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EVERYDAY PRACTICAL ARITHMETIC AND MATHEMATICS

Everyday Practical Arithmetic and Mathematics

IN THREE BOOKS

SUITABLE FOR PUPILS OF ELEVEN TO FIFTEEN YEARS

WORKING IN

(*a*) The higher classes of Elementary Schools
(*b*) Central Schools
(*c*) Junior Technical Schools
(*d*) Evening Schools; and
(*e*) Day Continuation Schools

BOOK I. For pupils of ages 11 *minus* to 12 years
„ II. For pupils of ages 12 to 13 years *plus*
„ III. For pupils of ages 13 *plus* to 15 years

EVERYDAY PRACTICAL ARITHMETIC AND MATHEMATICS

IN THREE BOOKS

BY

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BOOK I

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PREFACE

These three books provide a course of instruction for those pupils who do not stay at school long enough to profit by an extended study of mathematics.

There is much redundant matter taught to these young pupils, who often leave school at fourteen or fifteen years of age, wrestling with long division or G.C.M. in Algebra. Consequently, they never reach those branches of the subject that will be of any practical value to them: Formulae, Practical Trigonometry, Logarithms, and the plotting of Graphs.

These phases of mathematics are introduced in an interesting manner, by methods shorn of much unnecessary philosophising and explanation.

The scholar's initiative is brought into play and strengthened, whilst his interest is maintained throughout. Thus, in **Algebra**, only those parts of the subject that will be of use to the scholar in his everyday business life are included. **Trigonometry** is approached on practical lines (Book II.), by means of instruments made by the scholars themselves (sextants, angle boards, etc.). By the aid of this simple and inexpensive apparatus, the pupils make their own observations, and tabulate their own data for future use. Examples are included of observations on the sea coast, on elevated land, and on a level country.

Formulae and Equations are taught by means of *Eight Principles and Truths* (Books II. and III.), which enable a child to use most formulae at an early stage.

The setting out of the Chapters and Examples have been done with a view to individual work, the teacher allowing the scholars to proceed at their own pace. The worked examples are consequently more numerous than is usual (there are in the three books 250 worked examples, and 780 examples for the pupils to work), thus allowing the child to learn for himself under the guidance of the teacher.

The three books are profusely illustrated with drawings, graphs, and photographs, and are compiled from the writer's personal experience in teaching the subject in various parts of the country.

It is hoped they will meet a real need in the advanced education of the child from eleven years to fifteen years—one of the most important and urgent matters before teachers and administrators at the present time.

Acknowledgements are due to the members of my staff for help in checking the examples, and to Mr. J. F. Slim, B.Sc., and Mr. M. N. Leitch (Superintendents respectively of the Science Laboratory and Handicraft Centres adjoining the School) for their co-operation.

The photographs are by Mr. C. H. Wake.

J. O. W.

ST. THOMAS'S C.E. SCHOOL, GRANVILLE STREET,
BIRMINGHAM, *January*, 1927.

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BOOK I

ARITHMETIC

DECIMALS—REVISION

As many of the examples in this work will require the use of decimals, and necessitate a sound knowledge of the operations of multiplication and division of decimals, it has been considered advisable to include a short revision course in these two rules in Book I. This will give an opportunity to the pupil of strengthening any weaknesses in these often forgotten rules.

It is assumed that the pupil understands that $\cdot 5 = \frac{5}{10}$; that $\cdot 53 = 5$ **tenths** $+ 3$ **hundredths** or 53 **hundredths**, and so on.

Also, that if we multiply the 3 **hundredths** by 10, we obtain 3 **tenths**, and if we multiply the 5 **tenths** by 10 we obtain 5 **units**.

Thus $\qquad \cdot 53 \times 10 = 5 \cdot 3.$

This is equivalent to moving the decimal point *one place to the right*.

If we now multiply this 5·3 by 10, we obtain 53·.

And, as $10 \times 10 = 100$, the result is the same as

$$\cdot 53 \times 100 = 53 \cdot .$$

This is equivalent to moving the decimal point *two places to the right*.

In multiplying by 1000, we move the decimal point *three places to the right*.

Division being the reverse process of multiplication, if we divide by 10, 100 or 1000, we move the decimal point *one*, *two*, or *three places to the left*.

MULTIPLICATION OF DECIMALS

There is a very useful principle in connection with multiplication of decimals, which should be mastered at this stage.

If we multiply any number by a units figure (*i.e.* less than 10), the position of the decimal point in the product is directly under that in the multiplicand.

Example. *27·35 × 7.*

$$\begin{array}{r} 27{\cdot}35 \\ 7 \\ \hline 191{\cdot}45 \\ \hline \end{array}$$

Similarly, if we multiply 27·35 by 7·2 :

$$\begin{array}{r} 27{\cdot}35 \\ 7{\cdot}2 \\ \\ 191{\cdot}45 \\ 5{\cdot}470 \\ \\ 196{\cdot}92 \\ \hline \end{array}$$

As 7 is the more important digit, we multiply by that number first ; and as 7 is a units figure, *the position of the decimal point is easily fixed.* When we multiply by ·2, however, the result is $\frac{1}{10}$ of what we should have obtained had we multiplied by 2. Hence by our previous rule, every figure in the product must be moved *one place to the right* (this is equivalent to moving the *decimal point one place to the left*).

When, therefore, we multiply one number by another, it is expedient to rewrite the multiplier, so that there is one units figure to the *left* of the decimal point.

Example. *Multiply 73·65 by ·036.*

Rewriting, we obtain :

$$\cdot 7365 \times 3{\cdot}6.$$

Note. As we have *multiplied* ·036 by 100, and *divided* 73·65 by 100, we have not altered the value of the product.

$$\therefore \frac{73{\cdot}65}{100} \times \frac{\cdot 036 \times 100}{1} = 73{\cdot}65 \times \cdot 036,$$

as the 100's cancel with one another.

Proceeding with the example, we obtain :

```
 ·7365
   3·6
------
2·2095
 ·44190
------
2·65140
```

EXAMPLE. *106·34 × ·00085.*
Rewriting, we obtain :

```
·010634
    8·5
-------
 085072
   53170
--------
·0903890
```

DIVISION OF DECIMALS

Before working examples in Division of Decimals, the student is advised to grasp the following principle :

400 ÷ 200	1·2 ÷ ·3
= 40 ÷ 20	12 ÷ 3
= 4 ÷ 2	120 ÷ 30
= ·4 ÷ ·2.	1200 ÷ 300.

If we multiply or divide the *dividend and divisor* by the same number, we do not alter their quotient.

EXAMPLE. *Thus, in dividing 38·463 by 276·4, we rewrite the example :*

·38463 ÷ 2·764.

In dividing by a units figure, the position of the decimal point in the quotient comes directly over (or below) that in the dividend.

```
           ·13915
2·764 ) ·38463
         2764
         -----
         10823
          8292
          -----
          25310
          24876
          -----
            4340
            2764
            -----
            15760
            13820
            -----
             1940
```

EXAMPLE. *·0765 ÷ 936·4.*

Rewriting, we obtain :

```
              ·0000816
    9·364 ) ·00076500
              74912
              15880
               9364
               65160
               56184
                8976
```

NOTE the four ciphers after the decimal point in the quotient, and how easily we obtained them by this method.

APPROXIMATIONS

By adopting the *units figure* method in multiplication and division of decimals, we can approximate the result to the first or second figure of the product or quotient, and also give the *place values* of these figures correctly.

This constitutes a valuable asset to the practical worker, and makes the adoption of this rule of first importance.

Another valuable asset is that of being able to *approximate* a result before actually working the example.

Thus, the example $8·9 \times 9·1$ to the casual observer and worker, might appear to be $8 \times 9 = 72$; but a closer inspection tells him that 8·9 approaches as near to 9·0, as 9·1 is in excess of 9·0, and that the product will be approximately that of $9 \times 9 = 81$.

In the example : 274·64 tons @ £1 18s. 6d. per ton, the result cannot possibly reach £550, as that would mean £2 *per ton*. An expert observer will approximate his answer at about £530. He swiftly calculates 275 shillings = about £14, and reckons £7 for the 275 sixpences. This gives a total of £21, which, taken from £550, leaves about £530. (NOTE. £1 18s. 6d. is 1s. 6d. short of £2 ; and 274·64 tons approximates 275 tons.)

The student is strongly advised to examine all his examples in arithmetic and mathematics closely, before working them accurately, and endeavour to give *approximate* results in each case. The practice will be its own reward, by making the student expert in giving a rough check to his working.

NEAREST DECIMAL

The Practical Engineer rarely finds it necessary to bring out his answers and deductions to more than *four* decimal places. He may only require the result to *two* decimal places, but in either case, he prefers to have the *nearest* fourth decimal place or the *nearest* second decimal place.

To assist in these calculations, a rule has been devised as follows :

If an answer of two decimal places be required *to the nearest second decimal*, we bring our answer out to *three* decimal places. If the third decimal place be *5, or a number greater than 5*, we *add 1* to the second decimal place. If the third decimal place be a figure *less than 5*, we *leave the second decimal place unaltered.*

For four place decimals *to the nearest fourth decimal*, we bring our answer out to *five* decimal places, and then proceed in a similar manner to that stated in the above rule.

Thus :	·31725	= ·3	to the	nearest	first	decimal	place.
		= ·32	,,	,,	second	,,	,,
		= ·317	,,	,,	third	,,	,,
		= ·3173	,,	,,	fourth	,,	,,
Also	·695	= ·7	,,	,,	first	,,	,,
		= ·70	,,	,,	second	,,	,,
NOTE.	π = 3·14159625						
		= 3·142	,,	,,	third	,,	,,
		= 3·1416	,,	,,	fourth	,,	,,
NOTE.	Metre = 39·3708 inches						
		= 39·37	,,	,,	second	,,	,,
		= 39·371	,,	,,	third	,,	,,
NOTE.	$\cdot\dot{6}$ = ·6666...						
		= ·67	,,	,,	second	,,	,,
		= ·667	,,	,,	third	,,	,,
NOTE.	$\cdot 23\dot{1}$ = ·23111...						
		= ·23	,,	,,	second	,,	,,
		= ·231	,,	,,	third	,,	,,

MULTIPLICATION AND DIVISION BY CONTRACTED METHODS

(*a*) Multiplication

The following is a method of finding the product of two numbers to a given degree of approximations :

EXAMPLE. *Multiply 86·954241 by 72·069543 true to two places of decimals.*

Take one of the numbers as the multiplier, say 72·069543. Evidently the last of the digits in the multiplicand which can affect the third place of decimals is the second 4, for it represents $\frac{4}{100000}$ and the maximum value of any digit which multiplies it, is 7 × 10, giving a product of $\frac{28}{10000}$ or ·0028.

It is, however, more convenient to consider the 2, which represents 10,000ths, and the 7 which represents tens, as producing a product of 1000, which is the denomination of the third place of decimals. The effect of the 4 can be allowed for, by an addition (if necessary). We now proceed as follows :

Place the digit of maximum value in the multiplier under that digit in the multiplicand, multiplication by which will affect one place beyond the last decimal place required, and write the other digits of the multiplier in a reverse order as in the example below. When the multiplication by 7 has been completed, strike out the 7 and 2, and proceed to multiply the 4 by 2. Then strike out the 4 and the 2 and of course the 0 and 5 and proceed with the 6 and 9, not forgetting to carry 3 from the product of 6 and 5. Now strike out the 6 and 9 and proceed with the 9 and 6, not forgetting to carry 8 from the product of 9 and 9. Strike out now the 9 and 6 and proceed with the 5 and 8, not forgetting to carry 3 from the product of 5 and 6. The 4 will only affect the result as regards the 3 in the third decimal place and is obtained by carrying it from the product of 4 and 8, whilst the 3 does not affect the answer at all.

```
  86·954241
3459 6027

  6086·796
   173·908
     5·217
       782
        43
         3
  6266·749
```

The position of the decimal point is obtained by remembering that the 7 tens multiplied by the $\frac{2}{100000}$ gives a result of three decimal places. Thus, by counting three places from the right, we obtain this position, which is between the 7 and the 6.

(*b*) Division

EXAMPLE. *Divide 5623 by 1547.*

Here is the ordinary method on the left. All figures below the line *AB*, and to the right of the dotted line, are unnecessary; hence the merit of the method on the right-hand side in which we do not add 0's, but cut off figures in the division.

```
1547 ) 5623 ( 3634 : 77 etc.        1547 ) 5623 ( 3635
       4641                                4641

        982  0                              982
        928  2                              928

         53  80                              54
         46  41                              46

          7  390                              8
          6  188                              8
      A             B
          1  2020
          1  0829

             11910
             10829

              1081
```

NOTE. In multiplying by the figures in the quotient, we must not forget to add on figures carried forward in the same manner as we did in the contracted multiplication.

1. MISCELLANEOUS EXAMPLES ON DECIMALS

1. ·075 + ·345 + ·008.
2. ·175 − ·025 + ·3.
3. ·25 + ·095 + ·385.
4. ·975 − ·085.
5. ·025 − ·01.
6. ·875 − ·643.
7. ·384 − ·45 + ·265.
8. ·005 − ·2 + ·196.
9. ·137 + ·043 − ·055.
10. ·9 − ·009.
11. Find the sum of 165·706, 98·005, 309·294, 256·075 and 110·5, and take the result from 1000.

45. How many times is ·6875 lbs contained in 119·625 lbs ?

46. A certain class of goods costs 1·875 shillings per lb. How many lbs. can be purchased for £2·375 ?

47. How often can 2·35 pints of liquid be taken from a cask holding 9·4 gallons ?

Find the quotients *to the nearest fourth decimal place* :

48. 856·725 ÷ 96·4. **49.** ·00875 ÷ ·0793. **50.** 7465·0 ÷ ·428.

51. 213·3549 ÷ 20·7. **52.** 18·7295 ÷ 98·7.

Rewrite the following decimals to the *nearest third decimal place* :

53. ·0674. **54.** ·31614. **55.** ·71265. **56.** ·00897.

57. ·00123. **58.** ·049876. **59.** ·32768. **60.** ·00096.

61. ·24675. **62.** ·16972. **63.** ·00437. **64.** ·06825.

Find the products to the *nearest fourth decimal place* :

65. 8·56102 × 5·6039. **66.** 589·0067 × 3·1008.

67. 7·643 × ·8584. **68.** 17·649 × ·86742.

69. 121·642 × 9·4863. **70.** 73·647 × 66·38.

Find the quotients to the *nearest fourth decimal place* :

71. $6{\cdot}428 \div 84{\cdot}32$. **72.** $7{\cdot}562 \div 9{\cdot}342$. **73.** $10{\cdot}58 \div 6{\cdot}427$.

74. $89{\cdot}645 \div 76{\cdot}42$. **75.** ${\cdot}1064 \div 5{\cdot}782$. **76.** $5{\cdot}396 \div {\cdot}0068$.

77. $\dfrac{6{\cdot}784 \times 8{\cdot}56}{9{\cdot}87}$ £. Answer to the *nearest penny*.

78. The sum of £936 10s. 4d. is to be divided amongst 29 persons What is (*a*) the greatest amount each person can receive, if all receive alike, *calculated to the nearest penny?*

(*b*) What amount of money will be left over after such a division ?

79. State approximately what 150 loads each containing 3 tons 17 cwts. will weigh ?

80. What is an approximate cost of 199 articles at 10s. 1d. each ?

ALGEBRA

Arithmetic deals with numbers, which stand for definite and unchangeable values, in whatever examples they are employed.

Algebra enables us to work with symbols (which are mostly the letters of our alphabet, a, b, c, x, y, z, etc.) in addition to numbers, in the solving of problems. Each of these symbols may stand for a different number or value in different examples; but in the *same* example, each symbol *must stand for the same number or value.*

Thus, we may be told that a steamer sailed 26 miles in the first hour, and "a" miles in the second hour. If we are further told that the steamer sailed 50 miles in these two hours, we can ascertain easily that "a" = 24 miles.

Again, we may be informed that a man earned £6 in the first week that he worked, and £a in the second week. If we are further informed that the man earned £14 in the two weeks, we readily see that "a," in this example, equals £8.

We thus note that the symbol "a" has represented two different values: (1) 24 miles *in the first example*, and (2) £8 *in the second example*.

In other examples, the symbol "a" may stand for still different values. The point to remember, however, is, that in the *same* example, each symbol must stand for the same value.

Besides the symbols, which are represented by letters of the alphabet, and called **literal symbols**, we employ others, which are called **symbols of operation**. These latter are merely an easy form of shorthand, and instruct us to perform certain arithmetical operations.

Many of these symbols will be familiar.

Thus $+$ means plus or add.

$-$ means minus or subtract.

$\times$ means multiply, or find the product.

$\div$ means divide, or find the quotient.

$\therefore$ means therefore.

$=$ means equals.

$\equiv$ means identical to.

$\sqrt{a}$ means the square root of a.

$\sqrt[3]{a}$ means the cube root of a.

$a \div b$, or $a : b$, or a/b means a divided by b.

$a \times b$, or $a \,.\, b$, or ab means a multiplied by b.

$\sqrt[3]{a^2}$ means the cube root of a^2.

$a > b$ means a is greater than b.

$a < b$ means a is less than b.

ADDITION OF SYMBOLS

We can add together *like* literal symbols, and ascertain their total, but we cannot add *unlike* literal symbols together. Thus, if we have the terms 4a and 6a (which mean 4 times the value of "*a*," and 6 times the value of "*a*" respectively), we can say that the sum is 10a ; just as we can say that the sum of 6 *miles* and 4 *miles* is 10 *miles*.

We also know that if a person who has £5, earns £8, and afterwards spends £2, the final amount in his possession is £11. This is equivalent to the expression $5a + 8a - 2a = 11a$.

Take another example : A person who has £6, earns £4. If this £10 represents all that the person possesses, and he then contracts a debt of £12, it is obvious that he is £2 short of the payment. In other words, he will have to earn another £2 before he can pay the debt in full, and which will then leave him without any money. This is equivalent to the expression, $6a + 4a - 12a = -2a$.

From these examples, we see, that to find the sum of *like* literal symbols, we add together (1) All those with a *plus* sign before them ; (2) All those with a *minus* sign before them ; (3) Take the smaller number from the greater ; and (4) Place *the sign that is before the greater number, before the answer.*

EXAMPLE. *Find the sum of* $5d + 7d - 6d + 3d - 5d$.

(1) $+5d + 7d + 3d = +15d$.

(2) $-6d - 5d = -11d$.

(3) Difference between $15d$ and $11d = 4d$.

(4) Sign before the greater number is *plus*.

$\therefore$ Answer = $+4d$.

EXAMPLE. *Find the sum of* $16y - 28y + 10y + 2y - 18y + 6y - 9y$.

(1) $16y + 10y + 2y + 6y = +34y$.

(2) $-28y - 18y - 9y = -55y$.

(3) Difference between $55y$ and $34y = 21y$.

(4) Sign before the greater number is *minus*.

$\therefore$ Answer = $-21y$.

NOTE. We cannot add together **5a** and **6d** and say the result is **11a** or **11d** or **11ad** or **11da**, any more than we can add together **5** cats and **6** dogs and say the result is **11** cats or **11** dogs ; or **£5** and **8** shillings, and say the result is **£13** or **13** shillings.

We are sometimes asked to find the sum of a long string of terms, in which the same literal symbols recur, as for example :

Find the sum of :

$$5a + 6b - 3a + 2c - 4b + 7a - 18c + 5c - 2a - 3a + 10b.$$

The student will note that the symbols "*a*," "*b*," and "*c*" recur.

In working this example, we deal with it as three separate operations, the first dealing with the symbol "**a**," the second dealing with the symbol "**b**," and the third dealing with the symbol "**c**."

The symbol "a."

(1) $+5a + 7a = +12a$.

(2) $-3a - 2a - 3a = -8a$.

(3) Difference between $12a$ and $8a = 4a$.

(4) Sign before the greater number is *plus*. $\therefore$ Sum = $+4a$.

The symbol " b."

(1) $+6b + 10b = +16b$.

(2) $-4b = -4b$.

(3) Difference between $16b$ and $4b = 12b$.

(4) Sign before the greater number is *plus*. ∴ Sum = **+12b.**

The symbol " c."

(1) $+2c + 5c = +7c$.

(2) $-18c = -18c$.

(3) Difference between $7c$ and $18c = 11c$.

(4) Sign before the greater number is *minus*. ∴ Sum = **−11c.**

∴ the total result = **4a + 12b − 11c.**

This process is a protracted and tedious one, and is only included in the book for the pupil to visualise the various stages of the working. When these have been thoroughly mastered, the example may be set down in the following manner :

(a)	(b)	(c)
$+5a$	$+6b$	
$-3a$		$+2c$
	$-4b$	
$+7a$		$-18c$
		$+5c$
$-2a$		
$-3a$	$+10b$	
$+4a$	$+12b$	$-11c$

This setting down can be abbreviated still further :

(a)	(b)	(c)
$+5a$	$+6b$	$+2c$
$-3a$	$-4b$	$-18c$
$+7a$	$+10b$	$+5c$
$-2a$		
$-3a$		
$+4a$	$+12b$	$-11c$

The student now can take each column separately, *doing the four steps mentally.*

NOTE. The above abbreviated process is known as that of *collecting like literal symbols together.*

This means that the **a's**, the **b's**, and the **c's** are collected, each in its own vertical column.

