But the observations and experiments of the philosopher are commonly only a step towards a farther end. This end is, *first*,

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" *Experimenta* fiunt ponendo naturam in eas circumstantias, in quibus debeat agere et " nobis ostendere id quod quærimus, quod pertinet ad physicam experimentalem. Porro " et ferro et igni utimur, ac dissolvimus per vim compagem corporum, potissimum in " chemiâ, et naturam quodammodo velut torquentes cogimus revelare sua secreta."

I have elsewhere remarked, that the physical discoveries of the moderns have been chiefly owing to the skilful contrivance and conduct of *experiments*; and that this method of interrogating nature was, in a great measure, unknown to the ancients. (*Philosophical Essays*, p. xxxv.) Even Aristotle himself is acknowledged, by one of his most devoted admirers, to have confined himself chiefly to *observation*; and is, on this very ground, proudly contrasted with the empirical experimentalists of the present times. " Aristotle, " (says Dr Gillies) was contented with catching nature in the fact, without attempting, " after the modern fashion, to put her to the torture; and in rejecting experiments " operose, toilsome, or painful, either to their objects or their authors, he was justified " by the habits of thinking, almost universally prevalent in his age and country. Edu- " cated in free and martial republics, careless of wealth, because uncorrupted by luxu- " ry, the whole tribe of ancient Philosophers dedicated themselves to agreeable only " and liberal pursuits, with too proud a disdain of arts merely useful or lucrative. They " ranked with the first class of citizens; and as such, were not to be lightly subjected " to unwholesome or disgusting employments. To bend over a furnace, inhaling nox- " ious steams, to torture animals, or to touch dead bodies, appeared to them operations " not more misbecoming their humanity, than unsuitable to their dignity. For such " discoveries as the heating and mixing of bodies offers to inquisitive curiosity, the " naturalists of Greece trusted to slaves and mercenary mechanics, whose poverty or " avarice tempted them to work in metals and minerals; and to produce, by unweari- " ed labour, those coloured and sculptured ornaments, those gems, rings, cups, and " vases, and other admired but frivolous elegancies, of which (in the opinion of good " judges of art) our boasted chemistry cannot produce the materials; nor, were the " materials at hand, supply us with instruments fit to shape. The work-shops of " tradesmen then revealed those mysteries which are now sought for in colleges and " laboratories; and useful knowledge, perhaps, was not the less likely to be advanced, " while the arts were confined to artists only; nor facts the more likely to be perverted, " in order to support favourite theories, before the empiric had yet assumed the name,

to resolve particular facts into other facts more simple and comprehensive: and, *secondly*, to apply these general facts (or, as they are usually called, these *laws of nature*) to a synthetical explanation of particular phenomena. These two processes of the mind, together with that judicious employment of observation and experiment which they presuppose, exhaust the whole business of philosophical investigation; and the great object of the rules of philosophising is, to show in what manner they ought to be conducted.

I. For the more complete illustration of this fundamental doctrine, it is necessary for me to recur to what has been already

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" and usurped the functions of the philosopher." (Translation of Aristotle's Ethics and Politics, Vol. I. p. 161, 2d Ed.)

In another passage, we are told by the same author, that " the learning of Greece " properly terminates in the Stagirite, by whom it was finally embodied into one great " work; *a work rather impaired than improved by the labours of succeeding ages.*" ! (Ibid, p. x. of the Preface.)

Notwithstanding the length of this note, I must beg leave to add to it a short extract from one of the aphorisms of Lord Bacon.— " Of the *criteria* for guiding our judgment " among so many different and discordant schools, there is none more to be relied on; " than that which is exhibited by their *fruits*; for the fruits of any speculative doctrine, " or the inventions which it has really produced, are, as it were, sponsors or vouchers for " the truths which it contains. Now, it is well known, that from the philosophy of the " Greeks, with its numerous derivative schools, hardly one experimental discovery can " be collected which has any tendency to aid or to ameliorate the condition of man, " or which is entitled to rank with the acknowledged principles of genuine science."— " Wherefore, as in religion, faith is proved by its works, so in philosophy, it were to be " wished, that those theories should be accounted vain, which, when tried by their " fruits, are barren;—much more those, which, instead of grapes and olives, have pro- " duced only the thorns and thistles of controversy."—Nov. Org. Lib. i. Aph. lxiii.

stated with respect to our ignorance of *efficient causes*. As we can, in no instance, perceive the link by which two successive events are connected, so as to deduce, by any reasoning *a priori*, the one from the other as a consequence or effect, it follows, that when we see an event take place which has been preceded by a combination of different circumstances, it is impossible for human sagacity to ascertain whether the effect is connected with *all* the circumstances, or only with a *part* of them; and (on the latter supposition) which of the circumstances is essential to the result, and which are merely accidental accessories or concomitants. The only way, in such a case, of coming at the truth, is to repeat over the experiment again and again, leaving out all the different circumstances successively, and observing with what particular combinations of them the effect is conjoined. If there be no possibility of making this separation, and if, at the same time, we wish to obtain the same result, the only method of *insuring* success is to combine together *all* the various circumstances which were united in our former trials. It is on this principle, that I have attempted, in a former chapter of this work, to account for the superstitious observances which always accompany the practice of medicine among rude nations. These are commonly ascribed to the influence of imagination, and the low state of reason in the earlier periods of society; but the truth is, that they are the necessary and unavoidable consequences of a limited experience, and are to be corrected, not by mere force of intellect, but by a more enlarged acquaintance with the established order of nature.*

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* Elements of the Philosophy of the Human Mind, Vol. I. Chap. v. Part ii. sect. i.

Observations perfectly similar to those which I made with respect to medicine are applicable to all the other branches of philosophy. Wherever an interesting *change* is preceded by a combination of different circumstances, it is of importance to vary our experiments in such a manner as to distinguish what is essential from what is accessory; and when we have carried the decomposition as far as we can, we are entitled to consider this simplest combination of indispensable conditions, as the *physical cause* of the event.

When by thus comparing a number of cases, agreeing in some circumstances, but differing in others, and all attended with the same result, a philosopher connects, as a general law of nature, the event with its *physical cause*, he is said to proceed according to the method of *induction*. This, at least, appears to me to be the idea which, in general, Bacon himself annexes to the phrase*: although I will not venture to affirm, that he has always employed it with uniform precision. I acknowledge, also, that it is often used by very accurate writers, to denote the whole of that system of rules, of which the process just mentioned forms the most essential and characteristical part.

The same word *induction* is employed by mathematicians in a sense not altogether different. In that general *formula* (for

* Inductio, quæ ad inventionem et demonstrationem scientiarum et artium erit utilis, naturam separare debet, per rejectiones et exclusiones debitas, &c. &c.—Nov. Org. Lib. i. Aph. cv.

instance) known by the name of the Binomial Theorem, having found that it corresponds with the table of powers raised from a Binomial root, as far as it is carried by actual multiplication, we have no scruple to conclude, that it holds universally. Such a proof of a mathematical theorem is called a proof by *induction*;—a mode of speaking obviously suggested by the previous application of this term to our inferences concerning the laws of nature. There is, at the same time, notwithstanding the obvious analogy between the two cases, one very essential circumstance by which they are discriminated;—that, in mathematical induction, we are led to our conclusion (as I shall afterwards endeavour to shew) by a process of thought, which, although not conformable to the rules of legitimate demonstration, involves, nevertheless, a logical inference of the understanding with respect to an universal truth or theorem; whereas, in drawing a general physical conclusion from particular facts, we are guided merely by our instinctive expectation of the continuance of the laws of nature; an expectation which, implying little, if any, exercise of the reasoning powers, operates alike on the philosopher and on the savage.

To this belief in the permanent uniformity of physical laws, Dr Reid long ago gave the name of the *inductive principle*. "It "is from the force of this principle (he observed) that we imme- "diately assent to that axiom upon which all our knowledge of "nature is built, That effects of the same kind must have the "same cause. For effects and causes, in the operations of nature, "mean nothing but signs, and the things signified by them. "We perceive no proper causality or efficiency in any natural

" cause; but only a connection established by the course of " nature between it and what is called its effects *."

A late celebrated writer, more distinguished by the singular variety and versatility of his talents than by the depth or soundness of his understanding, was pleased to consider Reid's *inductive principle* as a fit subject of ridicule; asserting that the phenomenon in question was easily explicable by the common principles of *experience*, and the *association of ideas*. " Though " no man (says he) has had any experience of what *is* fu- " ture, every man has had experience of what *was* future †." Of the shallowness of this solution philosophers are, I believe, now very generally convinced; but even if the case were otherwise, the *fact* remarked by Reid would be equally entitled to the attention of logicians as the basis of all physical science, nor would it be easy to distinguish it by a name less liable to objection than that which he has selected.

In all Bacon's logical rules, the authority of this law of belief is virtually recognized, although it is nowhere formally stated in his writings; and although the doctrines connected with it do not seem to be easily reconcilable with some of his occasional expressions. It is indeed only of late that natural philosophers have been fully aware of its importance as the groundwork of the inductive logic; the earlier writers under whose re-

* Inquiry into the Human Mind, Chap. vi. Sect. 24.

† Priestley's Examination of Reid, Beattie and Oswald, p. 85. Some very judicious and decisive strictures on this theory of Priestley may be found in Dr Campbell's Philosophy of Rhetoric. See note at the end of the sixth Chapter of Book i.

view it fell having been led to consider it chiefly by its supposed subserviency to their metaphysical or to their theological speculations. Dr Reid and M. Turgot were, so far as I know, the first who recognized its existence as an original and ultimate law of the understanding;—the source of all that experimental knowledge which we begin to acquire from the moment of our birth, as well as of those more recondite discoveries which are dignified by the name of science. It is but justice to Mr Hume to acknowledge, that his Treatise of Human Nature furnished to Dr Reid all the premises from which his conclusions were drawn; and that he is therefore fairly entitled to the honour of having reduced logicians to the alternative of either acquiescing in his sceptical inferences, or of acknowledging the authority of some instinctive principles of belief, overlooked in Locke's Analysis *.

