INTERMEDIATE MATHEMATICS.

Univ. Corr. Coll. Tutorial Series.

INTERMEDIATE MATHEMATICS.

A GUIDE TO THE MATHEMATICS OF THE INTERMEDIATE EXAMINATIONS IN ARTS AND SCIENCE OF THE UNIVERSITY OF LONDON.

(Fifth Thousand.)

BY THE PRINCIPAL OF
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CONTENTS.

PREFACE TO THE SECOND EDITION.

The most gratifying testimony we could possibly have to the usefulness of this book is that a Second Edition has been called for within two years of the publication of the first. Although its use has been extended beyond the private student, for whom it was originally intended, to most of the schools from which candidates proceed, this demand for a work which appeals to such a small class of students is extremely exceptional.

In the present edition references have been given in the schemes of study to Lock's *Arithmetic* and Hall and Knight's *Algebra.* The latter has been plotted out for our "Weekly Schemes of Study" by one of the Authors, Mr. Hall, whose kind aid we thankfully acknowledge. Answers to the miscellaneous questions have been added by Mr. G. H. Bryan, Smith's Prizeman, Fellow of Peterhouse, Cambridge.

THIRD EDITION.

References have been added to Pendlebury's *Arithmetic* and Briggs and Bryan's *Coordinate Geometry.* A further series of fifteen Test Papers has been appended.

INTRODUCTION.

It is a well-known fact among successful coaches, that a knowledge of the requirements of an Examination can be got much better from a careful analysis of previous papers than from the regulations issued by the examining body; and a pupil under a skilful tutor has, perhaps unconsciously to himself, the leading parts of the subject exhibited in the strongest light by repetitions, numerous illustrations, and papers composed of examination types. It is hoped that the general student will be able to learn, from our graduated system of Test Papers (one of which should follow each week's reading), without a tutor's guidance, what the Examiners may fairly require of him; and that his attention will be drawn to the fundamental parts and landmarks of the subject, so that he may not lose sight of the important points in a mass of details—a common event with private students, and one which makes them lose interest in their work.

In working through the course as mapped out, the student should make a synopsis of each branch of the subject, paying special attention to the parts marked *important* in the Schemes of Reading, and of the formulæ, and write the questions neatly at the head of his Solutions to the Test Papers in a book, for revision just before the Examination.

As 79 of our students passed the last Intermediate Arts Examination, we can the more confidently recommend the course adopted.

PART I.

PLAN OF THIS BOOK.

The book is arranged on the following plan :—

(1) An account of the Text-Books commonly read for the Examination is given. A selection is made of those we can recommend by experience as most suited to the Examination. Others are mentioned which in many respects are as good for the general student, or which possess special merits to recommend them to students who have particular weaknesses.

(2) The whole of the work to be done has been distributed as equally as possible into 30 parts, including a first revision; so that the student may set himself one of these parts each week* or fortnight, and thus have the gratification of having done an aliquot part in the arranged time, and so counting sure progress. The distribution of the work into what has been proved in actual practice to be a fair amount for the average student ensures the complete preparation in any one by the appointed date without drawing more than is necessary from the time he has at his disposal for all subjects. For examination and educative purposes it is far better to get a thorough grounding which will secure three-fourths of the marks, than a shallow general knowledge of the subject, upon which so many self-deluded candidates rely; and, if the student plodding on alone feels that he has quite enough to do to obtain mediocrity, by all means let him avoid out-of-the-way parts of the subject and alternative methods,

* For the sake of convenience we designate the period to be spent over each part a *week*, although very often the longer time will be necessary.

which are really only short cuts for the advanced student.

(3) A Test Paper to be answered at the end of the week, compiled from questions formerly set at the Examination, comes next. This is in all cases on subjects which have been previously covered, and almost exclusively on the work prescribed in the scheme of study for that week. The Examination Papers from which these questions come are mainly those set since 1874, but wherever a somewhat older question has been thought specially suitable it has been selected, and in many cases the oldest papers have been used to furnish a type which is again becoming popular. As an example of this recurrence of fashion in Mathematics, we may mention questions on "Piles of shot and shell"; these will be found in the old books of Hind (1830), and Wood and Lund (1841), but not in Todhunter or his contemporaries; while we find them again in Hall and Knight's *Higher Algebra* (1888) and C. Smith's *Treatise on Algebra.*

(4) All the questions which have been set since 1874, and are not contained in the Test Papers, along with the whole of the questions in Conics from 1843, which come in the present University schedule, have been collected under the head of Miscellaneous Questions, or given in the Examination Papers. Many of these are extremely useful for additional practice, although they are amply illustrated by the types in the Test Papers.

(5) For the benefit of students who can find time to work more examples, a hundred and fifty have been arranged as fifteen additional Test Papers. Each of these corresponds to two weeks' work in the Schemes of Study. Answers to all the questions, and full solutions to the last Paper (1890), complete the plan of the book.

In working through the course the student should make a synopsis of all the important book-work in Geometry, draw the figure neatly, and write out in a few lines each proposition by indicating (after learning it) the most important steps of the proof. He should also keep a formula book.

A mark X, made in the margin opposite any part which he finds on reperusal has escaped his memory, and repeated at each revision, will soon show him his weaknesses, and help to economise his time in revising.

We do not intend giving the grandmotherly advice we find in some guides, as to diet, exercise, and keeping out of easy chairs when studying; the candidate will soon learn that self-denial is a very important factor in preparation for an Examination; as also is self-respect, in the broadest sense of the word—a qualification which all must possess who wish to win any position; but we will draw the attention of each student individually to the following:—

Never study Mathematics without a pen or pencil in your hand, and a sheet of paper before you.

Work out Illustrative Examples on paper.

Make your reading *connected;* don't dabble in many books. Get a firm grasp either of Euclid or its modern equivalent at one time; don't work the two simultaneously.

In reading, put mentally to yourself possible questions on the subject-matter.

Draw comparisons, and tabulate those parts of Mathematics which admit of such treatment.

Revise continually, but judiciously; don't go through all previous reading each time of revision, but through the parts especially worthy of attention, and those which you have marked as your weaknesses or as keystones.

It is not so much the amount of time spent in reading that will tell, as the amount of concentration of thought you can bring to bear upon your reading.

Part VII. of our book should not be used until all the work is completed.

The following questions in the Test Papers have presented great difficulty to many students, and are not of great importance as types:—

III.—3—an acquaintance with the sixth week's work may make this easier. IV.—2—laborious, unless you note the factors of $u^3+v^3+w^3-3uvw$. VII.—1. XII.—2—if possible, see a similar question worked out. XXIII.—1. XXIV.—9 & 10—difficult examples, but not altogether unimportant. XXVI.—1—worked out in Hall and Knight.—8. XXVII.—2. XXVIII.—part of 9.

We give a list of the following difficult questions in the Test Papers in order that the student may not be discouraged at failure in his first attempt to work them.

I.—5. II.—2, **5.** III.—2, **5.** IV.—4, 6. V.—7. VI.—**2.** VII.—1. VIII.—5, 7, 8. IX.—1, 2, **5 (ii.)**, **9.** X.—1, 2 (ii.), **3**, 5 (ii.), **10.** XI.—4, **8.** XII.—**3 (ii.)**, 8. XIII.—2—book-work, yet many find it difficult to understand, 6, 7, **9 (ii.)**, 10. XVI.—**2 (ii.)**, 7, 8. XVII.—**7.** XVIII.—4, 5, 9. XIX.—10. XX.—2 (ii.) (iii.), **3**, 8. XXI.—7. XXII.—10. XXIII.—2, 3—really a problem on arithmetico-geometric progressions, like XXIV., 2. XXIV.—1. XXVIII.—1—worthy of consideration, however; 7 and 8 involve somewhat difficult book-work. XXX.—**3**, 4, 5, **6**, **10**.

I. A—3 (ii.). III. A—3, **4.** V. A—1, 4, **9.** VI. A—2, **6**, 8. VII. A—5, 6, **8 (ii.)**. VIII. A—1, 5. IX. A—**1**, 3, 7, **8** (i.). X. A—1, 2 (ii.), **7**, **10**. XI. A.—5, 9. XII. A—**2**, 10. XIII. A—1 (ii.). XIV. A—1 (ii.), 2, 9. XV. A—5, 7.

The questions numbered in bold type have proved stumbling-blocks to the majority of students.

W. B.

BURLINGTON HOUSE,
CAMBRIDGE,
August 1st, 1890.

PART II.

TEXT-BOOKS.

ARITHMETIC.—**Pendlebury** (Bell, 4s. 6d.) contains all that is required. Lock, Brook Smith, Hamblin Smith, and Girdlestone are also good, but in some respects deficient. Annuities and Logarithms must be read up from an *Algebra*.

ALGEBRA.—**Hall and Knight's Elementary Algebra** (Macmillan, 4s. 6d.) can be recommended as a model of lucidity; but, if either it or Hamblin Smith's Algebra (Longmans, 3s.) be used, they must be supplemented by **Hall and Knight's Higher Algebra** (Macmillan, 7s. 6d.; Key, 10s. 6d.), or C. Smith's *Algebra for Schools and Colleges* (Macmillan, 7s. 6d.; Key, 10s. 6d.). These books are brought up to modern requirements, and have all the advantages of the old favourite Todhunter, issued by the same publisher. The *Higher Algebra* gives excellent practice in the applications of Progressions, a leading feature of the Intermediate Papers, and it is as well fitted as any other to enable a bright student to grapple with the involved questions on Annuities set at the examination. We shall not enter into the merits of Elsee, Colenso, Barnard Smith, Chambers, Atkins, Pryde, and Wood, because we think the private student cannot do better than get the Elementary book of Hall and Knight, and supplement by their *Higher* work, or C. Smith, keeping closely to the syllabus.

GEOMETRY.—(1) *Similar Figures and Planes.*—Class-book: any good edition of Euclid. One of the best with which we are acquainted is that by Dr. Casey, published by Hodges, Figgis, & Co., Dublin (Part II., 2s. 6d.; Key, 6s.). The riders are exceedingly numerous, and a number of useful exercises are followed by solutions. In Book VI., his alternative proofs to Propositions XII., XVI., XIX., and XXXI. are excellent, and the Appendix on the Prism, Pyramid, Cylinder, Sphere, and Cone, makes the book of special service to the Intermediate student. The student who possesses Hall and Stevens' Euclid (Macmillan, 4s. 6d.) can scarcely better it. H. Smith's *Geometry*, Todhunter's *Euclid* (Macmillan, 3s. 6d.; Key, 6s. 6d.), and Potts' *Euclid* are often recommended. The first section of Wilson's *Solid Geometry* may be used instead of Book XI.

MENSURATION, AND PROPERTIES OF THE SPHERE.—Class-book: **Wilson's Solid Geometry** (Macmillan, 3s. 6d.). Sections II., III., and IV., about forty pages in all, provide the student with a clear, compact, and complete course. In the treatment of the Sphere, Wilson stands alone: but by those who prefer it, Watson's *Plane and Solid Geometry* may be read for all but the latter part. The student who possesses Casey's *Euclid* will be able to distinguish the fundamental parts of Mensuration; yet, although Wilson may be pruned slightly, the average student will not find forty pages very laborious, and it is better to err on the safe side. Wormell's *Solid Geometry* is sometimes recommended (Murby, 2s. 6d.); although in itself not exactly the style of book required for the examination, it is useful for practice in the applications of the parts given in Wilson and for its happy illustrations, the formation of a mental picture of the figures in Solid Geometry being somewhat difficult to beginners.

CO-ORDINATE GEOMETRY.—The most satisfactory book on this subject for the general student who has to go beyond the *Line and Circle* is C. Smith's *Conic Sections* (Macmillan, 7s. 6d.; *Key*, 10s. 6d), but it is rather too full to be used for this examination, as only parts of Chapters I., II. and IV., with perhaps the first two or three paragraphs of Chapter III., are necessary; all articles marked *, and those on oblique axes, should be omitted, and, by the general student, the determinant notation (that is, where the involved quantities are arranged between two vertical lines), and the exercises, both illustrative and unworked, at the ends of the chapters. **Briggs and Bryan's Coordinate Geometry, Part I.*** (Univ. Corr. Coll. Press Warehouse, 2s.), was prepared specially to help beginners in this difficult subject of the Intermediate Examination; it contains a very large number of Illustrative Examples, and aims at simplicity so far as that is compatible with completeness. Great care has been bestowed on the arrangement of the book, and on the diagrams, which are over sixty in number. Vyvyan's *Coordinate Geometry* (Bell & Sons, 4s. 6d.) is as useful a book in many respects as C. Smith, for which it may be substituted; the first three chapters, omitting oblique axes, quite cover the syllabus. Those who find a difficulty in working problems should procure **Briggs and Threlfall's Worked Examples in Coordinate Geometry** (Univ. Corr. Coll. Press Warehouse, 1s. 6d.), a graduated course of exercises on the *Line and Circle.*

TRIGONOMETRY.—The class-book, **H. Smith** (Longmans, 4s. 6d.), is just the kind of book a private student requires, as it represents almost exactly the amount of reading expected of a candidate for the Intermediate Arts, graduated gently and with a copious supply of worked examples. The Key to it makes it of special value to private students.

The pupil who wishes to extend his reading, and acquire more of the geometrical proofs of formulæ, should get Lock's *Elementary Trigonometry* (Macmillan, 4s. 6d.; Key, 8s. 6d.), in our opinion the best elementary book extant. Vyvyan, Casey, Pinkerton, Beasley, and Walmsley are often recommended for this examination; but, for reasons previously stated, we omit them and some of the most popular treatises from our account. Briggs' *Synopsis of Elementary Trigonometry* (Univ. Corr. Coll. Press Warehouse, 1s.) will prove of great service to most candidates.

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* A specimen copy will be presented, on application, to any teacher who is willing to adopt it in his classes if found suitable.

PART III.

WEEKLY SCHEMES OF STUDY.

ABBREVIATIONS for names of Authors—Pendlebury (P.), Hamblin Smith (H. S.), Hall and Knight (H. K.), Lock (L.), C. Smith (C. S.), Vyvyan (V.), Briggs and Bryan (B. B.).

The most important articles are printed in thicker type.

First Week's Work.

ALG.—Quadratics. H. S., 230–248; H. K., 189–202.

TRIG.—Meaning of π. Measurement of Angles. H. S., 5–34, **13, 15, 22, 27, 30, 31, 32**; L., 19–64, **29, 34, 40, 46** (omit 47–50 and 53), **59, 60, 63, 64.**

EUC. VI.—Def. and Props. 1, **2**, 3, and 3A.

ARITH.—Fractions. P., 98–138, 140–148; L., 76–99.

Second Week's Work.

ALG.—Simultaneous Quadratics. H. S., 249–258; H. K., 203–208. All important; work very carefully through the Illustrative Examples, the harder types especially, viz., H. K., Arts. **206, 207, 208.** A question is given in almost every Exam. Paper.

TRIG.—Angular Systems and Direction Signs. H. S., 35–50, **38, 39**; L., 65–72, 120–125, **69, 70, 122**; and C. Smith's Conics, 1–3.

EUC. VI.—**4, 5,** and 6.

ARITH.—Decimals. P., 149–176; L., 103–115, 118–120.

Third Week's Work.

Alg.—Quadratic Problems. Revise Lessons I. and II., and work Illustrative Problems in H. S., Chap. xxi.; H. K., Chap. xxvii.

Trig.—Trigonometrical Ratios for acute angles. H. S., 1–4, 51–64, 75–81, **51** and **52**; **Ex. 2, p. 36, 78, 79,** and **80**; L., 1–18, 73–96, **75, 78, 80**; **Ex. xii.** (5), (6) **90, 91, 92, 96,** meaning of ∞.

Euc. VI.—7, 8, 9, 10.

