case with geometry: And it is also manifestly the case with arithmetic, another science to which, in common with geometry, we apply the word mathematical. The simple arithmetical equations $2 + 2 = 4$; $2 + 3 = 5$, and other elementary propositions of the same sort, are (as was formerly observed) mere *definitions* *; perfectly analogous, in this respect, to those at the beginning of Euclid; and it is from a few fundamental principles of this sort, or at least from principles which are essentially of the same description, that all the more complicated results in the science are derived.

To this general conclusion, with respect to the nature of mathematical demonstration, an exception may perhaps be, at first sight, apprehended to occur, in our reasonings concerning geometrical *problems*; all of these reasonings (as is well known) resting ultimately upon a particular class of principles called *postulates*, which are commonly understood to be so very nearly akin to *axioms*, that both might, without impropriety, be comprehended under the same name. "The definition of a "postulate (says the learned and ingenious Dr Hutton) will "nearly agree also to an axiom, which is a self-evident theo-"rem, as a postulate is a self-evident problem †." The same

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"the knowledge of the truth of things." "Of this sort (he adds) a man may find an "infinite number of propositions, reasonings, and conclusions, in books of metaphysics, "school-divinity, and some sort of natural philosophy; and, after all, know as little of "God, spirits, or bodies, as he did before he set out."—*Essay on Human Understanding*, Book IV. chap. viii.

* See page 32.

† Mathematical Dictionary, Art. *Postulate*.

author, in another part of his work, quotes a remark from Dr Barrow, that "there is the same affinity between postulates "and problems, as between axioms and theorems *." Dr Wallis, too, appears, from the following passage, to have had a decided leaning to this opinion: "According to some, the "difference between axioms and postulates is analogous to that "between theorems and problems; the former expressing "truths which are self-evident, and from which other proposi-"tions may be deduced; the latter, operations which may be "easily performed, and by the help of which more difficult "constructions may be effected." He afterwards adds, "This "account of the distinction between postulates and axioms seems "not ill adapted to the division of mathematical propositions "into problems and theorems. And indeed, if both postulates "and axioms were to be comprehended under either of these "names, the innovation would not, in my opinion, afford much "ground for censure †."

In opposition to these very high authorities, I have no hesitation to assert, that it is with the *definitions* of Euclid and not with the *axioms* that the *postulates* ought to be compared, in respect of their logical character and importance;—inasmuch as all the *demonstrations* in plane geometry are ultimately founded on the former, and all the *constructions* which it recognizes as legitimate, may be resolved ultimately into the latter.

* Ibid. Art. *Hypothesis.*

† Wallisii Opera, Vol. II. pp. 667, 668.

To this remark it may be added, that, according to Euclid's view of the subject, the problems of geometry are not less hypothetical and speculative (or, to adopt the phraseology of some late writers, not less objects of *pure reason*) than the theorems; the possibility of drawing a *mathematical* straight line, and of describing a *mathematical* circle, being assumed in the construction of every problem, in a way quite analogous to that in which the enunciation of a theorem assumes the *existence* of straight lines and of circles corresponding to their *mathematical* definitions. The reasoning, therefore, on which the solution of a problem rests, is not less *demonstrative* than that which is employed in proof of a theorem. Grant the possibility of the three operations described in the postulates, and the correctness of the solution is as mathematically certain, as the truth of any property of the triangle or of the circle. The three postulates of Euclid are, indeed, nothing more than the definitions of a circle and a straight line thrown into a form somewhat different; and a similar remark may be extended to the corresponding distribution of propositions into theorems and problems. Notwithstanding the many conveniences with which this distribution is attended, it was evidently a matter of choice rather than of necessity; all the truths of geometry easily admitting of being moulded into either shape, according to the fancy of the mathematician. As to the *axioms*, there cannot be a doubt (whatever opinion may be entertained of their utility or of their insignificance) that they stand precisely in the same relation to both classes of propositions *.

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* In farther illustration of what is said above, on the subject of postulates and of

II.

Continuation of the Subject.—How far it is true that all Mathematical Evidence is resolvable into Identical Propositions.

I HAD occasion to take notice, in the first section of the preceding chapter, of a theory with respect to the nature of ma-

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problems, I transcribe, with pleasure, a short passage from a learned and interesting memoir, just published, by an author intimately and critically conversant with the classical remains of Greek geometry.

"The description of any geometrical line from the data by which it is defined, must "always be assumed as possible, and is admitted as the legitimate means of a geome-"trical construction: it is therefore properly regarded as a *postulate*. Thus, the de-"scription of a straight line and of a circle are the postulates of plane geometry as-"sumed by Euclid. The description of the three *conic sections*, according to the de-"finitions of them, must also be regarded as postulates; and though not formally stated "like those of Euclid, are in truth admitted as such by Apollonius, and all other "writers on this branch of geometry. The same principle must be extended to all su-"perior lines.

"It is true, however, that the properties of such superior lines may be treated of, "and the description of them may be assumed in the solution of problems, without an "actual delineation of them.——For it must be observed, that no lines whatever, not "even the straight line or circle, can be truly represented to the senses according to the "strict mathematical definitions; but this by no means affects the theoretical conclusions "which are logically deduced from such definitions. It is only when geometry is applied "to practice, either in mensuration, or in the arts connected with geometrical princi-"ples, that accuracy of delineation becomes important."—See an *Account of the Life and Writings of Robert Simson*, M. D. By the Rev. William Trail, LL. D. Published by G. and W. Nicol, London, 1812.

thematical evidence, very different from that which I have been now attempting to explain. According to this theory (originally, I believe, proposed by Leibnitz) we are taught, that all mathematical evidence ultimately resolves into the perception of identity; the innumerable variety of propositions which *have* been discovered, or which *remain* to be discovered in the science, being only diversified expressions of the simple formula, a = a. A writer of great eminence, both as a mathematician and a philosopher, has lately given his sanction, in the strongest terms, to this doctrine; asserting, that all the prodigies performed by the geometrician are accomplished by the constant repetition of these words,—*the same is the same*. "Le géomêtre "avance de supposition en supposition. Et rétournant sa "pensée sous mille formes, c'est en répétant sans cesse, *le même* "*est le même*, qu'il opère tous ses prodiges."

As this account of mathematical evidence is quite irreconcilable with the scope of the foregoing observations, it is necessary, before proceeding farther, to examine its real import and amount; and what the circumstances are from which it derives that plausibility which it has been so generally supposed to possess.

That all mathematical evidence resolves ultimately into the perception of identity, has been considered by some as a consequence of the commonly received doctrine, which represents the axioms of Euclid as the *first principles* of all our subsequent reasonings in geometry. Upon this view of the subject I have nothing to offer, in addition to what I have already stated.

The argument which I mean to combat at present is of a more subtile and refined nature; and, at the same time, involves an admixture of important truth, which contributes not a little to the specious verisimilitude of the conclusion. It is founded on this simple consideration, that the geometrical notions of *equality* and of *coincidence* are the same; and that, even in comparing together spaces of different figures, all our conclusions ultimately lean with their whole weight on the imaginary application of one triangle to another;—the object of which imaginary application is merely to *identify* the two triangles together, in every circumstance connected both with magnitude and figure *.

Of the justness of the assumption on which this argument proceeds, I do not entertain the slightest doubt. Whoever has the curiosity to examine any one theorem in the elements

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* It was probably with a view to the establishment of this doctrine, that some foreign elementary writers have lately given the name of *identical triangles* to such as agree with each other, both in sides, in angles, and in area. The differences which may exist between them in respect of place, and of relative position (differences which do not at all enter into the reasonings of the geometer) seem to have been considered as of so little account in discriminating them as separate objects of thought, that it has been concluded they only form *one and the same triangle*, in the contemplation of the logician.

This idea is very explicitly stated, more than once, by Aristotle: ισα ὧν το ποσον ἑν. "Those things are equal whose quantity is the same;" (Met. iv. c. 16.) and still more precisely in these remarkable words, εν τουτοις ἡ ισοτης ἑνοτης; "In mathematical quantities, equality is identity." (Met. x. c. 3.)

For some remarks on this last passage, see Note (F.)

of plane geometry, in which different spaces are compared together, will easily perceive, that the demonstration, when traced back to its first principles, terminates in the fourth proposition of Euclid's first book: a proposition of which the proof rests entirely on a supposed application of the one triangle to the other. In the case of equal triangles which differ in figure, this expedient of ideal superposition cannot be directly and immediately employed to evince their equality; but the demonstration will nevertheless be found to rest at bottom on the same species of evidence. In illustration of this doctrine, I shall only appeal to the thirty-seventh proposition of the first book, in which it is proved that triangles on the same base, and between the same parallels, are equal; a theorem which appears, from a very simple construction, to be only a few steps removed from the fourth of the same book, in which the supposed application of the one triangle to the other, is the only medium of comparison from which their equality is inferred.

In general, it seems to be almost self-evident, that the equality of two spaces can be demonstrated only by showing, either that the one might be applied to the other, so that their boundaries should exactly coincide; or that it is possible, by a geometrical construction, to divide them into compartments, in such a manner, that the sum of parts in the one may be proved to be equal to the sum of parts in the other, upon the principle of superposition. To devise the easiest and simplest constructions for attaining this end, is the object to which the skill and invention of the geometer is chiefly directed.

