

ing so may be easily collected from the tenor of the preceding remarks.

It is time, however, to proceed to the examination of those discursive processes, the different steps of which admit of being distinctly stated and enunciated in the form of logical arguments ; and which, in consequence of this circumstance, furnish more certain and palpable data for our speculations. I begin with some remarks on the Power of General Reasoning ; for the exercise of which (as I formerly endeavoured to shew) the use of language, as an instrument of thought, is indispensably requisite.

SECTION II.

Of General Reasoning.

I.

Illustrations of some Remarks formerly stated in treating of Abstraction.

I SHOULD scarcely have thought it necessary to resume the consideration of Abstraction here, if I had not neglected, in my first volume, to examine the force of an objection to Berkeley's doctrine concerning abstract general ideas, on which great stress is laid by Dr Reid, in his *Essays on the Intellectual Powers of Man* ; and which some late writers seem to have considered as not less conclusive against the view of the question

which I have taken. Of this objection I was aware from the first; but was unwilling, by replying to it in form, to lengthen a discussion which savoured so much of the schools; more especially, as I conceived that I had guarded my own argument from any such attack, by the cautious terms in which I had expressed it. Having since had reason to believe that I was precipitate in forming this judgment, and that Reid's strictures on Berkeley's theory of General Signs have produced a deeper impression than I had expected *, I shall endeavour to obviate them, at least as far as they apply to myself, before entering on any new speculations concerning our reasoning powers; and shall, at the same time, introduce some occasional illustrations of the principles which I formerly endeavoured to establish.

To prevent the possibility of misrepresentation, I state Dr Reid's objection in his own words.

“Berkeley, in his reasoning against abstract general ideas, seems unwillingly or unwarily to grant all that is necessary to support abstract and general conceptions.

“A man (says Berkeley) may consider a figure merely as

* See a book entitled, *Elements of Intellectual Philosophy*, by the late learned and justly regretted Mr Scott, of King's College, Aberdeen, p. 118. et seq. (Edinburgh, 1805.) I have not thought it necessary to reply to Mr Scott's own reasonings, which do not appear to me to throw much new light on the question; but I thought it right to refer to them here, that the reader may, if he pleases, have an opportunity of judging for himself.

“triangular, without attending to the particular qualities of the angles, or relations of the sides. So far he may abstract. But this will never prove that he can frame an abstract general inconsistent idea of a triangle.”

Upon this passage Dr Reid makes the following remark: *“If a man may consider a figure merely as triangular, he must have some conception of this object of his consideration; for no man can consider a thing which he does not conceive. He has a conception, therefore, of a triangular figure, merely as such. I know no more that is meant by an abstract general conception of a triangle.”*

“He that considers a figure merely as triangular (continues the same author) must understand what is meant by the word triangular. If to the conception he joins to this word, he adds any particular quality of angles or relation of sides, he misunderstands it, and does not consider the figure merely as triangular. Whence I think it is evident, that he who considers a figure merely as triangular, must have the conception of a triangle, abstracting from any quality of angles or relations of sides*.”

For what appears to myself to be a satisfactory answer to this reasoning, I have only to refer to the first volume of these Elements. The remarks to which I allude are to be found in

* Reid's Intellectual Powers, p. 483, 4to edit.

the third section of chapter fourth*; and I must beg leave to recommend them to the attention of my readers, as a necessary preparation for the following discussion.

In the farther prosecution of the same argument, Dr Reid lays hold of an acknowledgement which Berkeley has made, "That we may consider *Peter* so far forth as man, or so far forth as animal, inasmuch as all that is perceived is not considered."—"It may here (says Reid) be observed, that he who considers *Peter* so far forth as man, or so far forth as animal, must *conceive* the meaning of those abstract general words *man* and *animal*; and he who *conceives* the meaning of them, has an abstract general conception."

According to the definition of the word *conception*, which I have given in treating of that faculty of the mind, a *general conception* is an obvious impossibility. But, as Dr Reid has chosen to annex a more extensive meaning to the term than seems to me consistent with precision, I would be far from being understood to object to his conclusion, *merely* because it is inconsistent with an arbitrary definition of my own. Let us consider, therefore, how far his doctrine is consistent with itself; or rather, since both parties are evidently so nearly agreed about the principal fact, which of the two have adopted the more perspicuous and philosophical mode of stating it.

In the first place, then, let it be remembered as a thing admitted

on both sides, "that we have a power of *reasoning* concerning a "figure considered merely as triangular, without attending to the "particular qualities of the angles, or relations of the sides;" and also, that "we may reason concerning *Peter* or *John*, considered so far forth as *man*, or so far forth as *animal*." About these facts there is but one opinion; and the only question is, Whether it throws additional light on the subject, to tell us, in scholastic language, that "we are enabled to carry on these "general reasonings, in consequence of the power which the "mind has of forming abstract general conceptions." To myself it appears, that this last statement (even on the supposition that the word *conception* is to be understood agreeably to Dr Reid's own explanation,) can serve no other purpose than that of involving a plain and simple truth in obscurity and mystery. If it be used in the sense in which I have invariably employed it in this work, the proposition is altogether absurd and incomprehensible.

For the more complete illustration of this point, I must here recur to a distinction formerly made between the abstractions which are subservient to reasoning, and those which are subservient to imagination. "In every instance in "which imagination is employed in forming new wholes, by de- "compounding and combining the perceptions of sense, it is evidently necessary that the poet or the painter should be able to "state or represent to himself the circumstances abstracted, as "separate objects of conception. But this is by no means requisite in every case in which abstraction is subservient to "the power of reasoning; for it frequently happens, that we

“can reason concerning the quality or property of an object
“abstracted from the rest, while, at the same time, we find it
“impossible to conceive it separately. Thus, I can reason
“concerning extension and figure, without any reference to
“colour, although it may be doubted, if a person possessed of
“sight can make extension and figure steady objects of con-
“ception, without connecting with them the idea of one colour
“or another. Nor is this always owing (as it is in the instance
“just mentioned) merely to the association of ideas ; for there
“are cases, in which we can *reason* concerning things separate-
“ly, which it is impossible for us to suppose any mind so con-
“stituted as to conceive apart. Thus we can reason concern-
“ing length, abstracted from any other dimension ; although,
“surely, no understanding can make length, without breadth,
“an object of conception *.” In like manner, while I am stu-
dying Euclid’s demonstration of the equality of the three angles
of a triangle to two right angles, I find no difficulty in follow-
ing his train of reasoning, although it has no reference what-
ever to the specific *size* or to the specific *form* of the diagram
before me. I *abstract*, therefore, in this instance, from both of
these circumstances presented to my senses by the immediate
objects of my perceptions ; and yet, it is manifestly impracti-
cable for me either to delineate on paper, or to conceive in the
mind, such a figure as shall not include the circumstances from
which I abstract, as well as those on which the demonstration
hinges.

* Vol. I. pp. 157, 158, 3d edit.

In order to form a precise notion of the manner in which this process of the mind is carried on, it is necessary to attend to the close and inseparable connection which exists between the faculty of general reasoning, and the use of artificial language. It is in consequence of the aids which *this* lends to our natural faculties, that we are furnished with a class of signs, expressive of all the circumstances which we wish our reasonings to comprehend; and, at the same time, exclusive of all those which we wish to leave out of consideration. The word *triangle*, for instance, when used without any additional epithet, confines the attention to the *three* angles and *three* sides of the figure before us; and reminds us, as we proceed, that no step of our deduction is to turn on any of the specific varieties which that figure may exhibit. The notion, however, which we annex to the word *triangle*, while we are reading the demonstration, is not the less a *particular* notion, that this word, from its partial or abstracted import, is equally applicable to an infinite variety of other individuals*.

* " By this imposition of names, some of larger, some of stricter signification, we
" turn the reckoning of the consequences of things imagined in the mind, into a reck-
" oning of the consequences of appellations. For example, a man that hath no use of
" speech at all (such as is born and remains perfectly deaf and dumb) if he set before
" his eyes a triangle, and by it two right angles (such as are the corners of a square fi-
" gure) he may by meditation compare and find, that the three angles of that triangle,
" are equal to those right angles that stand by it. But if another triangle be shewn him,
" different in shape from the former, he cannot know, without a new labour, whether
" the three angles of that also be equal to the same. But he that hath the use of words,
" when he observes that such equality was consequent, not to the length of the sides,
" nor to any particular thing in this triangle; but only to this, that the sides were
" straight, and the angles three; and that that was all for which he named it a trian-
" gle; will boldly conclude universally, that such equality of angles is in all triangles

These observations lead, in my opinion, to so easy an explanation of the transition from *particular* to *general reasoning*, that I shall make no apology for prosecuting the subject a little farther, before leaving this branch of my argument.

