might be thought that, if " axioms were propositions perfectly identical, it would be im- " possible to advance a step by their means, beyond the simple " ideas first perceived by the mind. And it must be owned, if " the predicate of the proposition were nothing but a repetition " of the subject, under the same aspect, and in the same or " synonymous terms, no conceivable advantage could be made " of it for the furtherance of knowledge. Of such propositions, " for instance, as these,—seven are seven, eight are eight, and " ten added to eleven are equal to ten added to eleven, it is " manifest that we could never avail ourselves for the improve- " ment of science. Nor does the change of the term make any " alteration in point of utility. The propositions, twelve are a " dozen, twenty are a score, unless considered as explications " of the words *dozen* and *score*, are equally insignificant with " the former. But when the thing, though in effect coinciding, is " considered under a different aspect; when what is single in the " subject is divided in the predicate, and conversely; or when " what is a whole in the one is regarded as a part of something " else in the other; such propositions lead to the discovery of " innumerable and apparently remote relations. One added " to four may be accounted no other than a definition of the " word five, as was remarked above. But when I say, 'Two

" added to three are equal to five,' I advance a truth which, " though equally clear, is quite distinct from the preceding. " Thus, if one should affirm, ' That twice fifteen make thirty,' " and again, that ' thirteen added to seventeen make thirty,' " nobody would pretend that he had repeated the same pro- " position in other words. The cases are entirely similar. In " both cases, the same thing is predicated of ideas which, taken " severally, are different. From these again result other equa- " tions, as ' one added to four are equal to two added to " three,' and ' twice fifteen are equal to thirteen added to " seventeen.'

" Now, it is by the aid of such simple and elementary prin- " ciples, that the arithmetician and algebraist proceed to the " most astonishing discoveries. Nor are the operations of the " geometrician essentially different."

I have little to object to these observations of Dr Campbell, as far as they relate to arithmetic and to algebra; for, in these sciences, all our investigations amount to nothing more than to a comparison of different expressions of the same thing. Our common language indeed frequently supposes the case to be otherwise; as when an equation is defined to be, " A proposition " asserting the equality of two quantities." It would, however, be much more correct to define it, " A proposition asserting " the equivalence of two expressions of the same quantity;" for algebra is merely a *universal arithmetic;* and the names of numbers are nothing else than collectives, by which we are enabled to express ourselves more concisely than could be done by enumerating all the units that they contain. Of this doctrine, the

passage now quoted from Dr Campbell shews that he entertained a sufficiently just and precise idea.

But, if Dr Campbell perceived that arithmetical equations, such as "one and four make five," are no other than definitions, why should he have classed them with the axioms he quotes from Euclid, "That the whole is greater than a part," and that "Things equal to the same thing are equal to one "another;" propositions which, however clearly their truth be implied in the meaning of the terms of which they consist, cannot certainly, by any interpretation, be considered in the light of definitions at all analogous to the former? The former, indeed, are only explanations of the relative import of particular names; the latter are universal propositions, applicable alike to an infinite variety of instances*.

* D'Alembert also has confounded these two classes of propositions. "What do the "greater part of those axioms on which geometry prides itself amount to, but to an ex- "pression, by means of two different words or signs, of the same simple idea? He who "says that two and two make four, what more does he know than another who should "content himself with saying, that two and two make two and two?"——Here, a simple arithmetical equation (which is obviously a mere *definition*) is brought to illustrate a remark on the nature of geometrical axioms.——With respect to these last (I mean such axioms as Euclid has prefixed to his Elements) D'Alembert's opinion seems to coincide exactly with that of Locke, already mentioned. "I would not be under- "stood, nevertheless, to condemn the use of them altogether: I wish only to remark, "that their utility rises no higher than this, that they render our simple ideas more fa- "miliar by means of habit, and better adapted to the different purposes to which we "may have occasion to apply them."—"Je ne prétends point cependant en condamner "absolument l'usage: je veux seulement faire observer, à quoi il se reduit; c'est à "nous rendre les idées simples plus familières par l'habitude, et plus propres aux dif- "férens usages auxquels nous pouvons les appliquer."—*Discours Preliminaire*, &c. &c.