Arith.—Recurring Decimals. P., 177–185, **186**, 187–191, 195; L., 128–136, **138.**

The proof of the rule for the conversion of a recurring decimal into a vulgar fraction will be found in Algebra, and should be got up.

Fourth Week's Work.

Alg.—Results of Multiplication and Division. Revision of H. C. F. and L. C. M., H. S., 59–61, 83–85, 119–121, 124, 126, 132, 160–171; H. K., 54, 55, 114, 138–145, 159, 160, 163, **213, 214.** Add

$$a^3+b^3+c^3-3abc=(a+b+c)\ (a^2+b^2+c^2-ab-bc-ca)$$

and note

$$a^2+b^2+c^2-ab-bc-ca=\tfrac{1}{2}\ \{(a-b)^2+b-c^2+(c-a)^2\}.$$

Trig.—Changes in Sign, &c., in different quadrants. H. S., Chaps. vii. and viii., **66, 67, 68**; L., **114,** 126–136, **132, 133.** Remember *all—sin—tan—cos*, to know the ratios which are positive in the 1st, 2nd, 3rd, and 4th quadrants respectively.

Euc. VI.—11, **12,** 13, 14.

Arith.—Fractional Measures: P., 139, 192–194; L., 116, 117, 137. Approximations: P., 196–203; L., 139–154.

Fifth Week's Work.

ALG.—Revision of Fractions. H. S., Chap. v.; H. K., Chap. xix., 151–158, 168, 169, and all of Chap. xxii.

TRIG.—Relation between Trigonometrical Ratios for same angle. H. S., **88–91**; L., **102–108.**

EUC. VI.—15, 16, 17.

ARITH.—H.C.F. and L.C.M., P., 49–59; Appendix, 1–3; L., 67–75.

Sixth Week's Work.

ALG.—Harder Factors and Roots of Equations. H. S., 324–337; H. K., Chapters xxviii. and xxxvi. Of frequent application in Co-ordinate Geometry.

TRIG.—To express one ratio in terms of the others. H. S., 92–96, **94**; L., 109–116, **113**.

EUC. VI.—18, **19,** 20.

ARITH.—Interest, Present Worth, and Discount. P., 251–255, **256, 257, 259,** 260–266, **267, 268, 269, 270,** 271–276; L., 180–183, **184,** 185, 186.

Seventh Week's Work.

ALG.—Indices and Evolution. H. S., 220–225, 265–287; H. K., Chapters xxx. and xvi. (omitting cube root), also Arts. **217, 218.**

TRIG.—Comparison of Ratios for Relational Angles. H. S., Chap. x., **99, 101**; L., **117, 118,** 137–141.

EUC. VI.—21, **22,** 23, **24.**

ARITH.—Square Root. P., 315–324, 326; L., 199–205.

Eighth Week's Work.

ALG.—Surds, and Square Root of Surds. H. S., 289–317; H. K., 249–280: in particular **265, 266, 278, 279, 280.**

TRIG.—Trigonometrical Equations, H. S., **107, 108**; L., **119.**

EUC. VI.—**25,** 26, 27.

ARITH.—Square Root of Surds. Revise last lesson and read P., 325, or L., 206–209, in addition.

Ninth Week's Work.

ALG.—Equations involving Surds. H. S., 318–321; H. K., 281, 282 [this may be supplemented by H. K., *Higher Algebra*, 130, 131, 132].

TRIG.—General expressions for all angles with a given Trigonometrical Ratio, H. S., 108, 109, **110, 111, 112,** 113, and **114**; L., 142–144, **145,** 146, **147,** 148, **149,** 187.

EUC. VI.—30, **31, 32,** 33.

ARITH.—Stocks and Shares. P., 277–288, **289, 290, 291, 292, 293**; L., 196–198.

Tenth Week's Work.

Revise all Lessons I. to IX., H. S. and H. K. (pay special attention to the parts marked important).

Eleventh Week's Work.

Alg.—Ratio, H. S., 338–352 and 204–205; H. K., 283–294 (*Higher Algebra*, Art. **16**).

Trig.—$A+B$, Results. H. S., 115, 116, **117, 118, 119. Exs. 1 and 2** (most important Lesson of all); L., 153, **154** (find sin 75°).

Geom.—Euclid xi., Defs. 1–5 and Props. 1–3. Wilson, **Defs. 1–9** and Props. 1 and 2.

Conics.—Coordinates. Distance between two points in terms of their coords. Point dividing the line joining two points in a given ratio.

B. B.—Lesson I.

C. S.—1–5, **4, 5.**

V.—1–4, 11 (first part), 12. All parts relating to oblique axes must be omitted in Vyvyan.

Twelfth Week's Work.

Alg.—Ratio and Proportion. H. S., 353–362; H. K., 295–302.

Trig.—Sum and Difference Formulæ. H. S., **120** and **121**; L., 157, **158,** 159, 169, **170**, 171–174. These formulæ require an *exorbitant* amount of practice to get up *thoroughly*, without which they are of no use.

Geom.—Euc. xi., **4,** 5, **6,** 7, and **8.** Wilson, 3, 4, 5, 6, and **7.**

Conics.—Areas of Triangle and Quadrilateral in term of the coords. of their angular points.

B. B.—Lesson II.

C. S.—**6** and 7.

V.—13, first part.

Thirteenth Week's Work.

ALG.—Variation. H. S., 363–371; H. K., 303–311 (see also examples worked out, 22, 24, 25 of *Higher Algebra*).

TRIG.—Sum and Difference Formulæ. H. S., 120–123; L., 157–159, **160**, 161.

GEOM.—Euc. xi., 9, 10, and **11**; Wilson, 8, 9, 10, 11.

CONICS.—Locus of an Equation, and Polar Coordinates.

B. B.—Lesson III.

C. S.—**8, 9,** 10, 11, **12.**

V.—6–9, 5, 11, **13.**

Fourteenth Week's Work.

ALG.—Arithmetical Progression. H. S., **372–379** (all very important); H. K., 312–316, **314.**

TRIG.—Tan $(A \pm B)$ Formulæ. H. S., **124,** 125; L., **156.** Revise H. S., Chap. xii.; L., Chap. xi.

GEOM.—Euc. xi., 12, 13, 14, **15**; Wilson, **14.**

CONICS.—Equation to line in tangent form $y = mx + c$.

B. B.—Lesson IV.

C. S.—13, 14, **15,** 17.

V.—21, 24, 26.

Fifteenth Week's Work.

ALG.—Arithmetical Progression. H. S., **Chap. xxx.**; H. K., Chap. xxxiii. (also more fully in *Higher Algebra*, Chap. iv.).

TRIG.—Multiple Angles. H. S., **126–130,** 131, **132**; L., **162–167, 93.**

GEOM.—Euc. xi., 16, **17, 18.** Wilson, 12, 13, **15, 16,** 17, **18.**

CONICS.—Equations to line in *intercept* and *perpendicular* forms $\frac{x}{a} + \frac{y}{b} = 1$, $p = x \cos a + y \sin a$ respectively.

B. B.—Lesson V.

C. S.—**18, 19,** 20, 21.

Sixteenth Week's Work.

ALG.—**Geometrical Progression.** H. S., **Chap. xxxi.**; H. K., **Chap. xxxiv.**

TRIG.—Equations involving Multiple Angles. Revise H. S., 126–132; L., 162–168; and do as many examples as possible.

GEOM.—Euc. xi., **19**; Wilson, 19; and Defs. **10** and **11.**

CONICS.—Equations of lines drawn through one and two given points.

B. B.—Lesson VI.

C. S.—22, **23, 24.**

V.—23, 25–31.

Seventeenth Week's Work.

ALG.—Revise Geometrical Progression, and work as many examples as possible (a large number in *Higher Algebra*, Chap. v.).

TRIG.—Submultiple Angles: H. S., 132, **133, 134,** 135, 136; L., 175, **176,** 177–183. Inverse Functions: H. S., **137**; L., **187,** 188.

GEOM.—Euc. xi., **20** and 21. Wilson, **20** and 21. Proposition 20 is of very great importance.

CONICS.—Equation of a line through a given point in a given direction, $\frac{x-x'}{\cos\theta}=\frac{y-y'}{\sin\theta}=r$. Position of a point with regard to a line. Coords. of point of intersection of two lines. Condition that three lines may meet at a point.

B. B.—Lesson VII.

C. S.—**25,** 26, **27, 28.**

V.—32–34, 39, 42.

Eighteenth Week's Work.

Alg.—Revise all Progressions in *Higher Algebra*, Chapters **iv., v., and vi.**

Trig.—Transformations involving Relational Angles. H. S., 185 (revise Lessons XI.–XIV. first); L., 235.

Mens.—Wilson, Sec. II. The values of the angles, and of perpendicular in terms of a side, should be studied carefully in the regular tetrahedron.

Conics.—Angle between two given lines. Condition of perpendicularity. Distance of a point from a given line.

B. B.—Lesson VIII.

C. S.—29, **30, 31.**

V.—33, 34, 37.

Nineteenth Week's Work.

Alg.—Logarithms. H. S., **446–462** and **467, 455** and **456**; H. K., 371–385, **376, 377.**

Trig.—Logarithms, H. S., **138–154** and **159, 147** and **148**; L., **189–208, 192, 193.**

Mens.—Wilson. Sec. III. to **29,** and also 30 and 34, and Def. 28.

Conics.—Equation of bisectors. Equation of a line through point of intersection of two given lines.

B. B.—Lesson IX.

C. S.—**32,** 33, and 34. Do not spend too long over these if you find them difficult.

V.—43, 40.

Twentieth Week's Work.

Revise all Lessons XI.–XIX., paying special attention o parts marked important.

Twenty-First Week's Work.

ALG.—Logarithms. H. S., **463**, 464, **465, 466**; H. K., **386, 387** (see also *Higher Algebra*, Chap. xvi.)

TRIG.—Logarithms. H. S., 155–154, **155, 157, 162, 164**; L., 209–224, **209, 220, 224.**

MENS.—The Cylinder. All given in Wilson is important.

CONICS. — Interpretation of an Equation of Second Degree in Conics. Change of Axes. To find the equation of a circle.

B. B.—Lesson XI.

C. S.—35–37, 49, 50, **65.**

V.—44, 45.

Twenty-Second Week's Work.

ALG.—Application of Logarithms to Interest. H. S., **471–474** (very important); *Higher Algebra*, 229–235, and examples; L., "Trig.," pp. 190, (vi.), (vii.).

TRIG.—Formulæ for Solution of Triangles. (i.) The sine and cosine rules, H. S., 175–176, **178, 179**, 188, 199, 196, 200, 204–205, 207, (i.) 208, (i.) **210–212**, 213, 214; L., 231–236, 237, **238, 239, 240**, 247, 248, 250, 251, 255–257, **261, 262**, 263–267.

MENS.—The Cone. In Wilson too much time should not be given to the parts numbered (1) to (7), following Def. 27. Prop. 33 is of fundamental importance. Omit Cor. 3.

CONICS.—Constants in equation of a circle.

B. B.—Lesson XII.

C. S.—**66.** Revise **65.**

V.—46. Revise 45.

Twenty-Third Week's Work.

ALG.—Annuities. Todhunter's *Large Algebra*, or *Higher Algebra*, 236–244.

TRIG.—The formulæ for solution of Triangles. (ii.) The Tangent Rule and Relations for Semi-Angles. H. S., **180–184, 200–203, 207–208**; L., **241, 242, 243–246, 249, 252–254, 258, 259.**

SPH. GEOM.—The Sphere to end of Wilson, 35.

CONICS.—The equations of the tangent and normal.

B. B.—Lesson XIII.

C. S.—68–71, **70.**

V.—49–53.

Twenty-Fourth Week's Work.

ALG.—Revise the last two important Lessons.

TRIG.—Solution of Right-Angled Triangles. H. S., Chap. xvii.; L., 236, Ex. xii. (6). Revise H. S., Chap. xvi.; L., Chap. xvi.

SPH. GEOM.—Wilson, 36, **37,** 38.

CONICS.—Points of intersection of a given line and a circle. Locus of middle points of a system of parallel chords of a circle. Tangents from a point within, on, or outside a circle.

B. B.—Lesson XIV.

C. S.—**72,** 73–75.

V.—The student who uses this book might read **54,** and then go on to 58 and 59.

Twenty-Fifth Week's Work.

ALG.—Permutations and Combinations. H. S., 402–406 (work *all* the Exs.); H. K., 344–350, **346, 348,** 349, or *Higher Algebra*, 139–148, **141, 144, 145.**

TRIG.—Solutions of Oblique-angled Triangles. Revise H. S., Chap. xviii.; L., Chap. xviii. H. S., 209 is scarcely elementary, and may almost safely be omitted.

SPH. GEOM.—Wilson, **39**, **Cor. 1, Cor. 2.**

CONICS.—Polar of a point with respect to a circle.

B. B.—Lesson XV.

C. S.—**76,** 77.

V.—58–60.

Twenty-Sixth Week's Work.

ALG.—Permutations and Combinations. H. S., 407–411, **408, 409**; H. K., 351–354, **352,** or *Higher Algebra*, 149–156, **151.**

TRIG.—Heights and Distances. H. S., 82–87, and Chap. xix.; L., 97–101, Chap. xviii. Work carefully through the lesson several times; do not cram any formulæ, but try a good many examples until you can do them without difficulty.

SPH. GEOM.—Wilson, **40,** and **Cor.**

CONICS.—Polars continued.

B. B.—Lesson XVI.

C. S.—78, 79, 80, and 82. If pressed for time, omit these.

V.—61–64.

Twenty-Seventh Week's Work.

ALG.—Revise Permutations and Combinations, and work all the examples in *Higher Algebra*, Chap. xi.

TRIG.—Areas of Triangles, &c. H. S., **219–222,** 223; L., **273–276,** 277–280. (See hints under Recapitulation and Structure of Papers.)

SPH. GEOM.—Spherical Triangles to Theorem 41 (exc.). *If* this lesson is to be of any use, it must be done thoroughly.

CONICS.—Length of a tangent. Radical Axis.

B. B.—Lesson XVII.

C. S.—**82,** 83, 86, and 87. Omit the two last if found difficult.

V.—55, 65, 66, 68, 69.

Twenty-Eighth Week's Work.

ALG.—Revise Lessons XIV. to XXIII. (all exceedingly important).

TRIG.—Limiting Values of Sine and Cosine, and Area a Circle. H. S., 229, 230, 224, 225, **226**; L., 288–291, 286, **287.**

SPH. GEOM.—Spherical Triangles. Wilson, 41 and **42.** The Corollary is extremely important. Remember the formula.

CONICS.—Polar equations.

B. B.—Lesson XVIII.

C. S.—**45,** 46, **81.** Revise 9, 10, 11, 12.

V.—35, 36, 48.

Twenty-Ninth Week's Work.

Revise all Lessons XXI. to XXVII., devoting special attention to the parts marked important.

Thirtieth Week's Work.

Revise all Lessons I. to XXX.

PART IV.

TEST PAPERS.

I.

'85. **1.** Solve the equations

$$\text{(i)}\quad \frac{a}{x-b}+\frac{b}{x-a}-\frac{2(a+b)}{2x-a-b}=0.$$

$$\text{(ii)}\quad \frac{x}{2x-1}-\frac{2x-1}{2x+1}=\frac{2}{3}.$$

'84. **2.** Solve the equation

$$\frac{2x-1}{3x+2}+\frac{2x-3}{2x-1}+\frac{2x-1}{6x^2+x-2}=0.$$

'83. **3.** Solve the equations

$$\text{(i)}\quad \frac{x-1}{x-2}-\frac{2}{x-1}=1. \qquad \text{(ii)}\quad x+\frac{1}{x}=2\tfrac{1}{1640}.$$

'76. **4.** If two straight lines which meet are divided proportionally by a series of secants, prove that the latter must be parallel.