II. There is another circumstance which frequently adds to the difficulty of tracing the laws of nature; and which imposes on the philosopher, while carrying on the process of induction, the necessity of following a still more refined logic than has been hitherto described.—When a uniformity is observed in a number of different events, the curiosity is roused by the coincidence, and is sometimes led insensibly to a general conclusion. In a few other cases, a multiplicity of events, which appear to common observers to be altogether anomalous, are found, upon a more accurate and continued examination of them, to be subjected to a regular law †. The

* Note (O.)

† Philosophy of the Human Mind, Vol. I. Chap. vi. Sect. iv.

cycles by which the ancients predicted eclipses of the sun and moon; the two laws inferred by Kepler from the observations of Tycho Brahe; the law of refraction inferred by Snellius from the tables of Kircher and Scheiner, are instances of very comprehensive and most important rules obtained by the mere examination and comparison of particulars. Such purely *empirical discoveries*, however, are confined almost entirely to optics and astronomy, in which the physical laws combined together are comparatively few, and are insulated from the influence of those incalculable accidents which, in general, disturb the regularity of terrestrial phenomena. In by far the greater number of instances, the appearances of nature depend on a variety of different laws, all of which are often combined together in producing one single event: And, wherever such a combination happens, although each law may take place with the most complete uniformity, it is likely that nothing but confusion will strike the mere observer. A collection of such results, therefore, would not advance us one step in the knowledge of nature; nor would it enable us to anticipate the issue of one new experiment. In cases of this description, before we can avail ourselves of our past experience, we must employ our reasoning powers in comparing a variety of instances together, in order to discover, by a sort of *analysis* or decomposition, the simple laws which are concerned in the phenomenon under consideration;—after which, we may proceed safely, in determining *a priori* what the result will be of any hypothetical combination of them, whether total or partial *.

* "Itaque naturæ facienda est prorsus solutio et separatio; non per iguem certe, sed

These observations have led us to the same conclusion with that which forms the great outline of Bacon's plan of philosophising; and which Newton has so successfully exemplified in his inquiries concerning gravitation and the properties of light. While they point out, too, the respective provinces and uses of the *analytic* and the *synthetic* methods, they illustrate the etymological propriety of the names by which, in the Newtonian School, they are contradistinguished from each other.

In fact, the meaning of the words *analysis* and *synthesis*, when applied to the two opposite modes of investigation in physics, is extremely analogous to their use in the practice of chemistry. The chief difference lies in this, that, in the former case, they refer to the logical processes of the understanding in the study of *physical laws*; in the latter, to the operative processes of the laboratory in the examination of material substances.

If the foregoing remarks are well founded, they lead to the

"per mentem, tanquam ignem divinum." Nov. Organ. Lib. II. Aphor. xvi. The remainder of the aphorism is equally worthy of attention; in reading which, however, as well as the rest of Bacon's philosophical works, I must request, for a reason afterwards to be mentioned, that the word *Law* may be substituted for *Form*, wherever it may occur.—An attention to this circumstance will be found of much use in studying the *Novum Organon*.

A similar idea, under other metaphorical disguises often occurs in Bacon. Considering the circumstances in which he wrote, logical precision was altogether impossible; yet it is astonishing with what force he conveys the *spirit* of the soundest philosophy of the eighteenth century. "Neque enim in plano via sita est, sed *ascendendo* "et *descendendo*; ascendendo primo ad axiomata, descendendo ad opera." Nov. Org. Lib. I. Aphor. ciii.

correction of an oversight which occurs in the ingenious and elegant sketch of the History of Astronomy, lately published among the posthumous works of Mr Smith; and which seems calculated to keep out of view, if not entirely to explode that essential distinction which I have been endeavouring to establish, between the inductive logic of Bacon's followers, and the hypothetical theories of their predecessors.

"Philosophy" (says Mr Smith) "is the science of the connecting principles of nature. Nature, after the largest experience that common observation can acquire, seems to abound with events which appear solitary and incoherent with all that go before them; which therefore disturb the easy movement of the imagination; which make its ideas succeed each other, if one may say so, by irregular starts and sallies; and which thus tend, in some measure, to introduce a confusion and distraction and giddiness of mind. Philosophy, by representing the invisible chains which bind together all these disjointed objects, endeavours to introduce order into this chaos of jarring and discordant appearances; to allay this tumult of the imagination; and to restore it, when it surveys the great revolutions of the universe, to that tone of tranquillity and composure, which is both most agreeable in itself, and most suitable to its nature. Philosophy, therefore, may be regarded as one of those arts which address themselves to the imagination, by rendering the theatre of nature a more coherent, and, therefore, a more magnificent spectacle, than otherwise it would have appeared to be."

That this is *one* of the objects of philosophy, and *one* of the

advantages resulting from it, I very readily admit.—But, surely, it is not the *leading* object of that plan of inductive investigation which was recommended by Bacon, and which has been so skilfully pursued by Newton. Of all philosophical systems, indeed, hypothetical or legitimate, it must be allowed, that, to a certain degree, they both please the imagination and assist the memory, by introducing order and arrangement among facts, which had the appearance, before, of being altogether unconnected and isolated. But it is the peculiar and exclusive prerogative of a system fairly obtained by the method of induction, that, while it enables us to arrange facts already known, it furnishes the means of ascertaining, by synthetic reasoning, those which we have no access to examine by direct observation. The difference, besides, among hypothetical theories, is merely a difference of *degree*, arising from the greater or less ingenuity of their authors; whereas legitimate theories are distinguished from all others, *radically and essentially*; And, accordingly, while the former are liable to perpetual vicissitudes, the latter are as permanent as the laws which regulate the order of the universe.

Mr Smith himself has been led by this view of the object of philosophy, into expressions concerning the Newtonian discoveries, which seem to intimate, that, although he thought them far superior, in point of ingenuity, to any thing the world had seen before, yet, that he did not consider them as so completely exclusive of a still happier system in time to come, as the Newtonians are apt to imagine. "The system of New-" ton" (he observes) "now prevails over all opposition, and

"has advanced to the acquisition of the most universal empire "that was ever established in philosophy. His principles, it "must be acknowledged, have a degree of firmness and so- "lidity that we should in vain look for in any other system. "The most sceptical cannot avoid feeling this. They not only "connect together most perfectly all the phenomena of the "heavens which had been observed before his time; but those "also which the persevering industry and more perfect instru- "ments of later astronomers have made known to us, have "been either easily and immediately explained by the appli- "cation of his principles, or have been explained in conse- "quence of more laborious and accurate calculations from these "principles, than had been instituted before. And *even we*, while "we have been endeavouring to represent all *philosophical sys- "tems as mere inventions of the imagination*, to connect together "the otherwise disjointed and discordant phenomena of nature, "have insensibly been drawn in to make use of language ex- "pressing the connecting principles of this one, as if they were "the real chains which nature makes use of, to bind together "her several operations."

If the view which I have given of Lord Bacon's plan of investigation be just, it will follow, That the Newtonian theory of gravitation can, in no respect whatever, admit of a comparison with those systems which are, in the slightest degree, the offspring of imagination; inasmuch as the principle employed to explain the phenomena is not a hypothesis, but a *general fact* established by induction; for which fact we have the very same evidence as for the various particulars comprehended under it.

The Newtonian theory of gravitation, therefore, and every other theory which rests on a similar basis, is as little liable to be supplanted by the labours of future ages, as the mathematical conclusions of Euclid and Archimedes. The doctrines which it involves may be delivered in different, and perhaps less exceptionable forms; but, till the order of the universe shall be regulated by new physical laws, their substance must for ever remain essentially the same. On the *chains, indeed, which nature makes use of to bind together her several operations*, Newton has thrown no light whatever; nor was it the aim of his researches to do so. The subjects of his reasonings were not occult connections, but particular phenomena, and general laws;—both of them possessing all the evidence which can belong to *facts* ascertained by observation and experiment. From the one or the other of these all his inferences, whether analytical or synthetical, are deduced: Nor is a single hypothesis involved in his *data*, excepting the authority of that Law of Belief which is tacitly and necessarily assumed in all our physical conclusions,—The stability of the order of nature.

SECTION II.

Continuation of the Subject.—The Induction of Aristotle compared with that of Bacon.

In this section I intend to offer a few slight remarks upon an assertion which has been hazarded with some confidence in various late publications, that the method of investigation, so

much extolled by the admirers of Lord Bacon, was not unknown to Aristotle.—It is thus very strongly stated by the ingenious author of a memoir in the Asiatic Researches *.

"From some of the extracts contained in this paper, it will "appear, 1st, That the mode of reasoning by *induction*, illus- "trated and improved by the great Lord Verulam in his *Orga-* "*num Novum*, and generally considered as the cause of the rapid "progress of science in later times, was *perfectly known* to "Aristotle, and was distinctly delineated by him, as a method "of investigation that leads to certainty or truth: and 2dly, "That Aristotle was likewise *perfectly acquainted*, not merely "with the form of induction, but with the proper materials to "be employed in carrying it on—facts and experiments. We "are therefore led to conclude, that all the blame of confining "the human mind for so long a time in chains, by the force of "syllogism, cannot be fairly imputed to Aristotle; nor all the "merit of enlarging it, and setting it free, ascribed to Lord "Verulam."