Nor is it the geometer alone who reasons upon this principle. If you wish to convince a person of plain understanding, who is quite unacquainted with mathematics, of the truth of one of Euclid's theorems, it can only be done by exhibiting to his eye, operations exactly analogous to those which the geometer presents to the understanding. A good example of this occurs in the sensible or experimental illustration which is sometimes given of the forty-seventh proposition of Euclid's first book. For this purpose, a card is cut into the form of a right angled triangle, and square pieces of card are adapted to the different sides; after which, by a simple and ingenious contrivance, the different squares are so dissected, that those of the two sides are made to cover the same space with the square of the hypothenuse. In truth, this mode of comparison by a superposition, actual or ideal, is the only test of equality which it is possible to appeal to; and it is from this (as seems from a passage in Proclus to have been the opinion of Apollonius) that, in point of logical rigour, the *definition* of geometrical equality should have been taken *. The subject is discussed at great

* I do not think, however, that it would be fair, on this account, to censure Euclid for the arrangement which he has adopted, as he has thereby most ingeniously and dextrously contrived to keep out of the view of the student some very puzzling questions, to which it is not possible to give a satisfactory answer till a considerable progress has been made in the elements. When it is stated in the form of a self-evident truth, that magnitudes which coincide, or which exactly fill the same space, are equal to one another; the beginner readily yields his assent to the proposition; and this assent, without going any farther, is all that is required in any of the demonstrations of the first six books: whereas, if the proposition were converted into a definition, by saying, "Equal magnitudes are "those which coincide, or which exactly fill the same space;" the question would immediately occur, Are *no* magnitudes equal, but those to which this test of equality can

length, and with much acuteness, as well as learning, in one of the mathematical lectures of Dr Barrow; to which I must refer those readers who may wish to see it more fully illustrated.

I am strongly inclined to suspect, that most of the writers who have maintained that all mathematical evidence resolves ultimately into the perception of identity, have had a secret reference, in their own minds, to the doctrine just stated; and that they have imposed on themselves, by using the words *identity* and *equality* as literally synonymous and convertible terms. This does not seem to be at all consistent, either in point of expression or of fact, with sound logic. When it is affirmed (for instance) that "if two straight lines in a circle "intersect each other, the rectangle contained by the segments "of the one is equal to the rectangle contained by the seg- "ments of the other;" can it with any propriety be said, that the relation between these rectangles may be expressed by the formula a = a? Or, to take a case yet stronger, when it is affirmed, that "the area of a circle is equal to that of a tri- "angle having the circumference for its base, and the radius "for its altitude;" would it not be an obvious paralogism to infer from this proposition, that the triangle and the circle are

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be applied? Can the relation of equality not subsist between magnitudes which differ from each other in figure? In reply to this question, it would be necessary to explain the definition, by adding, That those magnitudes likewise are said to be equal, which are capable of being divided or dissected in such a manner that the parts of the one may severally coincide with the parts of the other;—a conception much too refined and complicated for the generality of students at their first outset; and which, if it were fully and clearly apprehended, would plunge them at once into the profound speculation concerning the comparison of rectilinear with curvilinear figures.

one and the same thing? In this last instance, Dr Barrow himself has thought it necessary, in order to reconcile the language of Archimedes with that of Euclid, to have recourse to a scholastic distinction between *actual* and *potential coincidence*; and, therefore, if we are to avail ourselves of the principle of *superposition*, in defence of the fashionable theory concerning mathematical evidence, we must, I apprehend, introduce a correspondent distinction between *actual and potential identity* *.

That I may not be accused, however, of misrepresenting the opinion which I am anxious to refute, I shall state it in the words of an author, who has made it the subject of a particular dissertation; and who appears to me to have done as much justice to his argument as any of its other defenders.

"Omnes mathematicorum propositiones sunt identicæ, et "repræsentantur hac formula, a = a. Sunt veritates identi-"cæ, sub varia forma expressæ, imo ipsum, quod dicitur con-"tradictionis principium, vario modo enunciatum et involu-

* "Cum demonstravit Archimedes circulum æquari rectangulo triangulo cujus basis "radio circuli, cathetus peripheriæ exæquetur, nil ille, siquis propius attendat, aliud "quicquam quam aream circuli ceu polygoni regularis indefinite multa latera habentis, "in tot dividi posse minutissima triangula, quæ totidem exilissimis dicti trianguli trigo-"nis æquentur; eorum verò triangulorum æqualitas è sola congruentia demonstratur "in elementis. Unde consequenter Archimedes circuli cum triangulo (sibi quantum-"vis dissimili) congruentiam demonstravit.—Ita congruentiæ nihil obstat figurarum "dissimilitudo; verùm seu similes sive dissimiles sint, modò æquales, semper poterunt, "semper posse debebunt congruere. Igitur octavum axioma vel nullo modo conversum "valet, aut universaliter converti potest; nullo modo, si quæ isthic habetur congruentia "designet *actualem congruentiam*; universim, si de *potentiali* tantùm accipiatur."—*Lectiones Mathematicæ*. Sect. V.

"tum; siquidem omnes hujus generis propositiones reverâ in "eo continentur. Secundum nostram autem intelligendi fa-"cultatem ea est propositionum differentia, quod quædam "longa ratiociniorum serie, alia autem breviore via, ad pri-"mum omnium principium reducantur, et in illud resolvan-"tur. Sic v. g. propositio 2 + 2 = 4 statim huc cedit 1 + 1 "+ 1 + 1 = 1 + 1 + 1 + 1; i. e. idem est idem; et proprie "loquendo, hoc modo enunciari debet.—Si contingat, adesse "vel existere quatuor entia, tum existunt quatuor entia; nam "de existentia non agunt geometræ, sed ea hypothetice tantum "subintelligitur. Inde summa oritur certitudo ratiocinia per-"spicienti; observat nempe idearum identitatem; et hæc est "evidentia assensum immediate cogens, quam mathematicam "aut geometricam vocamus. Mathesi tamen sua natura priva "non est et propria; oritur etenim ex identitatis perceptione, "quæ locum habere potest, etiamsi ideæ non repræsentent "extensum *."

With respect to this passage I have only to remark (and the same thing is observable of every other attempt which has been made to support the opinion in question), that the author confounds two things essentially different;—the nature of the *truths* which are the objects of a science, and the

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* The above extract (from a dissertation printed at Berlin in 1764) has long had a very extensive circulation in this country, in consequence of its being quoted by Dr Beattie, in his Essay on Truth, (see p. 221. 2d edit.) As the learned author of the essay has not given the slightest intimation of his own opinion on the subject, the doctrine in question has, I suspect, been considered as in some measure sanctioned by his authority. It is only in this way that I can account for the facility with which it has been admitted by so many of our northern logicians.

nature of the *evidence* by which these truths are established. Granting, for the sake of argument, that all mathematical propositions may be represented by the formula a = a, it would not therefore follow, that every step of the reasoning leading to these conclusions, was a proposition of the same nature; and that, to feel the full force of a mathematical demonstration, it is sufficient to be convinced of this maxim, that *every thing may be truly predicated of itself*; or, in plain English, that *the same is the same.* A paper written in cypher, and the interpretation of that paper by a skilful decypherer, may, in like manner, be considered as, to all intents and purposes, one and the same thing. They are so, in fact, just as much as one side of an algebraical equation is the same thing with the other. But does it therefore follow, that the whole evidence upon which the art of decyphering proceeds, resolves into the perception of identity?

It may be fairly questioned too, whether it can, with strict correctness, be said even of the simple arithmetical equation 2 + 2 = 4, that it may be represented by the formula a = a. The one is a proposition asserting *the equivalence of two different expressions*;—to ascertain which equivalence may, in numberless cases, be an object of the highest importance. The other is altogether unmeaning and nugatory, and cannot, by any possible supposition, admit of the slightest application of a practical nature. What opinion then shall we form of the proposition a = a, when considered as the representative of such a *formula* as the binomial theorem of Sir Isaac Newton? When applied to the equation 2 + 2 = 4 (which from its extreme simplicity and familiarity is apt to be regarded in the light of an axiom) the paradox does not appear to be so manifestly extravagant; but, in the other case, it seems quite impossible to annex to it any meaning whatever.

I should scarcely have been induced to dwell so long on this theory of Leibnitz concerning mathematical evidence, if I had not observed among some late logicians (particularly among the followers of Condillac) a growing disposition to extend it to all the different sorts of evidence resulting from the various employments of our reasoning powers. Condillac himself states his own opinion on this point with the most perfect confidence. "*L'évidence de raison consiste uniquement* "*dans l'identité*: c'est ce que nous avons démontré. Il faut "que cette vérité soit bien simple pour avoir échappé à tous "les philosophes, quoiqu' ils eussent tant d'intérêt à s'assurer "de l'évidence, dont ils avoient continuellement le mot dans "la bouche *."

The *demonstration* here alluded to is extremely concise; and if we grant the two *data* on which it proceeds, must be universally acknowledged to be irresistible. The first is, "That the "evidence of every mathematical *equation* is that of identity:" The second, "That what are called, in the other sciences, *pro*-"*positions* or *judgments*, are, at bottom, precisely of the same "nature with *equations*."—But it is proper, on this occasion, to let our author speak for himself.

"Mais, dira-t-on, c'est ainsi qu'on raisonne en mathéma-

* La Logique, Chap. IX.

"tiques, ou le raisonnement se fait avec des équations. En "sera-t-il de même dans les autres sciences, où le raisonnement "se fait avec des propositions? Je réponds qu' *équations, pro-* "*positions, jugemens*, sont au fond la même chose, et que par "conséquent on raisonne de la même manière dans toutes les "sciences *."

Upon this demonstration I have no comment to offer. The truth of the first assumption has been already examined at sufficient length; and the second (which is only Locke's very erroneous account of *judgment*, stated in terms incomparably more exceptionable) is too puerile to admit of refutation. It is melancholy to reflect, that a writer who, in his earlier years, had so admirably unfolded the mighty influence of language upon our speculative conclusions, should have left behind him, in one of his latest publications, so memorable an illustration of his own favourite doctrine.

It was manifestly with a view to the more complete establishment of the same theory, that Condillac undertook a work, which has appeared since his death, under the title of *La Langue des Calculs*; and which, we are told by the editors, was only meant as a prelude to other labours, more interesting and more difficult. From the circumstances which they have stated, it would seem that the intention of the author was to extend to all the other branches of knowledge, inferences similar to those which he has here endeavoured to establish with respect

* Ibid. Chap. VIII.

to mathematical calculations; and much regret is expressed by his friends, that he had not lived to accomplish a design of such incalculable importance to human happiness. I believe I may safely venture to assert, that it was fortunate for his reputation he proceeded no farther; as the sequel must, from the nature of the subject, have afforded, to every competent judge, an experimental and palpable proof of the vagueness and fallaciousness of those views by which the undertaking was suggested. In his posthumous volume, the mathematical precision and perspicuity of his details appear to a superficial reader to reflect some part of their own light on the general reasonings with which they are blended; while, to better judges, these reasonings come recommended with many advantages and with much additional authority, from their coincidence with the doctrines of the Leibnitzian school.