'76. **5.** Prove that two isosceles triangles with the angles of their vertices equal have their altitudes proportional to their bases.

'84. **6.** Find the length of an arc on the sea which subtends an angle of one minute at the centre of the earth, supposing the earth a sphere of diameter of 7,920 miles.

'75. **7.** Assuming the numerical value of π to 5 decimal places, calculate to the nearest integer the number of seconds in the angle subtended at the centre of a circle by an arc of length equal to that of the radius of the circle.

'80. **8.** A steamer goes 9·6 miles per hour in still water. How long will it take to run 10 miles up a stream, and return, the velocity of the stream being 2 miles per hour?

'74. **9.** A person inquiring the time of day, is told that it is between 5 and 6, and that the hour and minute hands are together. What o'clock is it?

'86. **10.** Calculate the value of e to five places of decimals from the formula $e=1+\frac{1}{1}+\frac{1}{1.2}+\frac{1}{1.2.3}+\frac{1}{1.2.3.4}+\cdots$.

II.

'77. 1. Find the solution of the system of equations:

$$x^2+xy=(a-b)^2.$$
$$xy+y^2=4ab.$$

'84. 2. Find two numbers, such that their sum is 9 and the sum of their fourth powers 2417.

'82. 3. The sum of the squares of two numbers is 650, and their product is 323. What are they?

'79 and '75. 4. Two rectilinear triangles being supposed equiangular, show that their three pairs of homologous sides, opposite to their three pairs of equivalent angles, are proportional.

'74. 5. (1) Prove that the straight lines, which join the vertices of a triangle to the middle points of the opposite sides, meet in a point.

(2) Prove that this point divides each of the lines into segments which bear to one another the ratio 1 : 2.

'84. 6. Prove that to turn circular measure into seconds we must multiply by 206265.

'84. 7. Prove that to turn seconds into circular measure we must multiply by ·0000048.

'85. 8. Prove that, if 276·543 be divided by 1137·4651, until there are four figures after the decimal point in the quotient, the remainder of the dividend now left is ·02523419.

'83. 9. Reduce $\frac{627}{6250}$ and $\frac{19}{35}$ to decimals.

'83. 10. Divide ·06059 by ·073, and also by 1460.

III.

'85. **1.** There is a number consisting of two digits, such that the difference of the cubes of the digits is 109 times the difference of the digits. Also the number exceeds twice the product of its digits by the digit in the unit's place. Find the number.

'83. **2.** To do a piece of work, *A* requires $\frac{5}{6}$ as long as B, but *A*'s wages are 1 shilling a day more than *B*'s. The work costs 22s. more if *A* does it than if *B* does it. And if they work together it costs 10 guineas. Find *A*'s daily wage.

'80. **3.** A purchaser is to take a rectangular plot of land fronting a street; three times its frontage added to twice its depth is to be 96 yards. What is the greatest number of square yards he may take?

'77. **4.** Given two finite right lines, one divided in any manner into a number of segments, and the other undivided. Show how to divide the latter similarly to the former.

'78. **5.** On a straight line 3 points, *A*, *B*, *C*, are given, *B* lying between *A* and *C*. Find a point *D* which divides the line *AC* externally in the same ratio as *B* does internally.

'85 and '84. **6.** Define the trigonometrical ratios of an angle, illustrating their names by reference to a figure.

'86. **7.** Calculate the value of sec 30°.

'83. **8.** Reduce $\cdot 03375$ and $\cdot 007\dot{9}\dot{9}$ to vulgar fractions.

'81. **9.** Reduce to a vulgar fraction, in its simplest form, $\dfrac{0\cdot\dot{5}\times 1\cdot\dot{7}1428\dot{5}\times 1\cdot\dot{0}76923\dot{0}}{2\cdot\dot{8}5714\dot{2}\times 2\cdot\dot{3}0769\dot{2}\times 14\cdot\dot{5}}$.

'62. **10.** Express $\cdot 2\dot{0}012\dot{3}$ as a fraction, and $\cdot 0\dot{1}\dot{2}+\cdot 00\dot{1}3\dot{2}$ as a recurring decimal.

IV.

'85. 1. Find the factor of the highest dimensions in x and y which is common to

$$x^2-y^2+2y-1 \text{ and } x^3+x^2(y-1)-x(y^2+2y+1)-(y^3+y^2-y-1).$$

'84. 2. Express $u^3+v^3+w^3-3uvw$ in terms of, a, b, c; being given $u=b+c-a$, $v=c+a-b$, $w=a+b-c$.

'74. 3. Divide $a^2-b^2-c^2-2bc$ by $\dfrac{a+b+c}{a+b-c}$.

'83. 4. If a, b, c, d, respectively represent the lengths of four given right lines, construct the square whose area is represented by $\sqrt{abcd}$.

'80. 5. Find a fourth proportional to three given right lines.

'76. 6. Construct a triangle similar to a given triangle, and with a given perimeter.

'85. 7. Determine the height of a chimney, when it is found that walking towards it 100 feet in a horizontal line through the base changes the angular elevation of the top from 30° to 60°.

'70. 8. Define the cosine of an angle; and trace the variations in sign and magnitude of the cosine as the angle increases from 0° to 180°.

'68. 9. Reduce 2 days 9 hours to the decimal of a week.

'66. 10. Reduce 9s. 11¾d. to the fraction of ten shillings.

V.

'84. **1.** Reduce to its simplest form

$$\frac{x^2+3x+2}{x^2+4x+3}+\frac{x^2-3x+2}{x^2-4x+3}+\frac{x^2-12x+35}{x^2-2x-15}.$$

'83. **2.** Multiply together $x^2-(b-c)x-bc$, $x^2-(c-a)x-ca$ $x^2-(a-b)x-ab$; and divide the result by $x^3-(a+b+c)x^2$ $-(bc+ca+ab)x-abc$.

'77. **3.** Find the factors of the expressions:

(α) $3x^3+x^2-8x+4$ (β) $3x^3+7x^2-4$
(γ) x^3+2x^2-x-2 (δ) $3x^3-2x^2-3x+2$.

'80. **4.** If four straight lines be proportional, prove that the rectangle contained by the extremes is equal to the rectangle contained by the means.

'81. **5.** Prove that two rectangles have to one another a ratio which is compounded of the ratios of their sides.

'85. **6.** Express all the trigonometrical ratios in terms of the tangent.

'78. **7.** Determine the sine and cosine of an angle whose tangent equals -2, and whose sine is positive.

'75. **8.** Find the prime factors of 6930, 1470, and 5775, and use them for calculating (1) the sum of the reciprocals, and (2) the square root of the product of the three numbers.

'70. **9.** Prove the rule for finding the greatest common measure of two numbers.

'70. **10.** Reduce the fraction $\frac{17427}{24975}$ to its lowest terms.

VI.

'84. 1. Solve the equation $x^4+x^3-4x^2+x+1=0$.

'80. 2. Find a and b that $x^3+ax^2+11x+6$
and $x^3+bx^2+14x+8$
may have a common factor of the form x^2+Ax+B.

'75. 3. Write down a quadratic equation which has the roots $\alpha-\beta$ and $\alpha+\beta$.

'83 and '84. 4. Prove that similar rectilinear figures are to one another in the same ratio as that of the squares described on a pair of homologous sides.

'74. 5. Similar polygons may be divided into the same number of similar triangles having the same ratio to one another that the polygons have.

'84. 6. Construct a table which shall exhibit the value of any one trigonometrical ratio in terms of the others.

'82. 7. Show how to construct an angle when its sine is given, and apply to the construction of an angle whose sine is $\dfrac{3}{2+\sqrt{5}}$.

'83. 8. The present value of a bill of £479 6s. 6d. is £385, the discount being at the rate of $4\frac{1}{8}$ per cent. per annum. How long has the bill to run?

'77. 9. Find to the nearest shilling the present value of £273, payable after 3 years, the rate of interest being 3 per cent. per annum.

'66. 10. The difference between the simple and compound interest of a certain sum of money for 3 years at 5 per cent. is £1. Find the sum.

VII.

'81. **1.** Find x from the equation $(a^{\frac{1}{2}}+x^{\frac{1}{2}})^{\frac{1}{3}}=(a^{\frac{1}{3}}+x^{\frac{1}{3}})^{\frac{1}{2}}$.

'78. **2.** Extract by the ordinary process or otherwise, and arrange in ascending powers of x, supposed to be less than unity, the square root of the progression $1+x+x^2+x^3+x^4+ \ldots$ etc. to infinity.

'75. **3.** Extract the square root of 10 by the ordinary rules to four places of decimals. How many more places can you get by simple division? By aid of the value $\sqrt{10}$ thus obtained find $\sqrt{\cdot 004}$.

'78. **4.** Prove that two triangles are similar if the sides of one are respectively parallel to those of the other.

'79. **5.** Four rectilinear segments being supposed proportional, show that every four rectilinear figures, similar in form and similarly described on them, are also proportional.

'67. **6.** Show that whatever be the magnitude of the angle A, $\sin A=\cos(90°-A)$.

'61. **7.** Prove that $\tan(180°-A)=-\tan A$.

'85. **8.** Extract the square root of $\cdot 20967241$.

'83. **9.** Find the square root of 11943936, and the value of $\frac{1}{\sqrt{5}}$ to six places of decimals.

'82. **10.** Express $\sqrt{\frac{26\cdot 54\times 0\cdot 004321}{0\cdot 00001357}}$ correctly to the nearest integer.

VIII.

'85. 1. In expressing, as a decimal, a fraction whose denominator contains square roots of numbers, why is it convenient first to transform the fraction into another whose denominator contains no square roots?

'84. 2. Find to five decimal places the value of

$$\frac{\sqrt{7}-\sqrt{5}}{\sqrt{7}+\sqrt{5}}.$$

'74. 3. Simplify $\dfrac{\sqrt{a+b}+\sqrt{a-b}}{\sqrt{a+b}-\sqrt{a-b}}-\dfrac{\sqrt{a+b}-\sqrt{a-b}}{\sqrt{a+b}+\sqrt{a-b}}$.

'77. 4. Given two rectilinear figures, each of any magnitude and form, show how to construct a third which shall be similar to one of them in form and equal to the other of them in area.

'81 and '82. 5. Describe a rectilinear figure which shall be similar to a given rectilinear figure and equal to five times its area.

'69. 6. Solve the equation $\sin x+\cos x=1$.

'67. 7. Solve the equation $\tan\theta=2\sin\theta$.

'66. 8. Find $\sin A$ from the equation $\tan A+\sec A=a$.

'85. 9. Calculate $\dfrac{2}{3-\sqrt{7}}$ to four places of decimals.

'77. 10. Extract the square root of $2+\sqrt{3}$ and $2-\sqrt{3}$.

IX.

'74. **1.** Solve the equation $\sqrt{x^2-8x+31}+(x-4)^2=5$.

'62. **2.** Solve the equation $x^4+x^2-4\sqrt{x^4+x^2-25}=550$.

'73. **3.** Solve the equation

$$\frac{1}{\sqrt{x}-\sqrt{2-x}}-\frac{1}{\sqrt{x}+\sqrt{2-x}}=1.$$

'85. **4.** In a right-angled triangle prove that the rectilinear figure described upon the side opposite the right angle is equal to the similar, and similarly described, figures upon the sides containing the right angle.

'78. **5.** If two similar rectilinear figures be situated in such a manner that two sides of the one are parallel respectively to the two homologous sides of the other, prove that every side of the one is parallel to the homologous side of the other, and that the lines joining homologous vertices all pass through a common point.

'70 and '65. **6.** The sine of an unknown angle x being given, equal to $\sin \alpha$ where α is given, investigate a general expression for the angle x.

'60. **7.** Write down in one formula all the angles which have $\frac{1}{2}$ for their sine.

'61. **8.** Find an expression for all the angles which have the same tangent as a given angle A.

'84. **9.** A person sells out of the 3 per cents at $98\frac{2}{3}$ and invests in railway stock when the £100 shares are at 111; find the dividend which he should receive on each railway share in order to gain 1 per cent. per annum by the transaction.

'68. **10.** A person invested in the 3 per cents. at $94\frac{1}{2}$, and received as interest just £200 a year. What sum did he invest?

X.

'80 and '82. **1.** Prove that a number is divisible by nine, if and not unless, the sum of its digits is divisible by nine.

'80. **2.** What are ashes per 100 loads when 8 more loads for a sovereign lowers the price a penny a load? Of what problem is the negative answer a solution?

'79. **3.** The sum of the squares of two numbers is 1105, and their product is 552 times their difference. What are they?

'82. **4.** If a, b, c, represent the lengths of three given lines, construct the lines whose lengths are represented by $\frac{ab}{c}$ and $\sqrt{a^2+b^2+c^2}$ respectively.

'72. **5.** Prove that, if the four sides of any quadrilateral figure are bisected, the four points of bisection are the four vertices of a parallelogram, of which the area is one half of the area of the quadrilateral figure.

'67. **6.** Solve the equation $\sin^2\theta+\cos^2(90°-\theta)=1$.

'61. **7.** Solve the equation $3\tan^4\theta-10\tan^2\theta+3=0$.

'68. **8.** Determine the trignometrical ratios for an angle of 60°.

'79. **9.** Multiply together $1{\cdot}3\dot{4}$ and $2{\cdot}5\dot{6}\dot{7}$, and extract the square root of the result correctly to seven significant figures.

'80. **10.** An iron bar breaks under a tensile strain of 21 tons per square inch of section. What is this in grammes per square centimetre? [One metre = $39\frac{3}{8}$ inches; a cubic foot of water weighs 1000 ounces. A kilogramme is the weight of a cubic decimetre of water.]

XI.

'85. 1. Find the value of the ratios $x : z$ and $y : z$ which satisfy the equations $2x+3y-7z=0$, $5x-2y-8z=0$.

'61. 2. If $\frac{a}{b}=\frac{c}{d}$, prove that they are each $=\frac{a+c}{b+d}$.

'68. 3. What quantity must be added to each of the terms of the ratio $\frac{a}{b}$, that it may become the ratio $\frac{e}{f}$?

'76. 4. Find the locus of points in a plane equidistant from a given point without the plane.

'71. 5. When is a right line said to be perpendicular to a plane? What is the measure of the inclination of a right line to a plane, of two planes to each other, and of two right lines which do not meet to each other?

'85 and '72. 6. Prove that $\sin(A+B)=\sin A\cos B+\cos A\sin B$, drawing the figure for the case in which A and B are each less than 90°, but $A+B$ greater than 90°.

'86. 7. Prove that $\tan 15°=2-\sqrt{3}$.

'74. 8. Prove geometrically that $\cos(A-B)=\cos A\cos B+\sin A\sin B$, A and B being angles in the second quadrant.

'70. 9. Given the co-ordinates of two points A and B, obtain the co-ordinates of the point which divides the straight line $A\,B$ in a given ratio.

'62. 10. Give diagrams of the locus of the equation $x-y=4$.

NOTE.—Take low values of x or y, and find the corresponding values of y or x. Take them in pairs as co-ordinates of points on the locus, and join the points to give the graph.

XII.

'72. 1. If 4 quantities are proportionals, and the second of them is a mean proportional between the third and the fourth, prove that the third will be a mean proportional between the first and second.

'74. 2. If a, b, c, d, e, f, are all positive, prove that $\frac{a+c+e}{b+d+f}$ lies between the greatest and least of the fractions

$$\frac{a}{b}, \frac{c}{d}, \frac{e}{f}.$$

'71. 3. If 4 numbers are proportionals, prove that

(1) their reciprocals are proportionals;

(2) the greatest and least of them are together greater than the other two.