It will not, I apprehend, be denied, that when a learner first enters on the study of geometry, he considers the diagrams before him as individual objects, and as individual objects alone. In reading, for example, the demonstration just referred to, of the equality of the three angles of every triangle to two right angles, he thinks only of the triangle which is presented to him on the margin of the page. Nay, so completely does this particular figure engross his attention, that it is not without some difficulty he, in the first instance, transfers the demonstration to another triangle whose form is very different, or even to the same triangle placed in an inverted position. It is in order to correct this natural bias of the mind, that a judicious teacher, after satisfying himself that the student comprehends perfectly the force of the demonstration, as applicable to the particular triangle which Euclid has selected, is led to vary the diagram in different ways, with a view to show him, that the very same demonstration, expressed in the very same form of words, is equally applicable to

"whatsoever; and register his invention in these general terms, *Every triangle hath its three angles equal to two right angles*. And thus the consequence found in one particular, comes to be registered and remembered as an universal rule; and discharges our mental reckoning of time and place; and delivers us from all labour of the mind, saving the first; and makes that which was found true *here*, and *now*, to be true in *all times and places*."—*Hobbes, Of Man, Part I. Chap. iv.*

them all. In this manner he comes, by slow degrees, to comprehend the nature of general reasoning, establishing insensibly in his mind this fundamental logical principle, that when the enunciation of a mathematical proposition involves only a certain portion of the attributes of the diagram which is employed to illustrate it, the same proposition must hold true of any other diagram involving the same attributes, how much soever distinguished from it by other specific peculiarities*.

* In order to impress the mind still more forcibly with the same conviction, some have supposed that it might be useful, in an elementary work, such as that of Euclid, to omit the diagrams altogether, leaving the student to delineate them for himself, agreeably to the terms of the enunciation and of the construction. And were the study of geometry to be regarded merely as subservient to that of logic, much might be alleged in confirmation of this idea. Where, however, it is the main purpose of the teacher (as almost always happens) to familiarize the mind of his pupil with the fundamental principles of the science, as a preparation for the study of physics and of the other parts of mixed mathematics, it cannot be denied, that such a practice would be far less favourable to the memory than the plan which Euclid has adopted, of annexing to each theorem an appropriate diagram, with which the general truth comes very soon to be strongly associated. Nor is this circumstance found to be attended in practice with the inconvenience it may seem to threaten; inasmuch as the student, without any reflection whatever on logical principles, generalizes the particular example, according to the different cases which may occur, as easily and unconsciously as he could have applied to these cases the general enunciation.

The same remark may be extended to the other departments of our knowledge; in all of which it will be found useful to associate with every important general conclusion some particular example or illustration, calculated, as much as possible, to present an impressive image to the power of conception. By this means, while the example gives us a firmer hold, and a readier command of the general theorem, the theorem, in its turn, serves to correct the errors into which the judgment might be led by the specific peculiarities of the example. Hence, by the way, a strong argument in favour of the practice recommended by Bacon, of connecting *emblems* with *prænotions*, as the most

Of all the generalizations in geometry, there are none into which the mind enters so easily, as those which relate to diversities in point of *size* or *magnitude*. Even in reading the very first demonstrations of Euclid, the learner almost immediately sees, that the *scale* on which the diagram is constructed, is as completely out of the question as the *breadth* or the *colour* of the lines which it presents to his external senses. The demonstration, for example, of the fourth proposition, is transferred, without any conscious process of reflection, from the two triangles on the margin of the page, to those comparatively large ones which a public teacher exhibits on his board or slate to a hundred spectators. I have frequently, however, observed in beginners, while employed in copying such elementary diagrams, a disposition to make the copy, as nearly as possible, both in size and figure, a *fac simile* of the original.

The generalizations which extend to varieties of *form* and of *position*, are accomplished much more slowly; and, for this obvious reason, that these varieties are more strongly marked and discriminated from one another, as objects of vision and of conception. How difficult (comparatively speaking) in such instances, the generalizing process is, appears manifestly from

powerful of all *adminicles* to the faculty of memory; and hence the aid which this faculty may be expected to receive, in point of promptitude, if not of correctness, from a lively imagination. Nor is it the least advantage of this practice, that it supplies us at all times with ready and apposite illustrations to facilitate the communication of our general conclusions to others. But the prosecution of these hints would lead me too far astray from the subject of this section.

the embarrassment which students experience, in applying the fourth proposition to the demonstration of the fifth. The inverted position, and the partial coincidence of the two little triangles below the base, seem to render their mutual relation so different from that of the two separate triangles which had been previously familiarized to the eye, that it is not surprising this step of the reasoning should be followed, by the mere novice, with some degree of doubt and hesitation. Indeed, where nothing of this sort is manifested, I should be more inclined to ascribe the apparent quickness of his apprehension to a retentive memory, seconded by implicit faith in his instructor; than to regard it as a promising symptom of mathematical genius.

Another, and perhaps a better illustration of that natural logic which is exemplified in the generalization of mathematical reasonings, may be derived from those instances where the same demonstration applies, in the same words, to what are called, in geometry, the different *cases* of a proposition. In the commencement of our studies, we read the demonstration over and over, applying it successively to the different diagrams; and it is not without some wonder we discover, that it is equally adapted to them all. In process of time, we learn that this labour is superfluous; and if we find it satisfactory in one of the *cases*, can anticipate with confidence the justness of the general conclusion, or the modifications which will be necessary to accommodate it to the different forms of which the hypothesis may admit.

The algebraical *calculus*, however, when applied to geometry, places the foregoing doctrine in a point of view still more striking; "representing (to borrow the words of Dr Halley) "all the possible cases of a problem at one view; and often "in one general theorem comprehending whole sciences; which "deduced at length into propositions, and demonstrated after "the manner of the ancients, might well become the subject "of large treatises *." Of this remark, Halley gives an instance in a *formula*, which, when he first published it, was justly regarded "as a notable instance of the great use and comprehensiveness of algebraic solutions." I allude to his formula for finding universally the foci of optic lenses; an example which I purposely select, as it cannot fail to be familiarly known to all who have the slightest tincture of mathematical and physical science.

In such instances as these, it will not surely be supposed, that while we read the geometrical demonstration, or follow the successive steps of the algebraical process, our *general conceptions* embrace all the various possible cases to which our reasonings extend. So very different is the fact, that the wide grasp of the conclusion is discovered only by a sort of subsequent *induction*; and, till habit has familiarized us with similar discoveries, they never fail to be attended with a certain degree of unexpected delight. Dr Halley seems to have felt this strongly when the optical *formula* already mentioned first presented itself to his mind.

* Philos. Transact. No. 205. Miscell. Cur. Vol. I. p. 348.

In the foregoing remarks, I have borrowed my examples from mathematics, because, at the period of life when we enter on this study, the mind has arrived at a sufficient degree of maturity to be able to reflect accurately on every step of its own progress; whereas, in those general conclusions to which we have been habituated from childhood, it is quite impossible for us to ascertain, by any direct examination, what the processes of thought were, which originally led us to adopt them. In this point of view, the first doubtful and unassured steps of the young geometer, present to the logician a peculiarly interesting and instructive class of phenomena, for illustrating the growth and developement of our reasoning powers. The true theory, more especially of *general reasoning*, may be here distinctly traced by every attentive observer; and may hence be confidently applied (under due limitations) to all the other departments of human knowledge*.

* The view of general reasoning which is given above, appears to myself to afford (without any comment) a satisfactory answer to the following argument of the late worthy and learned Dr Price: "That the universality consists in the *idea*, and not merely in the *name*, as used to signify a number of particulars, *resembling* that which is the immediate object of reflection, is plain; because, was the idea to which the name answers, and which it recalls into the mind, only a particular one, we could not know to what other ideas to apply it, or what particular objects had the resemblance necessary to bring them within the meaning of the name. A person, in reading over a mathematical demonstration, certainly is conscious that it relates to somewhat else, than just that precise figure presented to him in the diagram. But if he knows not what else, of what use can the demonstration be to him? How is his knowledge enlarged by it? Or how shall he know afterwards to what to apply it?"

In a note upon this passage, Dr Price observes, that, "according to Dr Cudworth, abstract ideas are implied in the *cognoscitive power of the mind*; which, he says, contains in itself virtually (as the future plant or tree is contained in the seed) general no-

From what has been now said, it would appear, that, in order to arrive at a general conclusion in mathematics (and the same observation holds with respect to other sciences) *two* different processes of reasoning are necessary. The one is the *demonstration* of the proposition in question ; in studying which, we certainly think of nothing but the individual diagram before us. The other is, the train of thought by which we transfer the particular conclusion to which we have been thus led, to any other diagram to which the same enunciation is equally applicable. As this last train of thought is, in all cases, essentially the same, we insensibly cease to repeat it when the occasion for employing it occurs, till we come at length, without any reflection, to generalize our particular conclusion, the mo-

tions or exemplars of all things, which are exerted by it, or unfold and discover themselves, as occasions invite, and proper circumstances occur." "This no doubt (Dr Price adds) many will very freely condemn as whimsical and extravagant. I have, I own, a different opinion of it; but yet I should not care to be obliged to defend it." —*Review of the Principal Questions in Morals*, pp. 38, 39, 2d edit.

For my own part, I have no scruple to say, that I consider this fancy of Cudworth as not only *whimsical and extravagant*, but as altogether unintelligible; and yet it appears to me, that some confused analogy of the same sort must exist in the mind of every person who imagines that he has the power of forming *general conceptions* without the intermediation of language.