It would probably have been not a little mortifying to this most ingenious and respectable philosopher, to have discovered, that, in attempting to generalize a very celebrated theory of Leibnitz, he had stumbled upon an obsolete conceit, started in this island upwards of a century before. "When a man "reasoneth (says Hobbes) he does nothing else but conceive a "sum total, from addition of parcels; or conceive a remain-"der from subtraction of one sum from another; which (if it "be done by words) is conceiving of the consequence of the "names of all the parts, to the name of the whole; or from "the names of the whole and one part, to the name of the "other part.—These operations are not incident to numbers "only, but to all manner of things that can be added together, "and taken one out of another.—In sum, in what matter so-

"ever there is place for addition and subtraction, there also "is place for reason; and where these have no place, there rea- "son has nothing at all to do.

"Out of all which we may define what that is which is "meant by the word *reason*, when we reckon it amongst the "faculties of the mind. For *reason*, in this sense, is nothing "but *reckoning* (that is, adding and subtracting) of the conse- "quences of general names agreed upon, for the *marking* and "*signifying* of our thoughts;—I say *marking* them, when we "reckon by ourselves; and *signifying*, when we demonstrate, "or approve our reckonings to other men *."

Agreeably to this definition, Hobbes has given to the first part of his elements of philosophy, the title of Computatio, sive Logica; evidently employing these two words as precisely synonymous. From this tract I shall quote a short paragraph, not certainly on account of its intrinsic value, but in consequence of the interest which it derives from its coincidence with the speculations of some of our contemporaries. I transcribe it from the Latin edition, as the antiquated English of the author is apt to puzzle readers not familiarized to the peculiarities of his philosophical diction.

"Per ratiocinationem autem intelligo computationem. Com- "putare vero est *plurium rerum simul additarum summam colli- "gere, vel unâ re ab aliâ detractâ, cognoscere residuum*. Ratio- "cinari igitur idem est quod *addere* et *subtrahere*, vel si quis

* Leviathan, Chap. v.

"adjungat his *multiplicare* et *dividere*, non abnuam, cum *multi-* "*plicatio* idem sit quod æqualium *additio*, *divisio* quod æqua- "lium quoties fieri potest *subtractio*. Recidit itaque ratioci- "natio omnis ad duas operationes animi, *additionem* et *subtrac-* "*tionem* *." How wonderfully does this jargon agree with the assertion of Condillac, that all equations are propositions, and all propositions equations!

These speculations, however, of Condillac and of Hobbes relate to reasoning in general; and it is with mathematical reasoning alone, that we are immediately concerned at present. That the peculiar evidence with which *this* is accompanied is not resolvable into the perception of identity, has, I flatter myself, been sufficiently proved in the beginning of this article; and the plausible extension by Condillac of the very same theory to our reasonings in all the different branches of moral science, affords a strong additional presumption in favour of our conclusion.

From this long digression, into which I have been insensibly led by the errors of some illustrious foreigners concerning the

* The *Logica* of Hobbes has been lately translated into French, under the title of *Calcul ou Logique*, by M. Destutt-Tracy. It is annexed to the third volume of his *Elémens d'Idéologie*, where it is honoured with the highest eulogies by the ingenious translator. "L'ouvrage en masse (he observes in one passage) mérite d'être regardé "comme un produit precieux des méditations de Bacon et de Descartes sur le sys- "tême d'Aristote, et comme le germe des progrès ultérieures de la science." (*Disc. Prel.* p. 117.)

nature of mathematical demonstration, I now return to a further examination of the distinction between sciences which rest ultimately on facts, and those in which *definitions* or *hypotheses* are the sole principles of our reasonings.

III.

Continuation of the Subject.—Evidence of the Mechanical Philosophy, not to be confounded with that which is properly called Demonstrative or Mathematical.—Opposite Error of some late Writers.

Next to geometry and arithmetic, in point of evidence and certainty, is that branch of general physics which is now called mechanical philosophy;—a science in which the progress of discovery has been astonishingly rapid, during the course of the last century; and which, in the systematical concatenation and filiation of its elementary principles, exhibits every day more and more of that logical simplicity and elegance which we admire in the works of the Greek mathematicians. It may, I think, be fairly questioned, whether, in this department of knowledge, the affectation of mathematical method has not been already carried to an excess; the essential distinction between mechanical and mathematical truths being, in many of the physical systems which have lately appeared on the Continent, studiously kept out of the reader's view, by exhibiting both, as nearly as possible, in the same form. A variety of circumstances, indeed, conspire to identify in the imagination, and, of consequence, to assimilate in the mode of their statement, these two very different classes of propositions; but as this assimilation (beside its obvious tendency to involve experimental facts in metaphysical mystery) is apt occasionally to lead to very erroneous logical conclusions, it becomes the more necessary, in proportion as it arises from a *natural* bias, to point out the causes in which it has originated, and the limitations with which it ought to be understood.

The following slight remarks will sufficiently explain my general ideas on this important article of logic.

1. As the study of the mechanical philosophy is, in a great measure, inaccessible to those who have not received a regular mathematical education, it commonly happens, that a taste for it is, in the first instance, grafted on a previous attachment to the researches of pure or abstract mathematics. Hence a natural and insensible transference to physical pursuits, of mathematical habits of thinking; and hence an almost unavoidable propensity to give to the former science, that systematical connection in all its various conclusions, which, from the nature of its first principles, is essential to the latter, but which can never belong to any science which has its foundations laid in facts collected from experience and observation.

2. Another circumstance, which has co-operated powerfully with the former in producing the same effect, is that proneness to simplification which has misled the mind, more or less, in all its researches; and which, in natural philosophy, is peculiarly

encouraged by those beautiful analogies which are observable among different physical phenomena;—analogies, at the same time, which, however pleasing to the fancy, cannot always be resolved by our reason into one general law. In a remarkable analogy, for example, which presents itself between the equality of action and re-action in the collision of bodies, and what obtains in their mutual attractions, the coincidence is so perfect, as to enable us to comprehend all the various facts in the same theorem; and it is difficult to resist the temptation which it seems to offer to our ingenuity, of attempting to trace it, in both cases, to some common principle. Such trials of theoretical skill I would not be understood to censure indiscriminately; but, in the present instance, I am fully persuaded, that it is at once more unexceptionable in point of sound logic, and more satisfactory to the learner to establish the fact, in particular cases, by an appeal to experiment; and to state the law of action and re-action in the collision of bodies, as well as that which regulates the mutual tendencies of bodies towards each other, merely as general rules which have been obtained by induction, and which are found to hold invariably as far as our knowledge of nature extends *.

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* It is observed by Mr Robison, in his Elements of Mechanical Philosophy, that "Sir Isaac Newton, in the general scholium on the laws of motion, seems to consider "the equality of action and reaction, as an axiom deduced from the relations of ideas. "*But this* (says Mr Robison) *seems doubtful.* Because a magnet causes the iron to "approach towards it, it does not appear that we *necessarily* suppose that iron also at-"tracts the magnet." In confirmation of this he remarks, that notwithstanding the previous conclusions of Wallis, Wren, and Huyghens, about the mutual, equal, and contrary action of solid bodies in their collisions, "Newton himself only *presumed* that, be-"cause the sun attracted the planets, these also attracted the sun; and that he is at

An additional example may be useful for the illustration of the same subject. It is well known to be a general principle in mechanics, that when, by means of any machine, two heavy bodies counterpoise each other, and are then made to move together, the quantities of motion with which one descends, and the other ascends perpendicularly, are equal. This equilibrium bears such a resemblance to the case of two moving

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" much pains to point out phenomena to astronomers, by which this may be proved, " when the art of observation shall be sufficiently perfected." Accordingly, Mr Robison, with great propriety, contents himself with stating this third law of motion, as a fact " with respect to all bodies on which we can make experiment or observation fit " for deciding the question."

In the very next paragraph, however, he proceeds thus: " As it is an universal law, we " cannot rid ourselves of the persuasion that it depends on some general principle which " influences all the matter in the universe;"—to which observation he subjoins a conjecture or hypothesis concerning the nature of this principle or cause. For an outline of his theory I must refer to his own statement. (See *Elements of Mechanical Philosophy*, Vol. I. pp. 124, 125, 126.)

Of the fallaciousness of synthetical reasonings concerning physical phenomena, there cannot be a stronger proof, than the diversity of opinion among the most eminent philosophers with respect to the species of evidence on which the third law of motion rests. On this point, a direct opposition may be remarked in the views of Sir Isaac Newton, and of his illustrious friend and commentator, Mr Maclaurin; the former seeming to lean to the supposition, that it is a corollary deducible *a priori* from abstract principles; while the latter (manifestly considering it as the effect of an arbitrary arrangement) strongly recommends it to the attention of those who delight in the investigation of final causes*. My own idea is, that, in the present state of our knowledge, it is at once more safe and more logical, to consider it merely as an experimental truth; without venturing to decide positively on either side of the question. As to the doctrine of final causes, it fortunately stands in need of no aid from such dubious speculations.

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* Account of Newton's Philosophical Discoveries. Book II. Chap. 2. § 28.

bodies stopping each other, when they meet together with equal quantities of motion, that, in the opinion of many writers, the cause of an equilibrium in the several machines is sufficiently explained, by remarking, "that a body always "loses as much motion as it communicates." Hence it is inferred, that when two heavy bodies are so circumstanced, that one cannot descend without causing the other to ascend at the same time, and with the same quantity of motion, both of these bodies must necessarily continue at rest. But this reasoning, however plausible it may seem to be at first sight, is by no means satisfactory; for (as Dr Hamilton has justly observed *) when we say, that one body *communicates* its motion to another, we must suppose the motion to exist, *first* in the one, and *afterwards* in the other; whereas, in the case of the machine, the ascent of the one body cannot, by any conceivable refinement, be ascribed to a communication of motion from the body which is descending at the same moment; and, therefore, (admitting the truth of the general law which obtains in the collision of bodies) we might suppose, that in the machine, the superior weight of the heavier body would overcome the lighter, and cause it to move upwards with the same quantity of motion with which itself moves downwards. In perusing a pretended demonstration of this sort, a student is dissatisfied and puzzled; not from the difficulty of the subject, which is

* See Philosophical Essays, by Hugh Hamilton, D. D. Professor of Philosophy in the University of Dublin, p. 135, et seq. 3d edit. (London, 1772.)

obvious to every capacity, but from the illogical and inconclusive reasoning to which his assent is required *.