'75, '80 and '83. 4. If a right line be at right angles to each of two intersecting right lines, prove that it is at right angles to every other right line passing through the point of intersection and lying in the plane of these lines.

'68. 5. If two straight lines be parallel, and one of them be at right angles to a plane, the other also shall be at right angles to the same plane.

'76. 6. Prove $\cos A + \cos B = 2 \cos \frac{A+B}{2} \cos \frac{A-B}{2}$, and investigate a corresponding expression for $\cos A - \cos B$.

'80. 7. Find the simplest form of the expression

$$\frac{\sin 7\theta - \sin 5\theta}{\cos 7\theta + \cos 5\theta}.$$

'80. 8. Find the simplest form of the expression

$$\frac{\cos 6\theta - \cos 4\theta}{\sin 6\theta + \sin 4\theta}.$$

'79. 9. Find the area of the triangle PQR where P is the point (3,4), Q (5,6), R (7,8).

'72. 10. Find the area of the triangle OPQ, O being the origin, P (a,b), and Q (b,a).

XIII.

'73 and '62. **1.** (i) When is one quantity said to vary as another?

(ii) If A varies as B^2, B^3 as C^4, C^5 as D^6, and D^7 as E^4, show that $\frac{A}{E}\times\frac{B}{E}\times\frac{C}{E}\times\frac{D}{E}$ does not vary at all.

'70. **2.** If A varies as B when C is invariable, and A varies as C when B is invariable, prove that A varies as $B.C$ when B and C are both variable.

'61. **3.** If x vary as y, prove x^2+y^2 will vary as x^2-y^2.

'81, '77, '73. **4.** Draw a perpendicular to a given plane from a point which does not lie in the plane.

'74. **5.** If two parallel straight lines in one plane are parallel to two straight lines in another, are the planes necessarily parallel? If not, what can you say of their line of intersection?

63. **6.** Prove that

$$\sin A+\sin 5A+\sin 9A-\sin 15A$$
$$=4\sin 3A.\sin 5A.\sin 7A.$$

'82. **7.** Find the simplest value of the expression

$$\frac{\sin\theta+\sin 3\theta+\sin 5\theta+\sin 7\theta}{\cos\theta+\cos 3\theta+\cos 5\theta+\cos 7\theta}.$$

'61. **8.** Explain what is meant by the locus of an equation in x and y, when x and y are the co-ordinates of a point referred to fixed axes.

'60, '61, '62. **9.** Give diagrams of the loci of

(i) $x^2+y^2=0$, (ii) $x^2-y^2=0$, (iii) $x=3y$.

'69. **10.** Find the locus of a point P which moves so that the sum of the squares of its distances from two fixed points A and B is constant.

XIV.

'83. 1. Find the sum of an odd number of terms of an arithmetical progression, given the middle term and the number of terms.

'85. 2. Prove that the sum of n terms of an arithmetical progression whose first term is a, and common difference b, is $na+\frac{1}{2}n(n-1)b$.

'85. 3. Find the number of terms in the arithmetical progression 39, 33, 27......, whose sum is 144; and explain the existence of two answers to the question.

'74. 4. Two planes are parallel if two straight lines which meet in the one are parallel respectively to two straight lines in the other.

'78. 5. Two angles in space have the limits of one parallel respectively to those of the other. Prove that the angles are equal or supplemental, and that their planes are parallel.

'84. 6. Prove that $\tan(A+B)=\dfrac{\tan A+\tan B}{1-\tan A\tan B}$.

'77. 7. Assuming the formulæ for $\sin(A+B)$ and $\cos(A+B)$ in terms of the sines and cosines of A and B; deduce from them that for $\cot(A+B+C)$ in terms of the cotangents of A, B and C.

'79. 8. Given for three angles α, β, γ, that $\tan\alpha=a$, $\tan\beta=b$, $\tan\gamma=c$, find the value of $\tan(\alpha+\beta+\gamma)$ in terms of a, b, c.

'85. 9. Explain the geometrical meaning of the constants in the equation of a straight line in the form $y=mx+c$.

'58. 10. What is the general form of the equation of the first degree between two variables? Show that it represents a straight line in co-ordinate geometry.

XV.

'84. 1. In boring a well 400 feet deep, the cost for the first foot is 2s. 3d., and an additional penny for each foot following. What is the cost of boring the last foot, and also of boring the entire well ?

'77. 2. If y denote the expression $ax+b$, show that the values which y assumes, when values in arithmetical progression are substituted for x, are themselves in arithmetical progression.

'78. 3. Given that in an arithmetical progression of n terms commencing with unity, the sum is equal to the square of the number of terms ; find by any method the common difference.

'69. 4. If two straight lines be cut by parallel planes, they shall be cut in the same ratio.

'73. 5. If a plane is perpendicular to a straight line, prove that it is perpendicular to every plane passing through the straight line.

'75 and '73. 6. Assuming the fundamental trigonometrical formulæ for $\sin(A+B)$ and $\cos(A+B)$ in terms of sines and cosines of A and B, deduce from them those for $\sin 3A$ and for $\cos 3A$ in terms of $\sin A$ and $\cos A$ respectively.

'73. 7. Apply the formulæ $\cos 3A=4\cos^3 A-3\cos A$ to find $\sin 18°$ and $\cos 36°$.

'62. 8. Establish the equalities $\sin 3A \sin A=\sin^2 2A-\sin^2 A$ and $\dfrac{1+\cos 2A}{\sin 2A}=\cot A$.

'85. 9. Explain the geometrical meaning of the constants in the equation of a straight line in the form $x\cos\alpha+y\sin\alpha=p$.

'84. 10. Establish the equation of a straight line in the forms

$$\text{(i) } \frac{x}{a}+\frac{y}{b}=1. \qquad \text{(ii) } x\cos\alpha+y\sin\alpha=p.$$

XVI.

'82. 1. The first term of a geometrical progression is a, and the tenth term is b; find the n^{th} term.

'74. 2. Find the sum of $1+\frac{1}{3}+\frac{1}{9}+\frac{1}{27}+\ldots$ to infinity; and the sum of the least number of terms of the series differing by less than $\frac{1}{1000}$ from the sum to infinity.

'83. 3. Sum to n terms $a+b, a-b, \frac{(a-b)^2}{a+b}, \frac{(a-b)^3}{(a+b)^2}, \ldots$

'77. 4. Two perpendiculars being supposed let fall from a common point upon two intersecting planes, show that, whatever be the position of the point, the plane of connection of the perpendiculars is perpendicular to the line of intersection of the planes.

'79. 5. Three planes, not having a common line of intersection, being supposed such that two of their three lines of intersection are parallel; show that the whole three are parallel.

'69. 6. Solve the equation $\cos 2x = \cos^2 x$.

'62. 7. Solve the equation $\sin 2x = \cos 3x$.

'63. 8. Solve the equation $\sin 3\theta = \sin 4\theta$.

'82. 9. Find the equation of the line which passes through the intersection of the right lines $2x-3y=10$, $2y+x=6$, and through that of the lines $16x-10y=33$, $12x+14y+29=0$.

'72. 10. Find the equations of the lines OP, OQ, PQ, O being the origin, P the point (a, b) and Q (b, a).

XVII.

'84. 1. Find three numbers in geometrical progression, their product being 1728, and the sum of the extremes 51.

'78. 2. Given that in a geometrical progression of n terms, commencing with unity, the sum is a number consisting of n digits, each equal to unity, determine the common ratio.

'80. 3. Prove that the arithmetic mean of two numbers is greater than their geometric mean.

'86, '82, '75. 4. If a solid angle be contained by three plane angles, prove the sum of any two greater than the third.

'72. 5. If two planes which cut one another be each of them perpendicular to a third plane, their common section shall be perpendicular to the same plane.

'84. 6. Prove that $\tan^2\frac{A}{2}=\frac{1-\cos A}{1+\cos A}$.

'83. 7. Find the value of x which satisfies the equation

$$\sin^{-1}\frac{5}{x}+\sin^{-1}\frac{12}{x}=\frac{\pi}{2}.$$

'81. 8. If $\tan A=\frac{1}{2}$ and $\tan B=\frac{1}{3}$, find values of $\tan(2A+B)$ and $\tan(2A-B)$.

'85. 9. Explain the geometrical meaning of the constants in the equation of a straight line in the form

$$\frac{x-h}{\cos\theta}=\frac{y-k}{\sin\theta}=r.$$

'84. and establish the equation of the straight line in this form.

'81. 10. Find the equation of the right line which joins the point (3, 2) to the intersection of the lines $2x+3y=1$ and $3x-4y=6$.

XVIII.

'83. **1.** Sum to n terms the series

$$a+b+(a-b),\ a+b+2(a-b),\ a+b+3(a-b),\ \ldots.$$

'79. **2.** Sum the series

$$1+2x+3x^2+4x^3+\ldots+nx^{n-1}.$$

'77. **3.** Find the sum of

$$\frac{1}{8}+\frac{4}{8^2}+\frac{6}{8^3}+\frac{3}{8^4}+\frac{1}{8^5}+\frac{4}{8^6}+\frac{6}{8^7}+\frac{3}{8^8}+\frac{1}{8^9}+\ldots \text{ to infinity,}$$

the numerators being 1, 4, 6, 3, recurring.

'85. **4.** In a pyramidal pile of equal spheres on a square base, determine the tangent of the slope of a face and of an edge of the pyramid, the spheres being arranged in square order in horizontal layers.

'84. **5.** Find the sine of the angle between 2 faces, and the sine of the angle between an edge and a face, of a regular tetrahedron.

'78. **6.** A, B, C being the angles of a triangle, show that $\tan A \cdot \tan B \cdot \tan C = \tan A + \tan B + \tan C$.

'76. **7.** If A, B, C are the angles of a triangle, show that

$$\sin A \sin B \sin C = \sin A \cos B \cos C + \sin B \cos C \cos A + \sin C \cos A \cos B.$$

'84. **8.** Find the length of the perpendicular drawn from the point (x, y) on the straight line $x \cos \alpha + y \sin \alpha = p$.

'83. **9.** Find the equations of the perpendiculars from the vertices on the opposite sides of the triangle whose sides are represented by the equations: $3y-x=1$, $3x+y=7$, $x+7y+11=0$.

'78. **10.** Find the equation of the line at right angles to $x+y+1=0$, and through the intersection of $2x-3y+7=0$, and $x+4y+3=0$.

XIX.

'85. 1. Calculate $\log_{10}(\frac{70}{11})$ given $\log_{10} 7 = \cdot 8450980$; $\log_{10} 11 = 1 \cdot 0413927$.

'83. 2. Calculate log 15 and log ·0025, given log 2= ·3010300, log 6=·7781513.

'78. 3. Calculate log 4·5, log 6·75 and log 10·125 to 10 decimal places, if in the same system log 2=·3010299561, log 3=·4771212546.

'76. 4. Prove that two triangular pyramids on equal bases and of equal altitudes, are equal in volume.

'75. 5. When two triangular pyramids are similar, as solid figures, show that their volumes are to each other in the triplicate ratio of the lengths of their homologous sides.

'85 and '71. 6. Prove that the logarithm of the quotient of two numbers is equal to the logarithm of the dividend diminished by the logarithm of the divisor.

'83 and '79. 7. Prove that the logarithm of a product is the sum of the logarithms of the factors of the product.

'80. 8. Find the equations of the right lines which bisect the angles between the lines $2x+3y-5=0$, $3x+2y-7=0$.

'73. 9. Given the 2 right lines $ax+by=c$, $ay-bx=c$; determine their mutual inclination and point of intersection. Find also the equation to a line bisecting internally or externally the angle at which they meet.

'76. 10. Prove that the line whose equation is

$$6x+66y-11=0$$

bisects the angle between the lines

$$15x-18y+1=0$$

$$\text{and } 12x+10y-3=0.$$

XX.

'77. 1. If x, y, z be proportional to given quantities, A, B, C, respectively, and if the sum of their squares be 100, find their values.

'71. 2. Sum the series $1-\frac{1}{2}+\frac{1}{4}-\frac{1}{8}+\ldots\ldots$ to n terms; and show that the sum of any odd number of terms of this series is always greater, the sum of any even number of terms always less, than the sum to infinity. What is the least number of terms of the series which will give a sum differing from the sum to infinity by less than $\cdot 0001$?

'70. 3. Prove that if the squares of three quantities be in arithmetical progression, so also will be the reciprocals of their sums taken two and two together; and give a numerical illustration.

'73. 4. State the relation between the respective volumes of a prism and a pyramid having the same base and altitude; show that any triangular prism may be divided into three equal pyramids, having for a common edge any one of the six diagonals lying in the three rectangular faces of the prism.

'72. 5. OA, OB, and OC are three adjacent edges of a cube. Given $OA=OB=OC=a$, find the solid contents of the pyramid $OABC$, and the area of the triangle ABC.

'86. 6. Prove $\tan(A-B)=\dfrac{\tan A-\tan B}{1+\tan A\tan B}$, assuming the formula for $\sin(A-B)$ and $\cos(A-B)$.

'73. 7. Prove the formula $\cos 3\theta=4\cos^3\theta-3\cos\theta$.

'66. 8. Show geometrically that $\sin 2A$ is less than $2\sin A$.

'80. 9. Being given the bases and the sum of the areas of a number of triangles having a common vertex, show how to find the locus of the vertex by Co-ordinate Geometry.

'76. 10. Prove the following statements with regard to the lines

$$15x-18y+1=0,\ 12x+10y-3=0,\ 6x+66y-11=0,$$

(i.) they meet in a point; (ii.) the first two lines are perdicular to one another.

XXI.

'77. 1. Given log 193·06=2·2856923, log 19307= 4·2857148, find the 7th root of 100 to six decimal places.

'73. 2. Define a logarithm ('79 and '75).
Find log ·00625, given log 2.*

'79 and '85. 3. Wherein lies the convenience of our tables being calculated to base 10? What is the value of $\log_{10} 10$?

'73 and '75. 4. What is meant by a system of logarithms? Find log of $\frac{1}{24}$, having given log 2 and log 3.

'75 and '73. 5. Having given a system of logarithms to the base a, how may the logarithms to the base b be calculated? Find the logarithm of $(\cdot 0003)^5$, given log 3.

'85. 6. Determine the volume and whole surface of a cylinder one foot high, and with a base one foot in diameter.

'79. 7. Three cylinders being supposed to pass through the three vertices, and to have for axes the three opposite sides of a rectilinear triangle. Show that their three curved surfaces are equal in area.

'73. 8. Obtain the general equation to a circle referred to rectangular co-ordinates.

'81. 9. The equation of a circle in rectangular co-ordinates is $x^2+y^2+lx+my+n=0$. Find the co-ordinates of its centre and the length of its radius.

'75. 10. A circle has its centre at the point (a, b), and passes through the origin. Find its equation.

* Log 2=·301030. Log 3=·4771213.

XXII.

'84. 1. In what time would a sum of money, accumulating at 3 per cent. per annum, compound interest, come to five times its original amount?

[Given log 2=·30103, log 2575=3·41078.]

'82. 2. Find the rate of compound interest in order that a sum of money may double itself in 20 years.

[Log 2=·3010300, log 20705=4·3160752, log 20706=4·3160962.]

'81. 3. How many years will it take £100 to accumulate to £1,000 at 4 per cent compound interest?

[Log 2=·3010300, log 13=1·1139434.]

'80. 4. Find an expression for the lateral superficial area of a right cone, and show how to bisect the surface of a right cone by a plane drawn parallel to its base.

'78. 5. The altitude of a right circular cone equals the circumference of its base. Calculate the volume and area of the whole surface of the cone, the radius of the base being given.

'84 and '70. 6. Prove in any triangle

$$\frac{a}{\sin A}=\frac{b}{\sin B}=\frac{c}{\sin C}.$$

'67 and '60. 7. Show that in any triangle

$$\cos A=\frac{b^2+c^2-a^2}{2bc}.$$

'71. 8. What is meant by the 'ambiguous case' in the solution of triangles?