Another very obvious consideration might have satisfied Dr Campbell, that the simple arithmetical equations which he mentions, do not hold the same place in that science which Euclid's axioms hold in geometry. What I allude to is, that the greater part of these axioms are equally essential to all the different branches of mathematics. That "the whole is greater than a "part," and that "things equal to the same thing are equal "to one another," are propositions as essentially connected with our arithmetical computations, as with our geometrical reasonings; and, therefore, to explain in what manner the mind makes a transition, in the case of numbers, from the more simple to the more complicated equations, throws no light whatever on the question, *how* the transition is made, either in arithmetic or in geometry, from what are properly called axioms, to the more remote conclusions in these sciences.

The very fruitless attempt thus made by this acute writer to illustrate the importance of axioms as the basis of mathematical truth, was probably suggested to him by a doctrine which has been repeatedly inculcated of late, concerning the grounds of that peculiar evidence which is allowed to accompany mathematical demonstration. "All the sciences (it has been said) "rest ultimately on first principles, which we must take for "granted without proof; and whose evidence determines, both "in kind and degree, the evidence which it is possible to attain "in our conclusions. In some of the sciences, our first princi- "ples are intuitively certain; in others, they are intuitively "probable; and such as the evidence of these principles is, "such must that of our conclusions be. If our first principles "are intuitively certain, and if we reason from them conse-

"quentially, our conclusions will be demonstratively certain: "but if our principles be only intuitively probable, our con- "clusions will be only demonstratively probable. In mathe- "matics, the first principles from which we reason are a set of "axioms which are not only intuitively certain, but of which "we find it impossible to conceive the contraries to be true: "And hence the peculiar evidence which belongs to all the "conclusions that follow from these principles as necessary con- "sequences."

To this view of the subject Dr Reid has repeatedly given his sanction, at least in the most essential points; more particularly, in controverting an assertion of Locke's, that "no science "is, or hath been built on *maxims*."—"Surely (says Dr Reid) "Mr Locke was not ignorant of geometry, which hath been "built upon maxims prefixed to the Elements, as far back as "we are able to trace it. But though they had not been pre- "fixed, which was a matter of utility rather than necessity, "yet it must be granted, that every demonstration in geometry "is grounded, either upon propositions formerly demonstrated, "or upon self-evident principles *."

On another occasion, he expresses himself thus: "I take it "to be certain, that whatever can, by just reasoning, be infer- "red from a principle that is necessary, must be a necessary "truth. Thus, as the axioms in mathematics are all neces- "sary truths, so are all the conclusions drawn from them; that "is, the whole body of that science †."

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* Essays on Intell. Powers, p. 647, 4to edit. † Ibid. p. 577. See also pp. 560, 561, 606.

That there is something fundamentally erroneous in these very strong statements with respect to the relation which Euclid's axioms bear to the geometrical theorems which follow, appears sufficiently from a consideration which was long ago mentioned by Locke,—that from these axioms it is not possible for human ingenuity to deduce a single inference. "It was "not (says Locke) the influence of those maxims which are "taken for principles in mathematics, that hath led the mas- "ters of that science into those wonderful discoveries they have "made. Let a man of good parts know all the maxims gene- "nerally made use of in mathematics, never so perfectly, and "contemplate their extent and consequences as much as he "pleases, he will, by their assistance, I suppose, scarce ever "come to know, that 'the square of the hypothenuse in a "right angled triangle, is equal to the squares of the two other "sides.' The knowledge that 'the whole is equal to all its "parts,' and, 'if you take equals from equals, the remainders "will be equal,' helped him not, I presume, to this demon- "stration: And a man may, I think, pore long enough on "those axioms, without ever seeing one jot the more of ma- "thematical truths *." But surely, if this be granted, and if, at the same time, by the first principles of a science be meant those fundamental propositions from which its remoter truths are derived, the axioms cannot, with any consistency, be called the First Principles of Mathematics. They have not (it will be admitted) the most distant analogy to what are called the first principles of natural philosophy;—to those general facts, for example, of the gravity and elasticity of the air, from which

* Essay on Human Understanding, Book IV. chap. xii. § 15.

may be deduced, as consequences, the suspension of the mercury in the Torricellian tube, and its fall when carried up to an eminence. According to this meaning of the word, the principles of mathematical science are, *not* the axioms but the *definitions*; which definitions hold, in mathematics, precisely the same place that is held in natural philosophy by such general facts as have now been referred to *.