In the continuation of the same note, Dr Price seems disposed to sanction another remark of Dr Cudworth: in which he pronounces the opinion of the nominalists to be so *ridiculous and false, as to deserve no confutation*. I suspect, that when Dr Cudworth wrote this splenetic and oracular sentence, he was out of humour with some argument of Hobbes, which he found himself unable to answer. It is not a little remarkable, that the doctrine which he here treats with so great contempt, should, with a very few exceptions, have united the suffrages of all the soundest philosophers of the eighteenth century.

ment it is formed; or, in other words, to consider it as a proposition comprehending an indefinite variety of particular truths. When this habit is established, we are apt to imagine,—forgetting the slow steps by which the habit was acquired,—that the general conclusion is an *immediate* inference from a general demonstration; and that, although there was only one particular diagram present to our external senses, we must have been aware, at every step, that our thoughts were really conversant, *not* about this diagram, but about *general ideas*, or, in Dr Reid's language, *general conceptions*. Hence the familiar use among logicians of these scholastic and mysterious phrases, which, whatever attempts may be made to interpret them in a manner not altogether inconsistent with good sense, have unquestionably the effect of keeping out of view the real procedure of the human mind in the generalization of its knowledge.

Dr Reid seems to be of opinion, that it is by the power of forming *general conceptions*, that man is distinguished from the brutes; for he observes, that “Berkeley's system goes to destroy the barrier between the rational and animal natures.” I must own I do not perceive the justness of this remark, at least in its application to the system of the nominalists, as I have endeavoured to explain and to limit it in the course of this work. On the contrary, it appears to me, that the account which has been just given of *general reasoning*, by ascribing to a process of *logical deduction* (presupposing the previous exercise of *abstraction* or *analysis*) what Dr Reid attempts to explain by the scholastic, and not very intelligible phrase of *general*

conceptions, places the distinction between man and brutes in a far clearer and stronger light than that in which philosophers have been accustomed to view it. That it is to the exclusive possession of the faculty of *abstraction*, and of the other powers subservient to the use of general signs, that our species is chiefly indebted for its superiority over the other animals, I shall afterwards endeavour to show.

It still remains for me to examine an attempt which Dr Reid has made, to convict Berkeley of an *inconsistency*, in the statement of his argument against abstract general ideas. “Let us now consider (says he) the Bishop’s notion of generalizing. “An idea (he tells us) which, considered in itself, is particular, “becomes general, by being made to represent or stand for all “other particular ideas of the same sort. To make this plain by “an example: Suppose (says Berkeley) a geometrician is demonstrating the method of cutting a line into two equal parts. He “draws, for instance, a black line of an inch in length. This, “which is in itself a particular line, is nevertheless, with regard to “its signification, general, since, as it is there used, it represents “all particular lines whatsoever, so that what is demonstrated “of it, is demonstrated of all lines, or, in other words, of a line “in general. And as that particular line becomes general by “being made a sign, so the name *line*, which, taken absolutely, “ly, is particular, by being a sign, is made general.

“Here (continues Dr Reid) I observe, that when a particular idea is made a sign to represent and stand for *all of a sort*, “this supposes a distinction of things into sorts or species.

“To be of a *sort*, implies having those attributes which characterize the sort, and are common to all the individuals that belong to it. There cannot therefore be a *sort*, without general attributes; nor can there be any conception of a sort without a conception of those general attributes which distinguish it. The conception of a *sort*, therefore, is an abstract general conception.

“The particular idea cannot surely be made a sign of a thing of which we have no conception. I do not say, that you must have an *idea* of the sort; but surely you ought to *understand* or *conceive* what it means, when you make a particular idea a representative of it; otherwise your particular idea represents you know not what *.”

Although I do not consider myself as called upon to defend all the expressions which Berkeley may have employed in support of his opinion on this question, I must take the liberty of remarking, that, in the present instance, he appears to me to have been treated with an undue severity. By *ideas of the same sort*, it is plain he meant nothing more than *things called by the same name*, and, consequently, (if our illustrations are to be borrowed from mathematics) *comprehended under the terms of the same definition*. In such cases, the individuals thus classed together are completely *identified* as subjects of reasoning; insomuch, that what is proved with respect to one individual, must hold equally true of all the others. As it is an

* Pages 484, 485.

axiom in geometry, that things which are equal to one and the same thing, are equal to one another; so it may be stated as a maxim in logic, that whatever things have the same *name* applied to them, in consequence of their being comprehended in the terms of the same definition, may all be considered as *the same identical subject*, in every case where that definition is the principle on which our reasoning proceeds. In reasoning, accordingly, concerning any *sort* or *species* of things, our thoughts have no occasion to wander from the individual *sign* or *representative* to which the attention happens to be directed, or to attempt the fruitless task of grasping at those specific varieties which are avowedly excluded from the number of our premises. As every conclusion which is logically deduced from the definition must, of necessity, hold equally true of all the individuals to which the common name is applicable, these individuals are regarded merely as so many *units*, which go to the composition of the multitude comprehended under the collective or generic term. Nor has the power of conception any thing more to do in the business, than when we think of the *units* expressed by a particular number in an arithmetical computation.

The word *sort* is evidently transferred to our intellectual arrangements, from those distributions of material objects into separate heaps or collections, which the common sense of mankind universally leads them to make for the sake of the memory; or (which is perhaps nearly the same thing) with a view to the pleasure arising from the perception of order. A familiar instance of this presents itself in the shelves, and drawers,

and parcels, to which every shopkeeper has recourse for assorting, according to their respective denominations and prices, the various articles which compose his stock of goods. In one parcel (for example) he collects and incloses under one common *envelope*, all his *gloves* of a particular size and quality; in another, all his *gloves* of a different size and quality; and, in like manner, he proceeds with the stockings, shoes, hats, and the various other commodities with which his warehouse is filled. By this means, the attention of his shop-boy, instead of being bewildered among an infinitude of particulars, is confined to *parcels* or *assortments* of particulars; of each of which parcels a distinct idea may be obtained from an examination of any one of the individuals contained in it. These individuals, therefore, are, in his apprehension, nothing more than so many *units* in a multitude, any one of which units is perfectly equivalent to any other; while, at the same time, the parcels themselves, notwithstanding the multitude of units of which they are made up, distract his attention, and burden his memory as little, as if they were individual articles. The truth is, that they become to his mind *individual objects of thought*, like a *box* of counters, or a *rouleau* of guineas, or any of the other material aggregates with which his senses are conversant; or, to take an example still more apposite to our present purpose, like the phrases *one thousand*, or *one million*, when considered merely as simple *units* entering into the composition of a numerical sum.

The task which I have here supposed the tradesman to perform, in order to facilitate the work of his shop-boy, is exactly analogous, in its effect, to the aid which is furnished to the in-

fant understanding by the structure of its mother-tongue ; the generic words which abound in language assorting, and (if I may use the expression) *packing up*, under a comparatively small number of comprehensive terms, the multifarious objects of human knowledge *. In consequence of the generic terms to which, in civilized society, the mind is early familiarized, the vast multiplicity of things which compose the furniture of this globe are presented to it, *not* as they occur to the senses of the untaught savage, but as they have been arranged and distributed into parcels or assortments by the successive observations and reflections of our predecessors. Were these arrangements and distributions agreeable, in every instance, to sound philosophy, the chief source of the errors to which we are liable in all our general conclusions, would be removed ; but it would be too much to expect (with some late theorists) that, even in the most advanced state either of physical or of moral science, this supposition is ever to be realized in all its extent. At the same time, it must be remembered, that the obvious tendency of the progressive reason and experience of the species, is to diminish more and more the imperfections of the classifications which have been transmitted from ages of comparative ignorance ; and, of consequence, to render language more and more a safe and powerful organ for the investigation of truth.

The only science which furnishes an exception to these observations is *mathematics* ; a science essentially distinguished

* The same analogy had occurred to Locke. " To shorten its way to knowledge, and make each perception more comprehensive, the mind binds them into bundles."

from every other by this remarkable circumstance, that the precise import of its generic terms is fixed and ascertained by the *definitions* which form the basis of all our reasonings, and in which, of consequence, the very possibility of error in our classifications is precluded, by the virtual identity of all those hypothetical objects of thought to which the same generic term is applied.

I intend to prosecute this subject farther, before concluding my observations on general reasoning. At present, I have only to add to the foregoing remarks, that, in the comprehensive theorems of the philosopher, as well as in the assortments of the tradesman, I cannot perceive a single step of the understanding, which implies any thing more than the notion of *number*, and the use of a common name.

Upon the whole, it appears to me, that the celebrated dispute concerning abstract general ideas, which so long divided the schools, is now reduced, among correct thinkers, to this simple question of fact, Could the human mind, *without the use of signs of one kind or another*, have carried on general reasonings, or formed general conclusions? Before arguing with any person on the subject, I should wish for a categorical explanation on this preliminary point. Indeed, every other controversy connected with it turns on little more than the meaning of words.