3. To these remarks it may be added, that even when one proposition in natural philosophy is logically deducible from another, it may frequently be expedient, in communicating the elements of the science, to illustrate and confirm the consequence, as well as the principle, by experiment. This I should apprehend to be proper, wherever a consequence is inferred from a principle less familiar and intelligible than itself; a thing which must occasionally happen in physics, from the complete incorporation (if I may use the expression) which, in modern times, has taken place between physical truths, and the discoveries of mathematicians. The necessary effect of this incorporation was, to give to natural philosophy a mathematical form, and to systematize its conclusions, as far as possible, agreeably to rules suggested by mathematical method.

In pure mathematics, where the truths which we investigate

* The following observation of Dr Hamilton places this question in its true point of view. " However, as the theorem above mentioned is a very elegant one, it ought certainly to be taken notice of in every treatise of mechanics, and may serve as a very good *index* of an *æquilibrium* in all machines; but I do not think that we can from thence, or from any one general principle, explain the nature and effects of all the mechanic powers in a satisfactory manner."

To the same purpose, it is remarked by Mr Maclaurin, that " though it be useful and agreeable, to observe how uniformly this principle prevails in engines of every sort, throughout the whole of mechanics, in all cases where an *æquilibrium* takes place; yet that it would not be right to rest the evidence of so important a doctrine upon a proof of this kind only." *Account of Newton's Discoveries*, B. II. c. 3.

are all co-existent in point of time, it is universally allowed, that one proposition *is said* to be a consequencce of another, only with a reference to our established arrangements. Thus all the properties of the circle might be as rigorously deduced from any one general property of the curve, as from the equality of the radii. But it does not therefore follow, that all these arrangements would be equally convenient: on the contrary, it is evidently useful, and indeed necessary, to lead the mind, as far as the thing is practicable, from what is simple to what is more complex. The misfortune is, that it seems impossible to carry this rule universally into execution: and, accordingly, in the most elegant geometrical treatises which have yet appeared, instances occur, in which consequences are deduced from principles more complicated than themselves. Such inversions, however, of what may justly be regarded as the natural order, must always be felt by the author as a subject of regret; and, in proportion to their frequency, they detract both from the beauty, and from the didactic simplicity of his general design.

The same thing often happens in the elementary doctrines of natural philosophy. A very obvious example occurs in the different demonstrations given by writers on mechanics, from the resolution of forces, of the fundamental proposition concerning the lever;—demonstrations in which the proposition, even in the simple case when the directions of the forces are supposed to be parallel, is inferred from a process of reasoning involving one of the most refined principles employed in the mechanical philosophy. I do not object to this arrangement as illogical;

nor do I presume to say that it is injudicious*. I would only suggest the propriety, in such instances, of confirming and illustrating the conclusion, by an appeal to experiment; an appeal which, in natural philosophy, possesses an authority equal to that which is generally, but very improperly, considered as a mathematical demonstration of physical truths. In pure geometry, no reference to the senses can be admitted, but in the

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* In *some* of these demonstrations, however, there is a logical inconsistency so glaring, that I cannot resist the temptation of pointing it out here, as a good instance of that undue predilection for mathematical evidence, in the exposition of physical principles, which is conspicuous in many elementary treatises. I allude to those demonstrations of the property of the lever, in which, after attempting to prove the general theorem, on the supposition that the directions of the forces meet in a point, the same conclusion is extended to the simple case in which these directions are parallel, by the *fiction* (for it deserves no other name) of conceiving parallel lines to meet at an infinite distance, or to form with each other an angle infinitely small. It is strange, that such a proof should ever have been thought more satisfactory than the direct evidence of our senses. How much more reasonable and pleasing to begin with the simpler case (which may be easily brought to the test of experiment), and then to deduce from it, by the resolution of forces, the general proposition! Even Dr Hamilton himself, who has treated of the mechanical powers with much ingenuity, seems to have imagined, that by demonstrating the theorem, in all its cases, from the composition and resolution of forces alone, he had brought the whole subject within the compass of pure geometry. It could scarcely, however, (one should think) have escaped him, that every valid demonstration of the composition of forces must necessarily assume as *a fact*, that "when a body is acted "upon by a force parallel to a straight line given in position, this force has no effect "either to accelerate or to retard the progress of the body towards that line." Is not this fact much farther removed from common observation than the fundamental property of the lever, which is familiar to every peasant, and even to every savage? And yet the same author objects to the demonstration of Huyghens, that it depends upon a principle, *which* (he says) *ought not to be granted on this occasion*,—that "when two "equal bodies are placed on the arms of a lever, that which is furthest from the *fulcrum* "will preponderate."

way of illustration; and any such reference, in the most trifling step of a demonstration, vitiates the whole. But, in natural philosophy, all our reasonings must be grounded on principles for which no evidence but that of sense can be obtained; and the propositions which we establish, differ from each other only as they are deduced from such principles immediately, or by the intervention of a mathematical demonstration. An experimental proof, therefore, of any particular physical truth, when it can be conveniently obtained, although it may not always be the most elegant or the most expedient way of introducing it to the knowledge of the student, is as rigorous and as satisfactory as any other; for the intervention of a process of mathematical reasoning can never bestow on our conclusions a greater degree of certainty than our principles possessed *.

I have been led to enlarge on these topics by that unqualified application of mathematical method to physics, which has been fashionable for many years past among foreign writers; and which seems to have originated chiefly in the com-

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* Several of the foregoing remarks were suggested by certain peculiarities of opinion relative to the distinct provinces of experimental and of mathematical evidence in the study of physics, which were entertained by my learned and excellent friend, the late Mr Robison. Though himself a most enlightened and zealous advocate for the doctrine of final causes, he is well known to have formed his scientific taste chiefly upon the mechanical philosophers of the Continent, and, in consequence of this circumstance, to have undervalued *experiment*, wherever a possibility offered of introducing mathematical, or even metaphysical reasoning. Of this bias various traces occur, both in his Elements of Mechanical Philosophy, and in the valuable articles which he furnished to the Encyclopædia Britannica.

manding influence which the genius and learning of Leibnitz have so long maintained over the scientific taste of most European nations *. In an account, lately published, of the Life and Writings of Dr Reid, I have taken notice of some other inconveniences resulting from it, still more important than the introduction of an unsound logic into the elements of natural

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* The following very extraordinary passage occurs in a letter from Leibnitz to Mr Oldenburg :

" Ego id agere constitui, ubi primum otium nactus ero, ut rem omnem mechani-" cam reducam ad puram geometriam; problemataque circa elateria, et aquas, et pen-" dula, et projecta, et solidorum resistentiam, et frictiones, &c. definiam. Quæ hactenus " attigit nemo. Credo autem rem omnem nunc esse in potestate; ex quo circa regu-" las motuum mihi penitus perfectis demonstrationibus satisfeci; neque quicquam am-" plius in eo genere desidero. Tota autem res, quod mireris, pendet ex axiomate me-" taphysico pulcherrimo, quod non minoris momenti est circa motum, quam hoc, to-" tum esse majus parte, circa magnitudinem." (*Wallisii Opera*, Vol. III. p. 633.)

The *beautiful metaphysical axiom* here referred to by Leibnitz, is plainly the principle of *the sufficient reason;* and it is not a little remarkable, that the highest praise which he had to bestow upon it was, to compare it to Euclid's axiom, " That the " whole is greater than its part." Upon this principle of *the sufficient reason*, Leibnitz, as is well known, conceived that a complete system of physical science might be built, as he thought the whole of mathematical science resolvable into the principles of identity and of contradiction.—By the first of these principles (it may not be altogether superfluous to add) is to be understood the maxim, " Whatever is, is;" By the second, the maxim, that " It is impossible for the same thing to be, and not to be :"—two maxims which, it is evident, are only different expressions of the same proposition.

In the remarks made by Locke on the logical inutility of *mathematical* axioms, and on the logical danger of assuming *metaphysical* axioms as the principles of our reasonings in other sciences, I think it highly probable, that he had a secret reference to the philosophical writings and epistolary correspondence of Leibnitz. This appears to me to furnish a key to some of Locke's observations, the scope of which Dr Reid professes his inability to discover. One sentence, in particular, on which he has animadverted philosophy; in particular, of the obvious tendency which it has to withdraw the attention from that unity of design which

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with some severity, is, in my opinion, distinctly pointed at the letter to Mr Oldenburg, quoted in the beginning of this note.

" Mr Locke farther says (I borrow Dr Reid's own statement) that maxims are not " of use to help men forward in the advancement of the sciences, or new discoveries of " yet unknown truths: that Newton, in the discoveries he has made in his never enough " to be admired book, has not been assisted by the general maxim, whatever is, is; or " the whole is greater than a part, or the like."

As the letter to Oldenburg is dated in 1676, (twelve years before the publication of the Essay on Human Understanding) and as Leibnitz expresses a desire that it may be communicated to Mr Newton, there can scarcely be a doubt that Locke had read it; and it reflects infinite honour on his sagacity, that he seems, at that early period, to have foreseen the extensive influence which the errors of this illustrious man were so long to maintain over the opinions of the learned world. The truth is, that even then he prepared a reply to some reasonings which, at the distance of a century, were to mislead, both in physics and in logic, the first philosophers in Europe.

If these conjectures be well founded, it must be acknowledged that Dr Reid has not only failed in his defence of *maxims* against Locke's attack; but that he has totally misapprehended the aim of Locke's argument.