From what principle are the various properties of the circle derived, but from the definition of a circle? From what principle the properties of the parabola or ellipse, but from the

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* In order to prevent cavil, it may be necessary for me to remark here, that when I speak of mathematical axioms, I have in view only such as are of the same description with the *first nine* of those which are prefixed to the Elements of Euclid; for, in that list, it is well known, that there are several which belong to a class of propositions altogether different from the others. That "all right angles (for example) are equal to "one another;" that "when one straight line falling on two other straight lines makes "the two interior angles on the same side less than two right angles, these two straight "lines, if produced, shall meet on the side, where are the two angles less than two right "angles;" are manifestly principles which bear no analogy to such barren *truisms* as these, "Things that are equal to one and the same thing are equal to one another." "If equals be added to equals, the wholes are equal." "If equals be taken from equals, "the remainders are equal." Of these propositions, the two former (the 10th and 11th axioms, to wit, in Euclid's list) are evidently theorems which, in point of strict logical accuracy, ought to be demonstrated; as may be easily done, with respect to the first, in a single sentence. That the second has not yet been proved in a simple and satisfactory manner, has been long considered as a sort of reproach to mathematicians; and I have little doubt that this reproach will continue to exist, till the basis of the science be somewhat enlarged, by the introduction of one or two new definitions, to serve as additional principles of geometrical reasoning.

For some farther remarks on Euclid's Axioms, see note (A).

The edition of Euclid to which I uniformly refer, is that of David Gregory. Oxon. 1713.

definitions of these curves? A similar observation may be extended to all the other theorems which the mathematician demonstrates: And it is this observation (which, obvious as it may seem, does not appear to have occurred, in all its force, either to Locke, to Reid, or to Campbell,) that furnishes, if I mistake not, the true explanation of the peculiarity already remarked in mathematical evidence *.

The prosecution of this last idea properly belongs to the subject of mathematical demonstration, of which I intend to treat afterwards. In the meantime, I trust, that enough has been said to correct those misapprehensions of the nature of axioms, which are countenanced by the speculations, and still more by the phraseology, of some late eminent writers. On this article, my own opinion coincides very nearly with that of Mr Locke—both in the view which he has given of the nature and use of axioms in geometry, and in what he has so forcibly urged concerning the danger, in other branches of knowledge, of attempting a similar list of *maxims*, without a due regard to the circumstances by which different sciences are distinguished from one another. With Mr Locke, too, I must beg leave to guard myself against the possibility of being misunderstood in

* D'Alembert, although he sometimes seems to speak a different language, approached nearly to this view of the subject when he wrote the following passage:

"Finally, it is not without reason that mathematicians consider *definitions* as *princi-*
"*ples*; since it is on clear and precise definitions that our knowledge rests in those sci-
"ences, where our reasoning powers have the widest field opened for their exercise."—
"Au reste, ce n'est pas sans raison que les mathématiciens regardent les définitions comme
"des principes, puisque, dans les sciences ou le raisonnement a la meilleure part,
"c'est sur des définitions nettes et exactes que nos connoissances sont appuyées."—
Elemens de Phil. p. 4.

the illustrations which I have offered of some of his ideas: And for this purpose, I cannot do better than borrow his words. "In all that is here suggested concerning the little use of axi- "oms for the improvement of knowledge, or dangerous use in "undetermined ideas, I have been far enough from saying or "intending they should be laid aside, as some have been too "forward to charge me. I affirm them to be truths, self-evi- "dent truths; and so cannot be laid aside. As far as their in- "fluence will reach, it is in vain to endeavour, nor would I at- "tempt to abridge it. But yet, without any injury to truth or "knowledge, I may have reason to think their use is not an- "swerable to the great stress which seems to be laid on them, "and I may warn men not to make an ill use of them, for the "confirming themselves in error*."

After what has been just stated, it is scarcely necessary for me again to repeat, with regard to mathematical axioms, that although they are not the *principles* of our reasoning, either in arithmetic or in geometry, their truth is supposed or implied in all our reasonings in both; and, if it were called in question, our further progress would be impossible. In both of these respects, we shall find them analogous to the other classes of primary or elemental truths which remain to be considered.