A difference of opinion with respect to this question of *fact* (or rather, I suspect, a want of attention in some of the disputants to the great variety of *signs* of which the mind can avail

itself, independently of words) still continues to keep up a sort of distinction between the Nominalists and the Conceptualists. As for the Realists, they may, I apprehend, be fairly considered, in the present state of science, as having been already forced to lay down their arms.

That the doctrine of the nominalists has been stated by some writers of note in very unguarded terms, I do not deny*, nor

* Particularly by Hobbes, some of whose incidental remarks and expressions would certainly, if followed strictly out to their logical consequences, lead to the complete subversion of truth, as a thing real, and independent of human opinion. It is to this, I presume, that Leibnitz alludes, when he says of him, "*Thomas Hobbes, qui ut verum fatear, mihi plus quam nominalis videtur.*"

I shall afterwards point out the mistake by which Hobbes seems to me to have been misled. In the meantime, it is but justice to him to say, that I do not think he had any intention to establish those sceptical conclusions which, it must be owned, may be fairly deduced as corollaries from some of his principles. Of this I would not wish for a stronger proof than his favourite maxim, that "words are the *counters* of wise men, "but the *money* of fools;" a sentence which expresses, with marvellous conciseness, not only the proper function of language, as an instrument of reasoning, but the abuses to which it is liable, when in unskilful hands.

Dr Gillies, who has taken much pains to establish Aristotle's claims to all that is valuable in the doctrine of the nominalists, has, at the same time, represented him as the only favourer of this opinion, by whom it has been taught without any admixture of those errors which are blended with it in the works of its modern revivers. Even Bishop Berkeley himself is involved with Hobbes and Hume in the same sweeping sentence of condemnation. "The language of the nominalists seems to have been extremely liable to be perverted to the purposes of scepticism, as taking away the specific distinctions of things; and is in fact thus perverted by Hobbes, Berkeley, Hume, and their innumerable followers. But Aristotle's language is not liable to this abuse." (*Gillies's Aristotle*, Vol. I. p. 71, 2d edit.)

Among these sceptical followers of Berkeley, we must, I presume, include the late

am I certain that it was ever delivered by any one of the schoolmen in a form completely unexceptionable; but after the luminous, and, at the same time, cautious manner in which it has been unfolded by Berkeley and his successors, I own it appears to me not a little surprising, that men of talents and candour should still be found inclined to shut their eyes against the light, and to shelter themselves in the darkness of the middle ages. For my own part, the longer and the more attentively that I reflect on the subject, the more am I disposed to acquiesce in the *eulogium* bestowed on Roscellinus and his followers by Leibnitz; one of the very few philosophers, if not the only philosopher, of great celebrity, who seems to have been fully aware of the singular merits of those by whom this theory was originally proposed: "SECTA NOMINALIUM, OMNIUM INTER SCHOLASTICAS PROFUNDISSIMA, ET HODIERNÆ REFORMATÆ PHILOSOPHANDI RATIONI CONGRUENTISSIMA." It is a theory, indeed, much more congenial to the spirit of the eighteenth than of the eleventh century; nor must it be forgotten, that it was proposed and maintained at a period when the algebraical art (or to express myself more precisely, *universal arithmetic*) from which we now borrow our best illustrations in explaining and defending it, was entirely unknown.

learned and ingenious Dr Campbell; whose remarks on this subject I will, nevertheless, venture to recommend to the particular attention of my readers. Indeed, I do not know of any writer who has treated it with more acuteness and perspicuity. (See *Philosophy of Rhetoric*, Book II. chap. vii.)

II.

Continuation of the Subject.—Of Language considered as an Instrument of Thought.

HAVING been led, in defence of some of my own opinions, to introduce a few additional remarks on the controversy with respect to the theory of *general reasoning*, I shall avail myself of this opportunity to illustrate a little farther *another* topic, (intimately connected with the foregoing argument) on which the current doctrines of modern logicians seem to require a good deal more of explanation and restriction than has been commonly apprehended. Upon this subject I enter the more willingly, that, in my first volume, I have alluded to these doctrines in a manner which may convey, to some of my readers, the idea of a more complete acquiescence, on *my* part, in their truth, than I am disposed to acknowledge.

In treating of abstraction, I endeavoured to show that we *think*, as well as *speak*, by means of words, and that, without the use of language, our reasoning faculty (if it could have been at all exercised) must necessarily have been limited to *particular* conclusions alone. The effects, therefore, of ambiguous and indefinite terms are not confined to our communications with others, but extend to our private and solitary speculations. Dr Campbell, in his *Philosophy of Rhetoric*, has made some judicious and important observations on this subject; and, at a much earlier period, it drew the attention of Des Cartes; who, in the

course of a very valuable discussion with respect to the sources of our errors, has laid particular stress on those to which we are exposed from the employment of language as an instrument of thought. "And, lastly, in consequence of the habitual use of speech, all our ideas become associated with the words in which we express them; nor do we ever commit these ideas to memory, without their accustomed signs. Hence it is, that there is hardly any one subject, of which we have so distinct a notion as to be able to think of it abstracted from all use of language; and, indeed, as we remember words more easily than things, our thoughts are much more conversant with the former than with the latter. Hence, too, it is, that we often yield our assent to propositions, the meaning of which we do not understand; imagining that we have either examined formerly the import of all the terms involved in them, or that we have adopted these terms on the authority of others upon whose judgment we can rely *."

* "Et denique, propter loquelæ usum, conceptos omnes nostros verbis, quibus eos exprimimus, alligamus, nec eos, nisi simul cum istis verbis, memoriæ mandamus. Cumque facilius postea verborum quam rerum recordemur, vix unquam ullius rei conceptum habemus tam distinctum, ut illum ab omni verborum conceptu separemus; cogitationesque hominum fere omnium, circa verba magis quam circa res versantur; adeo ut persæpe vocibus non intellectis præbeant assensum, quia putant se illos olim intellexisse, vel ab aliis qui eas recte intelligebant, accepisse."—*Princ. Phil.* Pars Prima, lxxiv.

I have quoted a very curious passage, nearly to the same purpose, from Leibnitz, in a note annexed to my first volume (see note L.) I was not then aware of the previous attention which had been given to this source of error by Des Cartes; nor did I expect to find so explicit an allusion to it in the writings of Aristotle, as I have since observed in the following paragraph:

To these important considerations, it may be worth while to add, that whatever improvements may yet be made in language by philosophers, they never can relieve the student from the indispensable task of analyzing with accuracy the complex ideas he annexes to the terms employed in his reasonings. The use of general terms, as Locke has remarked, is learned,

Διο και των παρα την λεξιν ουτος ο τροπος θετερος· πρῶτον μιν, ὅτι μαλλον ἢ ἀπατη γινεται μετ' ἄλλων σκοπουμενοις ἢ καθ' αὐτους· ἢ μιν γαρ μετ' ἄλλων σκι-φης δια λογῶν· ἢ δὲ καθ' αὐτους, ὅχ' ἡττον δι' αὐτῶν τῶν πραγμάτων· εἴτα, καὶ καθ' αὐτους ἀπατασθαι συμβαίνει, ὅταν ἐπὶ τῷ λόγῳ ποιηται τὴν σκι-φιν· ἐπὶ δ' ἢ μιν ἀπατῇ ἐκ τῆς ὁμοιοτητος· ἢ δὲ ὁμοιοτης, ἐκ τῆς λεξίως.—*De Sophist. Elenchis*, Lib. I. cap. vii.

“Quocirca inter eos (Paralogismos) qui in dictione consistunt, hic fallendi modus “est ponendus. Primum, quia magis decipimur considerantes cum aliis, quam apud “nosmetipsos: nam consideratio cum aliis per sermonem instituitur; apud nosmetipsos “autem non minus fit per rem ipsam. Deinde et per nosmetipsos ut fallamur accidit, “cum in rebus considerandis sermo adhibetur: Præterea deceptio est ex similitudine: “similitudo autem ex dictione.”—Edit. *Du Val*. Vol. I. p. 289.

Lest it should be concluded, however, from this detached remark, that Aristotle had completely anticipated Locke and Condillac in their speculations with respect to language, considered as an instrument of thought, I must beg of my readers to compare it with the previous enumeration given by the same author, of those paralogisms or fallacies which lie in the diction, (*De Sophist. Elenchis*, Lib. i. cap. 4.);—recommending to them, at the same time, as a useful comment on the original, the twentieth chapter of the third book of a work entitled *Institutio Logica*, by the learned and justly celebrated Dr Wallis of Oxford. I select this work in preference to any other modern one on the same subject, as it has been lately pronounced, by an authority for which I entertain a sincere respect, to be “a complete and accurate treatise of logic, strictly according to “the Aristotelian method;” and as we are farther told that it is, “still used by many “in the university to which Wallis belonged, as the lecture-book in that department of “study.” I intend to quote part of this chapter on another occasion. At present, I shall only observe, that it does not contain the slightest reference to the passage which has led me to introduce these observations; and which, I believe, will be now very generally allowed to be of greater value than all those puerile distinctions put together, which Dr Wallis has been at so much pains to illustrate and to exemplify.

in many cases, before it is possible for us to comprehend their meaning ; and the greater part of mankind continue to use them through life, without ever being at the trouble to examine accurately the notions they convey. This is a study which every individual must carry on for himself ; and of which no rules of logic (how useful soever they may be in directing our labours) can supersede the necessity.