2 (a). EXAMPLES IN ADDITION

Add together:

1. $3x - 2y$; $4x + 7y$; $2x + 3y$; $x - 5y$.
2. $9b^2 + 7c^2$; $-3b^2 + 4c^2$; $b^2 + c^2$; $4b^2 - 12c^2$.
3. $a + b + c$; $3a + 2b + 3c$; $-4a + 7b - c$; $2b + 5c$.
4. $x - y - z$; $y - x - z$; $z - x - y$; $x + y + z$.
5. $3a^2 - 4ab + 6b^2$; $7ab - a^2 - b^2$; $2a^2 - 3ab - 4b^2$; $4a^2 + ab - b^2$.
6. $2x^4 - 7x^2 + 3$; $-4x^3 + 6x^2 - 2x + 7$; $x^4 - 2x^2 - 4x$; $6x^3 - 9x - 12$.
7. $2a^2 + 7ab + 3b^2 - 6a - 5b - 2$; $a^2 + 3a - 2b + 9$; $9ab - 2a - 3b + 4$; $-3a^2 - 12ab - 3b^2 + 5a + 10b - 15$.
8. $x^3 - x^2y - x^2z + xy^2 - xyz + xz^2$; $x^2y - xy^2 - xyz + y^3 - y^2z + yz^2$; $x^2z - xyz - xz^2 + y^2z - yz^2 + z^3$.
9. $x^4 + x^2y^2 + x^3y$; $-x^3y - x^2y^2 - xy^3$; $y^4 + xy^3 + x^2y^2$.
10. $a^3 + ab^2 + ac^2 - 2a^2b - 2a^2c - 2abc$; $a^2b + b^3 + bc^2 + 2ab^2 - 2abc - 2b^2c$; $a^2c + b^2c + c^3 + 2abc - 2ac^2 - 2bc^2$.
11. $x^4 - xy^3 + xz^3 + 3x^3y + 3x^3z$; $3x^2y^2 + 3x^2z^2 + 3xy^2z$; $-3xyz^2 - 6x^2yz$; $y^4 - x^3y - yz^3 + 3x^2y^2 - 3x^2yz$; $-3xy^3 - 3xyz^2 - 3y^3z + 3y^2z^2 + 6xy^2z$; $z^4 + x^3z - y^3z - 3x^2yz + 3x^2z^2$; $3xy^2z + 3xz^3 + 3y^2z^2 - 3yz^3 - 6xyz^2$.
12. $a^4 - a^3b + 3a^2c^2 + ab^2c - 3abc^2 - b^3c$;
    $a^3b - a^3c + 3abc^2 - 3ac^3 - b^2c^2 + b^3c$;
    $a^3c - a^3d + 3ac^3 - 3ac^2d + b^2c^2 - b^2cd$;
    $-a^4 + a^3d - 3a^2c^2 + 3ac^2d - ab^2c + b^2cd$.

SUBTRACTION OF SYMBOLS

If a motor car travel 40 miles east, and then a further 50 miles *in the same direction*, the total distance that it has travelled *from the starting point* is 90 miles.

If a second car travel 40 miles east, and then 50 miles *in the reverse direction* (*i.e.* west), it will *not* have travelled a distance of 90 miles east from the starting point, but will, in fact, be 10 miles to the rear of it.

In other words, whilst the first car is 90 miles east of the starting point, the second car is 10 miles to the west of it, which

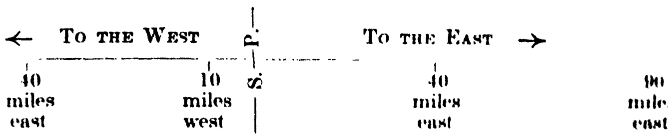


FIG. 1.

means that its distance *east* is *a negative* 10 *miles*. The distance between the two cars is 100 miles (see Fig. 1).

From this data we can ascertain certain rules for subtraction.

(1) The first car travelled 40 miles east and then completed the full 90 miles to the east.

The difference between 40 miles east and 90 miles east is 50 miles east.

This is equivalent to saying, that the difference between

$$+90 \text{ m. and } +40 \text{ m.} = +50 \text{ m.}$$

(2) The second car completed its journey 10 miles *behind the starting point*, which is *a negative* 10 *miles east*.

If the second car were to travel over the distance separating it from the first car, it would travel in a *positive* direction (to the east) for 100 miles.

We can, therefore, say that the difference between

$$+90 \text{ m. and } -10 \text{ m.} = +100 \text{ m.}$$

If, however, instead of this taking place, the *first car* had travelled the intervening 100 miles in a *negative* direction (to the west) to the second car, it would have illustrated that the difference between

$$-10 \text{ m. and } +90 \text{ m.} = -100 \text{ m.}$$

Let us assume for a further example, that the two cars are together, 10 miles behind the original starting point, that is 10 miles *to the west*, or a *negative* 10 miles *to the east*. If the first car travel another 30 miles *to the west*, the two cars will now be 40 miles and 10 miles respectively to the west of the starting point, or a negative 40 miles, and a negative 10 miles east respectively. The distance of the first car from the second car is obviously *a negative 30 miles*, which means that the difference between

$$-40 \text{ m. and } -10 \text{ m.} = -30 \text{ m.}$$

The distance of the second car from the first car is, however, *plus* 30 miles, because the second car is 30 miles to the *east*, or 30 miles in a *positive* direction from the first car. This means that the difference between

$$-10 \text{ m. and } -40 \text{ m.} = +30 \text{ m.}$$

Let us tabulate these five results in subtraction :

(1)	(2)	(3)	(4)	(5)
+90 m.	+ 90 m.	− 10 m.	− 40 m.	− 10 m.
+40 m.	− 10 m.	+ 90 m.	− 10 m.	− 40 m.
+50 m.	**+100 m.**	**−100 m.**	**−30 m.**	**+30 m.**

If we examine these results carefully, we shall see, in each case, that had we changed the signs before the five bottom quantities, *and then proceeded as in addition*, the five results would be unaltered.

We therefore learn this rule for subtraction :

Change mentally all the symbols of operation in the bottom quantity, and proceed as in addition.

EXAMPLE. *From* $6a + 5b - 7c + 10d$
Take $-5a + 2b - 6c + 7d$

Let us consider each symbol in connection with the motor cars.

The difference between $+6a$ and $-5a$ is equivalent to that of the distance between one car going 6 miles east, and the second car going 5 miles west, *i.e.* +11 miles.

The difference between $+5b$ and $+2b$ is equivalent to the distance between the two cars, when one has travelled 5 miles east, and the other 2 miles east, *i.e.* +3 miles.

The difference between $-7c$ and $-6c$ is equivalent to the distance between the two cars, when one has travelled 7 miles west, and the other 6 miles west, *i.e.* 1 mile west or − 1 mile east.

The difference between $+10d$ and $+7d$ is equivalent to the distance between the two cars when one has travelled 10 miles east, and the other 7 miles east, *i.e.* $+3$ miles.

The total difference is therefore :

$$+11a + 3b - c + 3d.$$

EXAMPLE. *From* $16a^2 + 5b + 6c^2 + 10d - 5ab$
Take $-18d + 5c^2 - 6a^2 + 4ab - 2b$

The first step will be to set the "from" line as it stands, and place the "take" line underneath with *like* symbols under *like* symbols, thus :

$$\begin{array}{l} +16a^2 + 5b + 6c^2 + 10d - 5ab \\ -\ 6a^2 - 2b + 5c^2 - 18d + 4ab \\ \hline +22a^2 + 7b + c^2 + 28d - 9ab \end{array}$$

EXAMPLE. *From* $19a + 12b - 7c + 18a^2 + 17ab$.
Take $15d + 16c - 17a - 9a^2$

Setting *like* symbols under *like* symbols, we obtain :

$$\begin{array}{l} +19a + 12b - 7c + 18a^2 + 17ab \\ -17a \qquad + 16c - 9a^2 \qquad + 15d \end{array}$$

It will be noticed that there is no "d" symbol in the "from" line, and there are no "b" or "ab" symbols in the "take" line. These symbols consequently are alone in their respective lines.

The total difference is therefore :

$$+36a + 12b - 23c + 27a^2 + 17ab - 15d.$$

The only difficulty in the example is the $+15d$ being changed to $-15d$ in the answer. This change comes about because we are subtracting $+15d$ from zero or **nothing**. If 15d be taken from 15d the result is **nil**. If 15d be taken from 14d (or in other words, if a person incurs a liability of 15 pence with only 14 pence to meet it, his deficit or debt is 1 penny), the result is $-d$.

If 15d be taken from 2d, the result is $-13d$, and consequently if 15d be taken from nil (or, in other words, if a person incurs a liability of 15 pence with nothing to meet it, his debt or deficit is 15 pence), the result is $-15d$.

2 (b). EXAMPLES IN SUBTRACTION

1. From $6a - 7b + 3c$, take $2a + 5b - 2c$.
2. From $2x - 3y - 8z$, take $6x - 5y - 2z$.

3. Take $5a^2+3ab+4b^2+3a+7b+8$, from $6a^2+3b^2-2a$.

4. Take $6a^4+8a^2x^2+x^4$, from $8a^4+6a^2x^2+2x^4$.

5. Subtract the sum of $a^4+2a^2b^2+b^4$ and $a^4-2a^2b^2+b^4$, $6a^4+8a^2b^2+6b^4$.

6. From $x^3+y^3+z^3-3xyz$, take $4x^3+y^3+4z^3+3x^2z+3xy^2-3xyz$.

7. From $3x^4+3ax^3-9a^2x^2+a^3x-a^4$, take $2x^4+4ax^3+4a^3x+a^4$.

8. Take $a^3-5a^2b+7ab^2-2b^3$, from the sum of $2a^3-9a^2b+11ab^2-3b^3$ and $b^3-4ab^2+4a^2b-a^3$.

9. Subtract $a+b+c+d$, from $e+f+g+h$.

10. Take $x^4-4x^3y+6x^2y^2-4xy^3+y^4$, from $x^4+4x^3y+6x^2y^2+4xy^3+y^4$, and subtract the result from their sum.

11. Add together the given quantities in the last example, and subtract the result from $3x^4+10x^2y^2+3y^4$.

12. Take $a^2+b^2+c^2+2ab+2ac+2bc$, from $2a^2+2b^2+4ab-c^2$.

SUBSTITUTION

The process of substitution of numerical values for the algebraical literal symbols is of the utmost importance in a study of **Practical Mathematics**, and will be found absolutely essential when the student deals with **Formulae**. It should be grasped and mastered at an early stage.

We have already found that a literal symbol can stand for any numerical value.

Let us suppose that $a=3$, $b=4$, $c=5$,

then,

$$a+b+c = 3+4+5 = 12,$$
$$a+b-c = 3+4-5 = 2,$$
$$a-b+c = 3-4+5 = 4,$$
$$a-b-c = 3-4-5 = -6,$$
$$-a-b+c = -3-4+5 = -2,$$

and so on.

If $x=2$, $y=4$, $z=6$,

then

$$5x = 5\times 2 = 10,$$
$$7y = 7\times 4 = 28,$$
$$3z = 3\times 6 = 18;$$
$$\therefore\ 5x+7y+3z = 10+28+18 = 56,$$

and

$$5x+7y-3z = 10+28-18 = 20.$$

Example. ***Given that $m=3$, $n=-2$, $p=-\frac{1}{2}$; find the numerical value of $7m+5n-6p$.***

Before proceeding with this example, we must consider two fundamental truths in connection with multiplication.

(1) *Like* signs, multiplied together, give plus.

(2) *Unlike* signs, multiplied together, give minus.

This means that

(1) *plus* × *plus* gives *plus*,
minus × *minus* gives *plus*.

(2) *plus* × *minus* gives *minus*,
minus × *plus* gives *minus*.

(1) Thus, $+5a\times(+4)=+20a$,

and $-5a\times(-4)=+20a$,

(because *like* signs give *plus*).

(2) Again, $+5a\times(-4)=-20a$,

$-5a\times(+4)=-20a$,

(because *unlike* signs give *minus*).

Returning to the example, we find that

$$+7m=+7\times(+3)=+21,$$
$$+5n=+5\times(-2)=-10,$$
$$-6p=-6\times(-\tfrac{1}{2})=+3;$$
$$\therefore\ +7m+5n-6p=+21-10+3$$
$$=\mathbf{14}.$$

Example. *Given that $a=3$, $b=-2$, $c=-4$, $d=-5$; find the numerical value of:*

$$15ab+6cd-8bc+10ad$$

$$+15ab=+15\times(+3)\times(-2)=-90,$$
$$+6cd=+6\times(-4)\times(-5)=+120.$$

(Note. $+6\times(-4)$ gives -24; $-24\times(-5)$ gives $+120$.)

$$-8bc=-8\times(-2)\times(-4)=-64,$$
$$+10ad=+10\times(+3)\times(-5)=-150;$$
$$\therefore\ +15ab+6cd-8bc+10ad$$
$$=-90+120-64-150$$
$$=+120-304$$
$$=\mathbf{-184}.$$

Example. *When $x = 2$, and $y = -2$, find the value of:*

$$\frac{x^2 - xy}{x^2 + y^2} + \sqrt{x^3 + y^3 - 3x^2y + 1}$$

$$= \frac{4 - (-4)}{4 + 4} + \sqrt{8 - 8 - (-24) + 1}$$

$$= \frac{8}{8} + \sqrt{25}$$

$$= 1 + 5$$

$$= 6.$$

Note. Why does $4 - (-4) = 8$? (Remember rule for subtraction.)

Example. *Find the value of:*

$$\frac{a^2 + 4b - 10c}{a^2 - 10b + 100c}, \text{ when } a = 1,\ b = \cdot 1,\ c = \cdot 01,$$

$$= \frac{1 + \cdot 4 - \cdot 1}{1 - 1 + 1}$$

$$= \frac{1 \cdot 3}{1}$$

$$= 1 \cdot 3.$$

2 (c). EXAMPLES IN SUBSTITUTION

If $a = 1$, $b = 2$, $c = 3$, $d = 0$, $e = 4$, find the values of the following:

1. $4a + 2b$. 2. $3b + 7c$. 3. $6a + 4d$.
4. $4c - 7e$. 5. $a + b + c$. 6. $a - b + c$.
7. $b + c - a$. 8. $a + b - c$. 9. $3a + 7b - 4c$.
10. $2a + 7d + 3e$. 11. $7a - 10b + 2c$.

If $x = 2$, $y = 3$, $z = 4$, find the values of the following:

12. $3x - 4y + 3y - 4x$. 13. $3x - 4y - 3y + 4x$.
14. $3x - 7y - 4z + 8y + 5z - 3x$. 15. $3x - 7y - 4z - 8y - 5z + 3x$.
16. $x - y - z + x - y + z$. 17. $x + y - z - x - y - z$.

If $a = 2$, $b = 3$, $c = 0$, $d = 1$, find the values of:

18. $6a^2 + 3b^2 - 5c^2$. 19. $ab + ac + bc$.
20. $bc + bd + cd$. 21. $a^3 + 3a^2b - 3ab^2 + b^3$.
22. $a^3 + b^3 + c^3 - 3abc$. 23. $\dfrac{a^4 - d^4}{a^3 + a^2d + ad^2 + d^3}$.
24. $\dfrac{c^3 + c^2d + 3cd^2 + d^3}{b^3 - 3b^2c + 3bc^2 - c^3}$.

FORMULAE

One of the fundamentals in the study of Algebra, is to employ symbols, so as to make the statements of arithmetical rules as short and concise as possible.

For example, the rule for finding the area of a triangle, given the base and the height, is, "Multiply the base by the height and divide the result by 2."

This rule is long and cumbersome, though short in comparison with other rules of mensuration. We must endeavour to state it in a more concise form. Instead of *multiply by*, we can substitute the symbol $\times$; instead of the word *base*, we can substitute the symbol "b"; instead of the word *height*, we can substitute the symbol "h"; and for the words *divide by*, we can substitute the symbol $\div$

Our rule is now shortened into

$$b \times h \div 2.$$

The symbol $\times$ is often replaced by a dot, thus, $b \cdot h$, and is often omitted entirely, thus bh; whilst instead of the symbol $\div$ we can place a horizontal line between bh and 2.

Thus, the rule can now be written in its simplest form: $\frac{bh}{2}$, and still convey the same meaning.

If we introduce the symbol **A** to stand for *Area of the Triangle*, and place the symbol = between A and $\frac{bh}{2}$, we obtain:

$$\mathbf{A} = \frac{\mathbf{bh}}{\mathbf{2}}.$$

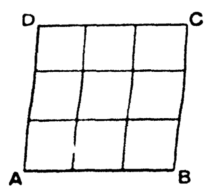


FIG. 2.

Note. $3 \times 3 = 9$.

EXAMPLE. *Given S = 4·5 ins., find the area.*

Formula. $\mathbf{A} = \mathbf{S}^2$.

$\mathbf{A} = (4{\cdot}5)^2$

$= \mathbf{20{\cdot}25}$ **sq. inches.**

NOTE. If $\mathbf{S}^2 = \mathbf{A}$, then $\mathbf{S} = \sqrt{\mathbf{A}}$.

EXAMPLE. *Find the side of a square whose area is 14·44 sq. inches.*

Formula. $\mathbf{S} = \sqrt{\mathbf{A}}$.

$= \sqrt{14{\cdot}44}$

$= \mathbf{3{\cdot}8}$ **inches.**

NOTE. It is assumed that the pupil has been taught the extraction of the square root.

To find the area of a rectangle, given the length and the breadth.

Rule. Multiply the length by the breadth. (Fig. 3.)

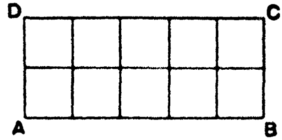


FIG. 3.

NOTE. $5 \times 2 = 10$.

Formula. $A = lb$,

where A = area of rectangle,

l = length.

b = breadth.

EXAMPLE. *Find the area of a rectangle whose length is 10·4 ins. and the breadth is 3·6 ins.*

Formula. $A = lb$

$= 10{\cdot}4'' \times 3{\cdot}6''$

$=$ **37·44 square inches.**

NOTE. $12 = 3 \times 4$;

$\therefore \frac{12}{3} = 4$,

and $\frac{12}{4} = 3$.

If $A = lb$,

$\therefore \frac{A}{l} = b$ .. (Formula for *breadth* of a rectangle, when area and length are given),

and $\frac{A}{b} = l$ .. (*Formula for length of a rectangle, when area and breadth are given*).

To find the total area of the four walls of a room.

In the accompanying diagram (Fig. 4), the four walls of a room are shown in one line, as one huge wall. It will be seen

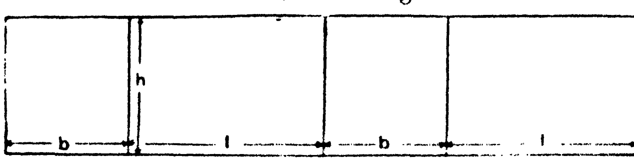


FIG. 4.

that the total length = twice the length of the room plus twice the breadth.

$$L = 2(+ bl).$$

the height (h) remains constant throughout.

The *formula* is $\therefore$ $A = 2h(l + b)$.

EXAMPLE. *Find the area of the four walls of a room, whose length is 15′ 9″, breadth is 9′ 3″, and height is 12′.*

Formula.

$$
\begin{aligned}
A &= 2h(l+b)\\
&= 2\times 12(15'9'' + 9'\,3'')\\
&= 24 \times 25\\
&= 600 \text{ square feet.}
\end{aligned}
$$

EXAMPLE. *The area of a rectangular field is an acre. If the length be 73 yards 1 foot, find the breadth.*

Formula.

$$\frac{A}{l} = b.$$

$$\therefore \frac{4840 \text{ sq. yds.}}{73\tfrac{1}{3} \text{ yds.}} = b.$$

$$\therefore 66 \text{ yards} = b.$$

EXAMPLE. *Find the length of a rectangular courtyard, whose area is 329 square yards, and breadth 15 yards 2 feet.*

Formula.

$$\frac{A}{b} = l.$$

$$\therefore \frac{329 \text{ sq. yds.}}{15\tfrac{2}{3} \text{ yds.}} = l.$$

$$\therefore 21 \text{ yards} = l.$$

The area of a triangle, given the base and the perpendicular height, has already been dealt with.

Formula.

$$A = \frac{bh}{2}.$$

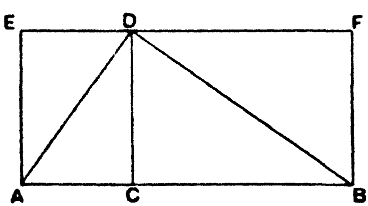


FIG. 5.

NOTE. *DAC* = ½ *ACDE*. *DBC* = ½ *BCDF*. ∴ *DAB* = ½ *ABFE*.

Example. *If the base of a triangle is 8 feet and the perpendicular height is 10·5 feet, find the area.*

$$\mathbf{A} = \frac{\mathbf{bh}}{\mathbf{2}}$$

$$= \frac{8' \times 10{\cdot}5'}{2}$$

$$= 42 \text{ square feet.}$$

Note. If $A = \dfrac{bh}{2}$,

then $2A = bh$.

$\therefore \dfrac{2A}{b} = h$ (Formula for *perpendicular height*, given area and base.)

$\therefore \dfrac{2A}{h} = b$ (Formula for *base*, given area and perpendicular height.)

Example. *Find the perpendicular height of a triangle whose area is 5·52 square inches, and the base 4·6 inches.*

Formula. $\mathbf{h} = \dfrac{\mathbf{2A}}{\mathbf{b}}$

$$= \frac{5{\cdot}52 \text{ sq. inches} \times 2}{4{\cdot}6 \text{ inches}}$$

$$= 2{\cdot}4 \text{ inches.}$$

Example. *The area of a triangular field is 1 acre 160 sq. yards. The perpendicular height is 80 yards. Find the base.*

Formula. $\mathbf{b} = \dfrac{\mathbf{2A}}{\mathbf{h}}$

$$= \frac{5000 \text{ sq. yards} \times 2}{80 \text{ yards}}$$

$$= 125 \text{ yards.}$$

To find the circumference of a circle, given the radius.

Rule. Double the radius and multiply the result by $3\frac{1}{7}$ (designated π (pi)).