The memoir from which this passage is copied, consists of extracts translated (through the medium of the Persian) from an Arabic treatise entitled the *Essence of Logic*. When it was first presented to the Asiatic Society, the author informs us, that he was altogether ignorant of the coincidence of his own conclusions with those of Dr Gillies; and he seems to have received much satisfaction from the subsequent perusal of the

* Asiatic Researches, Vol. VIII. p. 89, 90. London Edition.

proofs alleged in support of their common opinion by that learned writer. "From the perusal of *this wonderful book* (Dr "Gillies's exposition of the ethics and politics of Aristotle) I "have now the satisfaction to discover, that the conjectures I "had been led to draw from these scanty materials, are com- "pletely confirmed by the opinion of an author, who is pro- "bably better qualified than any preceding commentator on "Aristotle's works, to decide on this subject*."

It is observed by Bailly, in his History of Astronomy, that, although frequent mention is made of *attraction* in the writings of the ancients, we must not therefore "conclude that they had "any precise or just idea of that law into which Newton has "resolved the phenomena of the planetary revolutions. To "their conceptions, this word presented the notion of an occult "*sympathy* between different objects; and if any of them ex- "tended it from the descent of terrestrial bodies to explain the "manner in which the moon was retained in her orbit, it was "only an exhibition upon a larger scale of the popular error†." The same author has remarked, on a different occasion, that, in order to judge of the philosophical ideas entertained at a particular period, it would be necessary to possess the dictionary of the age,—exhibiting the various shades of meaning derived from fashion or from tradition. "The import of words (he "adds) changes with the times: their signification enlarging "with the progress of knowledge. Languages are every moment

* Ibid.

† Hist. de l'Astronomie Moderne, Tome II. p. 555, 556.

" perishing in detail from the variations introduced by custom: " they grow old like those that speak them, and like them " gradually alter their features and their form *."

If this observation be just, with respect to the *attraction* of the ancients, when compared with the *attraction* of Newton, it will be found to apply with still greater force to the *induction* of Aristotle †, considered in contrast with the *induction* of Bacon.

It is well known to those who are at all conversant with Bacon's writings, that, although he borrowed many expressions from the scholastic phraseology then in vogue, he has, in general, not only employed them in new acceptations, consonant to the general spirit of his own logic, but has, by definitions or explanations, endeavoured to guard his readers against the mistakes to which they might be exposed, from a want of attention to the innovations thus introduced in the use of consecrated terms. How far he judged wisely in adopting this plan, (which has certainly much injured his style in point of perspicuity) I do not presume to decide; I wish only to state the fact:—his motives may be judged of from his own words.

" Nobis vero ex altera parte (quibus, quantum calamo vale- " mus, inter vetera et nova in literis fœdus et commercium con- " trahere, cordi est) decretum manet, antiquitatem comitari *us-*

* Ibid. p. 184.

† Επαγωγη. Translated *Inductio* by Cicero.

"*que ad aras*; atque vocabula antiqua retinere, quanquam "sensum eorum et definitiones sæpius immutemus; secun- "dum moderatum illum et laudatum, in Civilibus, novandi "modum, quo rerum statu novato, verborum tamen solennia "durent; quod notat Tacitus; *eadem magistratuum vocabula* *."

Of these double significations, so common in Bacon's phraseology, a remarkable instance occurs in the use which he makes of the scholastic word *forms*. In one passage, he approves of the opinion of Plato, that the investigation of *forms* is the proper object of science; adding, however, that this is not true of the *forms* which Plato had in view, but of a different sort of *forms*, more suited to the grasp of our faculties †. In another passage, he observes, that when he employs the word *forms*, in speaking of natural philosophy, he is always to be understood

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* De Aug. Scient. Lib. iii. Cap. iv.

The necessity under which the anti-Aristotelians found themselves, in the earlier part of the 17th century, of disguising their attack on the prevailing tenets, is strongly illustrated in a letter from Des Cartes to Regius. "Pourquoi rejettez-vous publiquement "*les qualités réelles* et *les formes substantielles*, si cheres aux scholastiques: J'ai déclaré, "que je ne prétendois pas les nier, mais que je n'en avois pas besoin pour expliquer "mes pensées."

† Manifestum est, Platonem, virum sublimis ingenii (quique veluti ex rupe excelsa omnia circumspiciebat) in sua de ideis doctrina, *formas esse verum scientiæ objectum*, vidisse; utcunque sententiæ hujus verissimæ fructum amiserit, *formas* penitus a materia abstractas, non in materia determinatas contemplando et prensando. Quod si diligenter, serio, et sincere, ad actionem, et usum, et oculos convertamus; non difficile erit disquirere, et notitiam assequi, quæ sint illæ *formæ*, quarum cognitio res humanas meris modis locupletare et beare possit.—De Augment. Scient. Lib. iii. Cap. iv.

as meaning the *laws of nature* *. Whether so accurate a reasoner as Locke would have admitted Bacon's general apology for so glaring an *abuse of words*, may perhaps be doubted: but, after comparing the two foregoing sentences, would Locke (notwithstanding his ignorance of the syllogistic art) have inferred, that Bacon's opinion of the proper object of science was the same with that of Plato? The attempt to identify Bacon's induction with the induction of Aristotle, is (as I trust will immediately appear) infinitely more extravagant. It is like confounding the Christian Graces with the Graces of Heathen Mythology.

The passages in which Bacon has been at pains to guard against the possibility of such a mistake are so numerous, that it is surprizing how any person, who had ever turned over the pages of the Novum Organon, should have been so unlucky as not to have lighted upon some one of them. The two following will suffice for my present purpose.

"In constituendo autem axiomate, forma inductionis alia "quam adhuc in usu fuit, excogitanda est. Inductio enim "quæ procedit per *enumerationem simplicem* res puerilis est, et "precario concludit. At *inductio*, quæ ad inventionem et de-"monstrationem scientiarum et artium erit utilis, naturam se-

* Nos quum de formis loquimur, nil aliud intelligimus, quam leges illas, quæ naturam aliquam simplicem ordinant et constituunt; ut calorem, lumen, pondus, *in* omnimoda materia et subjecto susceptibili. Itaque eadem res est *forma* calidi, aut *forma* luminis, et *lex* calidi, sive *lex* luminis.—Nov. Org. Lib. ii. Aph. xvii.

" parare debet, per rejectiones et exclusiones debitas; ac deinde " post negativas tot quot sufficiunt, super affirmativas conclu- " dere; quod adhuc factum non est, nec tentatum certe, nisi " tantummodo a Platone, qui ad excutiendas definitiones et " idæas, hac certe forma inductionis aliquatenus utitur. Ve- " rum ad hujus inductionis, sive demonstrationis instructionem " bonam et legitimam, quamplurima adhibenda sunt, quæ ad- " huc nullius mortalium cogitationem subiere; adeo ut in ea " major sit consumenda opera, quam adhuc consumpta est in " syllogismo. *Atque in hac certe inductione*, spes maxima sita " est *."

—— " Cogitavit et illud — Restare *inductionem*, tanquam ul- " timum et unicum rebus subsidium et perfugium. Verum et " hujus *nomen* tantummodo notum esse; *vim et usum* homines " hactenus latuisse †."

That I may not, however, be accused of resting my judgment entirely upon evidence derived from Bacon's writings, it may be proper to consider more particularly to what the induction of Aristotle

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* Nov. Org. Lib. i. Aph. cv.

† Cogitata et Visa. The short tract to which Bacon has prefixed this title, contains a summary of what he seems to have considered as the leading tenets of his philosophical works. It is one of the most highly finished of all his pieces, and is marked throughout with an impressive brevity and solemnity, which commands and concentrates the attention. Nor does it affect to disguise that consciousness of intellectual force, which might be expected from a man destined to fix a new æra in the history of human reason.—FRANCISCUS BACONUS SIC COGITAVIT, &c. &c.

really amounted, and in what respects it coincided with that to which Bacon has extended the same name.

"Our belief (says Aristotle in one passage) is, in every in-
"stance, founded either on syllogism or induction." To which observation he adds, in the course of the same chapter, that "induction is an inference drawn from *all* the particulars which "it comprehends*." It is manifest, that, upon this occasion, Aristotle speaks of that induction which Bacon, in one of the extracts quoted above, describes as proceeding by simple enumeration; and which he, therefore, pronounces to be "a puerile "employment of the mind, and a mode of reasoning leading to "uncertain conclusions." In confirmation of Bacon's remark, it is sufficient to mention, by way of illustration, a single example; which example, to prevent cavils, I shall borrow from one of the highest logical authorities,—Dr Wallis of Oxford.

"In an inference from induction, (says this learned writer) "if the enumeration be complete, the evidence will be equal "to that of a perfect syllogism; as if a person should argue, "that all the planets (the Sun excepted) borrow their light from "the Sun, by proving this separately of Saturn, Jupiter, Mars, "Venus, Mercury, and the Moon. It is, in fact, a syllogism "*in Darapti*, of which this is the form:

"*Saturn, Jupiter, Mars, Venus, Mercury, and the Moon,*
"*each borrow their light from the Sun:*

* First Analytics, Chap. xxiii. Vol. I. p. 126. Edit. Du Val.

" *But this enumeration comprehends all the Planets, the Sun*
" *excepted:*
" *Therefore all the Planets, (the Sun excepted,) borrow their*
" *light from the Sun* *."