Of the essential utility of a cautious employment of words, both as a medium of communication and as an instrument of thought, many striking illustrations might be produced from the history of science during the time that the scholastic jargon was current among the learned ; a technical phraseology, which was not only ill-calculated for the discovery of truth, but which was dexterously contrived for the propagation of error ; and which gave to those who were habituated to the use of it, great advantages in controversy (at least in the judgment of the multitude) over their more enlightened and candid opponents. “ A blind wrestler, by fighting in a dark chamber (to adopt an allusion of Des Cartes) may not only conceal his defect, but may enjoy some advantages over those who see. It is the light of day only that can discover his inferiority.” The imperfections of this philosophy, accordingly, have been exposed by Des Cartes and his followers, less by the force of their reasonings, than by their teaching men to make use of their own faculties, instead of groping in the artificial darkness of the schools ; and to perceive the folly of expecting to advance science by ringing changes on words to which they annexed no clear or precise ideas.

In consequence of the influence of these views, the attention of our soundest philosophers was more and more turned, during the course of the last century, to the cultivation of that branch of logic which relates to the use of words. Mr Locke's observations on this subject form perhaps the most valuable part of his writings; and, since his time, much additional light has been thrown upon it by Condillac and his successors.

Important, however, as this branch of logic is in its practical applications; and highly interesting, from its intimate connection with the theory of the human mind, there is a possibility of pushing, to an erroneous and dangerous extreme, the conclusions to which it has led. Condillac himself falls, in no inconsiderable a degree, under this censure; having, upon more than one occasion, expressed himself as if he conceived it to be possible, by means of precise and definite terms, to reduce reasoning, in all the sciences, to a sort of mechanical operation, analogous, in its nature, to those which are practised by the algebraist, on letters of the alphabet. "The art of reasoning" (he repeats over and over) is nothing more than a language "well arranged."—"L'art de raisonner se réduit à une langue bien faite."

One of the first persons, as far as I know, who objected to the vagueness and incorrectness of this proposition, was M. De Gerando; to whom we are farther indebted for a clear and satisfactory exposition of the very important *fact* to which it relates. To this fact Condillac approximates nearly in various parts of his works; but never, perhaps, without some degree

of indistinctness and of exaggeration. The point of view in which it is placed by his ingenious successor, strikes me as so just and happy, that I cannot deny myself the pleasure of enriching my book with a few of his observations.

“ It is the distinguishing characteristic of a lively and vigorous conception, to push its speculative conclusions somewhat beyond their just limits. Hence, in the logical discussions of this estimable writer, these maxims (stated without any explanation or restriction), ‘ *That the study of a science is no thing more than the acquisition of a language ;*’ and, ‘ *that a science properly treated is only a language well-contrived.*’ Hence the rash assertion, ‘ *That mathematics possess no advantage over other sciences, but what they derive from a better phraseology ; and that all of these might attain to the same characters of simplicity and of certainty, if we knew how to give them signs equally perfect* *.”

“ The same task which must have been executed by those who contributed to the first formation of a language, and which is executed by every child when he learns to speak it, is repeated over in the mind of every adult when he makes use of his mother-tongue ; for it is only by the decomposition of his thoughts that he can learn to select the signs which he ought to employ, and to dispose them in a suitable order. Accordingly, those external actions which we call *speaking* or *writing*, are always accompanied with a philosophical process of the understanding, unless we content ourselves, as too

* Des Signes et de l'Art de Penser, &c. Introd. pp. xx. xxi.

“often happens, with repeating over mechanically what has
“been said by others. It is in *this* respect that languages, with
“their forms and rules, conducting (so to speak) those who
“use them, into the path of a regular analysis; tracing out to
“them, in a well-ordered discourse, the model of a perfect de-
“composition, may be regarded, *in a certain sense*, as *analyti-
“cal methods*.—But I stop short; Condillac, to whom this
“idea belongs, has developed it too well to leave any hope of
“improving upon his statement.”

In a note upon this passage, however, M. De Gerando has certainly improved not a little on the statement of Condillac. “In asserting (says he) that languages may be regarded as
“analytical methods, I have added the qualifying phrase,
“*in a certain sense*, for the word *method* cannot be employed
“here with exact propriety. Languages furnish the *occasions*
“and the *means of analysis*; that is to say, they afford us as-
“sistance in *following* that method; but they are not the me-
“thod itself. They resemble signals or finger-posts placed on
“a road to enable us to discover our way; and if they help
“us to analyze, it is because they are themselves the results,
“and, as it were, the monuments of an analysis which has
“been previously made; nor do they contribute to keep us
“in the right path, but in proportion to the degree of judg-
“ment with which that analysis has been conducted*.”

I was the more solicitous to introduce these excellent remarks, as I suspect that I have myself indirectly contributed

* Ibid. pp. 158, 159, Tom. I.

to propagate in this country the erroneous opinion which it is their object to correct. By some of our later writers it has not only been implicitly adopted, but has been regarded as a conclusion of too great value to be suffered to remain in the quiet possession of the moderns. "Aristotle (says the author of a "very valuable analysis of his works) *well knew* that our knowledge of *things* chiefly depending on the proper application "of language as an INSTRUMENT OF THOUGHT, the true art "of reasoning is nothing but a language accurately defined "and skilfully arranged; an opinion which, after many idle "declamations against his barren generalities and verbal trifling, philosophers have begun very generally to adopt *."

* Aristotle's Ethics, &c. by Dr Gillies, Vol. I. p. 94, 2d edit.

The passage in my first volume, to which I suspect an allusion is here made, is as follows:

"The technical terms, in the different sciences, render the appropriate language of philosophy a still more convenient INSTRUMENT OF THOUGHT, than those languages which have originated from popular use; and in proportion as these technical terms improve in point of precision and of comprehensiveness, they will continue to render our intellectual progress more certain and more rapid. 'While engaged (says Mr Lavoisier) in the composition of my Elements of Chemistry, I perceived, better than I had ever done before, the truth of an observation of Condillac, that we think only through the medium of words, and that languages are true analytic methods. Algebra, which, of all our modes of expression, is the most simple, the most exact, and the best adapted to its purpose, is, at the same time, a language and an analytical method. *The art of reasoning is nothing more than a language well arranged.*' The influence (I have added) which these very enlightened and philosophical views have already had on the doctrines of chemistry, cannot fail to be known to most of my readers."

When this paragraph was first written, I was fully aware of the looseness and indistinctness of Lavoisier's expressions; but as my only object in introducing the quotation was to illustrate the influence of general logical principles on the progress of particular

After this strong and explicit assertion of the priority of Aristotle's claim to the opinion which we are here told "*philosophers begin very generally to adopt*," it is to be hoped, that M. De Gerando will be in future allowed to enjoy the undisputed honour of having seen a little farther into this fundamental article of logic than the Stagirite himself.

sciences, I did not think it necessary, in the *introduction* to my work, to point out in what manner Condillac's propositions were to be limited and corrected. I am truly happy, for the sake of M. De Gerando, that I happened to transcribe them in the same vague and very exceptionable terms in which I found them sanctioned by the names of Condillac, and of one of the most illustrious of his disciples.

It will not, I hope, be considered as altogether foreign to the design of this note, if I remark further, how easy it is for a translator of Aristotle (in consequence of the unparalleled brevity which he sometimes affects) to accommodate the sense of the original, by the help of paraphrastical clauses, expressed in the phraseology of modern science, to every progressive step in the history of human knowledge. In truth, there is not one philosopher of antiquity, whose opinions, when they are stated in any terms but his own, are to be received with so great distrust.

III.

Continuation of the Subject.—Visionary Theories of some Logicians, occasioned by their inattention to the Essential Distinction between Mathematics and other Sciences.

IN a passage already quoted from De Gerando, he takes notice of what he justly calls *a rash assertion of Condillac*, "That mathematics possess no advantage over other sciences, but what they derive from a better phraseology; and that all of them might attain to the same characters of simplicity and of certainty, if we knew how to give them signs equally perfect."

Leibnitz seems to point at an idea of the same sort, in those obscure and enigmatical hints (not altogether worthy, in my opinion, of his powerful and comprehensive genius) which he has repeatedly thrown out, about the miracles to be effected by a new art of his own invention; to which art he sometimes gives the name of *Ars Combinatoria Characteristica*, and sometimes of *Ars Combinatoria Generalis ac Vera*. In one of his letters to Mr Oldenburg, he speaks of a plan he had long been meditating, of treating of the science of mind by means of mathematical demonstrations. "Many wonderful things (he adds) of this kind have occurred to me; which, at some future period, I shall explain to the public with that logical precision which the subject requires*." In the same letter, he inti-

* "Multa in hoc genere mira à me sunt observata, quæ aliquando, quo par est rigore, exposita dabo."

mates his belief in the possibility of inventing an art, "which, "with an exactitude resembling that of mechanism, may render the operations of reason steady and visible, and, in their effects on the minds of others, irresistible*." After which he proceeds thus :

"Our common algebra, which we justly value so highly, is no more than a branch of that general art which I have here in view. But, such as it is, it puts it out of our power to commit an error, even although we should wish to do so ; while it exhibits truth to our eyes like a picture stamped on paper by means of a machine. It must at the same time be recollected, that algebra is indebted for whatever it accomplishes in the demonstration of *general* theorems to the suggestions of a higher science ; a science which I have been accustomed to call *characteristical combination* ; very different, however, in its nature, from that which these words are likely, at first, to suggest to the hearer. The marvellous utility of this art I hope to illustrate, both by precepts and examples, if I shall be so fortunate as to enjoy health and leisure.