Formula. $\mathbf{C} = 2 \times \mathbf{r} \times \pi$, or $2\mathbf{r}\pi$, or $2\pi\mathbf{r}$.

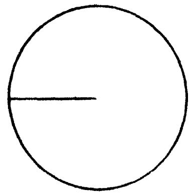


FIG. 6.

EXAMPLE. *Find the circumference of a circle, whose radius is 14". ($\pi = 3\frac{1}{7}$.)*

Formula.

$$\begin{aligned} \mathbf{C} &= 2\pi\mathbf{r} \\ &= 2 \times 3\tfrac{1}{7} \times 14 \\ &= \mathbf{88\ inches.} \end{aligned}$$

NOTE. If $C = 2\pi r$;

$$\therefore \frac{C}{2\pi} = r.$$

EXAMPLE. *Find the radius of a circular cycle track, which is 4 laps to the mile. (Obviously the circumference of the track $= \frac{1}{4}$ mile.) ($\pi = 3\frac{1}{7}$.)*

Formula.

$$\begin{aligned} \mathbf{r} &= \frac{\mathbf{C}}{2\pi} \\ &= \frac{440 \text{ yards}}{2 \times 3\frac{1}{7}} \\ &= \frac{440 \text{ yards}}{6\frac{2}{7}} \\ &= \mathbf{70\ yards.} \end{aligned}$$

To find the area of a circle, given the radius.

Rule. Square the radius and multiply the result by π.

Formula. $\mathbf{A} = \pi\mathbf{r}^2$.

EXAMPLE. *Find the area of a circle, whose radius is 7".* ($\pi = 3\frac{1}{7}$.)

$$\mathbf{A} = \pi \mathbf{r}^2$$
$$= 3\tfrac{1}{7} \times 7 \times 7$$
$$= \mathbf{154 \text{ sq. inches.}}$$

NOTE. If $A = \pi r^2$;

$$\therefore \frac{A}{\pi} = r^2;$$

$$\therefore \sqrt{\frac{A}{\pi}} = r$$ (Taking the square root on both sides).

EXAMPLE. *Find the radius of a circular room whose floor space is 34 sq. yards 2 sq. feet.* ($\pi = 3\frac{1}{7}$.)

Formula. $$\mathbf{r} = \sqrt{\frac{\mathbf{A}}{\pi}}$$

$$= \sqrt{\frac{308 \text{ sq. feet}}{3\frac{1}{7}}}$$

$$= \sqrt{98}$$

$$= \mathbf{9{\cdot}95 \text{ feet, or practically } 10 \text{ feet.}}$$

To find the length of the arc of a sector of a circle, given the radius of the circle, and the number of degrees in the sector.

NOTE. A sector of a circle is the part enclosed by two radii and the arc of the circle between them.

Rule. Find the circumference of the circle, and ascertain $\frac{x}{360}$ of this result (x being the number of degrees contained in the sector).

Formula. $$\frac{2\pi \mathbf{rx}}{360} \text{ or } \frac{\pi \mathbf{rx}}{180} = \mathbf{l},$$

where $\mathbf{r}$ = the radius of the circle,

$\mathbf{x}$ = the number of degrees in the sector,

$\pi = 3\frac{1}{7}$ or $3{\cdot}1416$,

360 or **180** = a constant.

NOTE. $\frac{2\pi r x}{360} = 2\pi r$ (circumference of circle) $\times \frac{x}{360}$.

Example. *Find the length of the arc of a sector of a circle of 10″ radius, the sector enclosing 54°.* ($\pi = 3{\cdot}1416$.)

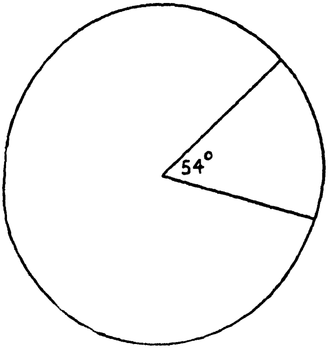


Fig. 7.

Formula. $\dfrac{\pi r x}{180} = l.$

(Substituting) $\therefore \dfrac{3{\cdot}1416 \times 10 \times 54}{180} = l$;

$\therefore$ $3{\cdot}1416 \times 3$ (cancelling);

$\therefore$ **9·425 inches.**

[Note. Another method is shown by a knowledge of **Radians** (see Chapter on Trigonometry, Book III.).]

Note. If $\dfrac{\pi r x}{180} = l$, $180l = \pi r x$, $\dfrac{180l}{\pi r} = x$.

Example. *Find the number of degrees in the sector of a circle, whose radius is 21″ and the length of the arc of the sector 15″.* ($\pi = 3\frac{1}{7}$.)

Formula. $x = \dfrac{180l}{\pi r}$

$= \dfrac{180 \times 15}{3\frac{1}{7} \times 21}$

$= \dfrac{2700}{66}$

$=$ **40° 55′, or nearly 41°.**

To find the area of a sector of a circle, given the radius of the circle, and the number of degrees contained in the sector.

Rule. Find the area of the circle and ascertain $\frac{x}{360}$ of this result (x being the number of degrees in the sector).

Formula. $\frac{\pi r^2 x}{360} = A$. (NOTE. $\pi r^2 \times \frac{x}{360}$)

where the letters stand for the same as in the last worked example.

EXAMPLE. *Find the area of a sector of a circle of 15" radius, the sector enclosing 63°.* ($\pi = 3{\cdot}1416$)

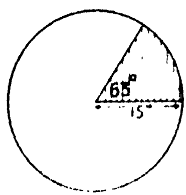


FIG. 8.

Formula. $A = \frac{\pi r^2 x}{360}$.

(Substituting) $= \frac{3{\cdot}1416 \times 15 \times 15 \times 63}{360}$

$= 123{\cdot}7$ square inches

NOTE. If $A = \frac{\pi r^2 x}{360}$,

$360A = \pi r^2 x$.

$\frac{360A}{\pi r^2} = x$.

EXAMPLE. *Find the number of degrees in a sector of a circle, whose radius is 14", the area of the sector being 88 square inches.* ($\pi = 3\frac{1}{7}$.)

Formula. $x = \frac{360A}{\pi r^2}$

$= \frac{360 \times 88}{3\frac{1}{7} \times 14 \times 14}$

$= 51\frac{3}{7}°$ or $51° 26'$.

SIMPLE INTEREST BY FORMULAE

Formulae for finding (*a*) the Simple Interest on a sum of money, (*b*) the time it will take for a sum of money to produce a certain amount of interest at a given rate per cent., (*c*) the rate per cent., given the time, the Principal, and the Interest, and (*d*) the Amount of Principal and Interest.

Let $\mathbf{P}$ = Principal.
$\mathbf{I}$ = Interest on the Principal.
$\mathbf{r}$ = Rate per cent.
$\mathbf{n}$ = Number of years.
$\mathbf{A}$ = Amount of Principal and Interest.

Then, the Interest (I) on £100 for one year is £r,

and ,, ,, (I) on £1 ,, ,, ,, $\frac{£r}{100}$;

∴ ,, ,, (I) on £1 for n years is $\frac{£nr}{100}$;

∴ ,, ,, (I) on £P ,, n ,, is $\frac{£Pnr}{100}$:

$$\therefore I = \frac{Pnr}{100}.$$

$$Pnr = 100I; \quad \therefore n = \frac{100I}{Pr}; \quad \therefore r = \frac{100I}{Pn}.$$

EXAMPLE. *Find the Simple Interest on £75 13s. 6d. for 9 months at 4½ per cent.*

Formula. $$I = \frac{Pnr}{100}$$

$$= \frac{75{\cdot}675 \times {\cdot}75 \times 4{\cdot}5}{100}$$

$$= {\cdot}75675 \times 3{\cdot}375 = £2\ 11s.\ 1d.$$

[NOTE. 13s. 6d. = ·675£; and 9 months = ·75 years.]

EXAMPLE. *Find the amount of £650 for 3½ years at 4½ per cent., Simple Interest.*

$$I = \frac{Pnr}{100};$$

$$\therefore A = \frac{Pnr}{100} + \frac{P}{1} \quad \left[\text{Proceed as in the addition of Vulgar Fractions.}\right]$$

$$= \frac{Pnr + 100P}{100}$$

Formula therefore $= \frac{P(100 + nr)}{100}$.

Substituting, we obtain :

$$A = \frac{650(100 + (3\tfrac{1}{2} \times 4\tfrac{1}{2}))}{100}$$

$$= \frac{650(100 + 15\tfrac{3}{4})}{100}$$

$$= \frac{650 \times 115\tfrac{3}{4}}{100}$$

$$= 6{\cdot}5 \times 115{\cdot}75$$

$$= £752\ 7s.\ 6d.$$

FORMULAE BASED ON THE WORK OF THE LEVER

This is a simple mechanical device to enable us to *lift* or *raise* heavy weights (hence the word "lever," from Latin

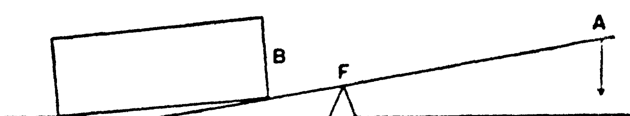


FIG. 9.

levare = to raise) with comparative ease. The simplest form of lever is a crow-bar (Fig. 9), and the longer the arm, at the end of which we exert force, the greater the mass we can raise.

This means that the *working power* round a fixed axis increases, when the *distance* of the force from the axis is increased.

(i) If we take a bar of iron, 20″ long, and balance it at *F*, we shall find that a 6 oz. weight, whose centre is 6″ from *F*, balances a 4 oz. weight, whose centre is 9″ from *F* (Fig. 10)

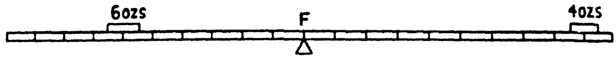


FIG. 10.

(ii) Try a 3 oz. weight 8″ from F, and a 4 oz. weight (on the further side) 6″ from *F*, and we shall see that they also balance.

In case (i), we found that a 4 oz. weight, placed 9″ from *F*, balanced a 6 oz. weight placed 6″ from *F*.

NOTE. $4 \times 9 = 6 \times 6$.

In case (ii), we found that a 3 oz. weight placed 8″ from *F*, balanced a 4 oz. weight placed 6″ from *F*.

NOTE. $3 \times 8 = 4 \times 6$.

The **turning power** is, therefore, measured by the product of the force, and its distance from *F*.

These simple examples explain fully the principle of the lever and its action.

Refer to Fig. 9. The load at *B* represents the weight to be raised, whilst the weight at *A* represents the pressure or effort to be exerted *in a downward direction*.

F is a support called a *fulcrum*.

The pressure (*P*) at *A*, sufficient to raise the weight, will be:

$$P \times (AF) = W \times (BF); \quad \therefore P = \frac{W \times (BF)}{(AF)}.$$

NOTE. *AF* and *BF* in these examples do not mean $A \times F$ or $B \times F$. They each represent one distance.

EXAMPLE. *What pressure will be required at A to raise the weight at B, if BF = 4 inches, AF = 36 inches, and the weight = 540 lbs.?*

Formula.

$$\mathbf{P} = \frac{\mathbf{W} \times (\mathbf{BF})}{(\mathbf{AF})}$$

$$= \frac{540 \times 4}{36}$$

= 60 lbs. pressure.

EXAMPLE. *A weight of 800 lbs. is resting on one end of a lever, the fulcrum of which is 3 inches from this point of contact. What pressure must be exerted 4 feet from the fulcrum, so as to raise the weight ?*

Formula.

$$P = \frac{W \times (BF)}{(AF)}$$

$$= \frac{800 \times 3}{48}$$

$$= \textbf{50 lbs. pressure.}$$

EXAMPLE. *The weight in a wheelbarrow is 9 inches from the axle of the wheel, whilst the handles are 2′ 9″ from the axle. There is a weight of 88 lbs. to be raised. How much upward pressure is necessary at each handle to raise this load ?*

Formula

$$P = \frac{W \times (BF)}{(AF)}$$

$$= \frac{88 \times 9}{33}$$

= **24 lbs.** on the *two* handles ;

∴ **12 lbs. upward pressure** must be exerted on *each* handle.

3. EXAMPLES ON FORMULAE

NOTE. All Examples are to be worked to the *nearest second decimal place.*

(*a*) The Square

Given the formulae : $A = S^2$; $S = \sqrt{A}$.

1. Find A, when $S = 2{\cdot}5$ ins.
2. Find A, when $S = 3{\cdot}8$ ft.
3. Find A, when $S = \frac{1}{4}$ a mile.
4. Find A, when $S = 7$ fur. 210 yds.
5. Find A, when $S = 3$ yds. 2 ft. 5 ins.
6. Find S, when $A = 1$ acre.
7. Find S, when $A = 69$ sq. inches.
8. Find S, when $A = 17$ sq. yards 6 sq. feet.
9. Find S, when $A = 10$ acres.
10. Find S, when $A = 156{\cdot}35$ sq. feet.

(b) The Rectangle

Given the formulae: $A = lb$; $l = \frac{A}{b}$; $b = \frac{A}{l}$.

11. Find A, when $l = 1{\cdot}4$ inches, and $b = 6$ inches.
12. Find A, when $l = 2{\cdot}8$ inches, and $b = 3{\cdot}6$ inches.
13. Find A, when $l = 12$ feet, and $b = 9{\cdot}4$ inches.
14. Find A, when $l = \frac{1}{4}$ mile, and $b = 85$ yards.
15. Find A, when $l = 5{\cdot}35$ inches, and $b = 4{\cdot}38$ inches.
16. Find l, when $A = 79{\cdot}6$ sq. inches, and $b = 9{\cdot}32$ inches.
17. Find b, when $A = 24$ sq. feet, and $l = 3$ feet $2{\cdot}7$ inches.
18. Find l, when $A = 75$ sq. yards 8 sq. feet, and $b = 10$ yards 2 feet.
19. Find b, when $A = 15$ acres, and $l = \frac{1}{4}$ mile.
20. Find l, when $A = 1$ sq. mile, and $b = 96{\cdot}5$ yards.

(c) The Triangle

Given the formula: $A = \frac{bh}{2}$.

(*Notice that* $2A = bh$.)

21. Find A, when $h = 7{\cdot}8$ inches, and $b = 4{\cdot}6$ inches.
22. Find A, when $h = 29$ yards 2 feet $3{\cdot}6$ inches, and $b = 20$ yards.
23. Find A, when $h = \frac{3}{4}$ mile, and $b = 5$ furlongs.
24. Find h, when $A = 77{\cdot}6$ sq. inches, and $b = 5{\cdot}7$ inches.
25. Find b, when $A = 48$ sq. yards, and $h = 5$ yards 2 feet.
26. Find h, when $A = 15$ acres, and $b = 70{\cdot}6$ yards.

(d) The Circle.

Given the formula for the circumference of a circle: $C = 2\pi r$, *and the formula for the radius of a circle*: $r = \frac{C}{2\pi}$.

27. Find C, when $r = 1'\ 10''$, and $\pi = 3\frac{1}{7}$.
28. Find C, when $r = 25{\cdot}7$ inches, and $\pi = 3{\cdot}1416$.
29. Find C, when $r = 10$ yards 2 feet 1 inch, and $\pi = 3\frac{1}{7}$.
30. Find r, when $C = 11$ feet, and $\pi = 3\frac{1}{7}$.
31. Find r, when $C = 89{\cdot}7$ yards, and $\pi = 3{\cdot}1416$.
32. Find r, when $C = 1$ mile, and $\pi = 3\frac{1}{7}$.

Given the formula for the area of a circle: *given the radius* $A = \pi r^2$, *and the formula for the radius, given the area*: $r = \sqrt{\frac{A}{\pi}}$.

33. Find A, when $r = 10\frac{1}{2}$ inches, and $\pi = 3\frac{1}{7}$.
34. Find A, when $r = 3$ yards 2 feet $1{\cdot}6$ inches, and $\pi = 3{\cdot}1416$.
35. Find A, when $r = 1$ furlong, and $\pi = 3{\cdot}1416$.

36. Find r, when $A = 15{\cdot}91$ sq. inches, and $\pi = 3{\cdot}1416$.

37. Find r, when $A = 1$ acre, and $\pi = 3\frac{1}{7}$.

38. Find r, when $A = 79{\cdot}61$ sq. yards, and $\pi = 3{\cdot}1416$.

39. A donkey is tethered to a post by a rope 14 feet long. How many square yards of pasture can he feed on? ($\pi = 3\frac{1}{7}$.)

40. A bicycle wheel is 28″ diameter. How many times will it revolve in a journey of 18 miles? ($\pi = 3\frac{1}{7}$.)

41. The large driving wheel of a locomotive is 10 feet diameter, whilst the small leading wheel is 1′ 9″ diameter. How many more revolutions will the small wheel make than the driving wheel on a journey from Birmingham to London, a distance of 110 miles? ($\pi = 3\frac{1}{7}$.)

42. A circular tower is 18 feet in diameter. What is the area of the ground that it stands on? ($\pi = 3{\cdot}1416$). Give the answer in sq. yards, sq. feet, and the nearest first decimal of a foot.

43. The large finger of a clock is 13″ long. How many yards does the extreme edge of the finger travel in a week of 7 days? ($\pi = 3{\cdot}1416$.)

44. A steam roundabout is erected with a radius of 4 yards. A boy, riding on one of the outer circle of horses, goes round 10 times for 1d. How far did he travel? ($\pi = 3\frac{1}{7}$.)

Given the formula for the length of the arc of a sector of a circle:

$$l = \frac{2\pi rx}{360}, \text{ or } l = \frac{\pi rx}{180}. \quad (\pi = 3\tfrac{1}{7}.)$$

45. Find the length of the arc of a sector of a circle, whose radius is 10·5 inches, the sector containing 50°.

46. Find the length of the arc of a sector of a circle, radius = 1 foot 9 inches, and the number of degrees in the sector = 65°.

47. A circular flower-bed is divided into five equal sectors. If the radius of the bed is 10 feet 6 inches, find the length of the outer edge of *two* of the sectors.

48. The large finger of a clock is 14″ long. How far does it move in 10 minutes?

Given the formula: $\frac{180l}{\pi r} = x$.

49. Find the number of degrees in the sector of a circle whose radius is 15″ and the length of the arc of the sector is 12″. ($\pi = 3\frac{1}{7}$.)

50. How many degrees does the large finger of a clock pass through in going 12 minutes?

Given the formula: $A = \frac{\pi r^2 x}{360}$. ($\pi = 3\frac{1}{7}$.)

51. Find the area of the sector of a circle, the radius being 18″, and the number of degrees in the sector 56°.

52. What is the area of one of the five sectors of the flower-bed mentioned in question 47?

53. Find the area covered by the clock finger in question 48.

54. A goat is tethered to a post in the centre of one of the fences of a rectangular field. If the rope is 12 yards long, and is attached to the post on the level of the ground, how many square yards of pasture can the goat graze on ?

55. Suppose that the goat be moved and tethered to the post forming one of the *corners* of the same field, how many square yards of pasture can the goat now graze on ?

Given the formula : $x = \frac{360A}{\pi r^2}$. $(\pi = 3\frac{1}{7}.)$

56. Find the number of degrees in the sector of a circle of 3·5 inches radius, the area of the sector being 25 sq. inches.

57. Find the number of degrees in the sector of a circle of 2 feet 4 inches radius, the area of the sector being 600 sq. inches.

58. A metal plate, $5\frac{1}{2}'' \times 4''$, has a circular hole of 3″ diameter, and a

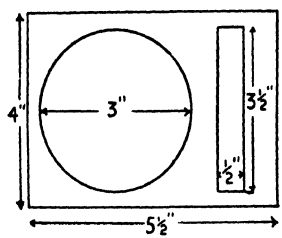


FIG. 11.

rectangular one, $3\frac{1}{2}'' \times \frac{1}{2}''$, punched out of it. How many square inches of metal remain ? $(\pi = 3\frac{1}{7}.)$

59. Measure a halfpenny across, and calculate its circumference. $(\pi = 3{\cdot}1416.)$

60. A triangular piece of wood, of 26″ base and 28″ perpendicular

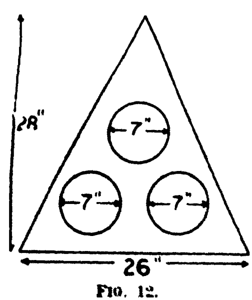


FIG. 12.

height, has three circular holes, each 7″ in diameter, bored through. What area of the surface of the board is left unbored ? $(\pi = 3\frac{1}{7}.)$

(e) The Lever

61. A steel rod is balanced at the centre of its length on a triangular wedge. A 4 oz. weight is placed 6 inches from the centre. How far from the centre must a 3 oz. weight be placed on the other side to balance it ?

62. On the same steel rod a weight of 6 ozs. is placed 3 ins. from the centre. What weight must be placed 9 ins. from the centre on the other side ?

63. A weight of 600 lbs. is resting on one end of a lever, the fulcrum of which is 3 inches from this point of contact. What pressure must be exerted $3\frac{1}{2}$ feet from the fulcrum, so as to raise the weight ?

64. The weight in a wheelbarrow acts 8 inches from the axle of the wheel, whilst the handles are 2′ 9″ from this axle. If there is a weight of 75 lbs. to be raised, what upward pressure must be exerted on each handle to do this ?

65. A spanner 6″ long requires a pressure of 56 lbs. to loosen a tight nut. What pressure must be exerted at the end of a 15″ spanner ?

(f) Simple Interest

66. *From the formula for calculating Simple Interest* : $I = \frac{Pnr}{100}$, find the Simple Interest on £760 15s for 8 years at 3 per cent. (to the nearest penny)

67. Find the Simple Interest on £84 10s. 6d. for $5\frac{1}{2}$ years at $4\frac{1}{2}$ per cent. (to the nearest penny).