If the object of Wallis had been to expose the *puerility* and the *precariousness* of such an argument, he could not possibly have selected a happier illustration. The *induction* of Aristotle, when considered in *this* light, is indeed a fit companion for his syllogism; inasmuch, as neither can possibly advance us a single step in the acquisition of new knowledge. How different from both is the induction of Bacon, which, instead of carrying the mind round in the same circle of words,

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* Institutio Logica, Lib. iii. Cap. 15. The reasoning employed by Wallis to shew that the above is a legitimate syllogism *in Darapti*, affords a specimen of the facility with which a logical conjuror can transform the same argument into the most different shapes. " Siquis objiciat, hunc non esse legitimum in *Darapti* syllogismum, eo quod " conclusionem habeat universalem; dicendum erit, hanc universalem (qualis qualis " est) esse *universalem collectivam*; quæ *singularis* est. Estque vox *omnis* hic loci (quæ " dici solet) pars *Categorematica*; utpote pars termini minoris (ut ex minori propositione " liquet) qui hic est (non *Planetæ* sed) *omnes Planetæ (excepto sole)*, seu tota collec- " tio reliquorum (excepto sole) Planetarum, quæ *collectio* unica est; adeoque conclusio " *singularis*. Quæ quidem (ut singulares aliæ) quamvis sit propositio *Universalis*, vi ma- " teriæ; non tamen talis est ut non possit esse conclusio in tertia figura. Quippe in tertia " figura, quoties minor terminus, seu prædicatum minoris propositionis (adeoque subjec- " tum conclusionis) est quid singulare, necesse estut conclusio ea sit (vi materiæ, non " formæ) ejusmodi universalis."

In justice to Dr Wallis, it is proper to subjoin to these quotations, a short extract from the dedication prefixed to this treatise.—" Exempla retineo, quæ apud lo- " gicos trita sunt; ex philosophia quam vocant Veterem et Peripateticam petita: quia " logicam hic trado, et quidem Peripateticam; non naturalem philosophiam. Adeo- " que, de quatuor elementis; de telluris quiete in universi medio; de gravium motu " deorsum, leviumque sursum; de septenario planetarum numero, aliisque; sic loquor, " ut loqui solent Peripatetici."

leads it from the *past* to the *future*, from the *known* to the *unknown*?*

Dr Wallis afterwards very justly remarks, "that inductions "of this sort are of frequent use in mathematical demonstra- "tions; in which, after enumerating all the possible cases, it "is proved, that the proposition in question is true of each of "these considered separately; and the general conclusion is "thence drawn, that the theorem holds universally. Thus, if "it were shewn, that, in all right-angled triangles, the three "angles are equal to two right angles, and that the same thing "is true in all acute-angled, and also in all obtuse-angled tri- "angles; it would necessarily follow, that in *every* triangle the "three angles are equal to two right angles; these three cases "manifestly exhausting all the possible varieties of which the "hypothesis is susceptible."

My chief motive for introducing this last passage, was to correct an idea, which, it is not impossible, may have contributed to mislead some of Wallis's readers. As the professed design of the treatise in question, was to expound the logic of Aristotle, agreeably to the views of its original author; and as all its examples and illustrations assume as truths the Peripate-

* "In arte judicandi (ut etiam vulgo receptum est) aut per Inductionem, aut per "Syllogismum concluditur. At quatenus ad judicium, quod fit per inductionem, nihil "est, quod nos detinere debeat: *uno siquidem eodemque mentis opere illud quod quæritur, "et invenitur et judicatur.*—At inductionis formam vitiosam prorsus valere jubemus; le- "gitimam ad Novum Organum remittimus."—De Aug. Scient. Lib. v. Cap. iv.

tic tenets, it was not unnatural to refer to the same venerated source, the few incidental reflections with which Wallis has enriched his work. Of this number is the foregoing remark, which differs so very widely from Aristotle's account of mathematical induction, that I was anxious to bring the two opinions into immediate contrast. The following is a faithful translation from Aristotle's own words:

"If any person were to show, by particular demonstrations, "that every triangle, separately considered, the equilateral, the "scalene, and the isosceles, has its three angles equal to two "right angles, he would not, *therefore*, know that the three "angles of a triangle are equal to two right angles, except af- "ter a *sophistical* manner. Nor would he know this as an uni- "versal property of a triangle, although, beside these, no other "triangle can be conceived to exist; for he does not know "that it belongs to it *quâ* triangle: Nor that it belongs to "every triangle, excepting in regard to number; his know- "ledge not extending to it as a property of the genus, although "it is impossible that there should be an individual which that "genus does not include*."

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* Διὰ τοῦτο οὐδ' ἂν τις δείξῃ καθ' ἕκαστον τὸ τρίγωνον ἀποδείξει ἢ μιᾷ ἢ ἑτέρᾳ, ὅτι δυὸ ὀρθὰς ἔχει ἕκαστον, τὸ ἰσοπλευρον χωρὶς, καὶ τὸ σκαληνον, καὶ τὸ ἰσοσκελὲς· οὔπω οἶδε τὸ τρίγωνον ὅτι δυὸ ὀρθαῖς ἴσον, εἰ μὴ τὸν σοφιστικὸν τρόπον· οὐδὲ καθόλου τρίγωνον, οὐδ' εἰ μηδὲν ἔστι παρὰ ταῦτα τρίγωνον ἕτερον· οὐ γὰρ, ἢ τρίγωνον οἶδεν· οὐδὲ πᾶν τρίγωνον, ἀλλ' ἢ κατ' ἀριθμὸν· κατ' εἶδος δὲ οὐ πᾶν, καὶ εἰ μηδέν ἐστιν ὃ οὐκ οἶδε.—Analyt. Poster. Lib. i. Cap. v.

I have rendered the last clause according to the best of my judgment; but in case of any misapprehension on my part, I have transcribed the author's words. It may be

For what reason Aristotle should have thought of applying to such an induction as this the epithet *sophistical*, it is difficult to conjecture. That it is more tedious, and therefore less elegant than a general demonstration of the same theorem, is undoubtedly true; but it is not on that account the less logical, nor, in point of form, the less rigorously geometrical. It is, indeed, precisely on the same footing with the proof of every mathematical proposition which has not yet been pushed to the utmost possible limit of generalization.

It is somewhat curious, that this hypothetical example of Aristotle is recorded as a historical fact by Proclus, in his commentary on Euclid. "One person we are told (I quote the "words of Mr Maclaurin) discovered, that the three angles of "an equilateral triangle are equal to two right angles; ano-"ther went farther, and shewed the same thing of those that "have two sides equal, and are called isosceles triangles: and "it was a third that found that the theorem was general, and "extended to triangles of all sorts. In like manner, when the "science was farther advanced, and they came to treat of the "conic sections, the plane of the section was always supposed "perpendicular to the side of the cone; the parabola was the "only section that was considered in the right-angled cone, "the ellipse in the acute-angled cone, and the hyperbola in the "obtuse-angled. From these three sorts of cones, the figures

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proper to mention, that this illustration is not produced by Aristotle as an instance of *induction*; but it obviously falls under his own definition of it, and is accordingly considered in that light by Dr Wallis.

" of the sections had their names for a considerable time; till, " at length, Apollonius shewed that they might all be cut out " of any one cone, and, by this discovery, merited in those days " the appellation of the Great Geometrician *."

It would appear, therefore, that, in mathematics, an inductive inference may not only be demonstratively certain, but that it is a natural, and sometimes perhaps a necessary step in the generalization of our knowledge. And yet it is of one of the most unexceptionable inductive conclusions in this science, (the only science in which it is easy to conceive an enumeration which excludes the possibility of any addition) that Aristotle has spoken,—as a conclusion resting on *sophistical* evidence.

So much with respect to Aristotle's *induction*, on the supposition that the enumeration is complete.

In cases where the enumeration is imperfect, Dr Wallis afterwards observes, " That our conclusion can only amount to " a probability or to a conjecture; and is always liable to be " overturned by an *instance* to the contrary." He observes also, " That this sort of reasoning is the principal instrument of " investigation in what is now called *experimental philosophy*; in " which, by observing and examining particulars, we arrive at " the knowledge of universal truths †." All this is clearly and correctly expressed; but it must not be forgotten, that it is the

* Account of Sir I. Newton's Phil. Discoveries, Book i. Chap. v.

† Institutio Logica.—See the Chapter De Inductione et Exemplo.

language of a writer trained in the schools of Bacon and of Newton.

Even, however, the induction here described by Dr Wallis, falls greatly short of the method of philosophising pointed out in the *Novum Organon*. It coincides exactly with those empirical inferences from mere experience, of which Bacon entertained such slender hopes for the advancement of science. "Restat experientia mera; quæ si occurrat, casus; si quæsita "sit, experimentum nominatur. Hoc autem experientiæ "genus nihil aliud est, quam mera palpatio, quali homines "noctu utuntur, omnia pertentando, si forte in rectam viam "incidere detur; quibus multo satius et consultius foret, diem "præstolari aut lumen accendere, deinceps viam inire. At "contra, verus experientiæ ordo primo lumen accendit, deinde "per lumen iter demonstrat, incipiendo ab experientia ordi-"nata et digesta, et minime præpostera aut erratica, atque ex ea "educendo axiomata, atque ex axiomatibus constitutis rursus ex-"perimenta nova, quum nec verbum divinum in rerum massam "absque ordine operatum sit *."

It is a common mistake, in the logical phraseology of the present times, to confound the words *experience* and *induction* as convertible terms †. There is, indeed, between them a very

* Nov. Org. Aph. lxxxii.