"It is impossible for me to convey an adequate idea of it in a short description. But this I may venture to assert, that no instrument (or organ) could easily be imagined of more powerful efficacy for promoting the improvement of the human understanding ; and that, supposing it to be adopted,

* "Quod velut mechanica ratione fixam et visibilem et (ut ita dicam) irresistibilem reddat rationem."

“as the common method of philosophizing, the time would very soon arrive, when we should be able to form conclusions concerning God and the Mind, with not less certainty than we do at present concerning figures and numbers*.”

The following passage is translated from another letter of Leibnitz to the same correspondent :

“The matter in question depends on another of much higher moment ; I mean, on a *general and true art of combination*, of the extensive influence of which I do not know that any person has yet been fully aware. This, in truth, does not differ from that sublime analysis, into the recesses of which Des Cartes himself, as far as I can judge, was not able to penetrate. But, in order to carry it into execution, an alphabet of human thoughts must be previously formed ; and for the invention of this alphabet, an analysis of axioms is indispensably necessary. I am not, however, surprised, that nobody has yet sufficiently considered it ; for we are, in general, apt to neglect what is easy ; and to take many things for granted, from their apparent evidence ; faults which, while they remain uncorrected, will for ever prevent us from reaching the summit of things intellectual, by the aid of a *calculus* adapted to moral as well as to mathematical science†.”

* Wallisii Opera, Vol. III. p. 621.

† Ibid. p. 633.

As these reveries of this truly great man are closely connected with the subsequent history of logical speculation in more than one country of Europe, I have been in-

In these extracts from Leibnitz, as well as in that quoted from Condillac, in the beginning of this article, the essential distinction between mathematics and the other sciences, in point of phraseology, is entirely overlooked. In the former science, where the use of an ambiguous word is impossible, it may be easily conceived how the solution of a problem may be reduced to something resembling the operation of a mill,—the conditions of the problem, when once translated from the common language into that of algebra, disappearing entirely from the view; and the subsequent process being almost mechanically regulated by general rules, till the final result is obtained. In the latter, *the whole* of the words about which our reasonings are conversant, admit, more or less, of different shades of meaning; and it is only by considering attentively the relation in which they stand to the immediate context, that the precise idea of the author in any particular instance is to be ascertained. In these sciences, accordingly, the constant and unremitting exercise of the attention is indispensably necessary, to prevent us, at every step of our progress, from going astray.

On this subject I have made various remarks in a volume lately published; to which I beg leave here to refer, in order to save the trouble of unnecessary repetitions*. From what I have there

duced to incorporate them, in an English version, with my own disquisitions. Some expressions, which, I am sensible, are not altogether agreeable to the idiom of our language, might have been easily avoided, if I had not felt it incumbent on me, in translating an author whose meaning, in this instance, I was able but very imperfectly to comprehend, to deviate as little as possible from his own words.

* Philosophical Essays, p. 153. et seq.

said, I trust it appears that, in following any train of reasoning, beyond the circle of the mathematical sciences, the mind must necessarily carry on, along with the logical deduction expressed in words, another logical process of a far nicker and more difficult nature;—that of fixing, with a rapidity which escapes our memory, the precise sense of every word which is ambiguous, by the relation in which it stands to the general scope of the argument. In proportion as the language of science becomes more and more exact, the difficulty of this task will be gradually diminished; but let the improvement be carried to any conceivable extent, not one step will have been gained in accelerating that æra, so sanguinely anticipated by Leibnitz and Condillac, when our reasonings in morals and politics shall resemble, in their mechanical regularity, and in their demonstrative certainty, the investigations of algebra. The improvements which language receives, in consequence of the progress of knowledge, consisting rather in a more precise distinction and classification of the various meanings of words, than in a reduction of these meanings in point of number, the task of mental induction and interpretation may be rendered more easy and unerring; but the necessity of this task can never be superseded, till every word which we employ shall be as fixed and invariable in its signification as an algebraical character, or as the name of a geometrical figure.

In the meantime, the intellectual superiority of one man above another, in all the different branches of moral and political philosophy, will be found to depend chiefly on the success

with which he has cultivated these *silent habits of inductive interpretation*,—much more, in my opinion, than on his acquaintance with those rules which form the great objects of study to the professed logician. In proof of this, it is sufficient for me to remind my readers, that the whole theory of syllogism proceeds on the supposition that the same word is always to be employed precisely in the same sense, (for otherwise, the syllogism would be vitiated by consisting of more than *three terms*) and, consequently, it takes for granted, in every rule which it furnishes for the guidance of our reasoning powers, that the nicest, and by far the most difficult part of the logical process has been previously brought to a successful termination.

In treating of a different question, I have elsewhere remarked, that although many authors have spoken of the wonderful *mechanism of speech*, none has hitherto attended to the far more wonderful *mechanism* which it puts into action behind the scene. A similar observation will be found to apply to what is commonly called the Art of Reasoning. The scholastic precepts which profess to teach it, reach no deeper than the very surface of the subject; being, all of them, confined to that part of the intellectual process which is embodied in the form of verbal propositions. On the most favourable supposition which can be formed with respect to them, they are superfluous and nugatory; but, in many cases, it is to be apprehended, that they interfere with the right conduct of the understanding, by withdrawing the attention from the cultivation of that mental logic on which the soundness of our conclusions essentially depends, and in the study of which (although some general

rules may be of use) every man must be, in a great measure, his own master*.

In the practical application of the foregoing conclusions, it cannot fail to occur, as a consideration equally obvious and important, that, in proportion as the objects of our reasoning are removed from the particular details with which our senses are conversant, the difficulty of these latent inductive processes must be increased. This is the real source of that incapacity for general speculation, which Mr Hume has so well described as a distinguishing characteristic of uncultivated minds. “General reasonings seem intricate, merely because they are general; nor is it easy for the bulk of mankind to distinguish, in a great number of particulars, that common circumstance in which they all agree, or to extract it, pure and unmixed, from the other superfluous circumstances. Every judgment or conclusion with them is particular. They cannot enlarge their views to those universal propositions which comprehend under them an infinite number of individuals, and include a whole science in a single theorem. Their eye is confounded with such an extensive prospect, and the conclusions deduced from it, even though clearly expressed, seem intricate and obscure†.”

Difficult, however, and even impossible as the task of gene-

* Those who are interested in this discussion, will enter more completely into my views, if they take the trouble to combine what is here stated with some observations I have introduced in the first volume of this work. See p. 177, et seq. 3d edit.

† Essay on Commerce.

ral speculation is to the bulk of mankind, it is nevertheless true, that it is the path which leads the cautious and skilful reasoner to all his most certain, as well as most valuable conclusions in morals and in politics. If a theorist, indeed, should expect, that these conclusions are in every particular instance to be realized, he would totally misapprehend their nature and application; inasmuch as they are only to be brought to an experimental test, by viewing them on an extensive scale, and continuing our observations during a long period of time. “When a man deliberates (says Mr Hume) concerning his conduct in any particular affair, and forms schemes in politics, trade, economy, or any business in life, he never ought to draw his arguments too fine, or connect too long a chain of consequences together. Something is sure to happen that will disconcert his reasoning, and produce an event different from what he expected. But when we reason upon general subjects, one may justly affirm, that our speculations can scarcely ever be too fine, provided they be just; and that the difference between a common man and a man of genius is chiefly seen in the shallowness or depth of the principles on which they proceed.” The same author afterwards excellently observes, “That general principles, however intricate they may seem, must always prevail, if they be just and sound, in the general course of things, though they may fail in particular cases; and that it is the chief business of philosophers to regard the general course of things.”—“I may add (continues Mr Hume) that it is also the chief business of politicians, especially in the domestic government of the state, where the public good, which is, or ought to be, their object, depends on the con-

“currence of a multitude of causes; not, as in foreign politics,
“on accidents and chances, and the caprices of a few persons*.”

To these profound reflections of Mr Hume, it may be added (although the remark does not bear directly on our present argument) that, in the systematical application of general and refined rules to their private concerns, men frequently err from calculating their measures upon a scale disproportionate to the ordinary duration of human life. This is one of the many mistakes into which projectors are apt to fall; and hence the ruin which so often overtakes them, while sowing the seeds of a harvest which others are to reap. A few years more might have secured to themselves the prize which they had in view; and changed the opinion of the world (which is always regulated by the accidental circumstances of failure or of success) from contempt of their folly, into admiration of their sagacity and perseverance.