68. *Given the formula* : $A = \frac{P(100 + nr)}{100}$

where A = amount of Principal and Interest,
P = Principal,
n = number of years,
and r = rate per cent.,

find the amount of £520 for 6 years at $3\frac{1}{2}$ per cent

69. Find the amount of £55 10s. 6d. for 8 months at 3 per cent. (to the nearest penny).

70. Find the amount of £550 10s. 6d. for 4 years at $3\frac{1}{2}$ per cent (to the nearest penny)

TRIGONOMETRY

THE word "trigonometry" is derived from two Greek words, one signifying "I measure," and the other "a triangle."

The subject embraces a much wider outlook than that of mensuration, and by introducing a new method of measuring *the angles of a triangle, the student is enabled to ascertain* and determine hitherto impossible data.

There are various ways of measuring an angle :

(i) We can measure it in **degrees, minutes and seconds**, and say the angle contains 40°, or 51° 30′, or 36° 10′ 12″.

(ii) We can measure it in **radians**, and say the angle contains π radians.

(iii) We can measure it by means of so-called **functions of an angle** (the **sine**, **cosine**, **tangent**, **cotangent**, **secant**, and **cosecant**).

These **Functions** are called the **Trigonometrical Ratios of an Angle.**

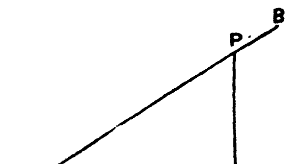


FIG. 13.

Let BAC be any acute angle (Fig. 13).

In AB, take any point P; and from P, draw PM, at right angles to AC.

We have now constructed a right-angled triangle, APM.

If we consider the original angle **BAC** (or **PAM**) in this triangle, then **PM** becomes the perpendicular height,

AM becomes the base,

and **AP** becomes the hypotenuse ;

and it is the relation of these three sides, taken in pairs (there will be six such selections) to one another, that constitutes the **Trigonometrical Ratios** of the angle PAM, or BAC, which we will designate **angle A**.

We will now tabulate the **Trigonometrical Ratios** of the angle A.

$$\frac{\mathbf{PM}}{\mathbf{AP}} = \frac{\text{perpendicular}}{\text{hypotenuse}} = \textbf{sine of A}, \text{ written } \textbf{sin A}.$$

$$\frac{\mathbf{AM}}{\mathbf{AP}} = \frac{\text{base}}{\text{hypotenuse}} = \textbf{cosine of A}, \text{ written } \textbf{cos A}.$$

$$\frac{\mathbf{PM}}{\mathbf{AM}} = \frac{\text{perpendicular}}{\text{base}} = \textbf{tangent of A}, \text{ written } \textbf{tan A}.$$

$$\frac{\mathbf{AM}}{\mathbf{PM}} = \frac{\text{base}}{\text{perpendicular}} = \textbf{cotangent of A}, \text{ written } \textbf{cot A}.$$

$$\frac{\mathbf{AP}}{\mathbf{AM}} = \frac{\text{hypotenuse}}{\text{base}} = \textbf{secant of A}, \text{ written } \textbf{sec A}.$$

$$\frac{\mathbf{AP}}{\mathbf{PM}} = \frac{\text{hypotenuse}}{\text{perpendicular}} = \textbf{cosecant of A}, \text{ written } \textbf{cosec A}.$$

If we consider the triangle PAM, as having $AM = 4$ units, $PM = 3$ units, and $AP = 5$ units, then

$$\textbf{sin A} = \frac{\text{perpendicular}}{\text{hypotenuse}} = \frac{3}{5} = \cdot 6000,$$

$$\textbf{cos A} = \frac{\text{base}}{\text{hypotenuse}} = \frac{4}{5} = \cdot 8000,$$

$$\textbf{tan A} = \frac{\text{perpendicular}}{\text{base}} = \frac{3}{4} = \cdot 7500,$$

$$\textbf{cot A} = \frac{\text{base}}{\text{perpendicular}} = \frac{4}{3} = 1 \cdot 3333\ldots,$$

$$\textbf{sec A} = \frac{\text{hypotenuse}}{\text{base}} = \frac{5}{4} = 1 \cdot 2500,$$

$$\textbf{cosec A} = \frac{\text{hypotenuse}}{\text{perpendicular}} = \frac{5}{3} = 1 \cdot 6666\ldots.$$

NOTE. In memorising these Trigonometrical Ratios (which is most essential) the student will only require to remember the first three (**sine, cosine, tangent**), as the remaining three ratios will be seen from the above table to be respectively the reciprocals (or inversions) of the first three ratios.

Thus: **cosec A is the Reciprocal of sin A,**
and **sec A is the Reciprocal of cos A,**
and **cot A is the Reciprocal of tan A;**

$$\therefore\ \text{cosec A} \times \sin \text{A} = 1,\ \text{cosec A} = \frac{1}{\sin \text{A}},\ \sin \text{A} = \frac{1}{\text{cosec A}};$$

$$\therefore\ \sec \text{A} \times \cos \text{A} = 1,\quad \sec \text{A} = \frac{1}{\cos \text{A}},\ \cos \text{A} = \frac{1}{\sec \text{A}}.$$

$$\therefore\ \cot \text{A} \times \tan \text{A} = 1,\quad \cot \text{A} = \frac{1}{\tan \text{A}},\ \tan \text{A} = \frac{1}{\cot \text{A}};$$

In the right-angled triangle PAM, we have, so far, considered only the Trigonometrical Ratios of the acute angle A. If we require those of the other acute angle, P, we must find them with the knowledge that, although the right-angled triangle is the same, the side AM is no longer the base, neither is the side PM the perpendicular height for the angle P. Of the two sides containing the right angle, in any right-angled triangle, the side *opposite* to the angle, whose trigonometrical ratios we require, is always the perpendicular height, whilst the other of these two sides forms the base.

Thus, **AM becomes the perpendicular height, and PM the base, when finding the trigonometrical ratios of the angle P.**

Consequently,
$$\frac{\text{AM}}{\text{AP}} = \sin \text{P},$$
$$\frac{\text{PM}}{\text{AP}} = \cos \text{P},$$
$$\frac{\text{AM}}{\text{PM}} = \tan \text{P},$$
$$\frac{\text{PM}}{\text{AM}} = \cot \text{P},$$
$$\frac{\text{AP}}{\text{PM}} = \sec \text{P},$$
$$\frac{\text{AP}}{\text{AM}} = \text{cosec P}.$$

The three right-angled triangles *ABC*, *DEF*, and *KMN* placed in various positions, will afford the student much practice in naming the respective perpendicular heights and bases for the six acute angles *A*, *C*, *D*, *F*, *K* and *N*.

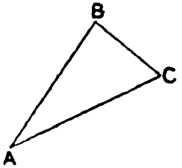


FIG. 14.

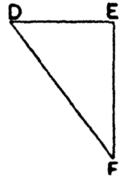


FIG. 15.

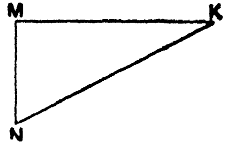


FIG. 16.

For example, *AB* is the perpendicular height and *BC* is the base of the right-angled triangle *ABC*, when we are finding the trigonometrical ratios of the angle *C*.

Those for the other five acute angles should be ascertained by the student, who must master this rule, if he wish to succeed.

The sine of one acute angle is the cosine of its complement.

In the right-angled triangle *PAM* (Fig. 17), the angles *A* and *P* are together equal to a right-angle;

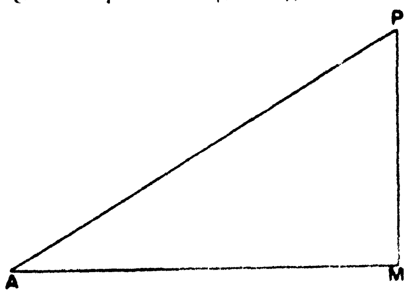


FIG. 17.

$\therefore$ angle **P** is the **complement** of angle **A**,

and angle **A** is the **complement** of angle **P**.

(NOTE. An angle of 31° is the complement of an angle of 59°, because 90° − 59° = 31°.)

Proof. $$\sin A = \frac{PM}{AP},$$

$$\cos P \text{ (the complement of angle } A) = \frac{PM}{AP};$$

$$\therefore \; \sin A = \cos P; \quad \text{and} \quad \sin P = \cos A.$$

Similarly, we can prove that

The tangent of an acute angle is the cotangent of its complement.

Proof. In the right-angled triangle PAM,

$$\tan A = \frac{PM}{AM},$$

and $$\cot P \text{ (the complement of angle } A) = \frac{PM}{AM};$$

$$\therefore \; \tan A = \cot P; \quad \text{and} \quad \tan P = \cot A.$$

We can also prove that

The secant of any acute angle is the cosecant of its complement.

Proof. In the right-angled triangle, PAM,

$$\sec A = \frac{AP}{AM},$$

and $$\text{cosec } P \text{ (the complement of angle } A) = \frac{AP}{AM};$$

$$\therefore \; \sec A = \text{cosec } P.$$

Tabulating all these proofs, for reference, we obtain the following :

$$\sin A = \frac{1}{\text{cosec } A}, \quad \text{and} \quad \text{cosec } A = \frac{1}{\sin A}.$$

$$\cos A = \frac{1}{\sec A}, \quad \text{and} \quad \sec A = \frac{1}{\cos A}.$$

$$\tan A = \frac{1}{\cot A}, \quad \text{and} \quad \cot A = \frac{1}{\tan A}.$$

$\sin A = \cos (90 - A)$ when A is an acute angle.

$\cos A = \sin (90 - A)$,, ,, ,, ,,

$\tan A = \cot (90 - A)$,, ,, ,, ,,

$\cot A = \tan (90 - A)$,, ,, ,, ,,

$\sec A = \text{cosec } (90 - A)$,, ,, ,, ,,

$\text{cosec } A = \sec (90 - A)$,, ,, ,, ,,

SIMILAR TRIANGLES

If we construct an acute angle PAM, and drop a number of lines from AP each perpendicular to the base AM, thus:

$$P_1M_1,\ P_2M_2,\ P_3M_3,\ \text{etc.};$$

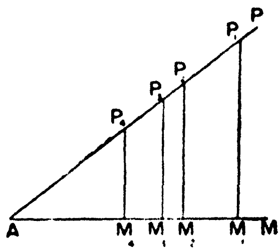


FIG. 18.

then the sine ratio

$$=\frac{P_1M_1}{AP_1}=\frac{P_2M_2}{AP_2}=\frac{P_3M_3}{AP_3}=\frac{P_4M_4}{AP_4},$$

and the cosine ratio

$$=\frac{AM_1}{AP_1}=\frac{AM_2}{AP_2}=\frac{AM_3}{AP_3}=\frac{AM_4}{AP_4},$$

and so on for all the trigonometrical ratios.

This is because AP_1M_1, AP_2M_2, etc. **are similar triangles**, or in other words, the angles are equal, each to each, and the sides are proportional, each to each.

The knowledge of the properties of similar triangles was made use of by Thales, the mathematician, who lived from 640 B.C. to 550 B.C. He visited Egypt, and showed the Egyptians how to find the height of one of the Pyramids.

Method. $OABCD$ = pyramid. (Fig. 19.)

CBF = shadow cast by sun shining from the direction SO.

OE = height of pyramid.

OFE = altitude of sun.

Thales took a rod $O'E'$ of known length (say 4 units of

Egyptian measurement), and stood it vertically by the side of the pyramid.

$E'F'$ is the shadow of the rod cast by the sun.

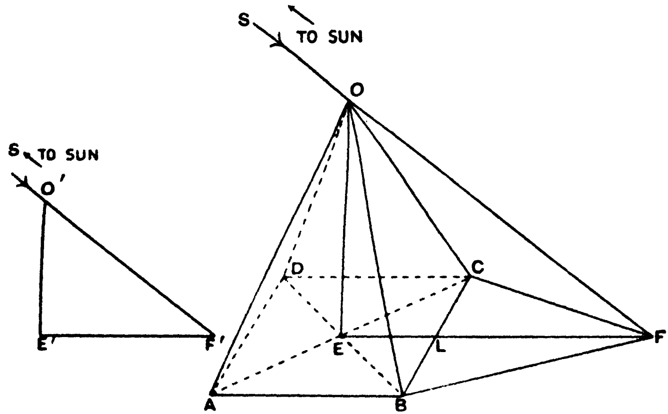


FIG. 19.

Angle $O'F'E'$ = altitude of sun = OFE;

$$\therefore \frac{OE}{EF} = \frac{O'E'}{E'F'}.$$

But Thales found that the shadow $E'F'$ was equal to 5·5 units, and that EF was equal to 150 units.

(NOTE. Thales could only measure that portion of the shadow represented by LF, but he knew that EL, the portion under the pyramid $= \frac{1}{2}AB$, which he measured, and added to LF, thus obtaining $EF = 150$ units.)

$$\therefore 4 : 5{\cdot}5 :: OE : 150;$$

$$\therefore \text{height of pyramid} = \frac{150 \times 4}{5{\cdot}5}$$

$$= \frac{600}{5{\cdot}5}$$

$$= 109{\cdot}1 \text{ units of Egyptian measure.}$$

The application of this proof is very useful in finding the heights of towers, steeples, trees, etc. *when the sun is shining.*

Example. *To find the height of a church tower, when the sun is shining.*

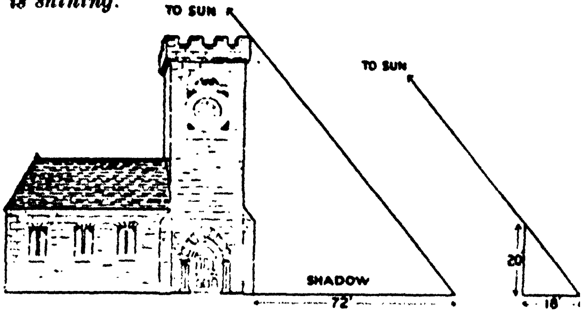


Fig. 20.

The shadow of the tower cast by the sun is measured, and found to be 72 feet. A 20′ pole gives a shadow of 18′;

$$\therefore\ 18 : 20 :: 72 : x.$$

$$\text{Height of church tower} = \frac{20 \times 72}{18}$$

$$= \mathbf{80\ feet.}$$

How to find the breadth of a stream by the principle of Similar Triangles.

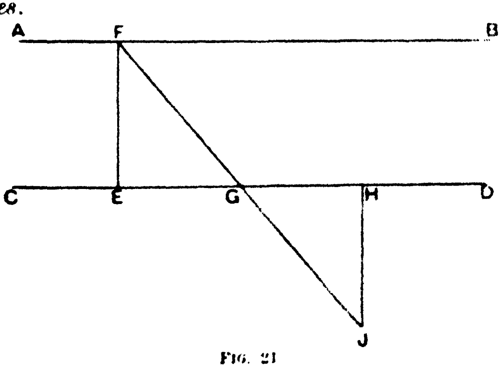


Fig. 21

AB and *CD* are the banks of a stream, whose breadth (or is required by persons on the bank *CD* (see Fig. 21).

A stick, or other object, is placed at E, vertically opposite *to a fixed point F,* on the opposite bank.

Another stick is placed at G (EG may be *any* reasonable distance).

The distance GH is carefully measured *exactly* equal to EG *(this is important).*

A distance is then stepped out, or measured, from H, in a direction at right angles to the bank CD.

The measurement ceases at the point J, which is exactly in line with G and F.

HJ is the same distance as the breadth (or width) across the stream from E to F.

NOTE. Not only are the triangles FEG and JHG similar, but they are also equal in every respect.

For examples of this class of work, the student must obtain the data outside the class-room. "Imaginary streams" marked out with chalk in the playground will serve this purpose.

HOW TO USE A TABLE OF SINES, ETC.

At the end of this book will be found a table of sines, cosines, tangents, cotangents, and radians, for angles, whose sizes range from 0° to 90°. The angles, lying between 0° and 45°, are tabulated on the extreme left, and their sine, cosine, and other values are placed horizontally opposite to their respective angles, and *under* the words sine, cosine, etc. The angles lying between 45° and 90° are tabulated on the extreme right, and their sines, cosines, and other values, are placed horizontally opposite to their respective angles and *over* the words sine, cosine, etc.

EXAMPLES. *To find the trigonometrical values (or ratios) of (a) an angle of 29°, and (b) an angle of 56°.*

(*a*) As the angle 29° lies between 0° and 45°, we find it tabulated on the *left* of the table, and its trigonometrical values placed horizontally *on the right*, and *under* the words sine, etc. Thus, the sine = ·4848, the tangent = ·5543, the co-tangent = 1·8040, and the cosine = ·8746.

(*b*) As the angle 56° lies between 45° and 90°, we find it tabulated on the *right* of the table, and its trigonometrical values placed horizontally *on the left*, and *over* the words sine, etc. Thus, the sine = ·8290, the tangent = 1·4826, the co-tangent = ·6745, and the cosine = ·5592.

Simple proof showing that as an acute angle increases from 0 to 90°, its sine value increases from 0 to 1.

In the accompanying diagram (Fig. 22), the lines OP', OP'', OP''', OP'''', being radii of the arc of the circle, whose centre is O, are all equal to one another.

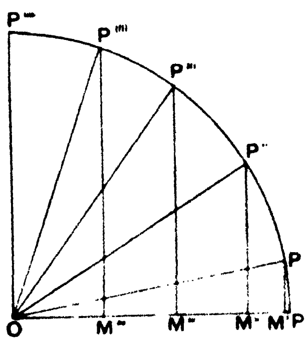


Fig. 22

It will be seen that they form in each case the hypotenuse of the right-angled triangles: $OP'M'$, $OP''M''$, $OP'''M'''$, $OP''''M''''$.

Whilst the hypotenuse in each triangle does not vary in length, it is obviously clear that as the angles at O ($P'OM'$, $P''OM''$, etc.) increase, the length of the perpendiculars $P'M'$, $P''M''$, etc. increase, as they approach nearer and nearer to $P''''O$.

In other words, each succeeding perpendicular approaches nearer to the length of the hypotenuse.

As the sine of an angle

$$= \frac{\text{perpendicular height}}{\text{hypotenuse}}$$

it is obvious that the greater the length of the perpendicular height, the greater is the value of the fraction representing the *sine* value of the angle.

There are the two extreme cases to prove.

1. In an angle of 0°, the hypotenuse OP (see Fig. 22) lies on the base OP. *There is no perpendicular height.*

Consequently, the sine value of 0°

$$= \frac{\text{perpendicular height}}{\text{hypotenuse}}$$

$$= \frac{0}{OP}$$

$$= 0.$$

2. In an angle of 90°, the perpendicular height and the hypotenuse coincide as $P'''''O$ (see Fig. 22). *There is no base.*

Consequently, the sine value of 90°

$$= \frac{\text{perpendicular height}}{\text{hypotenuse}}$$

$$= \frac{P'''''O}{P'''''O}$$

$$= 1.$$

The same illustration will prove to us, that as an acute angle increases from 0° to 90°, the *cosine value decreases from 1 to 0.*

As the angles at O ($P'OM'$, $P''OM''$, etc.) increase, the *bases* of the right-angled triangles become smaller and smaller.

Thus, OM'''' is much smaller than OM'.

Consequently, the fractional values of the cosines will decrease as the *bases* decrease, for these bases form the ***numerators*** of these fractional values, which have in each case ***the same denominator*** (the radius of the arc).

In these circumstances, the smaller the numerator, the smaller will be the fraction.

10. Without using your table of cosines, write down the sizes (in degrees) of the eight angles whose *cosines* are equal to those of the *sines* mentioned in Question 9.

11. From your table of *cosines*, find the angles (in degrees) whose *cosines* are :

(*a*) ·9994, (*b*) ·9925, (*c*) ·9816, (*d*) ·9336,
(*e*) ·7547, (*f*) ·6293, (*g*) ·1045, (*h*) ·0523.

12. Without using your table of *sines*, write down the sizes (in degrees) of the eight angles whose *sines* are equal to those of the cosines mentioned in Question 11.

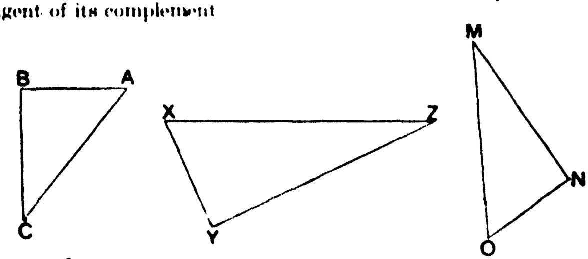
13. Prove that the cosine of an acute angle is equal to the sine of its complement.

14. From the table of *tangents*, find the angles (in degrees) whose *tangents* are :

(*a*) = ·1228, (*b*) ·2126, (*c*) – ·3057, (*d*) – ·4452,
(*e*) – 1·2799, (*f*) 2·0503, (*g*) = 3·2709, (*h*) – 6·3138.

15. Without using the table of *co-tangents*, write down the sizes (in degrees) of the eight angles, whose *co-tangents* are equal to those of the *tangents* mentioned in Question 14

16. Prove that the tangent of an acute angle is equal to the co-tangent of its complement



1 2 3

FIG. 23.

17. Study the above three right-angled triangles, and write down the names of the three right-angles

18. In triangle number 1, find out :

(*a*) Which is the *base*, when dealing with angle *A* ?
(*b*) Which is the *base*, when dealing with angle *C* ?
(*c*) Which is the *perpendicular height*, when dealing with angle *A* ?
(*d*) Which is the *perpendicular height*, when dealing with angle *C* ?

19. In triangle number 2, find out :

(*a*) Which is the *base*, when dealing with angle *X* ?
(*b*) Which is the *base*, when dealing with angle *Z* ?
(*c*) Which is the *perpendicular height*, when dealing with angle *X* ?
(*d*) Which is the *perpendicular height*, when dealing with angle *Z* ?