'82. 9. Find the equation of a circle whose centre is at the point (2, 3), and whose circumference passes through the centre of the circle $x^2+y^2+8x+10y=53$.

'84. 10. Find the co-ordinates of the centre and the radius of the circle whose equation, referred to rectangular co-ordinates, is $x^2+y^2-2ax\cos\alpha-2by\sin\alpha-a^2\sin^2\alpha=0$.

XXIII.

'85. **1.** How much should be paid for an annuity of £250 to last for 9 years, reckoning compound interest at 4 per cent.? Express the answer in pounds and decimals of a pound. [Log 1·04 = ·0170333, log 7·02587 = ·8467003.]

'75. **2.** Find the present worth of an annuity of £20 for 5 years, at $3\frac{1}{4}$ per cent., to commence at the end of 20 years. [Log 1·0325 = ·0138901, log 4·495184 = ·6527475, log 5·274702 = ·722198.]

'81. **3.** What is the present worth of a perpetual annuity of £10 payable at the end of first year, £20 at the end of the second, £30 at the end of the third, and so on, increasing £10 each year, interest 5 per cent.?

'80, '79, '78. **4.** If a plane intersect a sphere, prove that their curve of intersection is a circle, and show how to determine its centre.

'78. **5.** Determine the radius of the circle in which a plane cuts a sphere of radius 13 inches, if the distance of the plane from the centre be 12 inches.

'81. **6.** Prove $\tan\frac{A}{2} = \sqrt{\frac{(s-b)(s-c)}{s(s-a)}}$, and write down the corresponding logarithmic equation.

'70. **7.** Prove $\tan\frac{A-B}{2} = \frac{a-b}{a+b}\cot\frac{C}{2}$.

'82. **8.** Prove the formula for the sine of an angle of a triangle in terms of the sides of the triangle.

'79. **9.** The equations, in rectangular co-ordinates, of a circle in a plane, and of a straight line through its centre, being respectively $x^2+y^2=r^2$ and $x+y=0$; find by any method those of the two tangents to the circle which are parallel to the line.

'75. **10.** A circle in a plane has its centre at the point whose rectangular co-ordinates are a and b, and passes through the origin; find the equation of the tangent to it at the origin, and the lengths of the intercepts it cuts off on the axes.

XXIV.

'79. **1.** At what rate of compound interest will a given sum be increased eleven-fold in 100 years?

[Log 11=1·0413927, log 1·1266=0·0517697, log 1·1267=0·0518083.]

'82. **2.** What is the present worth of a perpetual annuity, £10 payable at the end of the first year, £11 at the end of the second year, and so on, increasing £1 per year, interest at 4 per cent.?

'74. **3.** (i.) What is the present value of an annuity of a given amount per annum in perpetuity, when the rate of interest is r per cent.? (ii.) If such an annuity is worth 25 years' purchase, what is the value of an annuity of £1 at the end of the first year, £2 at the end of the second year, £3 at the end of the third year, and so continued for ever?

'74 and '72. **4.** If two spheres intersect, their line of intersection will be a circle which has its centre in the straight line joining the centres of the spheres, and its plane perpendicular to this line.

'72. **5.** Given a circle and a point not in the plane of the circle, find the centre of the sphere which passes through the given point and through the circumference of the circle.

'85. **6.** In a triangle given $a=35$, $b=84$, $c=91$, find tan A, tan B, tan C.

'83. **7.** In a triangle $a=25$, $b=52$, $c=63$, find $\tan \frac{C}{2}$.

'84, '77, '82. **8.** $a=18$, $b=24$, $c=30$; $a=13$, $b=14$, $c=15$; $a=125$, $b=123$, $c=62$; find sin A, sin B, sin C.

'85. **9.** Prove that the straight line $(x-a)\cos\theta+(y-b)\sin\theta=c$ touches the circle $(x-a)^2+(y-b)^2=c^2$, and determine the co-ordinates of the point of contact.

'84. **10.** Find the value of p, in order that the straight line whose equation is $x\cos\theta+y\sin\theta=p$ should touch the circle $x^2+y^2-2ax\cos\alpha-2by\sin\alpha-a^2\sin^2\alpha=0$.

XXV.

'66. **1.** The contents of a basket containing 12 pears are to be distributed amongst 12 persons, so that each person is to have one. In how many ways can this be done? If the largest pear be always given to one particular person, in how many ways could the distribution be effected?

'80. **2.** Find the number of permutations of the letters of the word '*proportion.*' In how many ways can you arrange ten things round the circumference of a circle?

'81. **3.** How many words of three letters, in which the first and third letters are different consonants, and the second a vowel, can be formed with an alphabet of 20 consonants and 5 vowels?

'82. **4.** Draw a small circle on a sphere, so that the area of the spherical cap cut off equals the lateral area of the cone which stands on the circle and has its vertex at the centre of the sphere.

'82. **5.** Prove that the superficial area of a sphere is four times the area of one of its great circles.

'79 and '75. **6.** In a triangle given c, A and B, find a and b.

'77. **7.** In a triangle given a, b, and C, find c, A and B.

'74. **8.** Show that the line $4x-y=17$ passes through the centre of the circle $x^2+y^2-8x+2y=0$. Find the equation to the diameter at right angles to that line, and the co-ordinates of the points where it cuts the circle.

'71. **9.** Obtain the equation to the line joining the centres of the two circles

$$x^2+y^2+2ax+2by+c=0$$
$$x^2+y^2-2bx-2ay+c=0'$$

and find the relation between a, b and c when these circles touch.

'83. **10.** Find the equation of the polar of the point (4, 3) with respect to the circle whose equation is $x^2+y^2-6x-2y=8$.

XXVI.

'82. 1. Find the number of ways in which mn different things can be distributed amongst m persons so that each person may have n of them.

'77. 2. Three persons have four coats, five vests, and six hats between them, in how many different ways can they dress themselves with them?

'69. 3. Show that the number of combinations of n things taken r at a time, is the same as $n-r$ at a time.

'85. 4. Determine the volume and whole surface of a sphere 1 foot in diameter.

'82. 5. If the height of a right cone be equal to half the radius of its base, find the radius of the sphere whose volume equals that of the right cone.

'79. and '75. 6. Given in a triangle the base c, and the two base angles A and B, find in terms of them the altitude h of the triangle.

'78. 7. A vertical pole (more than 100 feet high), consists of two parts, the lower being $\frac{1}{3}$ of the whole. From a point in a horizontal plane through the foot of the pole, and 40 feet from it, the upper part subtends an angle whose tangent is $\frac{1}{2}$. Find the height of the pole.

'80. 8. The lengths of the lines joining 3 points A, B, C, are observed. At any point P in the plane A, B, C, the angles APC and BPC are observed; it is required to find the distance of P from each of the points A, B, C.

'78. 9. Find the equations of the tangents to the circle $x^2+y^2+2Ax+2By+C=0$ which are parallel to the line $x+2y-6=0$.

'77. 10. The equation of a circle in rectangular co-ordinates being $x^2+y^2+2r\cos\phi.x+2r\sin\phi.y+r^2=0$, find the co-ordinates of the centre, the square of its radius, and the common length of the two equal tangents from the origin.

XXVII.

'78. **1.** The operatives in a factory being supposed to consist of a men, b women, c boys, and d girls, required the entire number of different combinations of p men, q women, r boys, and s girls that can be told off from among them for any particular work.

'75. **2.** With five dice how many different throws are possible? How many throws in which no two dice show the same number of eyes? and distinguishing the different dice, in how many different ways may one of the latter throws be obtained?

'85. **3.** How many groups of 4 men can be selected from 12 men so as always to include a particular man?

'84. **4.** Prove that if the vertical angle of a spherical triangle is equal to the sum of the 2 angles at the base, the centre of the circumscribed circle is at the middle point of the base, and the chord triangle is right-angled.

'75. **5.** When of 2 great circles of a sphere, either passes through the 2 poles of the other, show that reciprocally the latter also passes through the two poles of the former.

'79 and '75. **6.** Given in a triangle the base c and the two base angles A and B, find in terms of them the area of the triangle.

'80, '76, '77. **7.** Prove the formulæ for the area of a triangle in terms of (i.) The three sides, (ii.) Two sides and the included angle.

'85, '84, '80. **8.** Find the area of the triangle having given (i.) $a=35$; $b=84$; $c=91$. (ii.) $a=18$; $b=24$; $c=30$.

'80. **9.** Show that the locus of a point from which the tangents to 2 circles are of equal length is a right line, and determine its equation when the equations of the circles are given.

'83. **10.** Find an expression for the length of the tangent drawn from the point (α, β) to the circle $x^2+y^2+ax+by+c=0$. Determine the length of the tangent from the point (13, 8) to the circle $x^2+y^2+22x-2y=278$.

XXVIII.

'79. 1. A sum of £1,000 bearing interest at 5 per cent. is to be paid off in three annual instalments, the payments, including the interest due, to be the same each year, and the first payment to be due at the end of the first year, what must the yearly payment be ?

'73. 2. The arithmetical mean between two numbers is $1+a^2$. And the geometrical mean $1-a^2$. What are the numbers ?

'73. 3. Three numbers are in geometric progression ; the common ratio is equal to the first, and also to nine-tenths of the sum of the second and third. Find the three numbers.

'83. 4. Prove that the area of a spherical triangle is proportional to the excess of the sum of its angles above two right angles.

'83. 5. If this excess be 2^0, state the proportion which the area of the triangle bears to the surface of the entire sphere.

'83. 6. Show in any manner that the area of a circle is equal to half the rectangle under its radius and circumference ; and construct a circle whose area is equal to the difference between the areas of two given circles.

'81. 7. Explain the method of calculating the numerical value of the sine of a small angle, and find the value of $\sin 1^0$ to 6 decimal places.

'83. 8. Explain the method of calculating the numerical value of the cosine of a small angle.

'85. 9. Draw the curves whose equations in polar co-ordinates are : (i.) $r=\sin\theta$; (ii.) $r=\cos\theta$; (iii.) $r=\sec\theta$; (iv.) $r=\operatorname{cosec}\theta$.

'86. 10. Find the radius and the polar co-ordinates of the centre of the circle whose equation is

$$r^2-2\,r\,c\cos(\theta-a)+c^2-a^2=0.$$

XXIX.

'70. **1.** Find the value of a perpetual annuity of £225 per annum at $3\frac{1}{2}$ per cent. rate of interest.

'70. **2.** If the mantissæ of logarithms of 9450, 9451, to the base 10 are 9754318, 9754778, respectively, find the complete logarithm to the same base of 9450666 by the method of proportional parts.

'72. **3.** With 17 consonants and 5 vowels, how many words can be formed having 2 different vowels in the middle and 1 consonant (repeated or different) at each end?

'81. **4.** Being given the length of a right cone and the radius of its base, show how to construct a circle whose area shall be equal to the entire superficial area of the cone.

'81. **5.** Find an expression for the area of the portion of the surface of a sphere cut off by any plane section; and show how to divide the surface of a sphere into any given number of equal parts by drawing parallel planes.

'83 and '80. **6.** Find the areas of the triangles of which the sides are respectively 25, 52, and 63, and 114, 101, and 25.

'84. **7.** Prove that in any triangle
$\frac{a}{\sin A}=\frac{b}{\sin B}=\frac{c}{\sin C}=$ diameter of the circle circumscribing the triangle.

'74. **8.** Given the area of a triangle and two of its sides, show how to find the angles and the third side.

'82. **9.** Find the locus of a point if the sum of its distances from three given right lines be given.

'84. **10.** Prove that the circle

$$x^2+y^2-2ax \cos a-2by \sin a-a^2 \sin^2 a=0$$

intercepts a length $2a$ on the axis of x.

XXX.

'79. **1.** A man 25 years of age can insure his life for £1,000 by paying an annual premium of £18. Taking interest at 5 per cent., what will be an equitable composition for an annual subscription of £3 ?

'78. **2.** The sum of two numbers is 1878, and their product is 880821 ; determine them completely each to the last integer figure.

'78. **3.** Given that the two quadratic functions $a_1x^2+2h_1x+b_1$, and $a_2x^2+2h_2x+b_2$ have a common factor ; prove by any method the equation of condition

$$(a_2b_1+a_1b_2-2h_1h_2)^2=4(h_1^2-a_1b_1)\,(h_2^2-a_2b_2):$$

'79. **4.** Under what conditions will x^3+ax^2+bx+c be divisible by x^2+px+q ?

'79. **5.** There are m white men and n black men, n being greater than m. Find the number of ways in which each white man may have one black servant. If a white man may have any number of servants, in how many ways may each black man have a master ?

'81. **6.** Sum the series $1+2^2x+3^2x^2+\ldots\ldots+(n+1)^2x^n$.

'79. **7.** Ten English labourers can do as much in 6 days as 9 French labourers can do in 7 days ; a Frenchman receives one franc per cubic metre ; how many pence must an Englishman receive per cubic yard that his daily earnings may be 5 per cent. more than a Frenchman's ? [A metre may be taken as equal to $39\frac{3}{8}$ inches, and a franc to tenpence.]

'79. **8.** A cone and a hemisphere being supposed to have equal bases and altitudes, determine the ratios (a) of their convex surfaces, (b) of their entire volumes.

'78, '76, '70. **9.** Prove by elementary or co-ordinate geometry that the locus of points whose distances from two fixed points have a constant ratio, is a circle.

'81. **10.** Find by any method the locus of a point, being given, the sum of the squares of its distances from a number of fixed points.

TEST PAPERS.—SECOND SERIES.

I.—A.

1. Solve the equations :—

$$\text{(i.)}\quad \frac{3x^3+3x^2+2x+6}{3x^2+3x+24}=\frac{x^2+x-4}{x+1}.$$

$$\text{(ii.)}\quad \frac{(x-a)(x-b)}{(x-4a)(x-4b)}=\frac{(x+a)(x+b)}{(x+4a)(x+4b)}.$$

$$\text{(iii.)}\quad \frac{1}{x+3}+\frac{1}{x-1}-\frac{3}{4x-12}=0.$$

2. The sum of the squares of two numbers is 505, and their product is 168. Find the numbers.

3. Solve the systems of equations :—(i.) $yz=p^2$; $zx=q^2$; $xy=r^2$; and (ii.) $x^2+xy+x=12$; $y^2+xy+y=18$.

4. ABC is a triangle; any straight line parallel to BC meets AB at D, and AC at E; join BE and CD meeting at F; show that the triangles ADF and AEF are equal.

5. The sides about the equal angles of triangles which are equiangular to one another are proportionals; and those which are opposite to the equal angles are homologous sides, that is, are the antecedents or the consequents of the ratios.

6. Find the length of an arc on the sea which subtends an angle of 2 minutes 30 seconds at the centre of the earth, supposing the earth to be a sphere of radius 4000 miles ($\pi = 3{\cdot}14159$).

7. Express, in degrees, grades, and circular measure, the angle of a regular octagon.

8. At what time between 8 and 9 o'clock are the hands of a watch together?

9. (i.) Express as decimal fractions $\frac{37}{320}$ and $\frac{39}{800}$.
(ii.) Divide 4·4482 by ·0023, and 56·0917536 by ·128.

10. Calculate the value of the following series to five places of decimals :—

$$1+\frac{1}{1}+\frac{1}{1.3}+\frac{1}{1.3.5}+\frac{1}{1.3.5.7}+\dots.$$

II.—A.

1. There is a number consisting of two digits such that the difference of their cubes is 63 times their difference, and the difference of their squares is 3 times their sum. Find the number.

2. Find the highest common factor of the three expressions:— $3x^3-2x^2-11x+10$, $x^4+7x^3+9x^2-7x-10$, and $2x^3+9x^2+13x+6$.