It is observed by the Comte de Bussi, that “time remedies
“all mischances; and that men die unfortunate, only because
“they did not live long enough. Mareschal d’Estrée, who
“died rich at a hundred, would have died a beggar, had

* Essay on Commerce.

This contrast between the domestic and the foreign policy of a state, occurs more than once in Mr Hume’s writings; (See in particular the first paragraphs of his Essay on the Rise of Arts and Sciences). A similar observation had long before been made by Polybius. “There are two ways by which every kind of government is destroyed: either by some accident that happens from without; or some evil that arises within itself. What the first will be, it is not always easy to foresee; but the latter is certain and determinate.”—Book VI. Ex. 3. (Hampton’s Translation.)

"he lived only to eighty." The maxim, like most other apothegms, is stated in terms much too unqualified; but it may furnish matter for many interesting reflections, to those who have surveyed with attention the characters which have passed before them on the stage of life; or who amuse themselves with marking the trifling and fortuitous circumstances by which the multitude are decided, in pronouncing their verdicts of foresight or of improvidence.

IV.

Continuation of the Subject.—Peculiar and supereminent Advantages possessed by Mathematicians, in consequence of their definite Phraseology.

If the remarks contained in the foregoing articles of this section be just, it will follow, that the various artificial aids to our reasoning powers which have been projected by Leibnitz and others, proceed on the supposition (a supposition which is also tacitly assumed in the syllogistic theory) that, in all the sciences, the words which we employ have, in the course of our previous studies, been brought to a sense as unequivocal as the phraseology of mathematicians. They proceed on the supposition, therefore, that by far the most difficult part of the logical problem has been already solved. Should the period ever arrive, when the language of moralists and politicians shall be rendered as perfect as that of geometers and algebraists, then, indeed, may such contrivances as the *Ars Combinatoria* and the *Alphabet of human thoughts*, become interesting subjects of phi-

losophical discussion; although the probability is, that, even were that era to take place, they would be found nearly as useless in morals and politics, as the syllogistic art is acknowledged to be at present, in the investigations of pure geometry.

Of the peculiar and supereminent advantage possessed by mathematicians, in consequence of those fixed and definite relations which form the objects of their science, and the correspondent precision in their language and reasonings, I can think of no illustration more striking than what is afforded by Dr Halley's Latin version from an Arabic manuscript, of the two books of Appollonius Pergæus *de Sectione Rationis*. The extraordinary circumstances under which this version was attempted and completed (which I presume are little known beyond the narrow circle of mathematical readers) appear to me so highly curious, considered as matter of literary history, that I shall copy a short detail of them from Halley's preface.

After mentioning the accidental discovery in the Bodleian library, by Dr Bernard, Savilian Professor of astronomy, of the Arabic version of Appollonius, *περί λόγῳ ἀποτομῆς*, Dr Halley proceeds thus:

“Delighted, therefore, with the discovery of such a treasure, BERNARD applied himself diligently to the task of a Latin translation. But before he had finished a tenth part of his undertaking, he abandoned it altogether, either from his experience of its growing difficulties, or from the pressure of other avocations. Afterwards, when, on the death of Dr

“ Wallis, the Savilian professorship was bestowed on me, I was
“ seized with a strong desire of making a trial to complete what
“ Bernard had begun ;—an attempt, of the boldness of which
“ the reader may judge, when he is informed, that, in addition
“ to my own entire ignorance of the Arabic language, I had to
“ contend with the obscurities occasioned by innumerable pas-
“ sages which were either defaced or altogether obliterated.
“ With the assistance, however, of the sheets which Bernard
“ had left, and which served me as a key for investigating the
“ sense of the original, I began first with making a list of those
“ words, the signification of which his version had clearly as-
“ certained ; and then proceeded, by comparing these words,
“ wherever they occurred, with the train of reasoning in which
“ they were involved, to decypher, by slow degrees, the im-
“ port of the context ; till at last I succeeded in mastering the
“ whole work, and in bringing my translation (without the aid
“ of any other person) to the form in which I now give it to
“ the public *.”

When a similar attempt shall be made with equal success, in decyphering a moral or a political treatise written in an unknown tongue, *then*, and not till *then*, may we think of comparing the phraseology of these two sciences with the simple and rigorous language of the Greek geometers ; or with the more refined and abstract, but not less scrupulously logical system of *signs*, employed by modern mathematicians.

* Appollon. Perg. de Sectione Rationis, &c. Opera et Studio Edm. Halley. Oxon. 1706. In Præfat.

It must not, however, be imagined, that it is solely by the nature of the ideas which form the objects of its reasonings, even when combined with the precision and unambiguity of its phraseology, that mathematics is distinguished from the other branches of our knowledge. The *truths* about which it is conversant, are of an order altogether peculiar and singular; and the *evidence* of which they admit resembles nothing, either in degree or in kind, to which the same name is given, in any of our other intellectual pursuits. On these points also, Leibnitz and many other great men have adopted very incorrect opinions; and, by the authority of their names, have given currency to some logical errors of fundamental importance. My reasons for so thinking, I shall state as clearly and fully as I can, in the following section.

SECTION III.

Of Mathematical Demonstration.

I.

Of the Circumstance on which Demonstrative Evidence essentially depends.

THE peculiarity of that species of evidence which is called demonstrative, and which so remarkably distinguishes our mathematical conclusions from those to which we are led in other branches of science, is a fact which must have arrested the at-

tention of every person who possesses the slightest acquaintance with the elements of geometry. And yet I am doubtful if a satisfactory account has been hitherto given of the circumstance from which it arises. Mr Locke tells us, that “ what constitutes a demonstration is intuitive evidence at every “ step ;” and I readily grant, that if in a single step such evidence should fail, the other parts of the demonstration would be of no value. It does not, however, seem to me that it is on this consideration that the demonstrative evidence of the conclusion depends,—not even when we add to it another which is much insisted on by Dr Reid,—that, “ in demonstrative evidence, “ our first principles must be intuitively certain.” The inaccuracy of this remark I formerly pointed out when treating of the evidence of axioms ; on which occasion I also observed, that the first principles of our reasonings in mathematics are not *axioms*, but *definitions*. It is in this last circumstance (I mean the peculiarity of reasoning from *definitions*) that the true theory of mathematical demonstration is to be found ; and I shall accordingly endeavour to explain it at considerable length, and to state some of the more important consequences to which it leads.

That I may not, however, have the appearance of claiming, in behalf of the following discussion, an undue share of originality, it is necessary for me to remark, that the leading idea which it contains has been repeatedly started, and even to a certain length prosecuted, by different writers, ancient as well as modern ; but that, in all of them, it has been so blended with collateral considerations, altogether foreign to the point in

question, as to divert the attention both of writer and reader, from that single principle on which the solution of the problem hinges. The advantages which mathematics derives from the peculiar nature of those relations about which it is conversant ; from its simple and definite phraseology ; and from the severe logic so admirably displayed in the concatenation of its innumerable theorems, are indeed immense, and well entitled to a separate and ample illustration ; but they do not appear to have any necessary connection with the subject of this section. How far I am right in this opinion, my readers will be enabled to judge by the sequel.

It was already remarked, in the first chapter of this Part, that whereas, in all other sciences, the propositions which we attempt to establish, express facts real or supposed,—in mathematics, the propositions which we demonstrate only assert a connection between certain suppositions and certain consequences. Our reasonings, therefore, in mathematics, are directed to an object essentially different from what we have in view, in any other employment of our intellectual faculties ;—not to ascertain *truths* with respect to actual existences, but to trace the logical filiation of consequences which follow from an assumed *hypothesis*. If from this *hypothesis* we reason with correctness, nothing, it is manifest, can be wanting to complete the evidence of the result ; as this result only asserts a necessary connection between the supposition and the conclusion. In the other sciences, admitting that every ambiguity of language were removed, and that every step of our deductions were rigorously accurate, our conclusions would still be attend-

ed with more or less of uncertainty ; being ultimately founded on principles which may, or may not, correspond exactly with the fact *.

Hence it appears, that it might be possible, by devising a set of arbitrary definitions, to form a science which, although conversant about moral, political, or physical ideas, should yet be as certain as geometry. It is of no moment, whether the definitions assumed correspond with facts or not, provided they do not express impossibilities, and be not inconsistent with each other. From these principles a series of consequences may be deduced by the most unexceptionable reasoning ; and the results obtained will be perfectly analogous to mathematical propositions. The terms *true* and *false*, cannot be applied to them ; at least in the sense in which they are applicable to propositions relative to facts. All that can be said is, that they are or are not connected with the definitions which form the principles of the science ; and, therefore, if we choose to call our conclusions *true* in the one case, and *false* in the other, these epithets must be understood merely to refer to their connection with the *data*, and not to their correspondence with things actually existing, or with events

* This distinction coincides with one which has been very ingeniously illustrated by M. Prevost in his philosophical essays. *See his remarks on those sciences which have for their object *absolute truth*, considered in contrast with those which are occupied only about *conditional or hypothetical truths*. Mathematics is a science of the latter description ; and is therefore called by M. Prevost a *science of pure reasoning*. In what respects my opinion on this subject differs from his, will appear afterwards.—*Essais de Philosophie*, Tom. II. p. 9. et seq.

which we expect to be realized in future. An example of such a science as that which I have now been describing, occurs in what has been called by some writers *theoretical mechanics*; in which, from arbitrary hypotheses concerning physical laws, the consequences are traced which *would* follow, if such was really the order of nature.