20. In triangle number 3, find out :

(*a*) Which is the *base*, when dealing with angle M ?

(*b*) Which is the *base*, when dealing with angle O ?

(*c*) Which is the *perpendicular height*, when dealing with angle M ?

(*d*) Which is the *perpendicular height*, when dealing with angle O ?

21. Construct a right-angled triangle ABC of which the sides AB and BC, containing the right angle, are 3″ and 4″ respectively.

(*a*) Find the length of the hypotenuse.

(*b*) Find the value of the *sine* of the angle A.

(*c*) Find the value of the *tangent* of the angle C.

(*d*) Find the value of the *cosine* of the angle A.

(*e*) Find the value of the *tangent* of the angle A.

(*f*) Find the value of the *cosine* of the angle C.

(*g*) Find the value of the *sine* of the angle C.

22. Construct a right-angled triangle XYZ of which the sides XY and YZ containing the right angle are 2½″ and 6″ respectively.

(*a*) Find the length of the hypotenuse.

(*b*) Find the value of the *sine* of the angle X.

(*c*) Find the value of the *cosine* of the angle X.

(*d*) Find the value of the *tangent* of the angle Z.

(*e*) Find the value of the *cosine* of the angle Z.

(*f*) Find the value of the *tangent* of the angle X.

(*g*) Find the value of the *sine* of the angle Z.

23. Given that the *sine* of an angle is [illegible], write down, without using the tables, the cosecant of the same angle.

24. Given that the *cosine* of an angle is [illegible], write down, without using the tables, the value of the *secant* of the same angle.

25. Given that the *tangent* of an angle is [illegible], write down, without using the tables, the value of the cotangent of the same angle.

26. Prove by a simple diagram that as the value of an *acute* angle *increases* from 0° to 90°, the value of the *sine increases* from 0 to 1.

27. Prove by a simple diagram, that as the value of an *acute* angle increases from 0° to 90°, the value of the *cosine decreases* from 1 to 0.

28. The shadow cast by a tower is 77½ feet, whilst that of a 6 foot rod is 5 feet. What is the height of the tower ?

29. The shadow cast by a lamp-post, 12 feet high, is 10½ feet. What is the height of a house, nearby, which casts a shadow of 36 ft. 9 ins. ?

30. What will be the height of a tree, which casts a shadow of 85 ft. 6 ins., when a 15 foot pole casts a shadow of 13 ft. 6 ins. ?

31. A church steeple is casting a shadow of 73 feet 6 ins. What height is the steeple, if a 20 foot flagpole, nearby, is casting a shadow of 17 feet 6 ins. ?

32 to 34. Do the examples illustrated by Figs. 19, 20 and 21, without reference to the solutions in the text.

GRAPHS AND GRAPHIC METHODS OF SOLVING PROBLEMS IN MENSURATION, TRIGONOMETRY, AND ALGEBRA

THE graphic representation of facts and figures has now become one of the most common experiences of our life, and there is scarcely a day that passes, in which we do not come in contact with **graphs** of some form or other. The daily papers, magazines, advertisements, **posters**, text-books, prospectuses, annual reports of societies, statements of accounts—to mention but a few—all call in the aid of **graphs** to present to the reader statistics and data in a simple and comprehensive manner.

Take an example: The temperature of a hospital patient is required at certain intervals by the doctor. The nurse in attendance does not make a tabulation of these observations in figures, but marks the rise and the fall of the patient's temperature on a sheet of squared paper, specially ruled for the purpose. This paper shows, horizontally, the days of the week (subdivided, if necessary, into hours), and, vertically, the rise and fall in the temperature of the patient.

The doctor, on his visit, is thus able to see, almost at a glance, whether the temperature is rising or falling, and on what day, and at what hour. If a specially coloured line be drawn horizontally across the paper, at the degree of temperature for a normally healthy person, the deductions drawn by the doctor will be fuller and more useful still. *The whole record is clear on the face of it.*

The various temperatures recorded on the squared paper, when joined together by lines, constitute a **graph** ; and the act of recording the observations, is called " **plotting**."

By means of sheets of squared paper, it is also possible to perform and solve in a practical manner, many mathematical problems in trigonometry, mensuration, and higher algebra, speedily and effectively.

It is consequently important and necessary that the children in our schools should be made acquainted with the use of

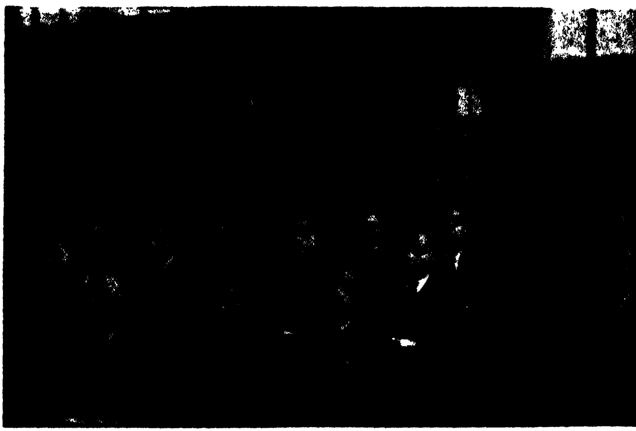


FIG. 24.

squared paper, and commence "plotting" their own simple graphs, at as early a stage as possible. They may begin with the daily attendances and the daily absences of the scholars in their own class, and plot them either on loose sheets, or, better still, in books made up of sheets of squared paper.

These books can now be purchased cheaply from most publishers, and are usually ruled in tenths of inches, because of our decimal system of notation. Every tenth line, horizontally and vertically drawn, is ruled heavier than the intervening nine lines, so as to facilitate counting, and the setting out of convenient scales. Every page in the squared paper book is ruled into about 90 small squares along its length, and into about 70 small squares along its breadth. If a child, therefore, counts the distance between each ruled line as one unit, he has sufficient space along the length of the page for more than two months' records of attendances (and absences), allowing for morning and afternoon school, *i.e.* two sets of records per day. The shorter edge of the page with the

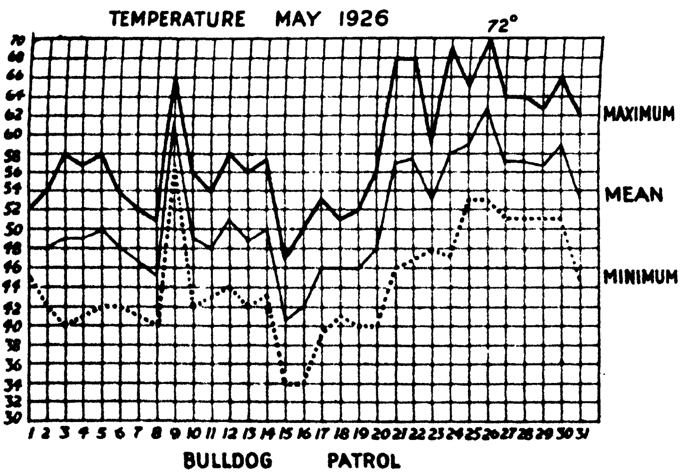


FIG. 25.

70 spaces will provide more than ample allowance for the daily attendances (and absences), as no class has now more than 55 on the books.

The above exercises may be followed by the recording of the daily thermometric readings, the daily barometric readings, the daily rain gauge readings, and the daily maximum and minimum thermometric readings. In addition to each scholar making records in his own book, a class may, with advantage, be divided into patrols or sections, each patrol or section being responsible for the plotting of one set of these observations on large sheets of squared paper, placed on the class-room walls.

Figure 24 provides an illustration of such records, kept by the boys of Class 2 in the writer's school, and "plotted" by the various patrols. The "Bulldog" patrol is held responsible for the thermometric readings, the "Lion" patrol

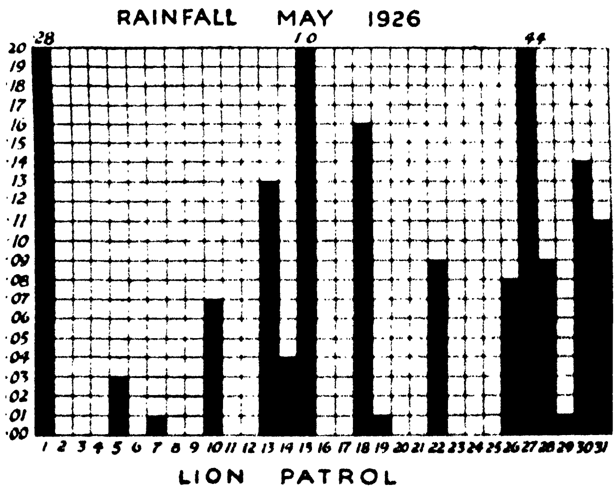


Fig. 26.

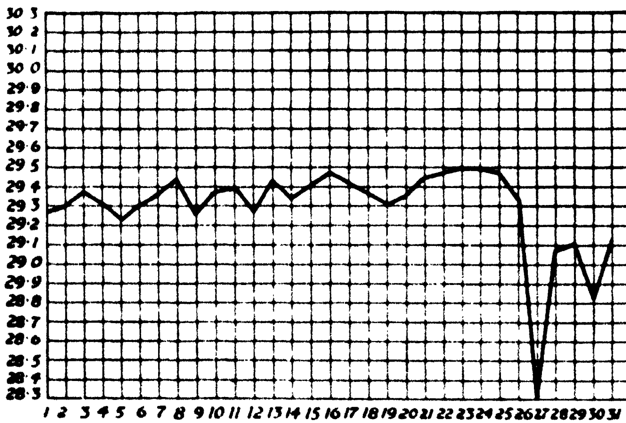
for the rain gauge readings, the "Kangaroo" patrol for the barometric readings, and the "Otter" patrol for the wind readings. Each patrol, furthermore, is responsible for the *accurate* plotting of the particular set of records observed by that patrol. The graphs illustrated in Figure 24 are all for the same month, January 1926, and as the records for several preceding months of January have been preserved by the patrols, valuable comparisons and contrasts can be made. These are entered in the pupils' note-books.

Emphasis has already been laid on the fact that the scholars make the records, and plot the graphs *from their own observations*—a most essential procedure, if lasting good is to accrue.

Enlarged illustrations are shown in Figs. 25, 26 and 27 of three of the records plotted by the various patrols for the month of May 1926.

NOTE. The patrols each consist of about 8 scholars—or $\frac{1}{6}$ of the class—the occupants of 4 dual desks, this being the

BAROMETER MAY 1926



KANGAROO PATROL

FIG. 27.

depth of the class from front to back. This is not, however, an arbitrary number, as each class should make its own divisions and patrols, according to the circumstances and conditions prevailing.

It is advisable for the children themselves to ascertain, by actual counting, the number present *and absent* each half-day in their own class. The two graphs can be shown on the one sheet of squared paper quite easily. Other simple statistics will readily commend themselves to the teacher and scholar, such as the total number of runs scored at each school cricket match during the season, and the total number of runs scored against the school at each match.

Children living at seaside resorts and seaports, can compile graphs showing the height of the tide at "high water" and "low water" each day These tides can be recorded to the *nearest foot*, for simplicity, in the earlier stages of compilation, whilst at a later stage, the *exact* number of feet and inches can be recorded, when the planning out of scales is more thoroughly grasped by the scholars.

An excellent plan is to allow each child to choose his own statistics, as this gives great scope for observation, research, and initiative.

RIGHT-ANGLED TRIANGLE

Another use of squared paper is to be found in connection with the right-angled triangle.

EXAMPLE. *The sides containing the right angle of a right-angled triangle, are 3·6 inches and 2·5 inches. Find the hypotenuse.*

The first thing to do is to fix on some convenient scale; say *let every 5 small squares represent an inch.*

Therefore, each small square = ·2 of an inch.

Now 3·6 inches on the scale will extend the length of three of these larger squares and three of the smaller squares. Therefore (as in Fig. 28) we set out the line AB to the above distance. In like manner, 2·5 inches will extend the length of two of the larger squares, and $2\frac{1}{2}$ of the smaller squares.

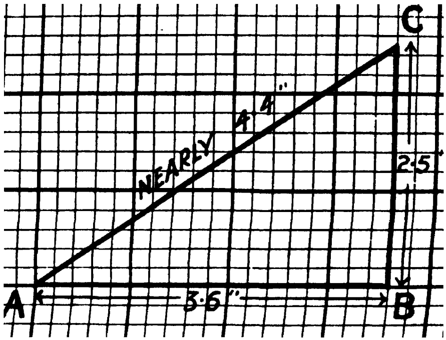
Therefore from B, we set out BC, equal to this latter distance, and at right angles to AB.

Join AC, and we get an **actual representation of our triangle to scale.**

We cannot judge the length of AC in its present position, as it lies crosswise over the squares. We, therefore, measure its length, by compasses, paper, etc., and then ascertain what this result is on one of the vertical or horizontal lines.

This will be found to be nearly 4·4 inches.

FIG. 28.



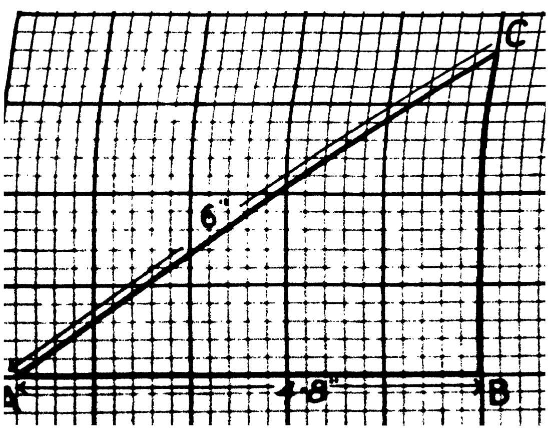


FIG. 29.

EXAMPLE. *The hypotenuse of a right-angled triangle = 6 inches, and the base = 4·8 inches. Find the height.*

By using the same scale set off (Fig. 29), the base AB to represent 4·8 ins. This AB will, of course, extend the length of 4 of the larger squares + 4 of the smaller ones. Now, by the aid of compasses, mark from A an arc having a radius equal to 6 of the larger squares.

From B draw BC at right angles to AB till it meets this arc. BC will represent the required perpendicular height, and will be found to be **3·6 inches.**

TRIANGLES

Our investigations of triangles are not, however, limited to right-angled triangles when dealing with squared paper, as the following examples will indicate.

EXAMPLE. *Two sides of a triangle equal 4 ft. and 3 ft. 6 ins. respectively. The angle between them is 60°. Find the length of the third side.*

Now 3 ft. 6 ins. = 3·5 ft., which we shall work by.

Using a scale in which 5 of the smaller squares (or one of the larger ones) = a foot, we commence (Fig. 30) by making a base AB to represent 4 feet.

At A we set off AD at an angle of 60° with AB.

Now from A and on the line AD we mark off by compasses a part AC, which represents 3·5 ft. = to $3\frac{1}{2}$ of the larger squares. We now join BC, which we find represents 2·8 ft. or **2 ft. 9·6 ins.**

EXAMPLE. *The equal sides of an isosceles triangle are each 4·5 inches and the angle at the apex = 30°. Find the length of the base.*

First set out two lines AX and AY containing an angle of 30°. From each of these, cut off portions AB and AC to represent 4·5 inches (Fig. 31). Join BC, which will be found to represent **2·35 inches.**

FIG. 30.

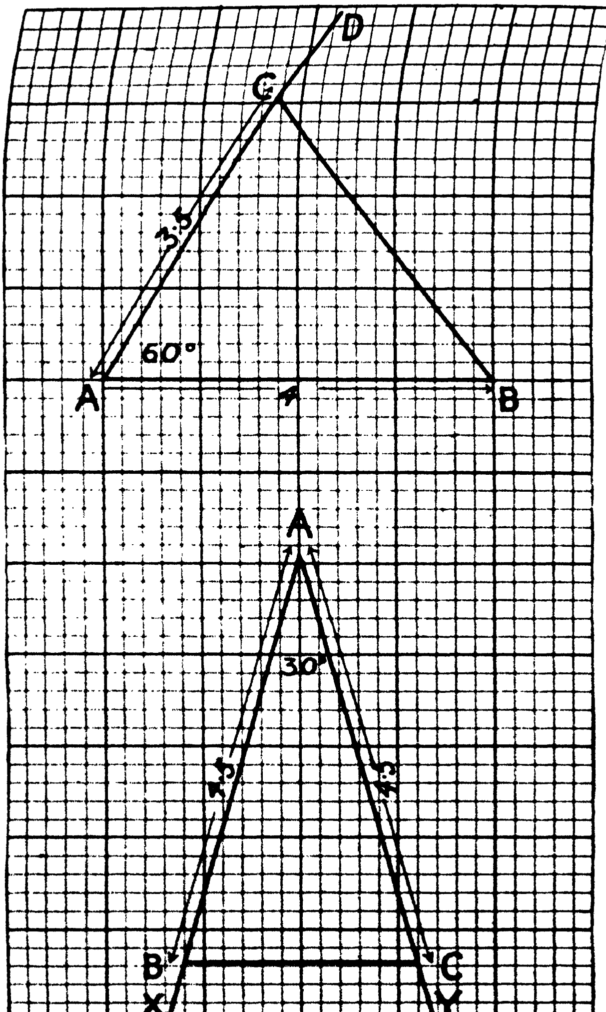


FIG. 31.

MULTIPLICATION AND DIVISION BY MEANS OF SQUARED PAPER

(*a*) Multiplication

EXAMPLE. *To multiply any number by 2.*

On a sheet of squared paper, mark out suitable scales, vertically and horizontally.

The most convenient ones for this example will be large scale ones, allowing 10 small squares to each unit (in each of the two directions), so that each small square = $\frac{1}{10}$ of a unit or ·1, and half the distance across a small square = ·05

NOTE. The two scales need not be of the same dimensions, and one may be double, treble, one half, etc. of the other.

To multiply 1 by 2, we note the *unit* 2 on the left hand vertical column, and draw a line horizontally to the right from it, till it meet P_1Q, a line drawn vertically from the unit 1 on the horizontal scale (Fig. 32).

Join OQ and produce it to Q_4. **This line OQ_4 can be used to multiply any number by 2.**

Thus, to multiply 4 by 2, we draw a vertical line P_4Q_3 from the **4** units on the horizontal scale, to the multiplying line OQ_4.

A horizontal line drawn from Q_3 to OX gives us 8 units as the product.

EXAMPLE. *To multiply 1·85 by 2.*

From the point P_2 (which is 1·85 units from O) draw a vertical line P_2Q_1 to OQ_4, and from Q_1 a horizontal line to OX. The answer is 3·7.

Note also, that 2·65 × 2 = 5·3 (Fig. 32).

EXAMPLE. *To multiply 1·3 by 2·7.*

On a conveniently scaled sheet of squared paper (Fig. 33) mark off from O, on the horizontal scale, OP_1 equal to 1 unit. On the vertical scale, find a distance equal to 2·7 units, and draw from it a horizontal line to meet a vertical one P_1Q_1 from the 1 unit mark on the horizontal scale. Join OQ_1, and produce it to Q. **Then OQ can be used to multiply any number by 2·7.**

From P_2 (which is 1·3 units from O) draw a vertical line

P_2Q_2 to OQ, and from Q_2 a horizontal line to OX. This point gives us **the product of 1·3 and 2·7 = 3·51.**

NOTE. Every new multiplier requires a new multiplying line.

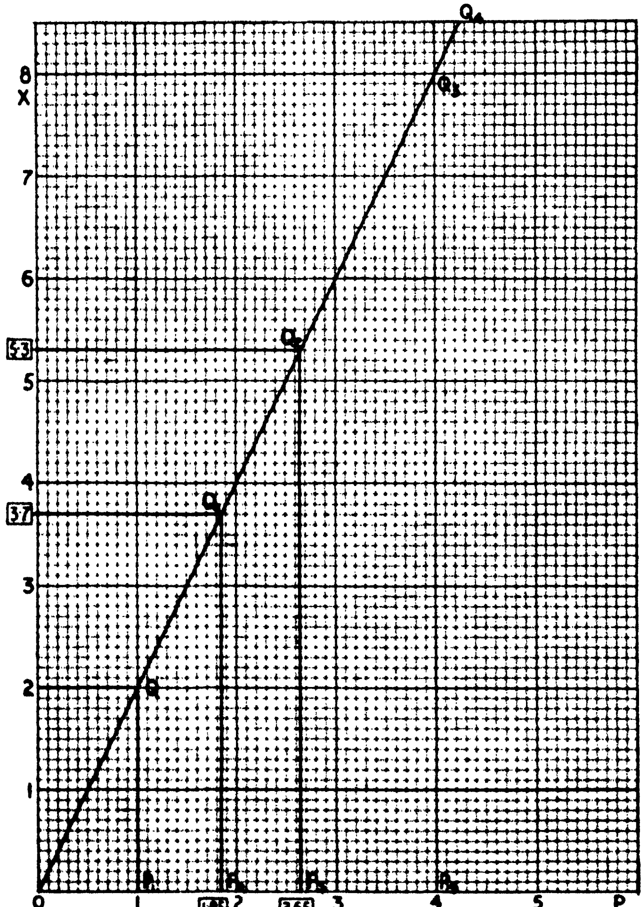


FIG. 32.