3. A piece of work can be done by A and B in 4 days, by A and C in 6 days, and by B and C in 3 days. In how many days can it be done if A, B, and C work together?

4. Show how to find a fourth proportional to three given straight lines.

5. *ABC* is a triangle, and a perpendicular is drawn from *A* to the opposite side, meeting it between *B* and *C*; show that, if *AD* is a mean proportional between *BD* and *CD*, the angle *BAC* is a right angle.

6. (i.) Define the trigonometrical ratios of an angle, illustrating their names by reference to a figure.
   (ii.) Calculate the value of cosec 60°.

7. Trace the variations in sign and magnitude of the tangent of an angle as the angle increases from 0° to 180°.

8. Reduce to its simplest form:—

$$\frac{\cdot\dot{5}\times 1\cdot\dot{7}1428\dot{5}\times 1\cdot\dot{0}7692\dot{3}\times 93\cdot\dot{5}7142\dot{8}}{2\cdot\dot{8}5714\dot{2}\times 2\cdot\dot{3}0769\dot{2}\times 14\cdot\dot{5}}.$$

9. Reduce one-third of 2*s*. 7½*d*. to the fraction of a guinea, and express the result as a decimal.

10. Simplify—

$$\frac{2\frac{1}{3}}{5\frac{4}{7}} \text{ of } 1\frac{1}{8} - \frac{15\frac{3}{4} - 4\frac{3}{7} \text{ of } 3\frac{1}{3}}{3\frac{5}{8} \text{ of } \frac{3}{8} + 2\frac{2}{3}}.$$

III.—A.

1. Simplify— $\dfrac{x^2-8x+12}{3x^2-17x-6} - \dfrac{2x^2+5x+2}{6x^2+x-1}$; and find its value when $3x = \sqrt{2}-1$.

2. Factorise the expressions—(i.) x^4+x^2+1;
(ii.) $2x^3-5x^2-14x+8$;
and (iii.) $x^{12}-1$.

3. Solve the equation $x^4+7x^3+9x^2-7x-10=0$.
If the roots of the equation $x^2-5x+6=0$ are α and β, where $\alpha > \beta$, write down the equation whose roots are $\alpha^3+\beta^3$ and $\alpha^3-\beta^3$.

4. Similar triangles are to one another in the duplicate ratio of their homologous sides.

5. A square is inscribed in a right angled triangle ABC, one side DE of the square coinciding with the hypotenuse AB of the triangle; show that the area of the square is equal to the rectangle $AD \cdot BE$.

6. Express all the trigonometrical ratios in terms of the sine.

7. Show how to construct an angle when its cosine is given, and apply to the construction of an angle whose cosine is $\dfrac{3}{1+\sqrt{13}}$.

8. (i.) Prove the rule for finding the greatest common measure of two numbers.
(ii.) Reduce the fraction $\frac{148995}{266805}$ to its lowest terms.

9. Find the present value of £1000 due 3 years hence at $3\frac{1}{4}$ per cent. per annum (Simple Interest).

10. The difference between the Simple and Compound Interest of a certain sum of money for 3 years at 5 per cent. is £6. Find the sum.

IV.—A.

1. Find the square root of

$$\text{(i.)}\ (x-y)^4 - 2(x^2+y^2)(x-y)^2 + 2(x^4+y^4),$$

$$\text{and (ii.)}\ a^2 + 2abx + (b^2-4ac)x^2 - 4bcx^3 + 4c^2x^4.$$

2. Find the value of $\sqrt{35}$ correctly to six places of decimals, and use the result to find the value of $\dfrac{6-\sqrt{35}}{6+\sqrt{35}}$.

3. Simplify $\dfrac{\sqrt{b^2+4ac}-\sqrt{b^2-4ac}}{\sqrt{b^2+4ac}+\sqrt{b^2-4ac}} + \dfrac{\sqrt{b^2+4ac}+\sqrt{b^2-4ac}}{\sqrt{b^2+4ac}-\sqrt{b^2-4ac}}$.

4. Parallelograms which are equiangular to one another have to one another the ratio which is compounded of the ratios of their sides.

5. Show how to describe a rectilinear figure which shall be similar to one given rectilinear figure and equal to another given rectilinear figure.

6. Show that (i.) $\cos(90°+A) = -\sin A$,
and (ii.) $\cos(180°-A) = -\cos A$.

7. Solve the equations:—(i.) $\sin 10\theta - \sin 4\theta = \sin 3\theta$,
and (ii.) $\sin 3\theta + \sin 2\theta + \sin\theta = 0$.

8. Find the square root of 3279051169.

9. Find the square root of $8+2\sqrt{15}$.

10. Calculate to four places of decimals the value of

$$\sqrt{\frac{\cdot 1\dot{7}\dot{4}}{\cdot 4 \times \cdot \dot{0}\dot{2}}}.$$

V.—A.

1. Solve the equations:—

$$\text{(i.)}\quad \sqrt{x^2+6x-23}+(x+3)^2 = 38,$$

$$\text{and (ii.)}\quad \frac{1}{\sqrt{x}-\sqrt{x-3}}+\frac{1}{\sqrt{x}+\sqrt{x-3}} = \tfrac{4}{3}.$$

2. Find, in its simplest form, the product of $\sqrt{6}-\sqrt{2}$ and $\sqrt{2}+\sqrt{3}+\sqrt{\tfrac{1}{3}}$.

3. A number consisting of two digits is such that the square of the sum of the digits exceeds their product by 28, and the square of the difference of the digits is equal to half their product. Find the number.

4. Show how to describe a rectilinear figure which shall be similar to a given rectilinear figure, and equal to seven times its area.

5. In any right-angled triangle, any rectilinear figure described on the hypotenuse is equal to the similar and similarly described figures on the sides containing the right angle.

6. Find an expression for all the angles which have the same cosine as a given angle A; and write down the expressions for all the angles whose cosines are (i.) 0, and (ii.) $\frac{\sqrt{3}}{2}$.

7. Determine the trigonometrical ratios for an angle of 30°.

8. Solve the equation $\tan^5\theta-4\tan^3\theta+3\tan\theta = 0$.

9. A man invests £713 in the 5 per Cents. at $114\frac{7}{8}$, and afterwards sells out at $135\frac{1}{8}$, and invests in the 4 per Cents. at $92\frac{7}{8}$. Find the change in his income (brokerage being $\frac{1}{8}$ per cent.).

10. Multiply together ·0562 and 53·27465, and find the square root of the result correctly to five places of decimals.

VI.—A.

1. If $\frac{a}{b} = \frac{c}{d} = \frac{e}{f}$, prove that

$$ace : bdf = la^3 + mc^3 + ne^3 : lb^3 + md^3 + nf^3;$$

also that $ac + ce + ea : bd + df + fb = pa^2 + qac + rc^2 : pb^2 + qbd + rd^2$.

2. What number must be subtracted from the antecedent and the consequent of the ratio 5 : 6, so that the resulting ratio may be equal to the duplicate ratio of 3 : 4?

3. If $\frac{a}{b} = \frac{c}{d}$, show that $\frac{a+b}{a-b} = \frac{c+d}{c-d}$;

and hence solve the equation $\frac{\sqrt{x+9}+\sqrt{x-7}}{\sqrt{x+9}-\sqrt{x-7}} = 4$.

4. When is a straight line said to be perpendicular to a plane? How are the angles between a straight line and a plane, and between two planes, measured?

5. If two straight lines are at right angles to the same plane, they will be parallel to one another.

6. Prove, geometrically, that

$$\sin(A+B) = \sin A \cos B + \sin B \cos A,$$

each of the angles A and B being greater than 90° and less than 180°, and the angle $A + B$ less than 270°.

7. Show that $\sin A - \sin B = 2 \cos \frac{A+B}{2} \sin \frac{A-B}{2}$,

and $\cos A - \cos B = -2 \sin \frac{A+B}{2} . \sin \frac{A-B}{2}$;

and simplify the expression $\frac{\sin A + \sin 2A + \sin 3A + \sin 4A}{\cos A + \cos 2A + \cos 3A + \cos 4A}$.

8. Show that

$$\sin 2A + \sin 4A + \sin 6A + \sin 8A = 4 \sin 5A \cos A \cos 2A.$$

9. Find the coordinates of the point which divides the line AB in the ratio 5 : 6, the coordinates of A and B being (5, 2) and (−4, 3), respectively.

10. Find the area of the triangle ABC, the coordinates of A, B, and C being (1, 2), (2, 3), (3, 4), respectively; also of the triangle PQR, the coordinates of P, Q, and R being (a, b), $(3a, b)$, and $(2a, 3b)$, respectively.

VII.—A.

1. (i.) When is one quantity said to vary as another? If a varies as b^2, b^4 as c^3, c^4 as d^5, d^7 as e^8, and e^9 as f^7, show that

$$\frac{a}{f} \times \frac{b^2}{f^2} \times \frac{c}{f} \times \frac{d^2}{f^2} \times \frac{e}{f}$$

is invariable.

(ii.) x varies as y directly, and as z inversely, and $x = 12$ when $y = 15$ and $z = 10$; find x when $y = 18$ and $z = 6$.

2. Find the sum of r terms of the Arithmetical Progression—

$$an, \ an + b, \ an + 2b, \ \ldots\ldots$$

3. Find the number of terms in the Arithmetical Progression, 45, 39, 33 ...... whose sum is 189; and explain the existence of two answers to the question.

4. Show how to draw a straight line perpendicular to a plane from a point without it.

5. From a point E draw EC, ED perpendicular to the two planes CAB, DAB, which intersect in AB, and from D draw DF perpendicular to the plane CAB, meeting it at F; show that the straight line CF, produced if necessary, is perpendicular to AB.

6. Find the simplest value of—

$$\frac{\sin 3\theta + \sin 5\theta - \sin 4\theta}{\cos 3\theta + \cos 5\theta - \cos 4\theta}.$$

7. Prove geometrically that $\tan(A + B) = \dfrac{\tan A + \tan B}{1 - \tan A \tan B}$; and find the value of $\tan(A + B + C)$ in terms of the tangents of A, B and C.

8. What is the locus of an equation in x and y, where x and y are the coordinates of a point referred to fixed axis? Give diagrams of the loci of $x^2 - 9y^2 = o$ and $x = 5y$.

9. Show how to get the equation of a straight line in the form $y = mx + c$.

10. Show how to find the distance between two points whose polar coordinates are given; and find the distance between the two points whose polar coordinates are (3, 25°) and (4, 85°).

VIII.—A.

1. The sum of an Arithmetical Progression of n terms, whose first term is unity, is equal to the cube of the number of terms; find the common difference.

2. Find the n^{th} term of a Geometrical Progression whose first term is a and r^{th} term b; also find its sum to n terms.

3. Sum to infinity the series $1 - \frac{1}{4} + \frac{1}{16} - \frac{1}{64} + \dots$.

4. If two straight lines be cut by parallel planes, they shall be cut in the same ratio.

5. If PQ, QR, and RS be three straight lines such that the angles PQR and QRS are right angles and PQ is perpendicular to the plane QRS; then RS will be perpendicular to the plane PQR.

6. Express $\cos 3A$ in terms of $\cos A$, and use the result to find $\sin 18°$. From the value of $\sin 18°$ thus obtained deduce the values of $\cos 18°$ and $\cos 54°$.

7. Solve the equations

$$\text{(i.) } \sin 3x = \cos 4x, \text{ and (ii.) } \cos 3x = \cos 4x.$$

8. Show how to obtain the equation of a straight line in the form $x \cos \alpha + y \sin \alpha - p = 0$.

9. Find the coordinates of the point of intersection of the three perpendiculars from the vertices of the triangle ABC on the opposite sides, the coordinates of A, B, and C being (4, 1), (1, 4), and (6, 6) respectively.

10. Find the equation of the line joining the points (4, 2) and (3, 3), and show that it passes through the point (8, −2). Draw a diagram of the line, marking the positions of the three points on it.

IX.—A.

1. Find the sum to n terms of the series

$$a+b+(a+2b)(a-b)+(a+3b)(a-b)^2+\dots;$$

also the sum to infinity of the series

$$\frac{1}{7}+\frac{3}{7^2}+\frac{5}{7^3}+\frac{1}{7^4}+\frac{3}{7^5}+\frac{5}{7^6}+\dots.$$

2. Prove that the Geometric Mean of two numbers is less than their Arithmetic Mean.

3. If a, b, c, d, e are in Geometrical Progression, show that

$$a^2(a-e)=(a^2-b^2)(a+c).$$

4. If a solid angle be contained by three plane angles, any two of them are together greater than the third.

5. Find the length of the perpendicular drawn from the vertex of a regular tetrahedron to the opposite face; also find the sine of the angle between an edge and a face.

6. Show that $\tan^2\left(45^\circ+\dfrac{A}{2}\right)=\dfrac{\sec A+\tan A}{\sec A-\tan A}$.

7. If A, B, and C are the angles of a triangle, show that

(i.) $\sin 2A+\sin 2B+\sin 2C=4\sin A\sin B\sin C$;

and (ii.) $\sin A+\sin B+\sin C=4\cos\dfrac{A}{2}\cos\dfrac{B}{2}\cos\dfrac{C}{2}$.

8. Find the value of $4\tan^{-1}\frac{1}{5}-\tan^{-1}\frac{1}{70}+\tan^{-1}\frac{1}{99}$. If $\tan A=\frac{1}{7}$ and $\tan B=\frac{1}{3}$, find the value of $\tan(2A+B)$ and $\tan(A+2B)$.

9. Find the equation of the straight line which joins the point $(2, 1)$ to the intersection of the lines $2x-3y=3$ and $x+4y=8$.

10. (i.) Find the length of the perpendicular drawn from the point $(h\ k)$ on the straight line $x\cos\alpha+y\sin\alpha-p=0$.

(ii.) Find the equation of the straight line which passes through the intersection of the lines $x-3y-3=0$ and $2x-5y-9=0$, and is perpendicular to $x-2y+1=0$.

X.—A.

1. If

$$(a+b-4c-4d)(3a-3b-5c+5d) = (3a+3b-5c-5d)(a-b-4c+4d),$$

show that $a : b = c : d$.

2. Sum to infinity the series $\sqrt{3} + \dfrac{1}{\sqrt{3}} + \dfrac{1}{3\sqrt{3}} + \dots$;

also find the sum to n terms of the Arithmetical Progression

$$(x+n)^2 + (x^2+n^2) + (x-n)^2 + \dots .$$

3. Calculate $\log \frac{49}{11}$, $\log 21$, and $\log \cdot 00231$ to base 10, having given $\log 3 = \cdot 4771213$, $\log 7 = \cdot 8450980$, and $\log 11 = 1 \cdot 0413927$ in the same system.

4. Show that a triangular prism may be divided into three equal triangular pyramids; and find the volume of a regular tetrahedron when the length of an edge is a.

5. OA, OB, OC are three adjacent sides of a rectangular parallelepiped, and $OA = a$, $OB = b$, $OC = c$. Find the volume of the pyramid $OABC$.

6. Prove that the logarithm of any power of a number is equal to the product of the logarithm of the number and the index denoting the power. Find $\log_{10} 1089$. (The necessary logarithms are given in question 3.)

7. (i.) Find $\sin 18^\circ$ and $\sin 54^\circ$, and show that they are the roots of the equation $4x^2 - 2x\sqrt{5} + 1 = 0$.

(ii.) If A, B, and C be the angles of a triangle, show that

$$4 \sin A \sin B \sin^2 \frac{C}{2} = \sin^2 C - (\sin A - \sin B)^2.$$

8. Find the angle between the lines $3x - 5y = 6$ and $25x + 12y = 12$, and also the bisectors of the angles between them.