† "Let it always be remembered, that the author who first taught this doctrine (*that "the true art of reasoning is nothing but a language accurately defined and skilfully ar-"ranged*), had previously endeavoured to prove, that all our notions, as well as the signs "by which they are expressed, originate in perceptions of sense; and that the prin-

close affinity; inasmuch as it is on experience alone that every legitimate induction must be raised. The process of induction therefore presupposes that of experience; but, according to Bacon's views, the process of experience does by no means imply any idea of induction. Of this method, Bacon has repeatedly said, that it proceeds "by means of rejections and exclusions" (that is, to adopt the phraseology of the Newtonians, in the way of *analysis*) to separate or decompose nature; so as to arrive at those axioms or general laws, from which we may infer (in the way of synthesis) other particulars formerly unknown to us, and perhaps placed beyond the reach of our direct examination *.

But enough, and more than enough, has been already said to enable my readers to judge, how far the assertion is correct, that the induction of Bacon was well known to Aristotle. Whether

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"ciples on which languages are first constructed, as well as every step in their progress "to perfection, all ultimately depend *on inductions from observation; in one word, on* "*experience merely.*"—Aristotle's Ethics and Politics by Gillies, Vol. I. pp. 94, 95.

In the latter of these pages, I observe the following sentence, which is of itself sufficient to shew what notion the Aristotelians still annex to the word under consideration. "Every kind of reasoning is carried on either by syllogism or by induction; the former "proving to us, that a particular proposition is true, because it is deducible from "a general one, already known to us; and the latter demonstrating a general truth, be-"cause it holds in ALL particular cases."

It is obvious, that this species of induction never can be of the slightest use in the study of nature, where the phenomena which it is our aim to classify under their general laws, are, in respect of number, if not infinite, at least incalculable and incomprehensible by our faculties.

* Nov. Org. Aph. cv. ciii.

it be yet *well known* to all his commentators, is a different question; with the discussion of which I do not think it necessary to interrupt any longer the progress of my work.

SECTION III.

Of the Import of the Words Analysis and Synthesis, in the Language of Modern Philosophy.

As the words Analysis and Synthesis are now become of constant and necessary use in all the different departments of knowledge; and as there is reason to suspect, that they are often employed without due attention to the various modifications of their import, which must be the consequence of this variety in their applications,—it may be proper, before proceeding farther, to illustrate, by a few examples, their true logical meaning in those branches of science, to which I have the most frequent occasions to refer in the course of these inquiries. I begin with some remarks on their primary signification in *that* science, from which they have been transferred by the moderns to Physics, to Chemistry, and to the Philosophy of the Human Mind.

I.

Preliminary Observations on the Analysis and Synthesis of the Greek Geometricians.

It appears from a very interesting relic of an ancient writer*, that, among the Greek geometricians, two different sorts of analysis were employed as aids or guides to the inventive powers; the one adapted to the solution of problems; the other to the demonstration of theorems. Of the former of these, many beautiful exemplifications have been long in the hands of mathematical students; and of the latter, (which has drawn much less attention in modern times) a satisfactory idea may be formed from a series of propositions published at Edinburgh about fifty years ago†. I do not, however, know, that any person has yet turned his thoughts to an examination of the deep and subtle logic displayed in these analytical investigations; although it is a subject well worth the study of those who delight in tracing the steps by which the mind proceeds in pursuit of scientific discoveries. This *desideratum* it is not my present purpose to make any attempt to supply; but only to convey such general notions as may prevent my readers from falling into the common error of confounding the analysis

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* Preface to the seventh book of the Mathematical Collections of Pappus Alexandrinus. An extract from the Latin version of it by Dr Halley may be found in Note (P.)

† Propositiones Geometricæ More Veterum Demonstratæ. Auctore Matthæo Stewart, S. T. P. Matheseos in Academia Edinensi Professore, 1763.

and synthesis of the Greek Geometry, with the analysis and synthesis of the Inductive Philosophy.

In the arrangement of the following hints, I shall consider, in the first place, the nature and use of analysis in investigating the demonstration of *theorems*.—For such an application of it, various occasions must be constantly presenting themselves to every geometer;—when engaged, for example, in the search of more elegant modes of demonstrating propositions previously brought to light; or in ascertaining the truth of dubious theorems, which, from analogy, or other accidental circumstances, possess a degree of verisimilitude sufficient to rouse the curiosity.

In order to make myself intelligible to those who are acquainted only with that form of reasoning which is used by Euclid, it is necessary to remind them, that the enunciation of every mathematical proposition consists of two parts. In the first place, certain suppositions are made; and secondly, a certain consequence is affirmed to follow from these suppositions. In all the demonstrations which are to be found in Euclid's Elements (with the exception of the small number of indirect demonstrations), the particulars involved in the hypothetical part of the enunciation are assumed as the principles of our reasoning; and from these principles, a series or chain of consequences is, link by link, deduced, till we at last arrive at the conclusion which the enunciation of the proposition asserted as a truth. A demonstration of this kind is called a Synthetical demonstration.

Suppose now, that I arrange the steps of my reasoning in the reverse order; that I assume hypothetically the truth of the proposition which I wish to demonstrate, and proceed to deduce from this assumption, as a principle, the different consequences to which it leads. If, in this deduction, I arrive at a consequence which I already know to be true, I conclude with confidence, that the principle from which it was deduced is likewise true. But if, on the other hand, I arrive at a consequence which I know to be false, I conclude, that the principle or assumption on which my reasoning has proceeded is false also.—Such a demonstration of the truth or falsity of a proposition is called an Analytical demonstration.

According to these definitions of Analysis and Synthesis, those demonstrations in Euclid which prove a proposition to be true, by shewing, that the contrary supposition leads to some absurd inference, are, properly speaking, analytical processes of reasoning.—In *every* case, the conclusiveness of an analytical proof rests on this general maxim, That *truth* is always consistent with itself; that a supposition which leads, by a concatenation of mathematical deductions, to a consequence which is true, must itself be true; and that what necessarily involves a consequence which is absurd or impossible, must itself be false.

It is evident, that, when we are demonstrating a proposition with a view to convince another of its truth, the synthetic form of reasoning is the more natural and pleasing of the two; as it leads the understanding directly from known truths to such as are unknown. When a proposition, however, is doubtful, and

we wish to satisfy our own minds with respect to it; or when we wish to discover a new method of demonstrating a theorem previously ascertained to be true; it will be found (as I already hinted) far more convenient to conduct the investigation analytically. The justness of this remark is universally acknowledged by all who have ever exercised their ingenuity in mathematical inquiries; and must be obvious to every one who has the curiosity to make the experiment. It is not, however, so easy to point out the principle on which this remarkable difference between these two opposite intellectual processes depends. The suggestions which I am now to offer appear to myself to touch upon the most essential circumstance; but I am perfectly aware that they by no means amount to a complete solution of the difficulty.

Let it be supposed, then, either that a new demonstration is required of an old theorem; or, that a new and doubtful theorem is proposed as a subject of examination. In what manner shall I set to work, in order to discover the necessary media of proof?—From the hypothetical part of the enunciation, it is probable, that a great variety of different consequences may be immediately deducible; from each of which consequences, a series of other consequences will follow: At the same time, it is possible, that only one or two of these trains of reasoning may lead the way to the truth which I wish to demonstrate. By what rule am I to be guided in selecting the line of deduction which I am here to pursue? The only expedient which seems to present itself, is merely tentative or experimental; to assume successively all the different *proximate*

consequences as the *first link* of the chain, and to follow out the deduction from each of them, till I, at last, find myself conducted to the truth which I am anxious to reach. According to this supposition, I merely grope my way in the dark, without rule or method: the object I am in quest of, may, after all my labour, elude my search; and even, if I should be so fortunate as to attain it, my success affords me no lights whatever to guide me in future on a similar occasion.

Suppose now that I reverse this order, and prosecute the investigation analytically; assuming (agreeably to the explanation already given) the proposition to be true, and attempting, from this supposition, to deduce some acknowledged truth as a necessary consequence. I have here one fixed point from which I am to set out; or, in other words, one specific *principle* or *datum* from which all my consequences are to be deduced; while it is perfectly immaterial in what particular conclusion my deduction terminates, provided this conclusion be previously known to be true. Instead, therefore, of being limited, as before, to *one conclusion exclusively*, and left in a state of uncertainty where to *begin* the investigation, I have one single supposition marked out to me, from which my departure must necessarily be taken; while, at the same time, the path which I follow, may terminate with equal advantage in a variety of different conclusions. In the former case, the procedure of the understanding bears some analogy to that of a foreign spy, landed in a remote corner of this island, and left to explore, by his own sagacity, the road to London. In the latter case, it may be compared to that of an inhabitant of the metropolis, who

wished to effect an escape, by *any one* of our sea-ports, to the continent. It is scarcely necessary to add, that as this fugitive,—should he happen, after reaching the coast, to alter his intentions,—would easily retrace the way to his own home; so the geometer, when he has once obtained a conclusion in manifest harmony with the known principles of his science, has only to return upon his own steps, (*cæca regens filo vestigia*) in order to convert his analysis into a direct synthetical proof.

A palpable and familiar illustration (at least in some of the most essential points) of the relation in which the two methods now described stand to each other, is presented to us by the operation of unloosing a difficult knot, in order to ascertain the exact process by which it was formed. The illustration appears to me to be the more apposite, that I have no doubt it was this very analogy, which suggested to the Greek geometers the metaphorical expressions of *analysis* and of *solution*, which they have transmitted to the philosophical language of modern times.