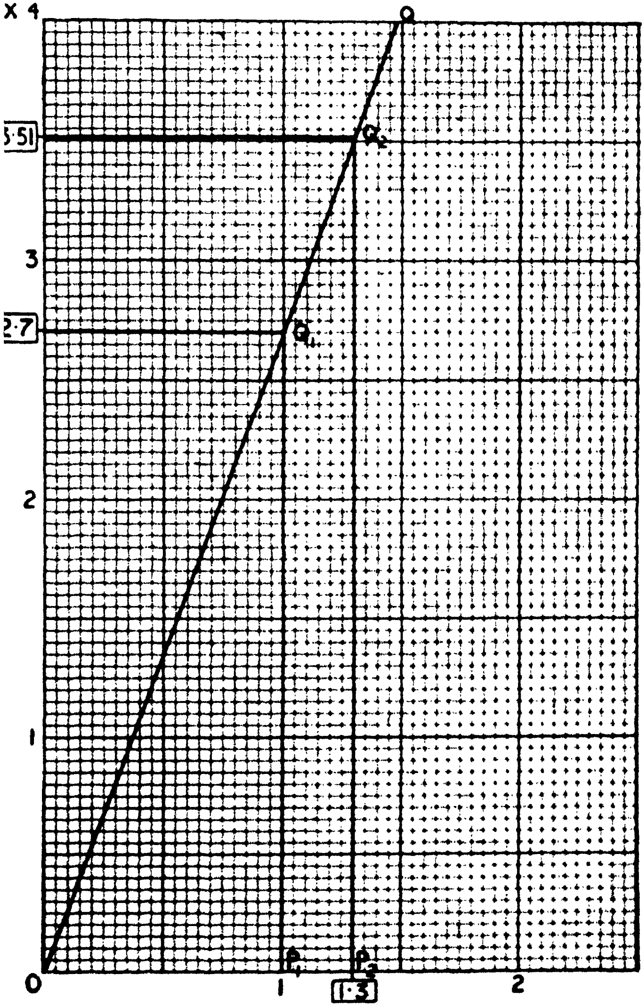


FIG. 33

(*b*) Division

Division is the reverse of multiplication, and in working examples in this rule, we must consequently reverse the process we adopted for multiplication on squared paper.

EXAMPLE. *Divide 3·25 by 2·5.*

Having marked out convenient scales on the horizontal and vertical axes, we first divide 2·5 by 2·5, by finding the point 2·5 on the vertical axis, drawing a horizontal line to a point vertically above the unit 1 (*i.e.* the quotient of 2·5 ÷ 2·5) on the horizontal axis, and dropping a vertical line on to OX. Through the point, where the two construction lines meet, draw **OQ, which can be used to divide any number by 2·5** (see Fig. 34).

Thus, to divide 3·25 by 2·5, we find the point representing 3·25 on the vertical axis, draw a horizontal line to OQ, and drop a vertical line on to OX. This will give us the required quotient, which is **1·3**.

TO PROVE THAT $\frac{3}{4}$ = ·75

The fraction $\frac{3}{4}$ means that 3 has been divided by 4, so we mark off (see Fig. 35) 4 units horizontally from O, and 3 units vertically from O. From the point Q', where the vertical line from number 4 and the horizontal line from number 3 meet, draw $Q'O$. From number 1 on the line OX draw a vertical line to Q on the line $Q'O$. A horizontal line from Q to OY gives us the required answer, **·75**.

GIVEN THE SINE TO FIND THE ANGLE

EXAMPLE. *The sine of an angle is $\frac{3}{7}$. Find the angle.*

Now, $$\text{sine} = \frac{\text{perpendicular}}{\text{hypotenuse}},$$

so we are required to construct a right-angled triangle whose perpendicular = 3 given units and hypotenuse = 7 given units.

First make BC = 3 given units, (Fig. 36.)

(3 larger squares),

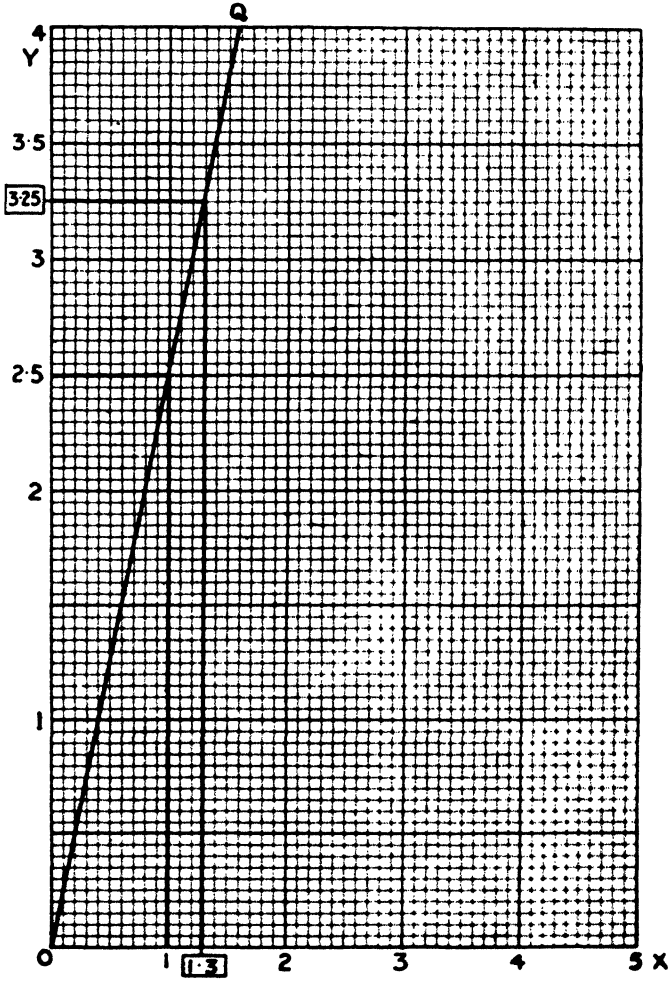


FIG. 34.

and from C describe an arc whose radius = 7 of same units. Make BA perpendicular to BC and meeting the arc at the point A. Join AC.

The angle CAB is the angle whose sine = $\frac{3}{7}$.

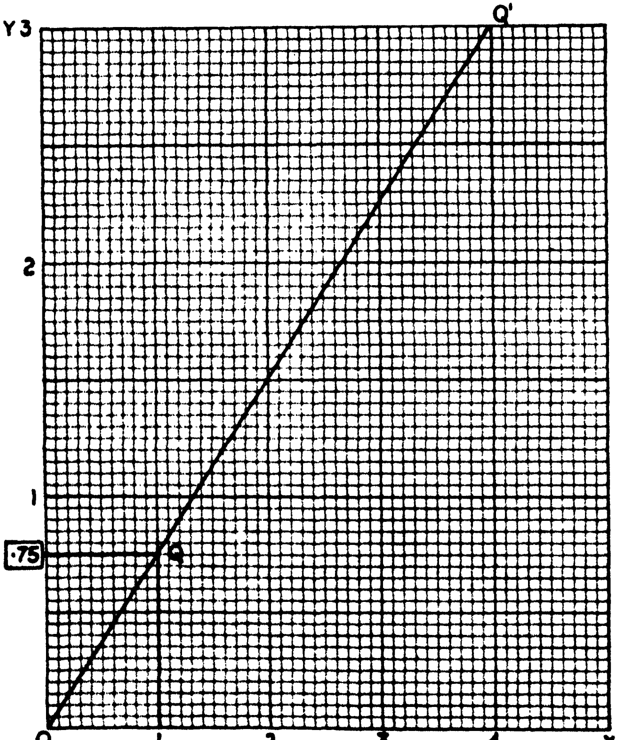


Fig. 35.

Example. *Find the angle whose sine is $\frac{2}{4}$ or $\frac{1}{2}$ or ·5.*

Proceeding as before, we make $BC = 2$ units (Fig. 37), and from C set out a line = to 4 units, which will meet the perpendicular BA from B at a point A.

The angle A is the one whose sine $= \frac{1}{2}$ or $\cdot 5$.

If we measure this angle with compasses or protractor, we shall find it equal to 30°, which is the angle given in the table as having a sine of ·5.

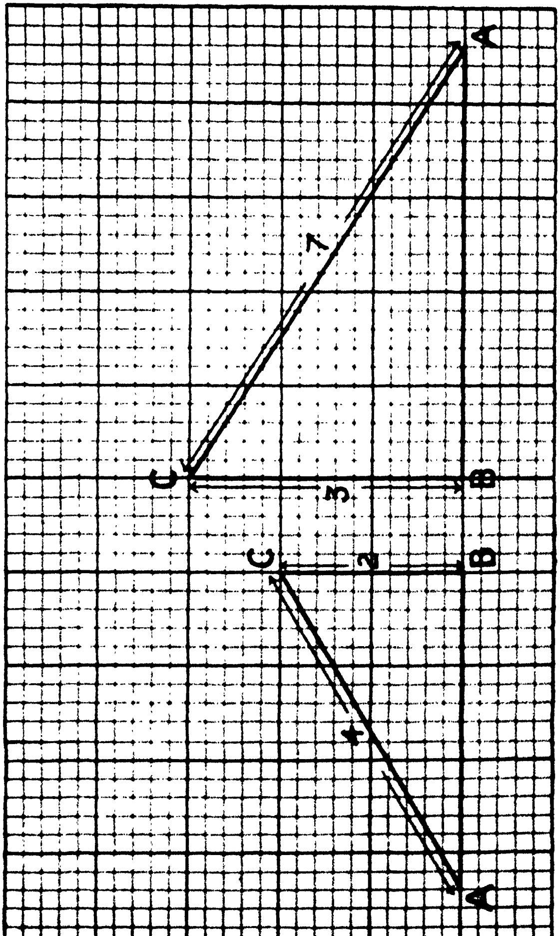


Fig. 36.

Fig. 37.

GIVEN THE COSINE, TO FIND THE ANGLE

EXAMPLE. *The cosine of an angle is $\frac{9}{10}$. Find the angle.*

$$\text{Cosine} = \frac{\text{base}}{\text{hypotenuse}}.$$

We are, therefore, required to construct a right-angled triangle, whose base = 9 units, and whose hypotenuse = 10 units.

Make $AB = 9$ units (see Fig. 38). At B, erect a perpendicular, and from A make an arc whose radius is equal to 10 of the same units. Join the point C (where the arc cuts the perpendicular) with A, and our triangle is completed;

∴ **A is the angle whose cosine $= \frac{9}{10}$.**

GIVEN THE TANGENT, TO FIND THE ANGLE

EXAMPLE. *The tangent of an angle is ·675. Find the angle.*

NOTE. $\cdot 675 = \frac{27}{40}$.

$$\text{Tangent} = \frac{\text{perpendicular}}{\text{base}}.$$

We are, therefore, required to construct a right-angled triangle whose base = 40 units, and whose perpendicular height = 27 units.

Mark off AC equal to 40 units (see Fig. 39), and BC (at right angles to AC) equal to 27 of the same units.

Join AB, and our triangle is completed;

∴ **A is the angle whose tangent $= \frac{27}{40}$ or ·675.**

SIMILAR TRIANGLES

Problems, similar to those given in the chapter on Trigonometry, in connection with similar triangles and the truths discovered by Thales, can be worked very effectively by the use of squared paper.

EXAMPLE. *A tower casts a shadow of 45 feet on the ground. At the same time, a 16 foot pole casts a shadow of 12 feet. Find, by squared paper, the height of the tower.*

First, mark out AB horizontally, to represent the shadow of the tower (45 feet), taking 45 small squares for the purpose

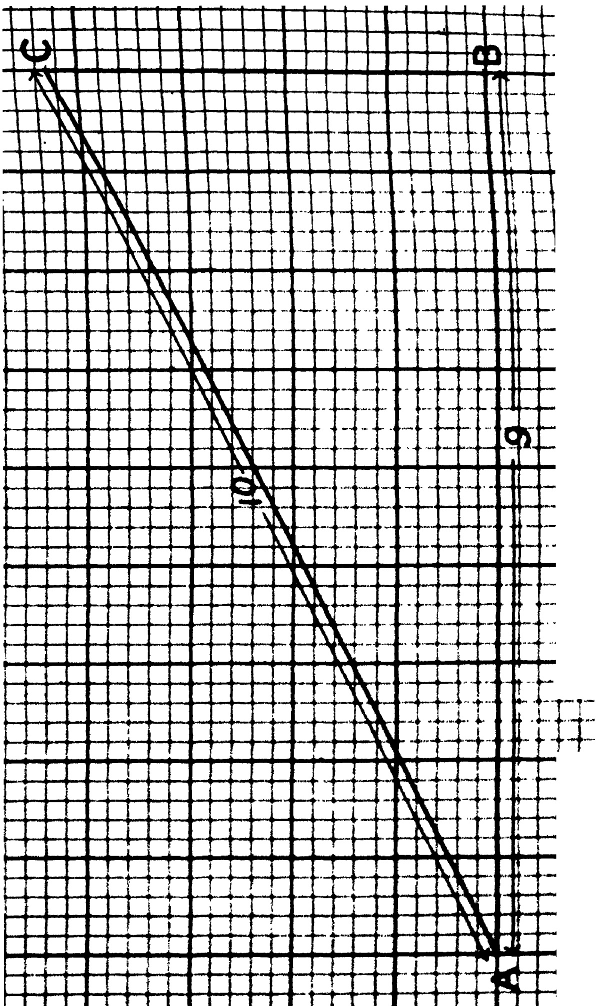


Fig. 38

(see Fig. 40). From *B*, mark off *BC* = 12 small squares, to represent the shadow of 12 feet cast by the pole. At *C*, erect *CD* at right angles to *AB*, and 16 small squares in length, to represent the 16 foot pole.

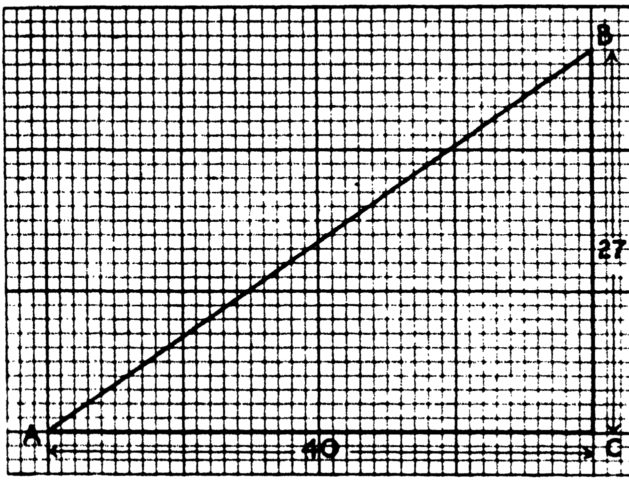


FIG. 39.

Join *BD*, and produce it till it meet a perpendicular line from *A* at *E*.

AE will represent the height of the tower, and will be found to be equal to 60 squares, which means that the tower is **60 feet high.**

EXAMPLE. *A man, 6 feet high, standing 15 feet from a lamp-post, observes that his shadow, cast by the light, is 5 feet long.*

(*a*) *Find how high is the light ;* (*b*) *how long the man's shadow will be if he move 8 feet nearer to the post ; and* (*c*) *the distance from A to H.*

(*a*) On a sheet of squared paper, mark off *AB* (Fig. 41) to represent 15 feet, say 45 small squares, the distance of the man from the post.

At A, erect the vertical line AD, to represent 6 feet (18 small squares), the height of the man.

Produce BA to C, making AC equal to 15 small squares, to represent the length of the man's shadow (5 feet).

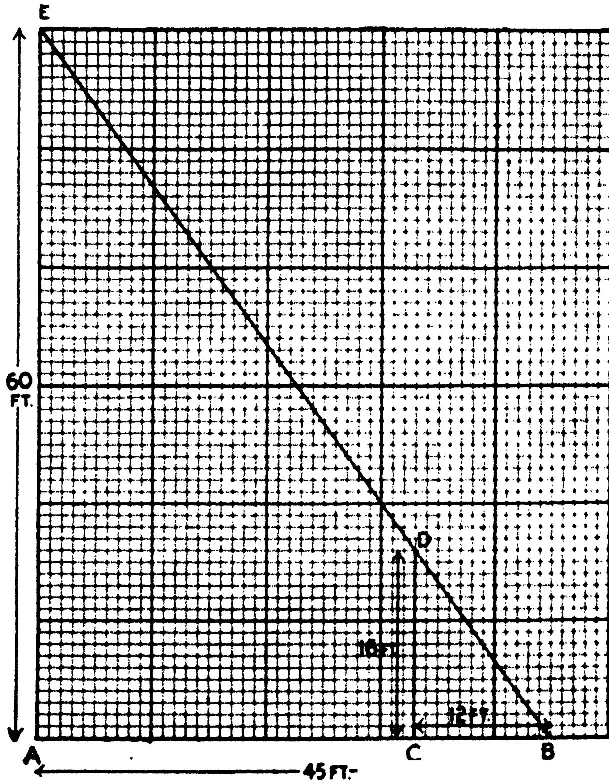


FIG. 40.

Join CD, and produce it to meet a vertical line from B, at E.

BE represents the height of the light, which will be found to be 72 small squares, representing 24 feet.

(*b*) From A mark off AF (in the direction of B) equal to 24 small squares, representing 8 feet.

At *F* erect the vertical line *FG*, equal to 18 small squares, representing the 6-foot man.

Join *EG*, and produce to *H*.

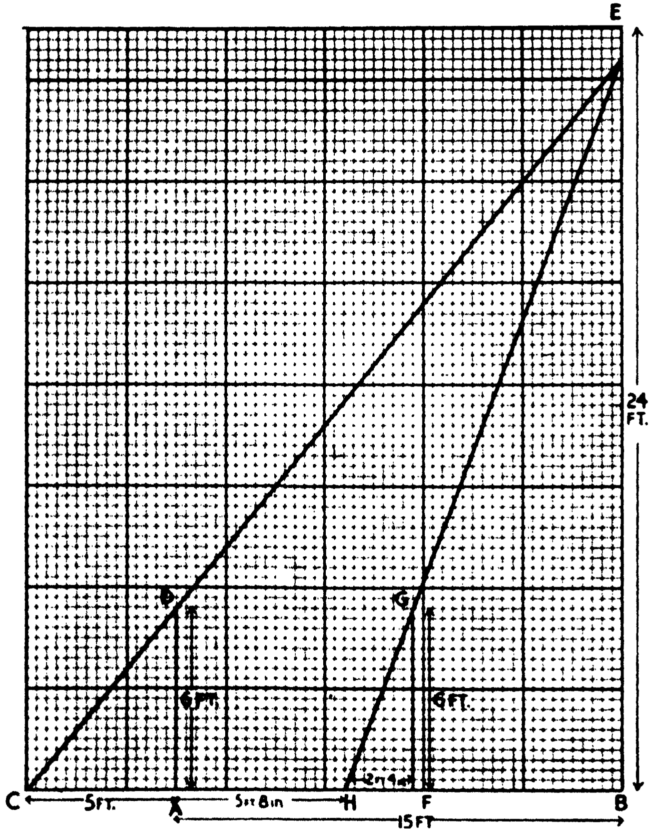


FIG. 41.

FH represents the length of the man's shadow, which will be found to be 7 small squares, representing **2 feet 4 inches.**

(*c*) The distance from *A* to *H* will be found to be equal to 17 small squares, representing **5 feet 8 inches.**

EXAMPLES IN GRAPHS AND SQUARED PAPER

1. Make a graph of the attendances of the scholars in your class for a week of 10 attendances.

2. Make also a graph (on the same sheet of squared paper) showing the number absent each ½ day of the same week.

3. Construct a graph showing the temperature of your classroom for a week, taking the temperature at 9 a.m., 12 noon, and 4 p.m. each day. (Note there will be 15 entries for the five school days.)

4. Compile a graph showing the height of the barometer each day for a period of a fortnight, or, if possible, a month.

5. Construct a graph showing the number of hours of sunshine in a selected watering-place (taken from a daily newspaper) for a month. Plot the exact hours, *e.g.* 8·6, 10·2 hours of sunshine.

6. Make a graph showing twice times table.

7. Make a graph showing three times table.

8. Make a graph showing four times table.

9. Continue these graphs up to, and including, twelve times table.

10. Make a graph showing 2·5 × 3·7.

11. Make a graph showing 18·5 ÷ 3·7.

12. Show, by means of a graph, that $\frac{1}{4}$ = ·25.

Construct right-angled triangles by means of squared paper, having given:

13. That $\sin A$ = [illegible].
14. $\sin A$ = [illegible].
15. $\sin A$ = [illegible].
16. $\cos A = \frac{7}{25}$.
17. $\cos A$ = [illegible].
18. $\cos A$ = [illegible].
19. $\tan A$ = [illegible].
20. $\tan A = \frac{24}{7}$.
21. $\tan A = \frac{12}{5}$.

In each case, find the size of the angle A, by means of your protractor, *to the nearest degree*.

22. The lengths of two sides of a triangle are 3·8 and 4·6 inches respectively, and the angle between them is 35°. Find, by means of squared paper, the length of the third side (to the nearest first decimal).

23. Do Question 28, in the trigonometry examples, on squared paper, and find the height of the tower.

24. Do Question 29, in the trigonometry examples, on squared paper, and find the height of the house.

25. Do Question 30, in the trigonometry examples, by means of squared paper, and find the height of the tree.

26. Do Question 31, in the trigonometry examples, by means of squared paper, and find the height of the steeple.

27. A ship sails 6 miles west, then 8 miles south. Find, by squared paper, how many miles, *in a straight line*, it is from the starting-point.

28. A ship sails 12 miles east, 10 miles south, and finally, another 12 miles to the east. Find, by means of squared paper, how many miles, *in a straight line*, it is from the starting-point.

29. A cyclist journeys 6 miles to the east, and then 4 miles to the north-east. How far is he, *in a straight line*, from the start (nearest first decimal place).

30. A ship leaves Lowestoft and sails 5 miles E., then 8 miles to the N.E., then 10 miles to the S.E., and finally, 4 miles to the S., where she anchors. Ascertain, by means of a graph, the course of the ship, and the distance that her anchorage is from Lowestoft, *in a straight line*.

31. Two ships start from the port A. One sails 70 miles to the north-east, and the second one 80 miles to the east. (*a*) How far are they apart, and (*b*) what is the bearing of C from B (show graphically)?