Nor let it be imagined, from this concession, that the dispute turns merely on the meaning annexed to the word *principle*. It turns upon an important question of fact; Whether the theo-

* Locke's Essay, Book IV. ch. vii. § 14.

rems of geometry rest on the *axioms*, in the same sense in which they rest on the *definitions?* or (to state the question in a manner still more obvious,) Whether axioms hold a place in geometry at all analogous to what is occupied in natural philosophy, by those sensible phenomena which form the basis of that science? Dr Reid compares them sometimes to the one set of propositions and sometimes to the other*. If the foregoing observations be just, they bear no analogy to either.

Into this indistinctness of language Dr Reid was probably led in part by Sir Isaac Newton, who, with a very illogical latitude in the use of words, gave the name of *axioms* to the *laws of motion*†, and also to those general experimental truths which form the ground-work of our general reasonings in catoptrics and dioptrics. For such a misapplication of the technical terms

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* "Mathematics, once fairly established on the foundation of a few *axioms and defi-* "*nitions*, as upon a rock, has grown from age to age, so as to become the loftiest and "the most solid fabric that human reason can boast."—*Essays on Int. Powers*, p. 561, 4to edition.

"Lord Bacon first delineated the only solid foundation on which natural philosophy "can be built; and Sir Isaac Newton reduced the principles laid down by Bacon into "three or four *axioms*, which he calls *regulæ philosophandi*. From these, *together with* "*the phenomena observed by the senses, which he likewise lays down as first principles*, "he deduces, by strict reasoning, the propositions contained in the third book of his "Principia, and in his Optics; and by this means has raised a fabric, which is not "liable to be shaken by doubtful disputation, but stands immoveable on the basis of "self-evident principles."—Ibid. See also pp. 647, 648.

† Axiomata, sive leges Motus. Vid. *Philosophiæ Naturalis Principia Mathematica.*

At the beginning, too, of Newton's Optics, the title of axioms is given to the following propositions:

of mathematics some apology might perhaps be made, if the author had been treating on any subject connected with moral science; but surely, in a work entitled "Mathematical Principles "of Natural Philosophy," the word *axiom* might reasonably have been expected to be used in a sense somewhat analogous to that which every person liberally educated is accustomed to annex to it, when he is first initiated into the elements of geometry.

The question to which the preceding discussion relates is of the greater consequence, that the prevailing mistake with respect to the nature of mathematical axioms, has contributed much to the support of a very erroneous theory concerning

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"AXIOM I.

"The angles of reflection and refraction lie in one and the same plane with the angle "of incidence.

"AXIOM II.

"The angle of reflection is equal to the angle of incidence.

"AXIOM III.

"If the refracted ray be turned directly back to the point of incidence, it shall be "refracted into the line before described by the incident ray.

"AXIOM IV.

"Refraction out of the rarer medium into the denser, is made towards the perpendi- "cular; that is, so that the angle of refraction be less than the angle of incidence.

"AXIOM V.

"The sine of incidence is either accurately, or very nearly in a given ratio to the "sine of refraction."

When the word *axiom* is understood by one writer in the sense annexed to it by Euclid, and by his antagonist in the sense here given to it by Sir Isaac Newton, it is not surprising that there should be apparently a wide diversity between their opinions concerning the logical importance of this class of propositions.

mathematical evidence, which is, I believe, pretty generally adopted at present,—that it all resolves ultimately into the perception of *identity*; and that it is this circumstance which constitutes the peculiar and characteristical cogency of mathematical demonstration.

Of some of the other arguments which have been alleged in favour of this theory, I shall afterwards have occasion to take notice. At present, it is sufficient for me to remark, (and this I flatter myself I may venture to do with some confidence, after the foregoing reasonings,) that in so far as it rests on the supposition that all geometrical truths are ultimately derived from Euclid's axioms, it proceeds on an assumption totally unfounded in fact, and indeed so obviously false, that nothing but its antiquity can account for the facility with which it continues to be admitted by the learned *.