Suppose *a knot*, of a very artificial construction, to be put into my hands as an exercise for my ingenuity, and that I was required to investigate a rule, which others, as well as myself, might be able to follow in practice, for making knots of the same sort. If I were to proceed in this attempt, according to the spirit of a geometrical *synthesis*, I should have to try, one after another, all the various experiments which my fancy could devise, till I had, at last, hit upon the particular knot I was anxious to tie. Such a process, however, would evidently be

so completely tentative, and its final success would, after all, be so extremely doubtful, that common sense could not fail to suggest immediately the idea, of tracing *the knot* through all the various complications of its progress, by cautiously *undoing* or *unknitting* each successive turn of the thread in a retrograde order, from the *last* to the *first*. After gaining this *first* step, were all the former complications restored again, by an inverse repetition of the same operations which I had performed in *undoing* them, an infallible rule would be obtained for solving the problem originally proposed; and, at the same time, some address or dexterity, in the practice of the *general method*, probably gained, which would encourage me to undertake, upon future occasions, still more arduous tasks of a similar description. The parallel between this obvious suggestion of reason, and the refined logic of the Greek *analysis*, undoubtedly fails in several particulars; but both proceed so much on the same cardinal principle, as to account sufficiently for a transference of the same expressions from the one to the other. That this transference has actually taken place in the instance now under consideration, the literal and primitive import of the words ανα and λυσις, affords as strong presumptive evidence as can well be expected in any etymological speculation.

In applying the method of analysis to geometrical problems, the investigation begins by supposing the problem to be solved; after which, a chain of consequences is deduced from this supposition, terminating at last in a conclusion, which either resolves into another problem, previously known to be within the reach of our resources; or which involves an ope-

ration known to be impracticable. In the former case, all that remains to be done, is to refer to the construction of the problem in which the analysis terminates; and then, by reversing our steps, to demonstrate synthetically, that this construction fulfils all the conditions of the problem in question. If it should appear, in the course of the composition, that in certain cases the problem is possible, and in others not, the specification of these different cases, (called by the Greek geometers the διορισμος or *determination*) becomes an indispensable requisite towards a complete solution.

The utility of the ancient analysis in facilitating the solution of problems, is still more manifest than in facilitating the demonstration of theorems; and, in all probability, was perceived by mathematicians at an earlier period. The steps by which it proceeds in quest of the thing sought, are faithfully copied (as might be easily shewn) from that natural logic which a sagacious mind would employ in similar circumstances; and are, in fact, but a scientific application of certain rules of method, collected from the successful investigations of men who were guided merely by the light of common sense. The same observation may be applied to the analytical processes of the algebraical art.

In order to increase, as far as the state of mathematical science then permitted, the powers of their analysis, the ancients, as appears from Pappus, wrote thirty-three different treatises, (known among mathematicians by the name of τοπος αναλυομενος) of which number there are twenty-four books,

whereof Pappus has particularly described the subjects and the contents. In what manner *some* of these were instrumental in accomplishing their purpose, has been fully explained by different modern writers; particularly by the late very learned Dr Simson of Glasgow. Of Euclid's *Data*, (for example) the first in order of those enumerated by Pappus, he observes, that "it is of the most general and necessary use in the solution of "problems of every kind; and that whoever tries to investi- "gate the solutions of problems geometrically, will soon find "this to be true; for the analysis of a problem requires, that "consequences be drawn from the things that are *given*, until "the thing that is sought be shewn to be *given* also. Now, "supposing that the *Data* were not extant, these consequences "must, in every particular instance, be found out and demon- "strated from the things given in the enunciation of the pro- "blem; whereas the possession of this elementary book super- "sedes the necessity of any thing more than a reference to the "propositions which it contains *."

With respect to some of the other books mentioned by Pappus, it is remarked by Dr Simson's biographer, that "they re- "late to general problems of frequent recurrence in geometri- "cal investigations: and that their use was for the more im- "mediate resolution of any proposed geometrical problem, "which could be easily reduced to a particular case of any "one of them. By such a reduction, the problem was con- "sidered as fully resolved; because it was then necessary only

* Letter from Dr Simson to George Lewis Scott, Esq. published by Dr Traill. See his Account of Dr Simson's Life and Writings, p. 118.

" to apply the analysis, composition, and determination of that " case of the general problem, to this particular problem which " it was shewn to comprehend *."

From these quotations it manifestly appears, that the greater part of what was formerly said of the utility of analysis in investigating the demonstration of theorems, is applicable, *mutatis mutandis*, to its employment in the solution of problems. It appears farther, that one great aim of the subsidiary books, comprehended under the title of τοπος αναλυομενος, was to multiply the number of such conclusions as might secure to the geometer a legitimate synthetical demonstration, by returning backwards, step by step, from a known or elementary construction. The obvious effect of this was, at once to abridge the analytical process, and to enlarge its resources;—on a principle somewhat analogous to the increased facilities which a fugitive from Great Britain would gain, in consequence of the multiplication of our sea-ports.

Notwithstanding, however, the immense aids afforded to the geometer by the ancient analysis, it must not be imagined that it altogether supersedes the necessity of ingenuity and invention. It diminishes, indeed, to a wonderful degree, the number of his tentative experiments, and of the paths by which he might go astray †; but (not to mention the *prospective* address which it

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* Ibid. pp. 159, 160.

† " Nihil a verâ et genuinâ analysi magis distat, nihil magis abhorret, quam tentandi " methodus; hanc enim amovere et certissimâ viâ ad quæsitum perducere, præcipuus " est analyseos finis."

Extract from a MS. of Dr Simson, published by Dr Traill. See his Account, &c. p. 127.

supposes, in preparing the way for the subsequent investigation, by a suitable *construction* of the diagram,) it leaves much to be supplied, at every step, by sagacity and practical skill; nor does the knowledge of it, till disciplined and perfected by long habit, fall under the description of that δυναμις αναλυτικη, which is justly represented by an old Greek writer*, as an acquisition of greater value than the most extensive acquaintance with particular mathematical truths.

According to the opinion of a modern geometer and philosopher of the first eminence, the genius thus displayed in conducting the *approaches* to a preconceived mathematical conclusion, is of a far higher order than that which is evinced by the discovery of new theorems. "Longe sublimioris ingenii est "(says Galileo) alieni Problematis enodatio, aut ostensio Theo- "rematis, quam novi cujuspiam inventio: hæc quippe fortunæ "in incertum vagantibus obviæ plerumque esse solent; tota "vero illa, quanta est, studiosissimam attentæ mentis, in unum "aliquem scopum collimantis, rationem exposcit†." Of the justness of this observation, on the whole, I have no doubt; and have only to add to it, by way of comment, that it is chiefly while engaged in the steady pursuit of a particular object, that those discoveries which are commonly considered as

* See the preface of Marinus to Euclid's Data. In the preface to the 7th book of Pappus, the same idea is expressed by the phrase δυναμις ευρετικη.

† Not having the works of Galileo at hand, I quote this passage on the authority of Guido Grandi, who has introduced it in the preface to his demonstration of Huyghens's Theorems concerning the Logarithmic Line.—Vid. Hugenii Opera Reliqua, Tom. I. p. 43.

entirely accidental, are most likely to present themselves to the geometer. It is the methodical inquirer alone, who is entitled to expect such fortunate occurrences as Galileo speaks of; and wherever invention appears as a characteristical quality of the mind, we may be assured, that something more than chance has contributed to its success. On this occasion, the fine and deep reflection of Fontenelle will be found to apply with peculiar force: "*Ces hasards ne sont que pour ceux qui jouent bien.*"

II.

Critical Remarks on the vague Use, among Modern Writers, of the Terms Analysis *and* Synthesis.

The foregoing observations on the *Analysis* and *Synthesis* of the Greek Geometers may, at first sight, appear somewhat out of place, in a disquisition concerning the principles and rules of the Inductive Logic. As it was, however, from the Mathematical Sciences, that these words were confessedly borrowed by the experimental inquirers of the Newtonian School, an attempt to illustrate their original technical import seemed to form a necessary introduction to the strictures which I am about to offer, on the loose and inconsistent applications of them, so frequent in the logical phraseology of the present times.

Sir Isaac Newton himself has, in one of his Queries, fairly brought into comparison the Mathematical and the Physical

Analysis, as if the word, in both cases, conveyed the same idea. "*As in Mathematics, so in Natural Philosophy, the investigation* "*of difficult things by the method of Analysis, ought ever to pre-* "*cede the method of Composition.* This analysis consists in "making experiments and observations, and in drawing con- "clusions from them by induction, and admitting of no ob- "jections against the conclusions, but such as are taken from "experiments, or other certain truths. For hypotheses are not "to be regarded in experimental philosophy. And although "the arguing from experiments and observations by induction "be no demonstration of general conclusions; yet it is the "best way of arguing which the nature of things admits of, "and may be looked upon as so much the stronger, by how "much the induction is more general. And if no exception "occur from phenomena, the conclusion may be pronounced "generally. But if, at any time afterwards, any exception "shall occur from experiments; it may then begin to be "pronounced, with such exceptions as occur. By this way "of analysis we may proceed from compounds to ingredients; "and from motions to the forces producing them; and, in "general, from effects to their causes; and from particular "causes to more general ones, till the argument end in the "most general. This is the method of analysis. And the "synthesis consists in assuming the causes discovered, and "established as principles, and by them explaining the "phenomena proceeding from them, and proving the expla- "nations*."

* See the concluding paragraphs of Newton's Optics.

It is to the first sentence of this extract (which has been repeated over and over by subsequent writers) that I would more particularly request the attention of my readers. Mr Maclaurin, one of the most illustrious of Newton's followers, has not only sanctioned it by transcribing it in the words of the author, but has endeavoured to illustrate and enforce the observation which it contains. "It is evident, that as in Mathe-"matics, so in Natural Philosophy, the investigation of diffi-"cult things by the method of analysis ought ever to precede "the method of composition, or the synthesis. For, in any "other way, we can never be sure that we assume the principles "which really obtain in nature; and that our system, after we "have composed it with great labour, is not mere dream or "illusion *." The very reason here stated by Mr Maclaurin, one should have thought, might have convinced him, that the parallel between the two kinds of analysis was not strictly correct; inasmuch as this reason ought, according to the logical interpretation of his words, to be applicable to the one science as well as to the other, instead of exclusively applying (as is obviously the case) to inquiries in Natural Philosophy.