32. A cyclist leaves the town B and travels 6 miles to the north-west, whilst a second cyclist leaves B and travels 5 miles due west. How far are they apart, and what are the "bearings" of the first cyclist with respect to the second one?

33 to 46. Do the examples illustrated by Figs. 28 to 41 without reference to the solutions in the text.

TABLE OF SINES, Etc.

ngle.	Radians.	Sine.	Tangent.	Co-tangent.	Cosine.		
0°	0	0	0	∞	1	1·5708	90°
1	·0175	·0175	·0175	57·2900	·9998	1·5533	89
2	·0349	·0349	·0349	28·6363	·9994	1·5359	88
3	·0524	·0523	·0524	19·0811	·9986	1·5184	87
4	·0698	·0698	·0699	14·3006	·9976	1·5010	86
5	·0873	·0872	·0875	11·4301	·9962	1·4835	85
6	·1047	·1045	·1051	9·5144	·9945	1·4661	84
7	·1222	·1219	·1228	8·1443	·9925	1·4486	83
8	·1396	·1392	·1405	7·1154	·9903	1·4312	82
9	·1571	·1564	·1584	6·3138	·9877	1·4137	81
10	·1745	·1736	·1763	5·6713	·9848	1·3963	80
11	·1920	·1908	·1944	5·1446	·9816	1·3788	79
12	·2094	·2079	·2126	4·7046	·9781	1·3614	78
13	·2269	·2250	·2309	4·3315	·9744	1·3439	77
14	·2443	·2419	·2493	4·0108	·9703	1·3265	76
15	·2618	·2588	·2679	3·7321	·9659	1·3090	75
16	·2793	·2756	·2867	3·4874	·9613	1·2915	74
17	·2967	·2924	·3057	3·2709	·9563	1·2741	73
18	·3142	·3090	·3249	3·0777	·9511	1·2566	72
19	·3316	·3256	·3443	2·9042	·9455	1·2392	71
20	·3491	·3420	·3640	2·7475	·9397	1·2217	70
21	·3665	·3584	·3839	2·6051	·9336	1·2043	69
22	·3840	·3746	·4040	2·4751	·9272	1·1868	68
23	·4014	·3907	·4245	2·3559	·9205	1·1694	67
24	·4189	·4067	·4452	2·2460	·9135	1·1519	66
25	·4363	·4226	·4663	2·1445	·9063	1·1345	65
26	·4538	·4384	·4877	2·0503	·8988	1·1170	64
27	·4712	·4540	·5095	1·9626	·8910	1·0996	63
28	·4887	·4695	·5317	1·8807	·8830	1·0821	62
29	·5061	·4848	·5543	1·8040	·8746	1·0647	61
30	·5236	·5000	·5774	1·7321	·8660	1·0472	60
31	·5411	·5150	·6009	1·6643	·8572	1·0297	59
32	·5585	·5299	·6249	1·6003	·8480	1·0123	58
33	·5760	·5446	·6494	1·5399	·8387	·9948	57
34	·5934	·5592	·6745	1·4826	·8290	·9774	56
35	·6109	·5736	·7002	1·4281	·8192	·9599	55
36	·6283	·5878	·7265	1·3764	·8090	·9425	54
37	·6458	·6018	·7536	1·3270	·7986	·9250	53
38	·6632	·6157	·7813	1·2799	·7880	·9076	52
39	·6807	·6293	·8098	1·2349	·7771	·8901	51
40	·6981	·6428	·8391	1·1918	·7660	·8727	50
41	·7156	·6561	·8693	1·1504	·7547	·8552	49
42	·7330	·6691	·9004	1·1106	·7431	·8378	48
43	·7505	·6820	·9325	1·0724	·7314	·8203	47
44	·7679	·6947	·9657	1·0355	·7193	·8029	46
45	·7854	·7071	1·0000	1·0000	·7071	·7854	45
			Co-tangent	Tangent	Sine	Radians	Angle

LOGARITHMS

	0	1	2	3	4	5	6	7	8	9	Mean Differences. 1	2	3	4	5	6	7	8	9
10	0000	0043	0086	0128	0170	0212	0253	0294	0334	0374	4	8	12	17	21	25	29	33	37
11	0414	0453	0492	0531	0569	0607	0645	0682	0719	0755	4	8	11	15	19	23	26	30	34
12	0792	0828	0864	0899	0934	0969	1004	1038	1072	1106	3	7	10	14	17	21	24	28	31
13	1139	1173	1206	1239	1271	1303	1335	1367	1399	1430	3	6	10	13	16	19	23	26	29
14	1461	1492	1523	1553	1584	1614	1644	1673	1703	1732	3	6	9	12	15	18	21	24	27
15	1761	1790	1818	1847	1875	1903	1931	1959	1987	2014	3	6	8	11	14	17	20	22	25
16	2041	2068	2095	2122	2148	2175	2201	2227	2253	2279	3	5	8	11	13	16	18	21	24
17	2304	2330	2355	2380	2405	2430	2455	2480	2504	2529	2	5	7	10	12	15	17	20	22
18	2553	2577	2601	2625	2648	2672	2695	2718	2742	2765	2	5	7	9	12	14	16	19	21
19	2788	2810	2833	2856	2878	2900	2923	2945	2967	2989	2	4	7	9	11	13	16	18	20
20	3010	3032	3054	3075	3096	3118	3139	3160	3181	3201	2	4	6	8	11	13	15	17	19
21	3222	3243	3263	3284	3304	3324	3345	3365	3385	3404	2	4	6	8	10	12	14	16	18
22	3424	3444	3464	3483	3502	3522	3541	3560	3579	3598	2	4	6	8	10	12	14	15	17
23	3617	3636	3655	3674	3692	3711	3729	3747	3766	3784	2	4	6	7	9	11	13	15	17
24	3802	3820	3838	3856	3874	3892	3909	3927	3945	3962	2	4	5	7	9	11	12	14	16
25	3979	3997	4014	4031	4048	4065	4082	4099	4116	4133	2	3	5	7	9	10	12	14	15
26	4150	4166	4183	4200	4216	4232	4249	4265	4281	4298	2	3	5	7	8	10	11	13	15
27	4314	4330	4346	4362	4378	4393	4409	4425	4440	4456	2	3	5	6	8	9	11	13	14
28	4472	4487	4502	4518	4533	4548	4564	4579	4594	4609	2	3	5	6	8	9	11	12	14
29	4624	4639	4654	4669	4683	4698	4713	4728	4742	4757	1	3	4	6	7	9	10	12	13
30	4771	4786	4800	4814	4829	4843	4857	4871	4886	4900	1	3	4	6	7	9	10	11	13
31	4914	4928	4942	4955	4969	4983	4997	5011	5024	5038	1	3	4	6	7	8	10	11	12
32	5051	5065	5079	5092	5105	5119	5132	5145	5159	5172	1	3	4	5	7	8	9	11	12
33	5185	5198	5211	5224	5237	5250	5263	5276	5289	5302	1	3	4	5	6	8	9	10	12
34	5315	5328	5340	5353	5366	5378	5391	5403	5416	5428	1	3	4	5	6	8	9	10	11
35	5441	5453	5465	5478	5490	5502	5514	5527	5539	5551	1	2	4	5	6	7	9	10	11
36	5563	5575	5587	5599	5611	5623	5635	5647	5658	5670	1	2	4	5	6	7	8	10	11
37	5682	5694	5705	5717	5729	5740	5752	5763	5775	5786	1	2	3	5	6	7	8	9	10
38	5798	5809	5821	5832	5843	5855	5866	5877	5888	5899	1	2	3	5	6	7	8	9	10
39	5911	5922	5933	5944	5955	5966	5977	5988	5999	6010	1	2	3	4	5	7	8	9	10
40	6021	6031	6042	6053	6064	6075	6085	6096	6107	6117	1	2	3	4	5	6	8	9	10
41	6128	6138	6149	6160	6170	6180	6191	6201	6212	6222	1	2	3	4	5	6	7	8	9
42	6232	6243	6253	6263	6274	6284	6294	6304	6314	6325	1	2	3	4	5	6	7	8	9
43	6335	6345	6355	6365	6375	6385	6395	6405	6415	6425	1	2	3	4	5	6	7	8	9
44	6435	6444	6454	6464	6474	6484	6493	6503	6513	6522	1	2	3	4	5	6	7	8	9
45	6532	6542	6551	6561	6571	6580	6590	6599	6609	6618	1	2	3	4	5	6	7	8	9
46	6628	6637	6646	6656	6665	6675	6684	6693	6702	6712	1	2	3	4	5	6	7	7	8
47	6721	6730	6739	6749	6758	6767	6776	6785	6794	6803	1	2	3	4	5	5	6	7	8
48	6812	6821	6830	6839	6848	6857	6866	6875	6884	6893	1	2	3	4	4	5	6	7	8
49	6902	6911	6920	6928	6937	6946	6955	6964	6972	6981	1	2	3	4	4	5	6	7	8
50	6990	6998	7007	7016	7024	7033	7042	7050	7059	7067	1	2	3	3	4	5	6	7	8
51	7076	7084	7093	7101	7110	7118	7126	7135	7143	7152	1	2	3	3	4	5	6	7	8
52	7160	7168	7177	7185	7193	7202	7210	7218	7226	7235	1	2	2	3	4	5	6	7	7
53	7243	7251	7259	7267	7275	7284	7292	7300	7308	7316	1	2	2	3	4	5	6	6	7
54	7324	7332	7340	7348	7356	7364	7372	7380	7388	7396	1	2	2	3	4	5	6	6	7

LOGARITHMS

	0	1	2	3	4	5	6	7	8	9	Mean Differences. 1	2	3	4	5	6	7	8	9
55	7404	7412	7419	7427	7435	7443	7451	7459	7466	7474	1	2	2	3	4	5	5	6	7
56	7482	7490	7497	7505	7513	7520	7528	7536	7543	7551	1	2	2	3	4	5	5	6	7
57	7559	7566	7574	7582	7589	7597	7604	7612	7619	7627	1	2	2	3	4	5	5	6	7
58	7634	7642	7649	7657	7664	7672	7679	7686	7694	7701	1	1	2	3	4	4	5	6	7
59	7709	7716	7723	7731	7738	7745	7752	7760	7767	7774	1	1	2	3	4	4	5	6	7
60	7782	7789	7796	7803	7810	7818	7825	7832	7839	7846	1	1	2	3	4	4	5	6	6
61	7853	7860	7868	7875	7882	7889	7896	7903	7910	7917	1	1	2	3	4	4	5	6	6
62	7924	7931	7938	7945	7952	7959	7966	7973	7980	7987	1	1	2	3	3	4	5	6	6
63	7993	8000	8007	8014	8021	8028	8035	8041	8048	8055	1	1	2	3	3	4	5	5	6
64	8062	8069	8075	8082	8089	8096	8102	8109	8116	8122	1	1	2	3	3	4	5	5	6
65	8129	8136	8142	8149	8156	8162	8169	8176	8182	8189	1	1	2	3	3	4	5	5	6
66	8195	8202	8209	8215	8222	8228	8235	8241	8248	8254	1	1	2	3	3	4	5	5	6
67	8261	8267	8274	8280	8287	8293	8299	8306	8312	8319	1	1	2	3	3	4	5	5	6
68	8325	8331	8338	8344	8351	8357	8363	8370	8376	8382	1	1	2	3	3	4	4	5	6
69	8388	8395	8401	8407	8414	8420	8426	8432	8439	8445	1	1	2	2	3	4	4	5	6
70	8451	8457	8463	8470	8476	8482	8488	8494	8500	8506	1	1	2	2	3	4	4	5	6
71	8513	8519	8525	8531	8537	8543	8549	8555	8561	8567	1	1	2	2	3	4	4	5	5
72	8573	8579	8585	8591	8597	8603	8609	8615	8621	8627	1	1	2	2	3	4	4	5	5
73	8633	8639	8645	8651	8657	8663	8669	8675	8681	8686	1	1	2	2	3	4	4	5	5
74	8692	8698	8704	8710	8716	8722	8727	8733	8739	8745	1	1	2	2	3	4	4	5	5
75	8751	8756	8762	8768	8774	8779	8785	8791	8797	8802	1	1	2	2	3	3	4	5	5
76	8808	8814	8820	8825	8831	8837	8842	8848	8854	8859	1	1	2	2	3	3	4	5	5
77	8865	8871	8876	8882	8887	8893	8899	8904	8910	8915	1	1	2	2	3	3	4	4	5
78	8921	8927	8932	8938	8943	8949	8954	8960	8965	8971	1	1	2	2	3	3	4	5	5
79	8976	8982	8987	8993	8998	9004	9009	9015	9020	9025	1	1	2	2	3	3	4	4	5
80	9031	9036	9042	9047	9053	9058	9063	9069	9074	9079	1	1	2	2	3	3	4	4	5
81	9085	9090	9096	9101	9106	9112	9117	9122	9128	9133	1	1	2	2	3	3	4	4	5
82	9138	9143	9149	9154	9159	9165	9170	9175	9180	9186	1	1	2	2	3	3	4	4	5
83	9191	9196	9201	9206	9212	9217	9222	9227	9232	9238	1	1	2	2	3	3	4	4	5
84	9243	9248	9253	9258	9263	9269	9274	9279	9284	9289	1	1	2	2	3	3	4	4	5
85	9294	9299	9304	9309	9315	9320	9325	9330	9335	9340	1	1	2	2	3	3	4	4	5
86	9345	9350	9355	9360	9365	9370	9375	9380	9385	9390	1	1	2	2	3	3	4	4	5
87	9395	9400	9405	9410	9415	9420	9425	9430	9435	9440	0	1	1	2	2	3	3	4	4
88	9445	9450	9455	9460	9465	9469	9474	9479	9484	9489	0	1	1	2	2	3	3	4	4
89	9494	9499	9504	9509	9513	9518	9523	9528	9533	9538	0	1	1	2	2	3	3	4	4
90	9542	9547	9552	9557	9562	9566	9571	9576	9581	9586	0	1	1	2	2	3	3	4	4
91	9590	9595	9600	9605	9609	9614	9619	9624	9628	9633	0	1	1	2	2	3	3	4	4
92	9638	9643	9647	9652	9657	9661	9666	9671	9675	9680	0	1	1	2	2	3	3	4	4
93	9685	9689	9694	9699	9703	9708	9713	9717	9722	9727	0	1	1	2	2	3	3	4	4
94	9731	9736	9741	9745	9750	9754	9759	9763	9768	9773	0	1	1	2	2	3	3	4	4
95	9777	9782	9786	9791	9795	9800	9805	9809	9814	9818	0	1	1	2	2	3	3	4	4
96	9823	9827	9832	9836	9841	9845	9850	9854	9859	9863	0	1	1	2	2	3	3	4	4
97	9868	9872	9877	9881	9886	9890	9894	9899	9903	9908	0	1	1	2	2	3	3	4	4
98	9912	9917	9921	9926	9930	9934	9939	9943	9948	9952	0	1	1	2	2	3	3	4	4
99	9956	9961	9965	9969	9974	9978	9983	9987	9991	9996	0	1	1	2	2	3	3	3	4

ANTILOGARITHMS

	0	1	2	3	4	5	6	7	8	9	Mean Differences. 1	2	3	4	5	6	7	8	9
·00	1000	1002	1005	1007	1009	1012	1014	1016	1019	1021	0	0	1	1	1	1	2	2	2
·01	1023	1026	1028	1030	1033	1035	1038	1040	1042	1045	0	0	1	1	1	1	2	2	2
·02	1047	1050	1052	1054	1057	1059	1062	1064	1067	1069	0	0	1	1	1	1	2	2	2
·03	1072	1074	1076	1079	1081	1084	1086	1089	1091	1094	0	0	1	1	1	1	2	2	2
·04	1096	1099	1102	1104	1107	1109	1112	1114	1117	1119	0	1	1	1	1	2	2	2	2
·05	1122	1125	1127	1130	1132	1135	1138	1140	1143	1146	0	1	1	1	1	2	2	2	2
·06	1148	1151	1153	1156	1159	1161	1164	1167	1169	1172	0	1	1	1	1	2	2	2	2
·07	1175	1178	1180	1183	1186	1189	1191	1194	1197	1199	0	1	1	1	1	2	2	2	2
·08	1202	1205	1208	1211	1213	1216	1219	1222	1225	1227	0	1	1	1	1	2	2	2	3
·09	1230	1233	1236	1239	1242	1245	1247	1250	1253	1256	0	1	1	1	1	2	2	2	3
·10	1259	1262	1265	1268	1271	1274	1276	1279	1282	1285	0	1	1	1	1	2	2	2	3
·11	1288	1291	1294	1297	1300	1303	1306	1309	1312	1315	0	1	1	1	2	2	2	2	3
·12	1318	1321	1324	1327	1330	1334	1337	1340	1343	1346	0	1	1	1	2	2	2	2	3
·13	1349	1352	1355	1358	1361	1365	1368	1371	1374	1377	0	1	1	1	2	2	2	3	3
·14	1380	1384	1387	1390	1393	1396	1400	1403	1406	1409	0	1	1	1	2	2	2	3	3
·15	1413	1416	1419	1422	1426	1429	1432	1435	1439	1442	0	1	1	1	2	2	2	3	3
·16	1445	1449	1452	1455	1459	1462	1466	1469	1472	1476	0	1	1	1	2	2	2	3	3
·17	1479	1483	1486	1489	1493	1496	1500	1503	1507	1510	0	1	1	1	2	2	2	3	3
·18	1514	1517	1521	1524	1528	1531	1535	1538	1542	1545	0	1	1	1	2	2	2	3	3
·19	1549	1552	1556	1560	1563	1567	1570	1574	1578	1581	0	1	1	1	2	2	3	3	3
·20	1585	1589	1592	1596	1600	1603	1607	1611	1614	1618	0	1	1	1	2	2	3	3	3
·21	1622	1626	1629	1633	1637	1641	1644	1648	1652	1656	0	1	1	2	2	2	3	3	3
·22	1660	1663	1667	1671	1675	1679	1683	1687	1690	1694	0	1	1	2	2	2	3	3	3
·23	1698	1702	1706	1710	1714	1718	1722	1726	1730	1734	0	1	1	2	2	2	3	3	4
·24	1738	1742	1746	1750	1754	1758	1762	1766	1770	1774	0	1	1	2	2	2	3	3	4
·25	1778	1782	1786	1791	1795	1799	1803	1807	1811	1816	0	1	1	2	2	2	3	3	4
·26	1820	1824	1828	1832	1837	1841	1845	1849	1854	1858	0	1	1	2	2	3	3	3	4
·27	1862	1866	1871	1875	1879	1884	1888	1892	1897	1901	0	1	1	2	2	3	3	3	4
·28	1905	1910	1914	1919	1923	1928	1932	1936	1941	1945	0	1	1	2	2	3	3	4	4
·29	1950	1954	1959	1963	1968	1972	1977	1982	1986	1991	0	1	1	2	2	3	3	4	4
·30	1995	2000	2004	2009	2014	2018	2023	2028	2032	2037	0	1	1	2	2	3	3	4	4
·31	2042	2046	2051	2056	2061	2065	2070	2075	2080	2084	0	1	1	2	2	3	3	4	4
·32	2089	2094	2099	2104	2109	2113	2118	2123	2128	2133	0	1	1	2	2	3	3	4	4
·33	2138	2143	2148	2153	2158	2163	2168	2173	2178	2183	0	1	1	2	2	3	3	4	4
·34	2188	2193	2198	2203	2208	2213	2218	2223	2228	2234	1	1	2	2	3	3	4	4	5
·35	2239	2244	2249	2254	2259	2265	2270	2275	2280	2286	1	1	2	2	3	3	4	4	5
·36	2291	2296	2301	2307	2312	2317	2323	2328	2333	2339	1	1	2	2	3	3	4	4	5
·37	2344	2350	2355	2360	2366	2371	2377	2382	2388	2393	1	1	2	2	3	3	4	4	5
·38	2399	2404	2410	2415	2421	2427	2432	2438	2443	2449	1	1	2	2	3	3	4	4	5
·39	2455	2460	2466	2472	2477	2483	2489	2495	2500	2506	1	1	2	2	3	3	4	5	5
·40	2512	2518	2523	2529	2535	2541	2547	2553	2559	2564	1	1	2	2	3	4	4	5	5
·41	2570	2576	2582	2588	2594	2600	2606	2612	2618	2624	1	1	2	2	3	4	4	5	5
·42	2630	2636	2642	2649	2655	2661	2667	2673	2679	2685	1	1	2	2	3	4	4	5	6
·43	2692	2698	2704	2710	2716	2723	2729	2735	2742	2748	1	1	2	3	3	4	4	5	6
·44	2754	2761	2767	2773	2780	2786	2793	2799	2805	2812	1	1	2	3	3	4	4	5	6
·45	2818	2825	2831	2838	2844	2851	2858	2864	2871	2877	1	1	2	3	3	4	5	5	6
·46	2884	2891	2897	2904	2911	2917	2924	2931	2938	2944	1	1	2	3	3	4	5	5	6
·47	2951	2958	2965	2972	2979	2985	2992	2999	3006	3013	1	1	2	3	3	4	5	5	6
·48	3020	3027	3034	3041	3048	3055	3062	3069	3076	3083	1	1	2	3	4	4	5	6	6
·49	3090	3097	3105	3112	3119	3126	3133	3141	3148	3155	1	1	2	3	4	4	5	6	6