" I answer (says he, in the paragraph immediately following that which was " quoted above), the first of these maxims (whatever is, is) is an identical proposition, " of no use in mathematics, or in any other science. The second (that the whole is " greater than a part) is often used by Newton, and by all mathematicians, and many " demonstrations rest upon it. In general, Newton, as well as all other mathematicians, " grounds his demonstrations of mathematical propositions upon the axioms laid down " by Euclid, or upon propositions which have been before demonstrated by help of " these axioms.

" But it deserves to be particularly observed, that Newton, intending in the third " book of his *Principia* to give a more scientific form to the physical part of astro- " nomy, which he had at first composed in a popular form, thought proper to follow " the example of Euclid, and to lay down first, in what he calls *Regulæ Philosophandi*, " and in his *Phenomena*, the first principles which he assumes in his reasoning.

" Nothing, therefore, could have been more unluckily adduced by Mr Locke to it is the noblest employment of philosophy to illustrate, by disguising it under the semblance of an eternal and necessary order, similar to what the mathematician delights to trace among the mutual relations of quantities and figures. The consequence has been, (in too many physical systems,) to level the study of nature, in point of moral interest, with the investigations of the algebraist;—an effect too which has taken place most remarkably, where, from the sublimity of the subject, it was least to be expected,—in the application of the mechanical philosophy to the phenomena of the heavens. But on this very extensive and important topic I must not enter at present.

In the opposite extreme to the error which I have now been endeavouring to correct, is a paradox which was broached, about twenty years ago, by the late ingenious Dr Beddoes; and which has since been adopted by some writers whose names are better entitled, on a question of this sort, to give weight to their opinions *. By the partisans of this new doctrine it seems to be imagined, that—so far from physics being a branch of mathematics,—mathematics, and more particularly geometry, is, in reality, only a branch of physics. "The ma-

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"support his aversion to first principles, than the example of Sir Isaac Newton." (*Essays on the Int. Powers*, pp. 647, 648, 4to edit.)

* I allude here more particularly to my learned friend, Mr Leslie, whose high and justly merited reputation, both as a mathematician and an experimentalist, renders it indispensably necessary for me to take notice of some fundamental logical mistakes which he appears to me to have committed in the course of those ingenious excursions,

" thematical sciences (says Dr Beddoes) are sciences of expe-" riment and observation, founded solely on the induction of " particular facts; as much so as mechanics, astronomy, op-" tics, or chemistry. In the kind of evidence there is no dif-" ference; for it originates from perception in all these cases " alike; but mathematical experiments are more simple, and " more perfectly within the grasp of our senses, and our per-" ceptions of mathematical objects are clearer *."

A doctrine essentially the same, though expressed in terms not quite so revolting, has been lately sanctioned by Mr Leslie; and it is to *his* view of the argument that I mean to confine my attention at present. " The whole structure of geometry " (he remarks) is grounded on the simple comparison of tri-" angles; and all the fundamental theorems which relate to " this comparison, derive their evidence from the *mere* superpo-" sition of the triangles themselves; a mode of proof which, in

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in which he occasionally indulges himself, beyond the strict limits of his favourite studies.

* Into this train of thinking, Dr Beddoes informs us, he was first led by Mr Horne Tooke's speculations concerning language. " In whatever study you are engaged, to " leave difficulties behind is distressing: and when these difficulties occur at your " very entrance upon a science, professing to be so clear and certain as geometry, your " feelings become still more uncomfortable; and you are dissatisfied with your own " powers of comprehension. I therefore think it due to the author of ΕΠΕΑ ΠΤΕ-" ΡΟΕΝΤΑ, to acknowledge my obligations to him for relieving me from this sort of " distress. For although I had often made the attempt, I could never solve certain dif-" ficulties in Euclid, till my reflections were revived and assisted by Mr Tooke's *disco-" veries*." (See *Observations on the Nature of Demonstrative Evidence*, London, 1793, pp. 5, and 15.)

"reality, is nothing but an ultimate appeal, though of the easiest "and most familiar kind, to external observation *." And, in another passage: "Geometry, like the other sciences which "are not concerned about the operations of mind, rests ulti-"mately on external observations. But those ultimate facts are "so few, so distinct and obvious, that the subsequent train of "reasoning is safely pursued to unlimited extent, without ever "appealing *again* to the evidence of the senses †."

Before proceeding to make any remarks on this theory,

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* Elements of Geometry and of Geometrical Analysis, &c. By Mr Leslie. Edinburgh, 1809.

The assertion that *the whole* structure of geometry is founded on the comparison of triangles, is expressed in terms too unqualified. D'Alembert has mentioned another principle as not less fundamental, the measurement of angles by circular arches. "Les "propositions fondamentales de géométrie peuvent être réduites à deux; la mesure des "angles par les arcs de cercle, et le principe de la superposition." (*Elemens de Philosophie*, Art. *Géométrie*.) The same writer, however, justly observes, in another part of his works, that the measure of angles by circular arches, is itself dependent on the principle of superposition; and that, consequently, however extensive and important in its application, it is entitled only to rank with what he calls *principles of a second order*. "La mesure des angles par les arcs de cercle décrit de leur sommet, est elle-même dé-"pendante du principe de la superposition. Car quand on dit que la mesure d'un an-"gle est l'arc circulaire décrit de son sommet, on veut dire que si deux angles sont "égaux, les angles décrits de leur sommet à même rayon, seront égaux; vérité qui se "démontre par le principe de la superposition, comme tout géometre tant soit peu ini-"tié dans cette science le sentira facilement." (*Eclaircissemens sur les Elémens de Philosophie*, § IV.)

Instead, therefore, of saying that the whole structure of geometry is grounded on the comparison of triangles, it would be more correct to say, that it is grounded on the principle of superposition.

† Elements of Geometry and of Geometrical Analysis, p. 453.

it is proper to premise, that it involves two separate considerations, which it is of material consequence to distinguish from each other. The first is, that extension and figure (the subjects of geometry) are qualities of body which are made known to us by our external senses alone, and which actually fall under the consideration of the natural philosopher, as well as of the mathematician. The second, that the whole fabric of geometrical science rests on the comparison of triangles, in forming which comparison, we are ultimately obliged to appeal (in the same manner as in establishing the first principles of physics) to a sensible and experimental proof.

1. In answer to the first of these allegations, it might perhaps be sufficient to observe, that in order to identify two sciences, it is not enough to state, that they are both conversant about the same objects; it is necessary farther to show, that, in both cases, these objects are considered in the same point of view, and give employment to the same faculties of the mind. The poet, the painter, the gardener, and the botanist, are all occupied in various degrees and modes, with the study of the vegetable kingdom; yet who has ever thought of confounding their several pursuits under one common name? The natural historian, the civil historian, the moralist, the logician, the dramatist, and the statesman, are all engaged in the study of man, and of the principles of human nature; yet how widely discriminated are these various departments of science and of art! how different are the kinds of evidence on which they respectively rest! how different the intellectual habits which they have a tendency to form! Indeed, if this mode of generalization

were to be admitted as legitimate, it would lead us to blend all the objects of science into one and the same mass; inasmuch as it is by the same impressions on our external senses, that our intellectual faculties are, in the first instance, roused to action, and all the first elements of our knowledge unfolded.

In the instance, however, before us, there is a very remarkable specialty, or rather singularity, which renders the attempt to identify the objects of geometrical and of physical science, incomparably more illogical than it would be to classify poetry with botany, or the natural history of man with the political history of nations. This specialty arises from certain peculiarities in the metaphysical nature of those *sensible qualities* which fall under the consideration of the geometer; and which led me, in a former work, to distinguish them from other sensible qualities (both primary and secondary), by bestowing on them the title of *mathematical affections of matter* *. Of these mathematical affections (*magnitude* and *figure*,) our first notions are, no doubt, derived (as well as of hardness, softness, roughness, and smoothness) from the exercise of our external senses; but it is equally certain, that when the notions of magnitude and figure have once been acquired, the mind is immediately led to consider them as attributes of space no less than of body; and (abstracting them entirely from the other sensible qualities perceived in conjunction with them), becomes impressed with an irresistible conviction, that their existence is necessary and eternal, and that it would remain unchanged if all the bodies in the

* Philosophical Essays, pp. 94, 95.

universe were annihilated. It is not our business here to inquire into the origin and grounds of this conviction. It is with the *fact* alone that we are concerned at present; and this I conceive to be one of the most obviously incontrovertible which the circle of our knowledge embraces. Let those explain it as they best can, who are of opinion, that all the judgments of the human understanding rest ultimately on observation and experience.

Nor is this the only case in which the mind forms conclusions concerning space, to which those of the natural philosopher do not bear the remotest analogy. Is it from experience we learn that space is infinite? or (to express myself in more unexceptionable terms), that no limits can be assigned to its immensity! Here is a fact, extending not only beyond the reach of our personal observation, but beyond the observation of all created beings; and a fact on which we pronounce with no less confidence, when in imagination we transport ourselves to the utmost verge of the material universe, than when we confine our thoughts to those regions of the globe which have been explored by travellers. How unlike those general laws which we investigate in physics, and which, how far soever we may find them to reach, may still, for any thing we are able to discover to the contrary, be only contingent, local, and temporary!