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* A late mathematician, of considerable ingenuity and learning, doubtful, it should seem, whether Euclid had laid a sufficiently broad foundation for mathematical science in the axioms prefixed to his Elements, has thought proper to introduce several new ones of his own invention. The first of these is, that "Every quantity is equal to itself;" to which he adds afterwards, that "A quantity expressed one way is equal to itself expressed any other way."—*See Elements of Mathematical Analysis, by Professor Vilant of St Andrew's.* We are apt to smile at the formal statement of these propositions; and yet, according to the theory alluded to in the text, it is in truths of this very description that the whole science of mathematics not only begins but ends. "Omnes mathematicorum propositiones sunt identicæ, et repræsentantur hac formula, "a = a." This sentence, which I quote from a dissertation published at Berlin about fifty years ago, expresses, in a few words, what seems to be now the prevailing opinion, (more particularly on the Continent) concerning the nature of mathematical evidence. The remarks which I have to offer upon it I delay till some other questions shall be previously considered.

Section I.

II.

Continuation of the same Subject.

The difference of opinion between Locke and Reid, of which I took notice in the foregoing part of this section, appears greater than it really is, in consequence of an ambiguity in the word *principle*, as employed by the latter. In its proper acceptation, it seems to me to denote an assumption (whether resting on fact or on hypothesis), upon which, as a *datum*, a train of reasoning proceeds; and for the falsity or incorrectness of which no logical rigour in the subsequent process can compensate. Thus the gravity and the elasticity of the air are *principles of reasoning* in our speculations about the barometer. The equality of the angles of incidence and reflection; the proportionality of the sines of incidence and refraction; are *principles of reasoning* in catoptrics and in dioptrics. In a sense perfectly analogous to this, the *definitions* of geometry (all of which are merely *hypothetical*) are the *first principles* of reasoning in the subsequent demonstrations, and the basis on which the whole fabric of the science rests.

I have called this the *proper* acceptation of the word, because it is that in which it is most frequently used by the best writers. It is also most agreeable to the literal mean-

ing which its etymology suggests, expressing the original point from which our reasoning sets out or commences.

Dr Reid often uses the word in this sense, as, for example, in the following sentence, already quoted: "From three or four "axioms, which he calls *regulæ philosophandi*, together with *the* "*phenomena observed by the senses, which he likewise lays down as* "*first principles*, Newton deduces, by strict reasoning, the pro- "positions contained in the third book of his Principia, and "in his Optics."

On other occasions, he uses the same word to denote those *elemental* truths (if I may use the expression,) which are virtually taken for granted or assumed, in every step of our reasoning; and without which, although no *consequences* can be directly inferred from them, a train of reasoning would be impossible. Of this kind, in mathematics, are the *axioms*, or (as Mr Locke and others frequently call them,) the *maxims;* in physics, a belief of *the continuance of the Laws of Nature;*—in all our reasonings, without exception, a belief in *our own identity*, and in *the evidence of memory*. Such truths are the *last elements* into which reasoning resolves itself, when subjected to a metaphysical analysis; and which no person but a metaphysician or a logician ever thinks of stating in the form of propositions, or even of expressing verbally to himself. It is to truths of this description that Locke seems in general to apply the name of *maxims;* and, in this sense, it is unquestionably true, that no science (not even geometry) is founded on maxims as its first principles.

In one sense of the word *principle*, indeed, maxims may be called principles of reasoning; for the words *principles* and *elements* are sometimes used as synonymous. Nor do I take upon me to say that this mode of speaking is exceptionable. All that I assert is, that they cannot be called *principles of reasoning*, in the sense which has just now been defined; and that accuracy requires, that the word, on which the whole question hinges, should not be used in both senses, in the course of the same argument. It is for this reason that I have employed the phrase *principles of reasoning* on the one occasion, and *elements of reasoning* on the other.

It is difficult to find unexceptionable language to mark distinctions so completely foreign to the ordinary purposes of speech; but, in the present instance, the line of separation is strongly and clearly drawn by this criterion,—that from *principles of reasoning* consequences may be deduced; from what I have called *elements of reasoning*, none ever can.

A process of logical reasoning has been often likened to a chain supporting a weight. If this similitude be adopted, the *axioms* or *elemental truths* now mentioned, may be compared to the successive concatenations which connect the different links immediately with each other; the *principles* of our reasoning resemble the hook, or rather the beam, from which the whole is suspended.

The foregoing observations, I am inclined to think, coincide with what was, at bottom, Mr Locke's opinion on this subject.