ANTILOGARITHMS

	0	1	2	3	4	5	6	7	8	9	Mean Differences 1	2	3	4	5	6	7	8	9
·50	3162	3170	3177	3184	3192	3199	3206	3214	3221	3228	1	1	2	3	4	4	5	6	7
·51	3236	3243	3251	3258	3266	3273	3281	3289	3296	3304	1	2	2	3	4	5	5	6	7
·52	3311	3319	3327	3334	3342	3350	3357	3365	3373	3381	1	2	2	3	4	5	5	6	7
·53	3388	3396	3404	3412	3420	3428	3436	3443	3451	3459	1	2	2	3	4	5	6	6	7
·54	3467	3475	3483	3491	3499	3508	3516	3524	3532	3540	1	2	2	3	4	5	6	6	7
·55	3548	3556	3565	3573	3581	3589	3597	3606	3614	3622	1	2	2	3	4	5	6	7	7
·56	3631	3639	3648	3656	3664	3673	3681	3690	3698	3707	1	2	3	3	4	5	6	7	8
·57	3715	3724	3733	3741	3750	3758	3767	3776	3784	3793	1	2	3	3	4	5	6	7	8
·58	3802	3811	3819	3828	3837	3846	3855	3864	3873	3882	1	2	3	4	4	5	6	7	8
·59	3890	3899	3908	3917	3926	3936	3945	3954	3963	3972	1	2	3	4	5	5	6	7	8
·60	3981	3990	3999	4009	4018	4027	4036	4046	4055	4064	1	2	3	4	5	6	6	7	8
·61	4074	4083	4093	4102	4111	4121	4130	4140	4150	4159	1	2	3	4	5	6	7	8	9
·62	4169	4178	4188	4198	4207	4217	4227	4236	4246	4256	1	2	3	4	5	6	7	8	9
·63	4266	4276	4285	4295	4305	4315	4325	4335	4345	4355	1	2	3	4	5	6	7	8	9
·64	4365	4375	4385	4395	4406	4416	4426	4436	4446	4457	1	2	3	4	5	7	7	8	9
·65	4467	4477	4487	4498	4508	4519	4529	4539	4550	4560	1	2	3	4	5	6	7	8	9
·66	4571	4581	4592	4603	4613	4624	4634	4645	4656	4667	1	2	3	4	5	6	7	9	10
·67	4677	4688	4699	4710	4721	4732	4742	4753	4764	4775	1	2	3	4	5	7	8	9	10
·68	4786	4797	4808	4819	4831	4842	4853	4864	4875	4887	1	2	3	4	6	7	8	9	10
·69	4898	4909	4920	4932	4943	4955	4966	4977	4989	5000	1	2	3	5	6	7	8	9	10
·70	5012	5023	5035	5047	5058	5070	5082	5093	5105	5117	1	2	4	5	6	7	8	9	11
·71	5129	5140	5152	5164	5176	5188	5200	5212	5224	5236	1	2	4	5	6	7	8	10	11
·72	5248	5260	5272	5284	5297	5309	5321	5333	5346	5358	1	2	4	5	6	7	9	10	11
·73	5370	5383	5395	5408	5420	5433	5445	5458	5470	5483	1	3	4	5	6	8	9	10	11
·74	5495	5508	5521	5534	5546	5559	5572	5585	5598	5610	1	3	4	5	6	8	9	10	12
·75	5623	5636	5649	5662	5675	5689	5702	5715	5728	5741	1	3	4	5	7	8	9	10	12
·76	5754	5768	5781	5794	5808	5821	5834	5848	5861	5875	1	3	4	5	7	8	9	11	12
·77	5888	5902	5916	5929	5943	5957	5970	5984	5998	6012	1	3	4	5	7	8	10	11	12
·78	6026	6039	6053	6067	6081	6095	6109	6124	6138	6152	1	3	4	6	7	8	10	11	13
·79	6166	6180	6194	6209	6223	6237	6252	6266	6281	6295	1	3	4	6	7	9	10	11	13
·80	6310	6324	6339	6353	6368	6383	6397	6412	6427	6442	1	3	4	6	7	9	10	12	13
·81	6457	6471	6486	6501	6516	6531	6546	6561	6577	6592	2	3	5	6	8	9	11	12	14
·82	6607	6622	6637	6653	6668	6683	6699	6714	6730	6745	2	3	5	6	8	9	11	12	14
·83	6761	6776	6792	6808	6823	6839	6855	6871	6887	6902	2	3	5	6	8	9	11	13	14
·84	6918	6934	6950	6966	6982	6998	7015	7031	7047	7063	2	3	5	6	8	10	11	13	15
·85	7079	7096	7112	7129	7145	7161	7178	7194	7211	7228	2	3	5	7	8	10	12	13	15
·86	7244	7261	7278	7295	7311	7328	7345	7362	7379	7396	2	3	5	7	8	10	12	13	15
·87	7413	7430	7447	7464	7482	7499	7516	7534	7551	7568	2	3	5	7	9	10	12	14	16
·88	7586	7603	7621	7638	7656	7674	7691	7709	7727	7745	2	4	5	7	9	11	12	14	16
·89	7762	7780	7798	7816	7834	7852	7870	7889	7907	7925	2	4	5	7	9	11	13	14	16
·90	7943	7962	7980	7998	8017	8035	8054	8072	8091	8110	2	4	6	7	9	11	13	15	17
·91	8128	8147	8166	8185	8204	8222	8241	8260	8279	8299	2	4	6	8	9	11	13	15	17
·92	8318	8337	8356	8375	8395	8414	8433	8453	8472	8492	2	4	6	8	10	12	14	15	17
·93	8511	8531	8551	8570	8590	8610	8630	8650	8670	8690	2	4	6	8	10	12	14	16	18
·94	8710	8730	8750	8770	8790	8810	8831	8851	8872	8892	2	4	6	8	10	12	14	16	18
·95	8913	8933	8954	8974	8995	9016	9036	9057	9078	9099	2	4	6	8	10	12	15	17	19
·96	9120	9141	9162	9183	9204	9226	9247	9268	9290	9311	2	4	6	8	11	13	15	17	19
·97	9333	9354	9376	9397	9419	9441	9462	9484	9506	9528	2	4	7	9	11	13	15	17	20
·98	9550	9572	9594	9616	9638	9661	9683	9705	9727	9750	2	4	7	9	11	13	16	18	20
·99	9772	9795	9817	9840	9863	9886	9908	9931	9954	9977	2	5	7	9	11	14	16	18	20

USEFUL CONSTANTS

1 inch = 25·4 millimetres.

1 gallon = ·1605 cubic foot = 10 lb. of water at 62° F.

1 knot = 6080 feet per hour.

One pound avoirdupois = 7000 grains = 453·6 grammes.

cubic foot of water weighs 62·3 lb.

cubic foot of air at 0° C. and 1 atmosphere, weighs ·0807 lb.

cubic foot of hydrogen at 0° C. and 1 atmosphere, weighs ·00557 lb.

foot-pound = $1·3562 \times 10^7$ ergs.

horse-power-hour = 33000 × 60 foot-pounds.

electrical unit = 1000 watt-hours.

horse-power = 33000 foot-pounds per minute = 746 watts.

Volts × amperes = watts.

1 atmosphere = 14·7 lb. per square inch = 2116 lb. per square foot = 760 mm. of mercury = 10^6 dynes per sq. cm. nearly.

A column of water 2·3 feet high corresponds to a pressure of 1 lb. per square inch.

π = 3·1416 ; or $3\frac{1}{7}$.

One radian = 57·3 degrees.

To convert common into Napierian logarithms, multiply by 2·3026.

The base of the Napierian logarithms is e = 2·7183.

ANSWERS TO EXAMPLES

1. MISCELLANEOUS EXAMPLES IN DECIMALS AND APPROXIMATIONS

1. ·428.	**2.** ·5.	**3.** ·73.	**4.** ·89.
5. ·015.	**6.** ·232.	**7.** ·190.	**8.** ·001.
9. ·125.	**10.** ·891.	**11.** 939·58, 60·42.	**12.** 58·172.
13. ·465.	**14.** 500 tons	**15.** 27·375 lbs.	**16.** 6·83 cwts
17. 3408·255	**18.** ·11808	**19.** 6628·125.	**20.** 503·75.
21. 42·96875.	**22.** 1194·375	**23.** 345·75.	**24.** 7533·2.
25. 1112·07.	**26.** ·3375.	**27.** ·0217.	**28.** 71·5716.
29. 278·61.	**30.** 180·6.	**31.** 2·92.	**32.** 830·875.
33. 14029·6875.	**34.** 328·5625.	**35.** 570·865 ins.	**36.** £192·375
37. 98·701.	**38.** 2·2482.	**39.** 1·0638	**40.** 11·7436.
41. 3·9463	**42.** 1620.	**43.** 24 iron rods	
44. 30 pieces of cord.		**45.** 174 times.	**46.** 25·333
47. 32 times.	**48.** 8·8872.	**49.** ·1103.	**50.** 17,441·5888
51. 10·3070.	**52.** ·1898.	**53.** ·067.	**54.** ·316.
55. ·713.	**56.** ·009.	**57.** ·001.	**58.** ·050.
59. ·328.	**60.** ·001.	**61.** ·247.	**62.** ·170
63. ·004.	**64.** ·068.	**65.** 47·9751.	**66.** 1826·3920.
67. 6·5608.	**68.** 15·3091.	**69.** 1153·9325.	**70.** 4888·6879.
71. ·0762.	**72.** ·8095.	**73.** 1·6462.	**74.** 1·1731.
75. ·0184.	**76.** 793·5294.	**77.** £5 17s. 8d.	
78. (*a*) £32 5s. 10d., (*b*) 1s. 2d.		**79.** 570 tons.	**80.** £100.

2. EXAMPLES IN ALGEBRA

(*a*) Addition

1. $10x + 3y$.	**2.** $11b^2$.	**3.** $12b + 8c$.	**4.** 0.
5. $8a^2 + ab$.	**6.** $3x^4 + 2x^3 - 3x^2 - 15x - 2$.		**7.** $4ab - 4$.

8. $x^3 + y^3 + z^3 - 3xyz.$

9. $x^4 + x^2y^2 + y^4.$

10. $a^3 + b^3 + c^3 + 3a^2b + 3ab^2 - a^2c - ac^2 - b^2c - bc^2 - 2abc.$

11. $x^4 + y^4 + z^4 - 4x^3y + 4x^3z - 4xy^3 - 4y^3z + 4xz^3 - 4yz^3 + 6x^2y^2 + 6x^2z^2 + 6y^2z^2 - 12x^2yz + 12xy^2z - 12xyz^2.$

12. 0.

(*b*) Subtraction

1. $4a + 2b + 5c.$

2. $-4x + 2y - 6z.$

3. $a^2 - 3ab - b^2 - 5a - 7b - 8.$

4. $2a^4 - 2a^2x^2 + x^4.$

5. $4a^4 + 8a^2b^2 + 4b^4.$

6. $-3x^3 - 3z^3 - 3x^2z - 3xy^2.$

7. $x^4 - ax^3 - 9a^2x^2 - 3a^3x - 2a^4.$

8. 0.

9. $-a - b - c - d + e + f + g + h.$

10. $2x^4 - 8x^3y + 12x^2y^2 - 8xy^3 + 2y^4.$

11. $x^4 - 2x^2y^2 + y^4.$

12. $a^2 + b^2 - 2c^2 + 2ab - 2ac - 2bc.$

(*c*) Substitution

1. 8.	2. 27.	3. 6.	4. −16.	5. 6.
6. 2.	7. 4.	8. 0.	9. 5.	10. 11.
11. −7.	12. −5.	13. −7.	14. 7.	15. −69.
16. −2.	17. −8.	18. 51.	19. 6.	20. 3.
21. 125.	22. 35.	23. 1.	24. $\frac{1}{27}$.	

3. EXAMPLES IN FORMULAE (INCLUDING MENSURATION)

(*a*) The Square

1. $A = 6{\cdot}25$ sq. ins.

2. $A = 14{\cdot}44$ sq. ft.

3. $A = 160$ acres.

4. $A = 632$ acres, 3620 sq. yds.

5. $A = 14$ sq. yds. 4 sq. ft. 49 sq. ins.

6. $S = 69{\cdot}57$ yards.

7. $S = 8{\cdot}31$ inches.

8. $S = 4$ yds. 0 ft. 7·31 ins.

9. $S = 220$ yards, or 1 furlong.

10. $S = 12{\cdot}50$ feet.

(*b*) The Rectangle

11. $A = 8{\cdot}4$ sq. ins.

12. $A = 10{\cdot}08$ sq. ins.

13. $A = 9{\cdot}4$ sq. ft.

14. $A = 7$ acres, 3520 sq. yds.

15. $A = 23{\cdot}43$ sq. ins.

16. $l = 8{\cdot}54$ ins.

17. $b = 7$ ft. 5·30 ins.

18. $l = 7$ yds. 0 ft. 4·13 ins.

19. $b = 165$ yards.

20. $l = 18$ mls. 1 fur. 199·48 yds.

(c) The Triangle

21. $A = 17{\cdot}94$ sq. ins. **22.** $A = 297$ sq. yds. 6 sq. ft.

23. $A = 150$ acres. **24.** $l = 27{\cdot}23$ ins.

25. $b = 16$ yds. 2 ft. 9·88 ins. **26.** $l = 9$ fur. 76·66 yds.

(d) The Circle

27. $C = 11$ ft. $6\frac{6}{7}$ ins. **28.** $C = 13$ ft. 5·48 ins.

29. $C = 67$ yds. 0 ft. 8 ins. **30.** $r = 1$ ft. 9 ins.

31. $r = 14{\cdot}28$ yards. **32.** $r = 280$ yards, or 1 fur. 60 yds.

33. $A = 2$ sq. ft. 58·5 sq. ins.

34. $A = 43$ sq. yds. 2 sq. ft. 58·29 sq. ins.

35. $A = 31{\cdot}42$ acres. **36.** $r = 2{\cdot}25$ ins.

37. $r = 39{\cdot}24$ yds., or 39 yds. 0 ft. 8·71 ins.

38. $r = 5{\cdot}03$ yds. or 5 yds. 0 ft. 1·08 ins.

39. $68\frac{2}{3}$ sq. yards. **40.** 12960 times.

41. 105,600 − 18,480 = 87,120 revolutions.

42. 28 sq. yds. 2·5 sq. ft. **43.** $381\frac{1}{3}$ yards (nearly).

44. $251\frac{3}{7}$ yards. **45.** $9\frac{1}{8}''$.

46. $1'\ 11\frac{5}{8}''$ (practically 2 feet). **47.** $26\frac{2}{7}$ feet.

48. $14\frac{2}{3}$ inches. **49.** $45\frac{9}{11}°$. **50.** 72°.

51. $158\frac{2}{3}$ sq. inches. **52.** $69\frac{7}{16}$ sq. feet. **53.** $102\frac{2}{3}$ sq. inches.

54. $226\frac{2}{7}$ sq. yards. **55.** $113\frac{1}{7}$ sq. yards. **56.** 23·4°.

57. 87·6° **58.** $13\frac{7}{32}$ sq. inches (practically $13\frac{1}{4}$ sq. inches).

59. 3·1416 inches. **60.** 1 sq. ft. $104\frac{1}{2}$ sq. inches.

(e) The Lever

61. 8 inches. **62.** 2 ounces. **63.** 42·9 lbs.

64. 9·1 lbs. on *each* handle. **65.** 22·4 lbs.

(f) Simple Interest

66. $I = £182$ 11s. 7d. **67.** $I = £20$ 18s. 5d. **68.** $A = £629$ 4s.

69. $A = £56$ 12s. 8d. **70.** $A = £627$ 12s. 0d.

4. EXAMPLES IN TRIGONOMETRY

1. (Construction). **2.** g is the largest angle.

3. h is the smallest angle.

4. $a = {\cdot}7660$, $b = {\cdot}8830$, $c = {\cdot}3256$, $d = {\cdot}4540$, $e = {\cdot}6157$, $f = {\cdot}9613$, $g = {\cdot}9986$, $h = {\cdot}1736$.

5. $a=40°$, $b=28°$, $c=71°$, $d=63°$, $e=52°$, $f=16°$, $g=3°$, $h=80°$.

6. $a=\cdot7660$, $b=\cdot8830$, $c=\cdot3256$, $d=\cdot4540$, $e=\cdot6157$, $f=\cdot9613$, $g=\cdot9986$, $h=\cdot1736$.

7. $a=\cdot6428$, $b=\cdot4695$, $c=\cdot9455$, $d=\cdot8910$, $e=\cdot7880$, $f=\cdot2756$, $g=\cdot0523$, $h=\cdot9848$.

8. $a=40°$, $b=28°$, $c=71°$, $d=63°$, $e=52°$, $f=16°$, $g=3°$, $h=80°$.

9. $a=15°$, $b=19°$, $c=23°$, $d=26°$, $e=31°$, $f=50°$, $g=64°$, $h=69°$.

10. $a=75°$, $b=71°$, $c=67°$, $d=64°$, $e=59°$, $f=40°$, $g=26°$, $h=21°$.

11. $a=2°$, $b=7°$, $c=11°$, $d=21°$, $e=41°$, $f=51°$, $g=84°$, $h=87°$.

12. $a=88°$, $b=83°$, $c=79°$, $d=69°$, $e=49°$, $f=39°$, $g=6°$, $h=3°$.

13. (See text in Book I.)

14. $a=7°$, $b=12°$, $c=17°$, $d=24°$, $e=52°$, $f=64°$, $g=73°$, $h=81°$.

15. $a=83°$, $b=78°$, $c=73°$, $d=66°$, $e=38°$, $f=26°$, $g=17°$, $h=9°$.

16. (See text in Book I.) 17. (1) ABC, (2) XYZ, (3) MNO.

18. (*a*) Base $=AB$, (*b*) Base $=BC$, (*c*) Perpendicular height $=BC$, (*d*) Perpendicular height $=AB$.

19. (*a*) Base $=XY$, (*b*) Base $=YZ$, (*c*) Perpendicular height $=YZ$, (*d*) Perpendicular height $=XY$.

20. (*a*) Base $=MN$, (*b*) Base $=ON$, (*c*) Perpendicular height $=ON$, (*d*) Perpendicular height $=MN$.

21. (*a*) Hypotenuse $=5$ inches, (*b*) $\sin A=\frac{4}{5}$ or $\cdot8$,
(*c*) $\tan C=\frac{3}{4}$ or $\cdot75$, (*d*) $\cos A=\frac{3}{5}$ or $\cdot6$,
(*e*) $\tan A=\frac{4}{3}$ or $1\cdot3333\ldots$, (*f*) $\cos C=\frac{4}{5}$ or $\cdot8$,
(*g*) $\sin C=\frac{3}{5}$ or $\cdot6$

22. (*a*) Hypotenuse $=6\frac{1}{2}$ inches, (*b*) $\sin X=\frac{12}{13}$ or $\cdot9231$,
(*c*) $\cos X=\frac{5}{13}$ or $\cdot3846$, (*d*) $\tan Z=\frac{5}{12}$ or $\cdot4167$,
(*e*) $\cos Z=\frac{12}{13}$ or $\cdot9231$, (*f*) $\tan X=\frac{12}{5}$ or $2\cdot4$,
(*g*) $\sin Z=\frac{5}{13}$ or $\cdot3846$.

23. Cosecant $=\frac{5}{3}$. 24. Secant $=\frac{5}{4}$. 25. Cotangent $=\frac{3}{4}$.

26. (See Text in Book I.) 27. (See Text in Book I.)

28. 93 feet. 29. 42 feet. 30. 95 feet. 31. 84 feet.

32, 33, 34. (See Figures 19, 20 and 21.)

5. EXAMPLES IN SQUARED PAPER WORK AND GRAPHS

1-5. (Various answers.) 6-9. (See Text in Book I.)

10. Answer $=9\cdot25$ (see also text in Book I.).

11. Answer $=5$ (see also text in Book I.).

12. (See text in Book I). **13.** 67°. **14.** 77°. **15.** 19°.
16. 74°. **17.** 10°. **18.** 41°. **19.** 53°. **20.** 74°.
21. 67°. **22.** 2·4 inches. **23.** 93 feet. **24.** 42 feet.

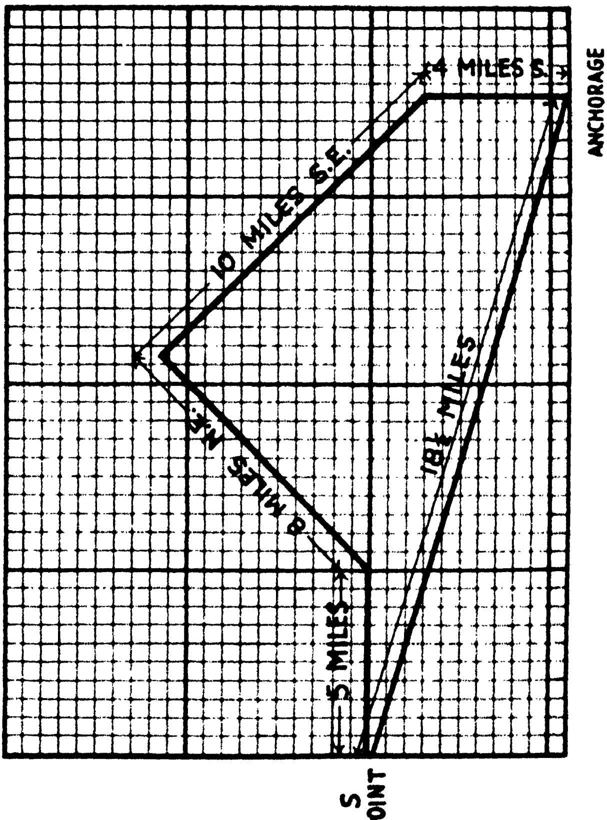


FIG. 42. ANSWER TO QUESTION 30.

25. 95 feet. **26.** 84 feet. **27.** 10 miles. **28.** 26 miles.
29. 9·3 miles. **30.** 18½ miles. **31.** (*a*) 58 miles, (*b*) W. 32° N.
32. (*a*) 4·3 miles (nearly), (*b*) N. 10° E.
33 to **46.** (See Figs. 28 to 41.)

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