9. Find the angle between the straight lines $2x + 3y = 7$ and $x - y = 1$; also their point of intersection, and the bisectors of the angles between them.

10. Show that the locus of a point which moves so that its distance from a fixed straight line bears a constant ratio to its distance from another straight line, is itself a straight line.

XI.—A.

1. Define a *logarithm*. What are the values of log 1 to any base, and $\log_{10} 10$? Find log ·0016875; given log 2 = ·30103 and log 3 = ·4771213. Calculate the value of the 9th root of 125; given log 17099 = 4·2329707 and log 171 = 2·2329961.

2. In what time would a sum of money amount to three times its original value, at 5 per cent. per annum? (Log 7 = ·845098; log 2 and log 3 given in Quest. 1.)

3. At what rate of Compound Interest will a sum of money double itself in 12 years? (Given log 10594 = 4·02506 and log 105·95 = 2·0251010.)

4. The area of the curved surface of a cone of height h and vertical angle 2α is equal to the area of the curved surface of a cylinder, the radius of whose base is $\frac{h}{\sqrt{3}}$; find the height of the cylinder. If, also, the volumes of the cone and cylinder are equal, find the vertical angle of the cone and the height of the cylinder.

5. Find how far from the base of a cone a plane parallel to the base must be drawn so as to divide the cone into two equal volumes.

6. Prove that, in any triangle, (i.) $\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$,

and (ii.) $\cos A = \frac{b^2 + c^2 - a^2}{2bc}$.

7. Find $\cos A$, $\cos B$, $\cos C$, when $a = 5$, $b = 6$, $c = 7$ are the sides of a triangle.

8. Show how to find the general equation to a circle referred to rectangular coordinates; and find the coordinates of the centre and the radius of the circle $x^2 - 2ax + y^2 = 3a^2$.

9. Find the equation of the circle whose centre is the point (1, 2), and which passes through the centre of the circle

$$x^2 - 4x + y^2 - 8y = 10.$$

10. Find the coordinates of the centres and the radii of the circles $x^2 - 2ax + y^2 - 2by = 0$ and $x^2 - 2bx + y^2 - 2ay = 0$; and determine the coordinates of the centre and the radius of the circle passing through the origin and the centres of these circles.

XII.—A.

1. Find the present value of an annuity of £320, to last for 14 years, reckoning Compound Interest at 4 per cent. (Log 1·04 = ·0170333, log 17316 = 4·2384476, and log 17317 = 4·2384727.)

2. What is the present value of a perpetual annuity of £100 payable at the end of the first year, £105 at the end of the second year, and so on, increasing £5 each year, Compound Interest at 5 per cent.?

3. How many years' purchase is an annuity of £100 worth, the annuity to last for 25 years, and the rate of interest $3\frac{1}{2}$ per cent.? (Log 1·035 = ·0149403, log 2·3632 = ·3735005, and log 2·3633 = ·3735189.)

4. Every section of a sphere by a plane is a circle. Find the radius of the circle in which a sphere of radius 5 inches is cut by a plane at a distance 4 inches from the centre.

5. Find the radius of the circle in which two spheres, whose radii are 9 inches and 12 inches respectively, intersect, the distance between their centres being 15 inches.

6. Prove $$\tan\frac{A-B}{2} = \frac{a-b}{a+b}\cot\frac{C}{2}.$$

Find the value of $\sin\frac{A}{2}$ and $\cos\frac{A}{2}$ in terms of the sides, and hence show that $$\sin A = \frac{2}{bc}\sqrt{s(s-a)(s-b)(s-c)}.$$

7. Find B, a and b in a triangle; given $c = 20$, $A = 30°$, $C = 90°$.

8. Find $\sin A$, $\sin B$, $\sin C$, having given (i.) $a = 11$, $b = 12$, $c = 13$, and (ii.) $a = 20$, $b = 25$, $c = 30$.

9. Find the equation to the tangent at any point of the circle
$$x^2 + y^2 + 2gx + 2fy + c = 0.$$
What is the equation of the tangent at the origin to the circle
$$x^2 - 2ax + y^2 - 2by = 0?$$

10. What is the value of c when $y = mx + c$ touches the circle $x^2 + y^2 = a^2$? Find the coordinates of the point at which the line touches the circle.

XIII.—A.

1. In how many ways can 10 prizes be distributed among ten boys so that each boy may receive one? If two particular boys are certain of carrying off the first and second prizes between them, in how many ways can the distribution be made?

2. How many numbers consisting of 10 digits can be made out of the digits 1111223999?

3. If twice the number of combinations of $2n$ things taken four at a time is equal to 35 times the number of combination of n things taken three at a time, find n.

4. Prove that the volume of a sphere is $\frac{2}{3}$ that of the circumscribing cylinder. If the height of a right cone be 4 times the radius of its base, find the radius of a sphere whose volume equals that of the cone.

5. A right cone, whose height is twice the radius of a sphere, has its volume equal to that of the sphere. Find the radius of the base of the cone.

6. What is the "Ambiguous Case" in the solution of triangles?

7. Given in a triangle $a = 234$, $b = 129$, $C = 84° 24'$, find A and B.

$$\text{Log } 2 = \cdot 3010300,\ \log 7 = \cdot 8450980,\ \log 11 = 1 \cdot 0413927;$$
$$L \cot 42° 12' = 10 \cdot 0425150;\ L \tan 17° 41' = 9 \cdot 5035459;$$
$$L \tan 17° 42' = 9 \cdot 5039822.$$

8. Find the equation of the diameter of the circle

$$x^2 - 4x + y^2 - 2y = 4,$$

which passes through the point (5, 4), and find the coordinates of the points where it meets the circle.

9. Find the equations of the tangents to the circle

$$x^2 - 2ax + y^2 - 2by = c^2,$$

which are parallel to $x + y = 0$.

10. Find the equation of the polar of the point $(5a, 4a)$ with respect to the circle $x^2 + y^2 - 8ax - 6ay + 22a^2 = 0$.

XIV.—A.

1. How many combinations of 10 men can be made out of a company of 15 men, (i.) when the choice is unrestricted, (ii.) when two particular men must be chosen each time?

2. Find the first term of a series of n consecutive odd numbers whose sum is n^n.

3. Sum to infinity the Geometric series $\frac{\sqrt{2}}{\sqrt{2}+1} + \frac{\sqrt{2}}{\sqrt{2}+2} + \ldots$

4. Show how to describe a small circle through three points on a sphere.

5. Show that the excess of the sum of the three angles of a spherical triangle over two right angles is a measure of the area. If the excess be 5°, find the ratio of the area of the triangle to the surface of the sphere.

6. (i.) Find the area of a triangle in terms of two of its sides and the included angle.

(ii.) Find the radii of the circumscribed and inscribed circles of a triangle whose sides are $3a$, $4a$, $5a$.

7. Show how to calculate the value of the sine of a small angle, and find the value of sin 20″ to 7 decimal places.

8. Find the length of the tangent from the point (4, 5) to the circle $x^2+y^2+4x-6y-18=0$.

9. Show that, if a point moves so that the tangents from it to two circles are equal, it will describe a straight line. Find the equation of this line when the circles are $x^2+y^2=0$ and $x^2+y^2+2x+3y-1=0$.

10. Draw the curves whose equations are (i.) $r=a\cos(\theta-\alpha)$ and (ii.) $r\cos(\theta-\alpha)=a$. Find the polar equation of the circle, the polar coordinates of whose centre are (c, α), and whose radius is a.

XV.—A.

1. Find the value of a perpetual annuity of £430 per annum at $3\frac{3}{4}$ per cent. rate of interest.

2. Given log 5632 = 3·7506626 and log 5633 = 3·7507398, find, by the method of proportional parts, log 5·63245.

3. Sum the series $1^2 + 2^2 + 3^2 + \ldots\ldots + n^2$.

4. The sum of two numbers is 842, and their product is 176841. Find the numbers.

5. A cone and a hemisphere stand on equal bases, and the height of the cone is twice that of the hemisphere. Find the ratios of their curved surfaces and of their volumes.

6. Solve the equations (i.) $3 \sin^2 \theta + 2 \sin \theta = 1$;
(ii.) $\left.\begin{array}{l} \sin \theta = \sqrt{2} \sin \phi \\ \text{and } \tan \theta = \sqrt{3} \tan \phi \end{array}\right\}$.

7. The elevation of a tower on a horizontal plane is observed; on advancing 100 feet nearer to the tower its elevation is found to be the complement of the former elevation; on advancing another 100 feet, its elevation is found to be double of the first elevation. Find the height of the tower and the distance of the last station from it.

8. Find the area of the trapezium formed by the lines

$$\frac{y}{a} - \frac{x}{c} = 1, \quad \frac{y}{a} + \frac{x}{b} = 1, \quad \frac{y}{d} - \frac{x}{b} = -1, \text{ and } \frac{y}{d} + \frac{x}{c} = -1$$

referred to rectangular axes.

9. Find the locus of a point which moves so that its distance from the point $(ka, 0)$, is k times its distance from the point $(a, 0)$.

10. Find the equation of the line joining the origin and the point of intersection of the tangents to the circle $x^2 + y^2 = c^2$ at the points $(x'\ y')$ and (x'', y'').

PART V.

MISCELLANEOUS QUESTIONS.

'85. **1.** Determine the volume and whole surface of a cone, a foot high and with base a foot in diameter.

'85. **2.** Prove that :

$$\text{Tan } A - \tan B = \sin (A - B) \sec A . \sec B.$$

'85. **3.** Assuming that the number of permutations of n things r at a time is $n (n-1) (n-2) \ldots\ldots (n-r+1)$, find the number of combinations of n things r at a time.

85. **4.** Find the values of x, y, z which will satisfy

$$2x+3y-7z=0, \quad 5x-2y-8z=0, \text{ and } 3x^2-4y^2+z^2=9.$$

'85. **5.** Being given ten consonants and five vowels, find the number of words that could be formed out of them, each word containing three of the consonants and two of the vowels.

'83. **6.** Reduce to its simplest form

$$\frac{x+1}{x^2+5x+6} - \frac{2(x+2)}{x^2+4x+3} + \frac{x+3}{x^2+3x+2}$$

'83. **7.** Reduce to its simplest form

$$\{a^2(b-c)+b^2(c-a)+c^2(a-b)\} \times \{a^2(b+c)+b^2(c+a)+ c^2(a+b)\} - \{a^4(b^2-c^2)+b^4(c^2-a^2)+c^4(a^2-b^2)\}$$

'83. **8.** A triangle makes a complete revolution round one of its sides; find in terms of the sides the volume and also the superficial area of the solid which is thus generated.

'83. **9.** The sides of a triangle A, B, C, are 25, 52, and 63, find the values of $\tan \frac{A}{2}$, $\tan \frac{B}{2}$

'82. **10.** Prove that a number is divisible by 11 if the sum of the odd digits (*i.e.* the 1st, 3rd, 5th, etc.) exceeds or is less than the sum of the even digits (*i.e.* the 2nd, 4th, 6th, etc.) by a number divisible by 11.

'82. **11.** Determine the condition that x^2+ax+b and $x^3+a'x+b'$ may have a common divisor, $x+c$; and prove that this common divisor will also divide $ax^2+(b-a')x-b'$.

'82. **12.** Reduce to its lowest terms

$$\frac{ax+2}{2a+(a^2-4)x-2ax^2}$$

'82. **13.** Reduce to its lowest terms

$$\frac{x^4+5x^3+6x^2+5x+1}{x^4+3x^3-2x^2+3x+1}.$$

'82. **14.** Solve the equation :

$$\frac{2}{x^2+2x-2}+\frac{3}{x^2-2x+3}=\frac{x}{2}$$

'82. **15.** Being given how to construct a figure similar to one given rectilinear figure and equal to another, apply to the construction of an equilateral triangle of given area.

'81. **16.** Supposing that a steam tug travels 10 miles an hour in still water when alone, but draws a barge 4 miles an hour. It has to take a barge 10 miles up a stream which runs 1 mile an hour, and then to return without the barge. How long will it take for the journey ?

'81. **17.** Find a, b, c, so that both $x^4+ax^3+bx^2+cx+1$ and $x^4+2ax^3+2bx^2+2cx+1$ are perfect squares.

'81. **18.** Assume the House of Commons to consist of Liberals, Conservatives, and Home Rulers. In one division half the Liberals, 100 Conservatives, and two-thirds of the Home Rulers were absent. The Liberals voted with, the Conservatives and Home Rulers against, the Government, which had a majority of 5. On another occasion, all the members voting, the Liberals had a majority of 40 over the Conservatives and Home Rulers combined. On a vote of urgency, 75 Liberals, half the Conservatives, and 10 Home Rulers being absent, the combined Liberals and Conservatives told 8 times as many as the Home Rulers. How many members of Parliament were there ?

'81. **19.** Find two numbers such that their sum may be 24 and the sum of their cubes 5256.

'81. **20.** If the sides of a rectilinear figure all touch the same circle, prove that the area of the figure is equal to the rectangle under its semiperimeter and the radius of the circle. Hence show that the area of a circle is equal to the rectangle under its radius and its semi-circumference.

'82. **21.** A man purchases the leasehold of a house for £3,000, the ground rent being £50 per annum, and the lease having 20 years to run. He lets the house for the whole term. What annual rent should he receive to make 5 per cent. on his outlay ?

[Log 2=0·3010300, log 3=0·4771213, log 7=0·8450980, log 11306=4·0533090, log 11307=4·0533474.]

'80. **22.** A man has in his pocket £10, in sovereigns, shillings, and pence; the total number of coins is 62, and the weight is $23\frac{1}{12}$ ounces. Assuming that (for the purpose of this question only) a sovereign weighs $\frac{1}{4}$ ounce, a shilling $\frac{1}{8}$ ounce, and a penny $\frac{1}{2}$ ounce; how many sovereigns, shillings, and pence has he respectively?

'80. **23.** In finding a fourth proportional to three given straight lines, if the first and second instead of being lines be given rectilinear figures, how can the fourth proportional be found?

'80. **24.** Prove that the volume of a sphere is to that of its circumscribing cylinder in the proportion of 2 to 3.

'79. **25.** Solve the equation

$$\frac{x-a}{b}+\frac{x-b}{a}=\frac{b}{x-a}+\frac{a}{x-b}$$

'79. **26.** What are oranges a gross when 50 more for a sovereign lowers the price twopence a score?

'79. **27.** Express the diameter, d, of the circle circumscribing a rectilinear triangle in terms of the three sides, a, b, c, of the triangle.

'78. **28.** Verify, by actual multiplication or otherwise, the algebraic identity

$$(y-z)^3+(z-x)^3+(x-y)^3=3(y-z)(z-x)(x-y).$$

'78. **29.** Find, by actual multiplication or otherwise, and arrange in ascending powers of x, supposed to be less than unity, the square of the progression

$$1+\frac{1}{2}x+\frac{1}{2}\cdot\frac{3}{4}x^2+\frac{1}{2}\cdot\frac{3}{4}\cdot\frac{5}{6}x^3+\frac{1}{2}\cdot\frac{3}{4}\cdot\frac{5}{6}\cdot\frac{7}{8}x^4 \text{ etc. } \ldots \text{ to}$$

infinity.

'78. **30.** Calculate, to the nearest halfpenny, the true present value of a bill for £152 8s., payable on December 31 and discounted on July 17, at $3\frac{1}{2}$ per cent. interest per annum.

'78. **31.** Find the values of x, y, z, which satisfy the algebraical equations:—

$$ax+by+cz=p^2$$
$$fx+gy+hz=q^2$$
$$x^2+y^2+z^2=r^2.$$

'78. **32.** Assuming the fundamental properties of logarithms, state and prove the ordinary rules by which the logarithm of a number not given in the tables, and conversely the number corresponding to a logarithm not given in the tables, may be calculated from the tables.