It must indeed be owned, with respect to the conclusions hitherto mentioned on the subject of space, that they are rather of a metaphysical than of a mathematical nature; but they are not, on that account, the less applicable to our purpose;

for if the theory of Beddoes had any foundation, it would lead us to identify with physics the former of these sciences as well as the latter; at least, all that part of the former which is employed about space, or extension,—a favourite object of metaphysical as well as of mathematical speculation. The truth, however, is, that some of our metaphysical conclusions concerning space are more nearly allied to geometrical theorems than we might be disposed at first to apprehend; being involved or implied in the most simple and fundamental propositions which occur in Euclid's Elements. When it is asserted, for example, that "if "one straight line falls on two other straight lines, so as to make "the two interior angles on the same side together equal to two "right angles, these two straight lines, though indefinitely pro- "duced, will never meet;"—is not the boundless immensity of space tacitly assumed as a thing unquestionable? And is not a universal affirmation made with respect to a fact which experience is equally incompetent to disprove or to confirm? In like manner, when it is said, that "triangles on the same base, and "between the same parallels are equal," do we feel ourselves the less ready to give our assent to the demonstration, if it should be supposed, that the one triangle is confined within the limits of the paper before us, and that the other, standing on the same base, has its vertex placed beyond the sphere of the fixed stars? In various instances, we are led, with a force equally imperious, to acquiesce in conclusions, which not only admit of no illustration or proof from the perceptions of sense, but which, at first sight, are apt to stagger and confound the faculty of imagination. It is sufficient to mention, as examples of this, the relation between the hyperbola and its asymptotes;

and the still more obvious truth of the infinite divisibility of extension. What analogy is there between such propositions as these, and that which announces, that the mercury in the Torricellian tube will fall, if carried up to the top of a mountain; or that the vibrations of a pendulum of a given length will be performed in the same time, while it remains in the same latitude? Were there, in reality, that analogy between mathematical and physical propositions, which Beddoes and his followers have fancied, the equality of the square of the hypothenuse of a right angled triangle to the squares described on the two other sides, and the proportion of 1, 2, 3, between the cone and its circumscribed hemisphere and cylinder, might, with fully as great propriety, be considered in the light of physical phenomena, as of geometrical theorems: Nor would it have been at all inconsistent with the logical unity of his work, if Mr Leslie had annexed to his Elements of Geometry, a scholium concerning the final causes of circles and of straight lines, similar to that which, with such sublime effect, closes the Principia of Sir Isaac Newton *.

* In the course of my own experience, I have met with *one* person, of no common ingenuity, who seemed seriously disposed to consider the truths of geometry very nearly in this light. The person I allude to was James Ferguson, author of the justly popular works on Astronomy and Mechanics. In the year 1768, he paid a visit to Edinburgh, when I had not only an opportunity of attending his public course of lectures, but of frequently enjoying, in private, the pleasure of his very interesting conversation. I remember distinctly to have heard him say, that he had more than once attempted to study the Elements of Euclid; but found himself quite unable to enter into that species of reasoning. The second proposition of the first book, he mentioned particularly as one of his stumbling-blocks at the very outset;—the circuitous process by which Euclid sets about an operation which never could puzzle, for a single moment, any man who had

2. It yet remains for me to say a few words upon that superposition of triangles which is the ground-work of all our geometrical reasonings concerning the relations which different spaces bear to one another in respect of magnitude. And here I must take the liberty to remark, in the first place, that the fact in question has been stated in terms much too loose and incorrect for a logical argument. When it is said, that "all the "fundamental theorems which relate to the comparison of tri-"angles, derive their evidence from the *mere* superposition of "the triangles themselves," it seems difficult, or rather impossible to annex to the adjective *mere*, an idea at all different from what would be conveyed, if the word *actual* were to be substituted in its place; more especially, when we attend to the

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seen a pair of compasses, appearing to him altogether capricious and ludicrous. He added, at the same time, that as there were various geometrical theorems of which he had daily occasion to make use, he had satisfied himself of their truth, either by means of his compasses and scale, or by some mechanical contrivances of his own invention. Of one of these I have still a perfect recollection;—his mechanical or experimental demonstration of the 47th proposition of Euclid's first Book, by cutting a card so as to afford an ocular proof, that the squares of the two sides actually filled the same space with the square of the hypothenuse.

To those who reflect on the disadvantages under which Mr Ferguson had laboured in point of education, and on the early and exclusive hold which experimental science had taken of his mind, it will not perhaps seem altogether unaccountable, that the refined and scrupulous logic of Euclid should have struck him as tedious, and even unsatisfactory, in comparison of that more summary and palpable evidence on which his judgment was accustomed to rest. Considering, however, the great number of years which have elapsed since this conversation took place, I should have hesitated about recording, solely on my own testimony, a fact so singular with respect to so distinguished a man, if I had not lately found, from Dr Hutton's Mathematical Dictionary, that *he* also had heard from Mr Ferguson's mouth, the most important of those particulars which I have now stated; and of which my own recollection is probably the more assertion which immediately follows, that "this mode of proof "is, in reality, nothing but an ultimate appeal, though of the "easiest and most familiar kind, to *external observation*." But if this be, in truth, the sense in which we are to interpret the statement quoted above, (and I cannot conceive any other interpretation of which it admits), it must appear obvious, upon the slightest reflection, that the statement proceeds upon a total misapprehension of the principle of *superposition*; inasmuch as it is not to an actual or *mere* superposition, but to an imaginary or ideal one, that any appeal is ever made by the geometer. Between these two modes of proof the difference is not only wide, but radical and essential. The one would, indeed, level geometry with physics, in point of evidence, by building the whole of its reasonings on *a fact* ascertained by mechanical measurement: The other is addressed to the understanding, and to the understanding alone, and is as rigorously conclusive as it is possible for demonstration to be*.

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lively and circumstantial, in consequence of the very early period of my life when they fell under my notice.

"Mr Ferguson's general mathematical knowledge (says Dr Hutton) was little or "nothing. Of algebra, he understood little more than the notation; and he has often "told me he could never demonstrate one proposition in Euclid's Elements; his con-"stant method being to satisfy himself, as to the truth of any problem, with a measure-"ment by scale and compasses." (*Hutton's Mathematical and Philosophical Dictionary, Article, Ferguson.*)

* The same remark was, more than fifty years ago, made by D'Alembert, in reply to some mathematicians on the Continent, who, it would appear, had then adopted a paradox very nearly approaching to that which I am now combating. "Le principe de la super-"position n'est point, comme l'ont prétendu plusieurs géometres, une méthode de démontrer "peu exacte et purement méchanique. La superposition, telle que les mathématiciens "la conçoivent, ne consiste pas à appliquer grossièrement une figure sur une autre, pour "juger par les yeux de leur égalité ou de leur différence, comme un ouvrier applique

That the reasoning employed by Euclid in proof of the fourth proposition of his first book is completely *demonstrative*, will be readily granted by those who compare its different steps with the conclusions to which we were formerly led, when treating of the nature of mathematical demonstration. In none of these steps is any appeal made to *facts* resting on the evidence of sense, nor indeed to any *facts* whatever. The constant appeal is to the *definition* of equality *. " Let the tri-

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" son pié sur une ligne pour la mesurer; elle consiste à imaginer une figure transportée " sur une autre, et à conclure de l'égalité supposée de certaines parties de deux figures, " la coincidence de ces parties entr'elles, et de leur coincidence la coincidence du " reste: d'où résulte l'égalité et la similitude parfaites des figures entières."

About a century before the time when D'Alembert wrote these observations, a similar view of the subject was taken by Dr Barrow; a writer who, like D'Alembert, added to the skill and originality of an inventive mathematician, the most refined, and, at the same time, the justest ideas concerning the theory of those intellectual processes which are subservient to mathematical reasoning.—" Unde meritò vir acutissimus Willebrordus " Snellius luculentissimum appellat geometriæ supellectilis instrumentum hanc ipsam " ἐφαρμοσιν. *Eam igitur in demonstrationibus mathematicis qui fastidiunt et respuunt,* " *ut mechanicæ crassitudinis ac* αὐτουργίας *aliquid redolentem, ipsissimam geometriæ basin* " *labefactare student; ast imprudenter et frustra.* Nam ἐφαρμοσιν geometræ suam non " manu sed mente peragunt, non oculi sensu, sed animi judicio æstimant. Supponunt " (id quod nulla manus præstare, nullus sensus discernere valet) accuratam et perfec- " tam congruentiam, ex eâque suppositâ justas et logicas eliciunt consequentias. Nul- " lus hîc regulæ, circini, vel normæ usus, nullus brachiorum labor, aut laterum con- " tentio, rationis totum opus, artificium et machinatio est; nil mechanicam sapiens " αὐτουργίαν exigitur; nil, inquam, mechanicum, nisi quatenus omnis magnitudo sit ali- " quo modo materiæ involuta, sensibus exposita, visibilis et palpabilis, sic ut quod mens " intelligi jubet, id manus quadantenus exequi possit, et contemplationem praxis utcun- " que conetur æmulari. Quæ tamen imitatio geometricæ demonstrationis robur ac dig- " nitatem nedum non infirmat aut deprimit, at validius constabilit, et atollit altius," &c. *Lectiones Mathematicæ*, Lect. III.

* It was before observed (see p. 168.) that Euclid's eighth *axiom (magnitudes which*

"angle A B C (says Euclid) be applied to the triangle D E F; "the point A to the point D, and the straight line A B to the "straight line D E; the point B *will* coincide with the point "E, because *A B is equal to D E.* And A B coinciding with "D E, A C *will* coincide with D F, *because the angle B A C* "*is equal to the angle E D F.*" A similar remark will be found to apply to every remaining step of the reasoning; and, therefore, this reasoning possesses the peculiar characteristic which distinguishes mathematical evidence from that of all the other sciences,—that it rests wholly on *hypotheses* and *definitions,* and in no respect upon any statement of *facts,* true or false. The ideas, indeed, of extension, of a triangle, and of equality, presuppose the exercise of our senses. Nay, the very idea of *superposition* involves that of *motion,* and consequently (as the parts of space are immoveable) of *a material triangle.* But where is there any thing analogous, in all this, to those *sensible facts,* which are the principles of our reasoning in physics; and which, according as they have been accurately or inaccurately ascertained, determine the accuracy or inaccuracy of our conclusions? The *material* triangle itself, as conceived by the mathematician, is the object, not of sense, but of intellect. It is not an actual *measure,* liable to expansion or contraction, from the influence of heat or of cold; nor does it require, in the ideal use which is made of

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coincide with each other are equal) ought, in point of logical rigour, to have been stated in the form of a *definition.* In our present argument, however, it is not of material consequence whether this criticism be adopted or not. Whether we consider the proposition in question in the light of an axiom or of a definition, it is equally evident, that it does not express *a fact* ascertained by observation or by experiment.

it by the student, the slightest address of hand or nicety of eye. Even in explaining this demonstration, for the first time, to a pupil, how slender soever his capacity might be, I do not believe that any teacher ever thought of illustrating its meaning by the actual application of the one triangle to the other. No teacher, at least, would do so, who had formed correct notions of the nature of mathematical science.