In those branches of study which are conversant about moral and political propositions, the nearest approach which I can imagine to a hypothetical science, analogous to mathematics, is to be found in a code of municipal jurisprudence; or rather might be conceived to exist in such a code, if systematically carried into execution, agreeably to certain general or fundamental principles. Whether these principles should or should not be founded in justice and expediency, it is evidently possible, by reasoning from them consequentially, to create an artificial or conventional body of knowledge, more systematical, and, at the same time, more complete in all its parts, than, in the present state of our information, any science can be rendered, which ultimately appeals to the eternal and immutable standards of truth and falsehood, of right and wrong. This consideration seems to me to throw some light on the following very curious parallel which Leibnitz has drawn (with what justness I presume not to decide) between the works of the Roman civilians and those of the Greek geometers. Few writers certainly have been so fully qualified as he was to pronounce on the characteristic merits of both.

“ I have often said, that, after the writings of geometricians,

“ there exists nothing which, in point of force and of subtilty,
“ can be compared to the works of the Roman lawyers. And,
“ as it would be scarcely possible, from mere intrinsic evi-
“ dence, to distinguish a demonstration of Euclid’s from one
“ of Archimedes or of Appollonius (the style of all of them ap-
“ pearing no less uniform than if reason herself was speaking
“ through their organs), so also the Roman lawyers all resem-
“ ble each other like twin-brothers ; insomuch that, from the
“ style alone of any particular opinion or argument, hardly
“ any conjecture could be formed with respect to the author.
“ Nor are the traces of a refined and deeply meditated system
“ of natural jurisprudence anywhere to be found more visible,
“ or in greater abundance. And, even in those cases where
“ its principles are departed from, either in compliance with
“ the language consecrated by technical forms, or in conse-
“ quence of new statutes, or of ancient traditions, the conclu-
“ sions which the assumed hypothesis renders it necessary to
“ incorporate with the eternal dictates of right reason, are de-
“ duced with the soundest logic, and with an ingenuity which
“ excites admiration. Nor are these deviations from the law
“ of nature so frequent as is commonly imagined *.”

I have quoted this passage merely as an illustration of the analogy already alluded to, between the systematical unity of mathematical science, and that which is *conceivable* in a system of municipal law. How far this unity is exemplified

* Leibnitz, Op. Tom. IV. p. 254.

in the Roman code, I leave to be determined by more competent judges*.

As something analogous to the hypothetical or conditional conclusions of mathematics may thus be fancied to take place in speculations concerning moral or political subjects, and actually does take place in theoretical mechanics; so, on the other hand, if a mathematician should affirm, of a general property of the circle, that it applies to a particular figure described on paper, he would at once degrade a geometrical theorem to the level of a fact resting ultimately on the evidence of our imperfect senses. The accuracy of his reasoning could never bestow on his proposition that peculiar evidence which is properly called *mathematical*, as long as the fact remained uncertain, whether all the straight lines drawn from the centre to the circumference of the figure were mathematically equal.

These observations lead me to remark a very common misconception concerning mathematical definitions; which are of a nature essentially different from the definitions employed in any of the other sciences. It is usual for writers on logic, after taking notice of the errors to which we are liable in consequence of the ambiguity of words, to appeal to

* It is not a little curious that the same code which furnished to this very learned and philosophical jurist, the subject of the *eulogium* quoted above, should have been lately stigmatized by an English lawyer, eminently distinguished for his acuteness and originality, as "an enormous mass of confusion and inconsistency." Making all due allowances for the exaggerations of Leibnitz, it is difficult to conceive that his opinion, on a subject which he had so profoundly studied, should be so very widely at variance with the truth.

the example of mathematicians, as a proof of the infinite advantage of using, in our reasonings, such expressions only as have been carefully defined. Various remarks to this purpose occur in the writings both of Mr Locke and of Dr Reid. But the example of mathematicians is by no means applicable to the sciences in which these eminent philosophers propose that it should be followed; and, indeed, if it were copied as a model in any other branch of human knowledge, it would lead to errors fully as dangerous as any which result from the imperfections of language. The real fact is, that it has been copied much more than it ought to have been, or than would have been attempted, if the peculiarities of mathematical evidence had been attentively considered.

That in mathematics there is no such thing as an ambiguous word, and that it is to the proper use of definitions we are indebted for this advantage, must unquestionably be granted. But this is an advantage easily secured, in consequence of the very limited vocabulary of mathematicians, and the distinctness of the ideas about which their reasonings are employed. The difference, besides, in *this* respect, between mathematics and the other sciences, however great, is yet only a difference in degree; and is by no means sufficient to account for the essential distinction which every person must perceive between the irresistible cogency of a mathematical demonstration, and that of any other process of reasoning.

From the foregoing considerations it appears, that, in mathematics, definitions answer two purposes; first, To prevent am-

biguities of language; and, secondly, To serve as the principles of our reasoning. It appears further, that it is to the latter of these circumstances (I mean to the employment of hypotheses instead of facts, as the data on which we proceed) that the peculiar force of demonstrative evidence is to be ascribed. It is however only in the *former* use of definitions, that any parallel can be drawn between mathematics and those branches of knowledge which relate to facts; and, therefore, it is not a fair argument in proof of their *general* utility, to appeal to the unrivalled certainty of mathematical science,—a pre-eminence which that science derives from a source altogether different, though comprehended under the same name, and which she will for ever claim as her own exclusive prerogative*.

Nor ought it to be forgotten, that it is in pure mathematics alone, that definitions can be attempted with propriety at the outset of our investigations. In most other instances, some previous discussion is necessary to shew, that the definitions which we lay down correspond with facts; and, in many cases, the formation of a just definition is the end to which our inquiries are directed. It is very judiciously observed by Mr Burke, in his Essay on Taste, that “when we define, we
“are in danger of circumscribing nature within the bounds of
“our own notions, which we often take up by hazard, or em-
“brace on trust, or form out of a limited and partial conside-
“ration of the object before us, instead of extending our ideas

* These two classes of definitions are very generally confounded by logicians; among others, by the Abbé de Condillac. See *La Logique, ou les premiers développemens de l'Art de Penser*, Chap. VI.

“ to take in all that nature comprehends, according to her manner of combining. We are limited in our inquiry by the strict laws to which we have submitted at our setting out.”

The same author adds, that “ a definition may be very exact, and yet go but a very little way towards informing us of the nature of the thing defined ;” and that, “ in the order of things, a definition (let its virtue be what it will) ought rather to follow than to precede our inquiries, of which it ought to be considered as the result.”

From a want of attention to these circumstances, and from a blind imitation of the mathematical arrangement, in speculations where facts are involved among the principles of our reasonings, numberless errors in the writings of philosophers might be easily traced. The subject is of too great extent to be pursued any farther here ; but it is well entitled to the examination of all who may turn their thoughts to the reformation of logic. That the ideas of Aristotle himself, with respect to it, were not very precise, must, I think, be granted, if the following statement of his ingenious commentator be admitted as correct.

“ Every general term (says Dr Gillies) is considered by Aristotle as the abridgement of a definition ; and every definition is denominated by him a *collection*, because it is the result always of observation and comparison, and often of many observations and of many comparisons*.”

* Gillies's Aristotle, Vol. I. p. 92. 2d edit.

These two propositions will be found, upon examination, not very consistent with each other. The first, "That every general term is the abridgement of a definition," applies indeed admirably to mathematics; and touches with singular precision on the very circumstance which constitutes (in my opinion) the peculiar cogency of mathematical reasoning. But it is to mathematics that it applies exclusively. If adopted as a logical maxim in other branches of knowledge, it would prove an endless source of sophistry and error.—The second proposition, on the other hand, "That every definition is the result of observation and comparison, and often of many observations and many comparisons;" however applicable to the definitions of natural history, and of other sciences which relate to *facts*, cannot, in one single instance, apply to the definitions of geometry; inasmuch as these definitions are neither the result of observations nor of comparisons, but the *hypotheses*, or first principles, on which the whole science rests.

If the foregoing account of demonstrative evidence be just, it follows, that no chain of reasoning whatever can deserve the name of a *demonstration* (at least in the mathematical sense of that word) which is not ultimately resolvable into hypotheses or definitions*. It has been already shown, that this is the

* Although the account given by Locke of what constitutes a *demonstration*, be different from that which I have here proposed, he admits the *converse* of this doctrine as manifest; viz. That if we reason accurately from our own definitions, our conclusions will possess *demonstrative evidence*; and "hence (he observes with great truth) it comes to pass, that one may often meet with very clear and coherent discourses, that amount yet to nothing." He afterwards remarks, that "one may make demonstrations and undoubted propositions in words, and yet thereby advance not one jot in