After the explanation which has been already given of geometrical and also of physical analysis, it is almost superfluous to remark, that there is little, if any thing in which they resemble each other, excepting this,—that both of them are methods of investigation and discovery; and that both happen to be called by the same name. This name is, indeed, from its literal or etymological import, very happily significant of the notions

* Account of Newton's Discoveries.

conveyed by it in both instances; but, notwithstanding this accidental coincidence, the wide and essential difference between the subjects to which the two kinds of analysis are applied, must render it extremely evident, that the analogy of the rules which are adapted to the one can be of no use in illustrating those which are suited to the other.

Nor is this all: The meaning conveyed by the word Analysis, in Physics, in Chemistry, and in the Philosophy of the Human Mind, is radically different from that which was annexed to it by the Greek Geometers, or which ever has been annexed to it, by any class of modern Mathematicians. In all the former sciences, it naturally suggests the idea of a decomposition of what is complex into its constituent elements. It is defined by Johnson, "a separation of a compound body "into the several parts of which it consists."—He afterwards mentions, as another signification of the same word, "a solu- "tion of any thing, whether corporeal or mental, to its first "elements; as of a sentence to the single words; of a com- "pound word, to the particles and words which form it; of a "tune, to single notes; of an argument, to single propositions." In the following sentence, quoted by the same author from Glanville, the word Analysis seems to be used in a sense precisely coincident with what I have said of its import, when applied to the Baconian method of investigation. "We can- "not know any thing of nature, but by an analysis of its true "initial causes*."

* By *the true initial causes of a phenomenon*, Glanville means, (as might be easily shewn by a comparison with other parts of his works) *the simple laws from the combination*

In the Greek geometry, on the other hand, the same word evidently had its chief reference to the *retrograde* direction of this method, when compared with the natural order of *dialectic demonstration*. Την τοιαυτην εφοδον (says Pappus) αναλυσιν καλουμεν, οιον αναπαλιν λυσιν; a passage which Halley thus translates; *hic processus Analysis vocatur, quasi dicas, inversa solutio.* That this is the primitive and genuine import of the preposition ανα, is very generally admitted by Grammarians; and it accords, in the present instance, so happily with the sense of the context, as to throw a new and strong light on the justness of their opinion *.

In farther proof of what I have here stated with respect to the double meaning of the words *analysis* and *synthesis*, as employed in physics and in mathematics, it may not be super-

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of which it results, and from a previous knowledge of which, it might have been synthetically deduced as a consequence.

That Bacon, when he speaks of those *separations of nature, by means of comparisons, exclusions, and rejections*, which form essential steps in the inductive process, had a view to the analytical operations of the chemical laboratory, appears sufficiently from the following words, before quoted. "Itaque naturæ facienda est prorsus solutio "et separatio; non per ignem certe, sed per mentem, tanquam ignem divinum."

* The force of this preposition, in its primitive sense, may perhaps, without any false refinement, be traced more or less palpably, in every instance to which the word analysis is with any propriety applied. In what Johnson calls (for example,) "the sepa- "ration of a compound body into the several parts of which it consists,"—we proceed on the supposition, that these parts have *previously* been combined, or put together, so as to make up the *aggregate whole*, submitted to the examination of the chemist; and consequently, that the analytic process follows an inverted or retrograde direction, in respect of that in which the compound is conceived to have been originally formed.—A similar remark will be found to apply *(mutatis mutandis)* to other cases, however apparently different.

fluous to add the following considerations. In mathematical analysis, we always set out from a hypothetical assumption, and our object is to arrive at some known truth, or some *datum*, by reasoning synthetically from which we may afterwards return, on our own footsteps, to the point where our investigation began. In all such cases, the synthesis is infallibly obtained by reversing the analytical process; and as both of them have in view the demonstration of the same theorem, or the solution of the same problem, they form, in reality, but different parts of one and the same investigation. But in natural philosophy, a synthesis which merely reversed the analysis would be absurd. On the contrary, our analysis necessarily sets out from *known facts;* and after it has conducted us to a general principle, the synthetical reasoning which follows, consists always of an application of this principle to phenomena, *different* from those comprehended in the original induction.

In some cases, the natural philosopher uses the word Analysis, where it is probable that a Greek geometer would have used the word Synthesis. Thus, in astronomy, when we attempt from the known phenomena to establish the truth of the Copernican system, we are said to proceed analytically. But the analogy of ancient geometry would apply this word to a process directly the reverse; a process which, assuming the system as true, should reason from it to the known phenomena: After which, if the process could be so reversed as to prove that this system, and this system alone, is consistent with these facts, it would bear some analogy to a geometrical synthesis.

These observations had occurred to me, long before I had remarked, that the celebrated Dr Hooke (guided also by what he conceived to be the analogy of the Greek geometry) uses the words analysis and synthesis in physics, precisely in the contrary acceptations to those assigned to them in the definitions of Sir Isaac Newton. "The methods (he observes) of "attaining a knowledge of nature may be two; either the "analytic or the synthetic. The first is the proceeding from "the causes to the effects. The second, from the effects to "the causes. The former is the more difficult, and supposes "the thing to be already done and known, which is the thing "sought and to be found out. This begins from the highest, "most general and universal principles or causes of things, "and branches itself out into the more particular and subor- "dinate. The second is the more proper for experimental in- "quiry, which from a true information of the effect by a due "process, finds out the immediate cause thereof, and so pro- "ceeds gradually to higher and more remote causes and "powers effective, founding its steps upon the lowest and "more immediate conclusions*."

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* Hooke's Posthumous Works, p. 330.

As this volume is now become extremely rare, I shall transcribe the paragraph which immediately follows the above quotation.

"An inquisition by the former (or analytic) method, is resembled fitly enough by "the example of an architect, who hath a full comprehension of what he designs to do, "and acts accordingly: But the latter (or synthetic) is more properly resembled to "that of a husbandman or gardener, who prepares his ground, and sows his seed, and "diligently cherishes the growing vegetable, supplying it continually with fitting mois- "ture, food, and shelter,—observing and cherishing its continual progression, till it "comes to its perfect ripeness and maturity, and yields him the fruit of his labour.

That Hooke was led into this mode of speaking by the phraseology of the ancient mathematicians, may, I think, be safely inferred from the following very sagacious and fortunate *conjecture* with respect to the nature of their analytical investigations, which occurs in a different part of the same volume. I do not know that any thing approaching to it is to be found in the works of any other English author prior to Dr Halley.

"What ways the ancients had for finding out these mediums, "or means of performing the thing required, we are much in "the dark; nor do any of them shew the way, or so much as

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"Nor is it to be expected, that a production of such perfection as this is designed, should "be brought to its complete ripeness in an instant; but as all the works of nature, if "it be naturally proceeded with, it must have its due time to acquire its due form and "full maturity, by gradual growth and a natural progression; not but that the other "method is also of excellent and necessary use, and will very often facilitate and hasten "the progress. An instance of which kind I designed, some years since, to have given "this honourable society, in some of my lectures upon the motions and influences of "the celestial bodies, if it had been then fit; but I understand, the same thing will "now be shortly done by Mr Newton, in a Treatise of his now in the press: But that "will not be the only instance of that kind which I design to produce, for that I have "diverse instances of the like nature, wherein, from a hypothesis being supposed, on a "premeditated design, all the phenomena of the subject will be *a priori* foretold, and "the effects naturally follow, as proceeding from a cause so and so qualified and limit-"ed. *And, in truth, the* SYNTHETIC *way, by experiments and observations, will be very "slow, if it be not often assisted by the* ANALYTIC, *which proves of excellent use, even "though it proceed by a false position; for that the discovery of a negative is one way "of restraining and limiting an affirmative.*"

Change the places of the words *analytic* and *synthetic* in this last sentence; and the remark coincides exactly with what Boscovich, Hartley, Le Sage, and many other authors, have advanced in favour of synthetical explanations from hypothetical theories. I shall have occasion afterwards to offer some additional suggestions in support of their opinion, and to point out the limitations which it seems to require.

"relate that they had such a one: Yet 'tis believed, they "were not ignorant of some kind of algebra, by which they "had a certain way to help themselves in their inquiries, "though that we now use be much confined and limited to a "few media. But I do rather conceive, that they had another "kind of analytics, which *went backwards* through almost all the "same steps by which their demonstrations *went forwards*, "though of this we have no certain account, their writings "being altogether silent in that particular. However, that "such a way is practicable, I may hereafter, upon some other "occasion, shew by some examples; whereby it will plainly "appear, how much more useful it is for the finding out the "ways for the solution of problems, than that which is now "generally known and practised by *species**."

The foregoing remarks, although rather of a critical than of a philosophical nature, may, I hope, be of some use in giving a little more precision to our notions on this important subject. They are introduced here, not with the most distant view to any alteration in our established language (which, in the present instance, appears to me to be not only unexceptionable, but very happily significant of its true logical import), but merely to illustrate the occasional influence of words over

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* Hooke's Post. Works, p. 68.

Of the illustrations here promised by Hooke of the utility of the analytical method in geometrical investigations, no traces, as far as I have observed, occur in his writings. And it would appear from the following note by the editor, on the passage last quoted, that nothing important on the subject had been discovered among his papers.