If the justness of these remarks be admitted, the *demonstration* in question must be allowed to be as well entitled to the name, as any other which the mathematician can produce; for as our conclusions relative to the properties of the circle (considered in the light of hypothetical theorems) are not the less rigorously and necessarily true, that no material circle may anywhere exist corresponding exactly to the definition of that figure, so the proof given by Euclid of the fourth proposition, would not be the less demonstrative, although our senses were incomparably less acute than they are, and although no material triangle continued of the same magnitude for a single instant. Indeed, when we have once acquired the ideas of equality and of a common measure, our mathematical conclusions would not be in the least affected, if all the bodies in the universe should vanish into nothing.

To many of my readers, I am perfectly aware, the foregoing remarks will be apt to appear tedious and superfluous. My only apology for the length to which they have extended is, my respect for the talents and learning of some of those writers who have lent the sanction of their authority to the logical errors which I have been endeavouring to correct;

and the obvious inconsistency of these conclusions with the doctrine concerning the characteristics of mathematical or demonstrative evidence, which it was the chief object of this section to establish *.

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* This doctrine is concisely and clearly stated by a writer whose acute and original, though very eccentric genius, seldom fails to redeem his wildest paradoxes by the new lights which he strikes out in defending them. " Demonstratio est syllogismus vel syl- " logismorum series à nominum definitionibus usque ad conclusionem ultimam deriva- " ta." (*Computatio sive Logica*, cap. 6.)

It will not, I trust, be inferred, from my having adopted, in the words of Hobbes, this detached proposition, that I am disposed to sanction any one of those conclusions which have been commonly *supposed* to be connected with it, in the mind of the author :— I say *supposed*, because I am by no means satisfied, (notwithstanding the loose and unguarded manner in which he has stated some of his logical opinions) that justice has been done to his views and motives in *this* part of his works. My own notions on the subject of evidence in general, will be sufficiently unfolded in the progress of my speculations. In the meantime, to prevent the possibility of any misapprehension of my meaning, I think it proper once more to remark, that the definition of Hobbes, quoted above, is to be understood (according to *my* interpretation of it) as applying solely to the word *demonstration* in pure mathematics. The extension of the same term by Dr Clarke and others, to reasonings which have for their object, not conditional or hypothetical, but absolute truth, appears to me to have been attended with many serious inconveniences, which these excellent authors did not foresee. Of the *demonstrations* with which Aristotle has attempted to fortify his syllogistic rules, I shall afterwards have occasion to examine the validity.

The charge of *unlimited* scepticism brought against Hobbes, has, in my opinion, been occasioned, partly by his neglecting to draw the line between absolute and hypothetical truth, and partly by his applying the word *demonstration* to our reasonings in other sciences as well as in mathematics. To these causes may perhaps be added, the offence which his logical writings must have given to the Realists of his time.

It is not, however, to Realists alone, that the charge has been confined. Leibnitz himself has given some countenance to it, in a dissertation prefixed to a work of Marius Nizolius; and Brucker, in referring to this dissertation, has aggravated not a little the censure of Hobbes, which it seems to contain. " Quin si illustrem Leibnitzium audi-

SECTION IV.

Of our Reasonings concerning Probable or Contingent Truths.

I.

Narrow Field of Demonstrative Evidence.—Of Demonstrative Evidence, when combined with that of SENSE, *as in Practical Geometry; and with those of Sense and of* INDUCTION, *as in the Mechanical Philosophy.—Remarks on a Fundamental Law of Belief, involved in all our Reasonings concerning Contingent Truths.*

IF the account which has been given of the nature of demonstrative evidence be admitted, the province over which it extends must be limited almost entirely to the objects of pure mathematics. A science perfectly analogous to this, in point of evidence, may indeed be conceived (as I have already remarked) to consist of a series of propositions relating to moral, to political, or to physical subjects; but as it could answer no other purpose than to display the ingenuity of the inventor, hardly any thing of the kind has been hitherto attempted. The only exception which I can think of, occurs in the speculations formerly mentioned under the title of *theoretical mechanics.*

"mus, Hobbesius quoque inter nominales referendus est, eam ob causam, quod *ipso* "*Occamo nominalior*, rerum veritatem dicat in nominibus consistere, ac, quod majus est, "pendere ab arbitrio humano." *Histor. Philosoph. de Ideis*, p. 209. Augustæ Vindelicorum, 1723.

But, if the field of mathematical demonstration be limited entirely to hypothetical or conditional truths, whence (it may be asked) arises the extensive and the various utility of mathematical knowledge, in our physical researches, and in the arts of life? The answer, I apprehend, is to be found in certain peculiarities of those objects to which the suppositions of the mathematician are confined; in consequence of which peculiarities, real combinations of circumstances may fall under the examination of our senses, approximating far more nearly to what his definitions describe, than is to be expected in any other theoretical process of the human mind. Hence a corresponding coincidence between his abstract conclusions, and those facts in practical geometry and in physics which they help him to ascertain.

For the more complete illustration of this subject, it may be observed, in the first place, that although the peculiar force of that reasoning which is properly called *mathematical*, depends on the circumstance of its principles being *hypothetical*, yet if, in any instance, the supposition could be ascertained as actually existing, the conclusion might, with the very same certainty, be applied. If I were satisfied, for example, that in a particular circle drawn on paper, all the *radii* were exactly equal, every property which Euclid has demonstrated of that curve might be confidently affirmed to belong to this diagram. As the thing, however, here supposed, is rendered impossible by the imperfection of our senses, the truths of geometry can never, in their practical applications, possess *demonstrative* evidence; but only that kind of evidence which our organs of perception enable us to obtain.

But although, in the practical applications of mathematics, the *evidence* of our conclusions differs essentially from that which belongs to the truths investigated in the theory, it does not therefore follow, that these conclusions are the less important. In proportion to the accuracy of our *data* will be that of all our subsequent deductions; and it fortunately happens, that the same imperfections of sense which limit what is physically attainable in the former, limit also, to the very same extent, what is practically useful in the latter. The astonishing precision which the mechanical ingenuity of modern times has given to mathematical instruments, has, in fact, communicated a nicety to the results of practical geometry, beyond the ordinary demands of human life, and far beyond the most sanguine anticipations of our forefathers *.

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* See a very interesting and able article, in the fifth volume of the Edinburgh Review, on Colonel Mudge's account of the operations carried on for accomplishing a trigonometrical survey of England and Wales. I cannot deny myself the pleasure of quoting a few sentences.

" In two distances that were deduced from sets of triangles, the one measured by " General Roy in 1787, the other by Major Mudge in 1794, one of 24.133 miles, and " the other of 38.688, the two measures agree within a foot as to the first distance, and " 16 inches as to the second. Such an agreement, where the observers and the instru- " ments were both different, where the lines measured were of such extent, and dedu- " ced from such a variety of *data*, is probably without any other example. Coinciden- " ces of this sort are frequent in the trigonometrical survey, and prove how much more " good instruments, used by skilful and attentive observers, are capable of performing, " than the most sanguine theorist could have ever ventured to foretel.—

" It is curious to compare the early essays of practical geometry with the perfection " to which its operations have now reached, and to consider that, while the artist had " made so little progress, the theorist had reached many of the sublimest heights of ma- " thematical speculation; that the latter had found out the area of the circle, and cal-

This remarkable, and indeed singular coincidence of propositions purely hypothetical, with facts which fall under the examination of our senses, is owing, as I already hinted, to the peculiar nature of the *objects* about which mathematics is conversant; and to the opportunity which we have (in consequence of that *mensurability* * which belongs to all of them) of adjusting, with a degree of accuracy approximating nearly to the truth, the *data* from which we are to reason in our practical operations, to those which are assumed in our theory. The only affections of matter which these objects comprehend are extension and figure; affections which matter possesses in common with space, and which may therefore be separated in fact, as well as abstracted in thought, from all its other sensible qualities. In examining, accordingly, the relations of *quantity* connected with these affections, we are not liable to be disturbed by those physical *accidents*, which, in the other applications of mathematical science, necessarily render the result, more or less, at variance with the theory. In measuring the height of a mountain, or in the survey of a country, if we are at due pains in ascertaining our data, and if we reason from them with mathematical strictness, the result may be depended on as accurate within very narrow limits; and as there is nothing but the incorrectness of our data by which the result can be

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"culated its circumference to more than a hundred places of decimals, when the former could hardly divide an arch into minutes of a degree; and that many excellent treatises had been written on the properties of curve lines, before a straight line of considerable length had ever been carefully drawn, or exactly measured on the surface of the earth."

* See note (G.)

vitiated, the limits of possible error may themselves be assigned. But, in the simplest applications of mathematics to mechanics or to physics, the abstractions which are necessary in the theory, must always leave out circumstances which are essentially connected with the effect. In demonstrating, for example, the property of the lever, we abstract entirely from its own weight, and consider it as an inflexible mathematical line;—suppositions with which the fact cannot possibly correspond; and for which, of course, allowances (which nothing but physical experience can enable us to judge of,) must be made in practice*.

Next to *practical geometry*, properly so called, one of the easiest applications of mathematical theory occurs in those branches of optics which are distinguished by the name of catoptrics and dioptrics. In these, the physical principles from which we reason are few and precisely definite, and the rest of the process is as purely geometrical as the Elements of Euclid.

In that part of astronomy, too, which relates solely to the phenomena, without any consideration of physical causes, our reasonings are purely geometrical. The *data* indeed on which we proceed must have been previously ascertained by observation; but the inferences we draw from these are connected with them by mathematical demonstration, and are accessible to all who are acquainted with the theory of spherics.

* See Note (H.)

In *physical* astronomy, the law of gravitation becomes also a principle or *datum* in our reasonings; but as in the celestial phenomena, it is disengaged from the effects of the various other causes which are combined with it near the surface of our planet, this branch of physics, as it is of all the most sublime and comprehensive in its objects, so it seems, in a greater degree than any other, to open a fair and advantageous field for mathematical ingenuity.