That he has not stated it with his usual clearness and distinctness, it is impossible to deny; at the same time, I cannot subscribe to the following severe criticism of Dr Reid:

"Mr Locke has observed, 'That intuitive knowledge is "necessary *to connect* all the steps of a demonstration.'

"From this, I think, it necessarily follows, that in every "branch of knowledge, we must make use of truths that are "intuitively known, in order to *deduce from them* such as re-"quire proof.

"But I cannot reconcile this with what he says (section "8th of the same chapter): 'The necessity of this intuitive "knowledge in every step of scientifical or demonstrative rea-"soning, gave occasion, I imagine, to that mistaken axiom, "that all reasoning was *ex præcognitis et præconcessis*, which "how far it is mistaken I shall have occasion to show more "at large when I come to consider propositions, and particu-"larly those propositions which are called maxims, and to "show that it is by a mistake that they are supposed to be the "foundation of all our knowledge and reasonings *."

The distinction which I have already made between *elements* of reasoning, and *first principles* of reasoning, appears to myself to throw much light on these apparent contradictions.

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* Essays on Int. Powers, p. 643, 4to edit.

That the seeming difference of opinion on this point between these two profound writers, arose chiefly from the ambiguities of language, may be inferred from the following acknowledgement of Dr Reid, which immediately follows the last quotation:

"I have carefully examined the chapter on maxims, which "Mr Locke here refers to, and though one would expect, from "the quotation last made, that it should run contrary to what "I have before delivered concerning first principles, I find only "two or three sentences in it, and those chiefly incidental, to "which I do not assent*."

Before dismissing this subject, I must once more repeat, that the doctrine which I have been attempting to establish, so far from degrading *axioms* from that rank which Dr Reid would assign them, tends to identify them still more than he has done with the exercise of our reasoning powers; inasmuch as, instead of comparing them with the *data*, on the accuracy of which that of our conclusion necessarily depends, it considers them as the *vincula* which give coherence to all the particular links of the chain; or, (to vary the metaphor) as *component elements*, without which the faculty of reasoning is inconceivable and impossible†.

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* Essays on Int. Powers, p. 643, 4to edit.

† D'Alembert has defined the word *principle* exactly in the sense in which I have used it; and has expressed himself (at least on one occasion) nearly as I have done, on the subject of axioms. He seems however on this, as well as on some other logical

Section II.

Of certain Laws of Belief, inseparably connected with the exercise of Consciousness, Memory, Perception, and Reasoning.

1. It is by the immediate evidence of consciousness that we are assured of the *present existence* of our various sensations,

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and metaphysical questions, to have varied a little in his views (probably from mere forgetfulness) in different parts of his writings.

"What then are the truths which are entitled to have a place in the elements of "philosophy? They are of two kinds; *those which form the head of each part of the "chain*, and those which are to be found at the points where different branches of the "chain unite together.

"Truths of the first kind are distinguished by this—that they do not depend on any "other truths, and that they possess within themselves the whole grounds of their evi- "dence. Some of my readers will be apt to suppose, that I here mean to speak of "*axioms*; but these are not the truths which I have at present in view. With respect "to this last class of *principles*, I must refer to what I have elsewhere said of them; "that, notwithstanding their truth, they add nothing to our information; and that the "palpable evidence which accompanies them, amounts to nothing more than to an ex- "pression of the same idea by means of two different terms. On such occasions, the "mind only turns to no purpose about its own axis, without advancing forward a sin- "gle step. Accordingly, *axioms* are so far from holding the highest rank in philoso- "phy, that they scarcely deserve the distinction of being formally enunciated."

["Or quelles sont les vérités qui doivent entrer dans des elemens de philosophie? "Il y en a de deux sortes; *celles qui forment la tête de chaque partie de la chaine*, et "celles qui se trouvent au point de reunion de plusieurs branches.

"Les vérités du premier genre ont pour caractère distinctif de ne dépendre d'au- "cune autre, et de n'avoir de preuves que dans elles-mêmes. Plusieurs lecteurs croi- "ront que nous voulons parler *des axioms*, et ils se tromperont; nous les renvoyons à ce "qui nous en avons dit ailleurs, que *ces sortes de principes* ne nous apprennent rien à