"I do not *anywhere* find, that this was ever done by Dr Hooke, and leave the usefulness therefore to be considered by the learned."

the most powerful understandings; and the vagueness of the reasonings into which they may insensibly be betrayed, by a careless employment of indefinite and ambiguous terms.

If the task were not ungrateful, it would be easy to produce numerous examples of this from writers of the highest and most deserved reputation in the present times. I must not, however, pass over in silence the name of Condillac, who has certainly contributed, more than any other individual, to the prevalence of the logical errors now under consideration. "I know well (says he, on one occasion) that it is customary "to distinguish different kinds of analysis; the logical analysis, "the metaphysical, and the mathematical; but there is, in fact, "only *one* analysis; and it is the same in all the sciences *." On another occasion, after quoting from the logic of Port Royal a passage in which it is said, "That analysis and syn-"thesis differ from each other only, as the road we follow in "ascending from the valley to the mountain differs from the "road by which we descend from the mountain into the "valley,"—Condillac proceeds thus: "From this comparison, "all I learn is, That the two methods are contrary to one "another, and consequently, that if the one be good, the other "must be bad. In truth, we cannot proceed otherwise than "from the known to the unknown. Now, if the thing un-"known be upon the mountain, it will never be found by "descending into the valley; and if it be in the valley, it "will not be found by ascending the mountain. There can-"not, therefore, be two contrary roads by which it is to be

* La Logique, Seconde Partie, Chap. vii.

"reached. Such opinions (Condillac adds) do not deserve a "more serious criticism*."

To this very extraordinary argument, it is unnecessary to offer any reply, after the observations already made on the analysis and synthesis of the Greek geometers. In the application of these two opposite methods to their respective functions, the theoretical reasoning of Condillac is contradicted by the universal experience of mathematicians, both ancient and modern; and is indeed so palpably absurd, as to carry along with it its own refutation, to the conviction of every person capable of comprehending the terms of the question.—Nor would it be found more conclusive or more intelligible, if applied to the analysis and synthesis of natural philosophers; or indeed to these words, in any of the various acceptations in which they have ever hitherto been understood. As it is affirmed, however, by Condillac, that "there neither is, nor can be, "more than *one* analysis," a refutation of his reasoning, drawn from any particular science, is, upon his own principle, not less conclusive, than if founded on a detailed examination of the whole circle of human knowledge. I shall content myself, therefore, on the present occasion, with a reference to the mathematical illustrations contained in the former part of this section.

With regard to the notion annexed to this word by Condillac himself, I am not certain, if, after all that he has written in

* Ibid, Chap. vi.

explanation of it, I have perfectly seized his meaning. "To "analyze, (he tells us, in the beginning of his Logic) is nothing "more than to observe in *a successive order* the qualities of an "object, with the view of giving them in the mind that *simul-* "*taneous order* in which they co-exist*." In illustration of this definition, he proceeds to remark, That "although with a "single glance of the eye, a person may discover a multitude "of objects in an open champaign which he has previously "surveyed with attention, yet that the prospect is never more "distinct, than when it is circumscribed within narrow bounds, "and only a small number of objects is taken in at once. We "always discern with accuracy but a part of what we see"

"The case (he continues) is similar with the intellectual "eye. I have, at the same moment, present to it, a great num- "ber of the familiar objects of my knowledge. I see the whole "group, but am unable to mark the discriminating qualities of "individuals. To comprehend with distinctness all that offers "itself simultaneously to my view, it is necessary that I should, "in the first place, decompose the mass;—in a manner analo- "gous to that in which a curious observer would proceed in "decomposing, by successive steps, the coexistent parts of a "landscape.—It is necessary for me, in other words, to *analyze* "my thoughts†."

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* La Logique, Première Partie, Chap. 2.

† Ibid. In this last paragraph, I have introduced one or two additional clauses, which seemed to me necessary for conveying clearly the author's idea. Those who take the trouble to compare it with the original, will be satisfied, that, in venturing on these slight interpolations, I had no wish to misrepresent his opinion.

The same author afterwards endeavours still farther to unfold his notion of analysis, by comparing it to the natural procedure of the mind in the examination of a machine. "If I wish "(says he) to understand a machine, I decompose it, in order to "study separately each of its parts. As soon as I have an exact "idea of them all, and am in a condition to replace them as "they were formerly, I have a perfect conception of the ma-"chine, having both decomposed and recomposed it*."

In all this, I must confess, there seems to me to be much both of vagueness and of confusion. In the two first quotations, the word analysis is employed to denote nothing more than that separation into parts, which is necessary to bring a very extensive or a very complicated subject within the grasp of our faculties;—a description, certainly, which conveys but a very partial and imperfect conception of *that* analysis which is represented as the great organ of invention in all the sciences and arts†. In the example of the machine, Condillac's language is somewhat more precise and unequivocal; but, when examined with attention, will be found to present an illustration equally foreign to his purpose. This is the more surprising, as the instance here appealed to might have been expected to suggest a juster idea of the method in question, than that which resolves into a literal *de*-composition and *re*-composition of the thing to be analyzed. That

* Ibid. Chap. iii.

† Ce qu'on nomme *méthode d'invention*, n'est autre chose que l'analyse. C'est elle qui a fait toutes les découvertes; c'est par elle que nous retrouverons tout ce qui a été trouvé. Ibid.

a man may be able to execute *both* of these manual operations on a machine, without acquiring any clear comprehension of the manner in which it performs its work, must appear manifest on the slightest reflection; nor is it less indisputable, that another person, without disengaging a single wheel, may gain, by a process purely intellectual, a complete knowledge of the whole contrivance. Indeed, I apprehend, that it is in this way alone that the theory of any complicated machine *can* be studied; for it is not the parts, separately considered, but the due combination of these parts, which constitutes the mechanism *. An observer, accordingly, of common sagacity, is here guided by the logic of nature, to a species of analysis, bearing as much resemblance to those of mathematicians and of natural philosophers, as the very different nature of the cases admits of. Instead of allowing his eye to wander at large over the perplexing mazes of such a labyrinth, he begins by remarking the ultimate *effect*: and thence proceeds to trace backwards, step by step, the series of intermediate movements by which it is connected with the *vis motrix*. In doing so, there is undoubtedly a sort of *mental* decomposition of the machine, inasmuch as all its parts are successively considered in detail; but it is not this decomposition which constitutes the *analysis*. It is the methodical *retrogradation* from the mechanical effect to the mechanical power †.

* If, on any occasion, a literal decomposition of a machine should be found necessary, it can only be to obtain a view of some of its parts, which, in their combined state, are concealed from observation.

† That this circumstance of *retrogradation* or *inversion*, figured more than any other in the imagination of Pappus, as the characteristical feature of geometrical analysis,

The passages in Condillac to which these criticisms refer, are all selected from his Treatise on Logic, written purposely to establish his favourite doctrine with respect to the influence of language upon thought. The paradoxical conclusions into which he himself has been led by an unwarrantable use of the words Analysis and Synthesis, is one of the most remarkable instances which the history of modern literature furnishes of the truth of his general principle.

Nor does this observation apply merely to the productions of his more advanced years. In early life, he distinguished himself by an ingenious work, in which he professed to trace *analytically* the history of our sensations and perceptions; and yet, it has been very justly remarked of late, that all the reasonings contained in it are purely *synthetical*. A very eminent mathematician of the present times has even gone so far as to mention it "as a model of *geometrical* synthesis*." He would, I apprehend, have expressed his idea more correctly, if, instead

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appears indisputably from a clause already quoted from the preface to his 7th Book;—Την τοιαυτην εφοδον αναλυσιν καλουμεν, οιον αναπαλιν λυσιν. To say therefore, as many writers have done, that the analysis of a geometrical problem consists in *decomposing* or *resolving* it in such a manner as may lead to the discovery of the *composition* or synthesis,—is at once to speak vaguely, and to keep out of view the cardinal principle on which the utility of the method hinges. There is indeed one species of *decomposition* exemplified in the Greek geometry;—*that* which has for its object to distinguish all the various cases of a general problem; but this part of the investigation was so far from being included by the ancients in their idea of analysis, that they bestowed upon it an appropriate name of its own;—the three requisites to a complete solution being (according to Pappus) αναλυσεα, ϗ συνθειναι, ϗ διοριζεσθαι κατα πτωσιν.

* M. Lacroix. See the Introduction to his Elements of Geometry.

of the epithet *geometrical*, he had employed, on this occasion, *logical* or *metaphysical*; in both of which sciences, as was formerly observed, the analytical and synthetical methods bear a much closer analogy to the experimental inductions of chemistry and of physics, than to the abstract and hypothetical investigations of the geometer.

The abuses of language which have been now under our review, will appear the less wonderful, when it is considered that mathematicians themselves do not always speak of analysis and synthesis with their characteristical precision of expression; the former word being frequently employed to denote *the modern calculus*, and the latter, *the pure geometry of the ancients*. This phraseology, although it has been frequently censured by foreign writers, whose opinions might have been expected to have some weight, still continues to prevail very generally upon the Continent. The learned and judicious author of the History of Mathematics complained of it more than fifty years ago; remarking the impropriety "of calling by the name of "the *synthetic method*, that which employs no algebraical "*calculus*, and which addresses itself to the mind and to the "eyes, by means of diagrams, and of reasonings expressed at "full length in ordinary language. It would be more exact "(he observes farther) to call it *the method of the ancients*, "which, (as is now universally known,) virtually supposes, in "all its synthetical demonstrations, the previous use of ana-"lysis. As to the algebraical calculus, it is only an abridged "manner of expressing a process of mathematical reason-"ing;—which process may, according to circumstances, be