In the instances which have been last mentioned, the evidence of our conclusions resolves ultimately not only into that of sense, but into another law of belief formerly mentioned; that which leads us to expect the continuance, in future, of the established order of physical phenomena. A very striking illustration of this presents itself in the computations of the astronomer; on the faith of which he predicts, with the most perfect assurance, many centuries before they happen, the appearances which the heavenly bodies are to exhibit. The same fact is assumed in all our conclusions in natural philosophy; and something extremely analogous to it in all our conclusions concerning human affairs. They relate, in both cases, not to necessary connections, but to *probable* or *contingent* events; of which (how confidently soever we may expect them to take place), the failure is by no means perceived to be impossible. Such conclusions, therefore, differ essentially from those to which we are led by the demonstrations of pure mathematics, which not only command our assent to the theorems they establish, but satisfy us that the contrary suppositions are absurd.

These examples may suffice to convey a general idea of the distinction between demonstrative and probable evidence; and I purposely borrowed them from sciences where the two are brought into immediate contrast with each other, and where the authority of both has hitherto been equally undisputed.

Before prosecuting any farther the subject of probable evidence, some attention seems to be due, in the first place, to the grounds of that fundamental supposition on which it proceeds,—*the stability of the order of nature.* Of this important subject, accordingly, I propose to treat at some length.

II.

Continuation of the Subject.—Of that Permanence or Stability in the Order of Nature, which is presupposed in our Reasonings concerning Contingent Truths.

I HAVE already taken notice of a remarkable principle of the mind, (whether coëval with the first exercise of its powers, or the gradual result of habit, it is not at present material to inquire), in consequence of which, we are irresistibly led to apply to future events, the results of our past experience. In again resuming the subject, I do not mean to add any thing to what was then stated concerning the origin or the nature of this principle; but shall confine myself to a few reflections on that established order in the succession of events, which it unconsciously assumes as a fact; and which, if it were not real, would render human life a continued series of errors and disappointments. In any incidental remarks that may occur on the principle itself, I shall consider its existence as a thing universally acknowledged, and shall direct my attention chiefly to its practical effects;—effects which will be found to extend equally to the theories of the learned, and to the prejudices of the vulgar. The question with regard to its *origin* is, in truth, a problem of mere curiosity; for of its actual influence on our belief, and on our conduct, no doubts have been suggested by the most sceptical writers.

Before entering, however, upon the following argument, it may not be superfluous to observe, with respect to this expectation, that, in whatever manner it at first arises, it cannot fail to be mightily confirmed and strengthened by habits of scientific research; the tendency of which is to familiarize us more and more with the simplicity and the uniformity of physical laws, by gradually reconciling with them, as our knowledge extends, those phenomena which we had previously been disposed to consider in the light of exceptions. It is thus that, when due allowances are made for the different circumstances of the two events, the ascent of smoke appears to be no less a proof of the law of gravitation than the fall of a stone. This simplification and generalization of the laws of nature is one of the greatest pleasures which philosophy yields; and the growing confidence with which it is anticipated, forms one of the chief incentives to philosophical pursuits. Few experiments perhaps, in physics, afford more exquisite delight to the novice, or throw a stronger light on the nature and object of that science, than when he sees, for the first time, the guinea and the feather drop together in the exhausted receiver.

In the language of modern science, the established order in the succession of physical events is commonly referred (by a sort of figure or metaphor) to *the general laws of nature*. It is a mode of speaking extremely convenient from its conciseness, but is apt to suggest to the fancy a groundless, and indeed absurd analogy between the material and the moral worlds. As the order of society results from the *laws* prescribed by the legislator, so the order of the universe is conceived to result from certain *laws* established by the Deity. Thus, it is customary to say, th , the fall of heavy bodies towards the earth's surface, the ebbing and flowing of the sea, and the motions of the planets in their orbits, are consequences of the *law* of gravitation. But although, in one sense, this may be abundantly accurate, it ought always to be kept in view, that it is not a literal but a metaphorical statement of the truth; a statement somewhat analogous to that poetical expression in the sacred writings, in which God is said "to have given his decree to the "seas, that they should not pass his commandment." In those political associations from which the metaphor is borrowed, the laws are addressed to rational and voluntary agents, who are able to comprehend their meaning, and to regulate their conduct accordingly; whereas, in the material universe, the subjects of our observation are understood by all men to be unconscious and passive, (that is, are understood to be unchangeable in their state, without the influence of some foreign and external force) and consequently *the order* so admirably maintained, amidst all the various *changes* which they actually undergo, not only implies *intelligence* in its first conception, but implies, in its continued existence, the incessant agency of *power*, executing the purposes of wise design. If the word *law*, therefore, be, in such instances, literally interpreted, it must mean a uniform mode of operation, prescribed by the Deity to himself; and it has accordingly been explained in this sense by some of our best philosophical writers, particularly by Dr Clarke *. In employing, however, the word with an exclusive reference to experimental philosophy, it is more correctly logical to consider it as merely a statement of some *general fact* with respect to the order of nature ;—a fact which has been found to hold uniformly in our past experience, and on the continuance of which, in future, the constitution of our mind determines us confidently to rely.

After what has been already said, it is hardly necessary to take notice of the absurdity of that opinion, or rather of that mode of speaking, which seems to refer the order of the universe to *general laws* operating as *efficient causes*. Absurd, however, as it is, there is reason to suspect, that it has, with many, had the effect of keeping the Deity out of view, while

* So likewise Halley, in his Latin verses prefixed to Newton's *Principia* :

" En tibi norma poli, et divæ libramina molis,
" Computus en Jovis ; et *quas, dum primordia rerum*
" *Pangeret, omniparens leges violare Creator*
" *Noluit.*"

they were studying his works. To an incautious use of the same very equivocal phrase, may be traced the bewildering obscurity in the speculations of some eminent French writers, concerning its metaphysical import. Even the great Montesquieu, in the very first chapter of his principal work, has lost himself in a fruitless attempt to explain its meaning, when, by a simple statement of the essential distinction between its literal and its metaphorical acceptations, he might have at once cleared up the mystery. After telling us that "laws, "in their most extensive signification, are the necessary re- "lations *(les rapports nécessaires)* which arise from the na- "ture of things, and that, in this sense, *all* beings have their "laws;—that the Deity has *his* laws; the material world *its* "laws; intelligences superior to man *their* laws; the brutes "*their* laws; man *his* laws;"—he proceeds to remark, "That "the moral world is far from being so well governed as the "material; for the former, although it has *its* laws, which are "invariable, does not observe these laws so constantly as the "latter." It is evident that this remark derives whatever plausibility it possesses from a play upon words; from confounding *moral* laws with *physical;* or, in plainer terms, from confounding laws which are addressed by a legislator to intelligent beings, with those general conclusions concerning the establish-ed order of the universe, to which, when legitimately inferred from an induction sufficiently extensive, philosophers have metaphorically applied the title of *Laws of Natur*[illegible]. In the one case, the conformity of the law with the nature of things, does not at all depend on its being observed or not, but on the reasonableness and moral obligation of the law. In the other case, the very definition of the word *law* supposes that it applies universally; insomuch that, if it failed in one single instance, it would cease to be a *law*. It is, therefore, a mere quibble to say, that the laws of the material world are better observed than those of the moral; the meaning of the word *law*, in the two cases to which it is here applied, being so totally different, as to render the comparison or contrast, in the statement of which it is involved, altogether illusory and sophistical. Indeed, nothing more is necessary to strip the proposition of every semblance of plausibility, but an attention to this verbal ambiguity *.

This metaphorical employment of the word *law*, to express a general fact, although it does not appear to have been adopted in the technical phraseology of ancient philosophy, is not unusual among the classical writers, when speaking of those physical arrangements, whether on the earth or in the heavens, which continue to exhibit the same appearance from age to age.

> " Hic segetes, illic veniunt felicius uvæ :
> " Arborei fetus alibi, atque injussa virescunt

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* I do not recollect any instance in the writings of Montesquieu, where he has reasoned more vaguely than in this chapter; and yet I am inclined to believe, that few chapters in the *Spirit of Laws* have been more admired. " Montesquieu (says a " French writer) paroissoit à Thomas le premier des écrivains, pour la force et l'étendue " des idées, pour la multitude, la profondeur, la nouveauté des rapports. Il est incroy- " able (disoit-il) tout ce que Montesquieu a fait apperçevoir dans ce mot si court, le " mot Loi." (*Nouveau Diction. Historique*, Art. *Thomas*. Lyon, 1804.)

For some important remarks on the distinction between moral and physical laws, see Dr Ferguson's Institutes of Moral Philosophy, last edit.

"Gramina. Nonne vides, croceos ut Tmolus odores,
"India mittit ebur, molles sua thura Sabæi?
"At Chalybes nudi ferrum, virosaque Pontus
"Castorea, Eliadum palmas Epiros equarum?
"Continuo has *leges*, æternaque fœdera certis
"Imposuit natura locis *."

The same metaphor occurs in another passage of the Georgics, where the poet describes the regularity which is exhibited in the economy of the bees:

"Solæ communes natos, consortia tecta
"Urbis habent, magnisque agitant *sub legibus* ævum †."

The following lines from Ovid's account of the Pythagorean philosophy, are still more in point:

"Et rerum causas, et quid natura docebat;
"Quid Deus: Unde nives: quæ fulminis esset origo:
"Jupiter, an venti, discussa nube tonarent:
"Quid quateret terras, quâ sidera *lege* mearent,
"Et quodcunque latet ‡."

I have quoted these different passages from ancient authors, chiefly as an illustration of the strength and of the similarity of the impression which the order of nature has made on the

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* Virg. I. Georg. 60. † Georg. IV. 153.

‡ Ovid. Met. XV. 68.

I shall only add to these quotations the epigram of Claudian on the instrument said to be invented by Archimedes for representing the movements of the heavenly bodies, in which various expressions occur coinciding remarkably with the scope of the foregoing observations.