'78. **33.** Through the point of intersection of the straight lines whose equations are

$$2x-3y+7=0$$
$$x+4y+3=0,$$

a straight line is drawn at right angles to the axis of x. Find its equation.

'77. **34.** If $k=x\sqrt{1+y^2}+y\sqrt{1+x}$ prove that $\sqrt{1+k^2}=xy+\sqrt{1+x^2}\sqrt{1+y^2}$

'77. **35.** Solve the equations

$$3x+9y-6z=8$$
$$4x-7y+12z=9$$
$$12x-3y-5z=11.$$

'77. **36.** Out of a cask containing 360 quarts of pure alcohol a quantity is drawn off and replaced by water. Of the mixture a second quantity, 84 quarts more than the first, is drawn off and replaced by water. The cask now contains as much water as alcohol. Find how many quarts were taken out the first time. Show that the problem has only one solution.

'77 **37.** Find the L.C.M. of $3x^3+x^2-8x+4$, $3x^3+7x^2-4$, $x+2x^2-x-2$, and $3x^3-2x^2-3x+2$; and also the factors of their sum. Find the sum of their reciprocals.

'77. **38.** Convert the circulating decimal fraction $1\cdot\dot{4}63\dot{1}$ into a vulgar fraction.

'77. **39.** An equilateral triangle being supposed to revolve round the line through its vertex perpendicular to its base; show that the area of the cone generated by either of its sides is double that of the circle generated by either half of its base.

'77. **40.** Three cylinders of equal altitudes being supposed to have for bases the three circles described on the three sides of a right-angled triangle as diameters; show that the volumes of the greatest of them is equal to the sum of those of the remaining two.

'77. **41.** A cone and hemisphere being supposed to have a common base, and to lie at opposite sides of it, required the ratio of the altitude of the cone to the radius of the hemisphere, in order that the volumes of both solids should be equal.

'77. **42.** If $x_1\, y_1$, $x_2\, y_2$, $x_3\, y_3$, be the Cartesian co-ordinates of three points P_1, P_2, P_3, in a plane with respect to any pair of rectangular axes in the plane; determine, to the same axes, the equations of the parallel and of the perpendicular through P_3 to the right line $P_1\, P_2$.

'77. **43.** Assuming the trigonometrical formulæ for sin $(A+B)$, and for cos $(A+B)$ in terms of the sines and cosines of A and B, deduce from them that for tan $(A+B+C)$ in terms of the tangents of A, B, and C.

'77. **44.** The measured lengths of the three sides a, b, c of a plane triangle are 13, 14, 15 feet respectively, calculate in square feet its area Δ.

'76 **45.** Simplify, by reduction, the algebraical sum

$$\frac{(x^2-yz)}{(x-y)\,(x-z)}+\frac{(y^2-xz)}{(y-z)\,(y-x)}+\frac{z^2-xy}{(z-x)\,(z-y)}.$$

'76. **46.** Verify, by multiplication, the algebraical identity
$(bcx+cay+abz-xyz)^2+(ayz+bzx+cxy-abc)^2=$
$(a^2+x^2)\,(b^2+y^2)\,(c^2+z^2)$.

'76. **47.** Extract the square root of $1+2x+3x^2+4x^3+$ etc. to infinity (where x is a proper fraction less than unity), by the ordinary process.

'76. **48.** Reduce to a simple equation and solve for x from:

$$\frac{x-1}{x-2}-\frac{x-2}{x-3}=\frac{x-4}{x-5}-\frac{x-5}{x-6}.$$

'76. **49.** Reduce to a quadratic equation, and solve for x from:

$$\frac{x+a}{x-a}+\frac{x+b}{x-b}+\frac{x+c}{x-c}=3.$$

'76. **50.** The sum of two numbers is 47·437, and their product is 486·7641. Determine them correctly, each to three places of decimals.

'76. **51.** Assuming that one rood of superficial area $=40$ square perches, and that one square perch$=30\frac{1}{4}$ square yards, calculate to the nearest integer the length in feet of each side of a square courtyard of one rood in extent.

'76. **52.** An alphabet being supposed to contain m consonants and n vowels, required the entire number of different words, each containing p of the former and q of the latter, that could be formed out of its letters.

'76. 53. The interest (x) on the less (a) of two given sums of money a and b being supposed equal to the discount (y) on the greater (b) for any fractional period (k) of one year; required in terms of a, b, and k, the value of x or y.

'76. 54. State and prove the ordinary formula for the present value at n per cent. compound interest of a deferred annuity to commence at the expiration p, and to continue for q years.

'76. 55. If x be a large positive number and $\pm\delta$ a comparatively small increase or diminution of it, prove the approximate formula $\log(x\pm\delta)=\log x\pm\mu\frac{\delta}{x}$ where μ=the modulus of the system employed.

'76. 56. Given that to five decimal places $\mu=\cdot43429$ in the ordinary system for which the base=10, calculate to that number of places the logarithms of 999 and of 1001 in that system.

'76. 57. A right cone has the same superficial area as the hemisphere on the same base, show that every plane passing through its axis intersects it in an equilateral triangle.

'75. 58. Given log 3796=3·5793262, log 2984=3·4747988,

$$\left.\begin{array}{l}\log 90714=4\cdot9576743\\ \quad\;\; 90715=4\cdot9576791\end{array}\right\}\text{Calculate to seven decimals } \sqrt[5]{\frac{(\cdot3796)^3}{(\cdot2984)^2}}.$$

'75. 59. Show that when the three pairs of corresponding sides of two rectilinear triangles are proportional, the three pairs of opposite angles are equal.

'75. 60. Solve the equations:

$$\left.\begin{array}{l}\left(\frac{24}{x}\right)^2+(y-4)^2=65\\ \left(\frac{12}{x}\right)^2+9=(5y-20)^2\end{array}\right\}$$

'75. 61. Simplify the expression $\left(\frac{y-z}{x}+\frac{z-x}{y}+\frac{x-y}{z}\right)$

$$\times\left(\frac{x}{y-z}+\frac{y}{z-x}+\frac{z}{x-y}\right)+\left(\frac{z-x}{x-y}-\frac{x-y}{z-x}\right)\left(\frac{y}{z}-\frac{z}{y}\right)$$
$$+\left(\frac{x-y}{y-z}-\frac{y-z}{x-y}\right)\left(\frac{z}{x}-\frac{x}{z}\right)+\left(\frac{y-z}{z-x}-\frac{z-x}{y-z}\right)\left(\frac{x}{y}-\frac{y}{x}\right)$$

75\. 62. Prove that, if the four fractions, $\frac{bx+cy+dz}{b+c+d-a}$, $\frac{cx+dy+az}{c+d+a-b}$, $\frac{dx+ay+bz}{d+a+b-c}$, $\frac{ax+by+cz}{a+b+c-d}$, are equal to one another, their common value will be equal to $\frac{x+y+z}{2}$ as long as $a+b+c+d$ does not vanish; but if $a+b+c+d=0$ the quantities x, y, z must be equal to one another; and then half their common value will be the common value of the fractions.

'75\. 63. Having given the first term, the common difference, and the sum of an arithmetical progression, find the number of terms. If the first term be equal to 27, the fourth term equal to 18, and the sum equal to 117, find the number of terms and the last term.

'75\. 64. Find the present value of an annuity of £10 payable half-yearly, continued for five years at $3\frac{1}{4}$ per cent., to commence at the end of twenty years. [Logs required.]

'75\. 65. Solve the equations

$$\left.\begin{aligned}\frac{x}{b+c}+\frac{y}{c-a}&=a+b\\ \frac{y}{c+a}+\frac{z}{a-b}&=b+c\\ \frac{z}{a+b}+\frac{x}{b-c}&=c+a\end{aligned}\right\}$$

'75\. 66. Solve the equations

$$\frac{1}{x+\frac{1}{y-\frac{1}{x}}}=\frac{1}{x-\frac{1}{y-\frac{1}{x}}},\quad \frac{1}{y}\left(1-\frac{1}{x}\right)=1.$$

'74\. 67. Solve the equations $\frac{x^2}{y}-\frac{y^2}{x}=28$, $x-y=8$.

'75\. 68. Find the condition that a quadratic equation $ax^2+2bx+c=0$ may have equal roots; and, this condition being satisfied, find the roots.

'73\. 69. The equation of a circle being

$$\sqrt{1+m^2}(x^2+y^2)-2cx-2mcy=0;$$

find its radius.

'71. **70.** Find the perpendicular distance of the point ξ, η from the line $y = ax + b$.

'71. **71.** Prove analytically that the three lines drawn from the angles of a triangle, whether to the middle points of the opposite sides or perpendicular to those sides, meet in a point.

'70. **72.** One vertex of a parallelogram is at the origin; the co-ordinates of the two vertices adjacent to this vertex are respectively (x_1, y_1) (x_2, y_2). Find the co-ordinates of the remaining vertex.

'70. **73.** Find the co-ordinates of the centre, and the radius, of the circle, $x^2 + y^2 + 2x - 6y = 0$. Trace this circle, and state in what points it cuts the axes.

'69. **74.** Find the equation to the straight line which passes through the point whose co-ordinates are a and b, and is parallel to $Ax + By + C = 0$.

'69. **75.** Give a diagram showing the position of the circle $x^2 + y^2 - 2x - 2y + 1 = 0$, and determine whether the straight line $x + y = 2 + \sqrt{2}$ is a tangent or not.

'68. **76.** Determine the equation to the straight line which is perpendicular to the straight line $\frac{x}{a} + \frac{y}{b} = 1$, and passes through the point $x = a$, $y = b$.

'68. **77.** Show that the equation of the form $x^2 + y^2 + Ax + By = C$ represents a circle.

'67. **78.** Find the equation to the straight line which joins the intersection of $2x + 3y - 4 = 0$, $x + 2y - 1 = 0$, to the point $x = 2$, $y = 3$; and give a diagram showing the positions of these three straight lines.

'67. **79.** Investigate the condition that the straight line $y = mx + c$ should be a tangent to the circle $x^2 + y^2 + Ax + By = C$.

'66. **80.** Show that the equation to the circle referred to an origin on the circle is of the form $x^2 + y^2 = 2ax + 2by$.

'66. **81.** If the straight line $y = mx + c$ cut the circle (of the last question), determine the equation which represents the two straight lines from the origin to the points of intersection.

'65. **82.** Give a diagram showing the position of the line $2x + 3y + 6 = 0$.

'64. 83. Find the equations to the straight lines which pass through a given point, and make a given angle with a given straight line.

'64. 84. Find the equations to the lines which pass through the origin, and are inclined at an angle of 75° to the straight line $x+y+\sqrt{3}(y-x)=a$.

'63. 85. Find the equation to a straight line which passes through a given point, and cuts off a given area from the co-ordinate axes, determining the condition that this may be possible.

'63. 86. Interpret the equation

$$(x-a)^2+(y-b)^2=c^2.$$

'63. 87. Interpret the equation

$$x^2+y^2+a^2+b^2=2ax+2by.$$

'63. 88. Find the equation to the circle passing through the origin and the points (a, b) (b, a); and determine the lengths of the chords it cuts from the axis.

'62. 89. Show that the equation

$$x \cos a+y \sin a-p=0$$

represents a straight line and a straight line only.

'62. 90. Show how to determine the position and magnitude of the curve represented by the equation

$$Ax^2+Ay^2+Bx+Cy+E=0.$$

'60. 91. A point moves so that the sum of the squares of its distances from the three angles of a triangle is constant. Prove that it moves along the circumference of a circle.

'62. 92. Find the equations to the two straight lines joining the origin to the points of intersection of $x^2+y^2=a^2$ and $y=bx+c$.

'61. 93. Find the conditions that the circle may cut off from the axes chords, of which the lengths are respectively a and b.

'60. 94. Find the equation to the straight line drawn from the given point (h, k) perpendicular to the given straight line $ax+by+c=0$.

'60. 95. Find the equation to the system of circles passing through the point (h, k) and touching the line $ax+by+c=0$.

'59. **96.** What angles do the straight lines $x-ky=0$ and $y-kx=0$ make with each other?

'59. **97.** Interpret the equation $x^2-y^2=0$.

'57. **98.** If $y=2x+3$ be the equation to a straight line, find the equation to the line perpendicular to it from the origin; find also the length of the perpendicular.

'47. **99.** Determine a and b in the equation to a straight line $y=ax+b$, when the straight line passes through two points whose co-ordinates are

$$(x', y') \quad \left(0, \frac{y'}{2}\right).$$

'40. **100.** Find the equation to a circle, when the origin is a point in the circumference, and the axis of x a diameter passing through that point.

PART VI.

ANSWERS TO TEST PAPERS.

I.

1. (i.) $a+b$. (ii.) 1 or $\frac{1}{14}$. **2.** $1\frac{1}{5}$ or $-\frac{1}{2}$.

3. (i.) 3. (ii). $\frac{40}{41}$, or $\frac{41}{40}$. **4.** Euc. vi. 2, 2nd part.

5. Ded. Euc. vi. 2. **6.** 1·15192 miles. **7.** 206265.

8. 2·18 or $2\frac{98}{551}$ hours. **9.** $27\frac{3}{11}$ mins., past 5. **10.** 2·71828.

II.

1. $x=\pm\frac{(a-b)^2}{a+b}$ $y=\pm\frac{4ab}{a+b}$. **2.** 7 and 2. **3.** ± 17 and ± 19.

4. Euc. vi. 4. **5.** Ded. Euc. vi. 4.

9. (i.) ·10032. (ii.) $\cdot 5\dot{4}2857\dot{1}$. **10.** (i.) ·83. (ii.) ·0000415.

III.

1. 75. **2.** 4s. **3.** 384. **4.** Euc. vi. 10. **5.** Ded. Euc. vi. 10.

7. $\frac{2}{\sqrt{3}}$. **8.** $\frac{27}{800}$, $\frac{1}{125}$. **9.** $\frac{7}{655}$.

10. (i.) $\frac{66701}{333300}$. (ii.) $9\cdot\dot{1}735537190082644628099\dot{9}$.

IV.

1. $x+y-1$. **2.** $4(a^3+b^3+c^3-3abc)$. **3.** $a^2-2ac+c^2-b^2$.

4. Find mean proportionals of a and b, and c and d. Construct a square equal to the rectangle under these two lines.

5. Euc. vi. 12. **6.** Take fourth proportionals to the perimeter of the given triangle, the given perimeter, and each side of given triangle in turn. Construct the triangle from these.

7. $50\sqrt{3}$. **8.** See Text Book. **9.** $.339\dot{2}85714\dot{}$. **10.** $\frac{479}{480}$

V.

1. $\dfrac{3x^2-10x+9}{x^2-9}$.

2. $x^6-(a^2+b^2+c^2)\,x^4+(a^2b^2+a^2c^2+b^2c^2)\,x^2-a^2b^2c^2$,
and $x^3+(a+b+c)\,x^2+(ab+ac+bc)\,x+abc$.

3. (α) $(3x-2)(x+2)(x-1)$, (β) $(x+1)(3x-2)(x+2)$,
(γ) $(x-1)(x+1)(x+2)$, (δ) $(x-1)(x+1)(3x-2)$.

4. Euc. vi. 16. **5.** A particular case of Euc. vi. 23.

6. See table on following page. **7.** $\sin=\dfrac{2}{\sqrt{5}}$, $\cos=-\dfrac{1}{\sqrt{5}}$.

8. $2\times3^2\times5\times7\times11$, $2\times3\times5\times7^2$, $3\times5^2\times7\times11$.

9. (1) $\dfrac{11}{11025}$. (2) 242550. **10.** $\dfrac{157}{225}$.

VI.

1. 1 and $\dfrac{-3\pm\sqrt{5}}{2}$. **2.** $a=6$, $b=7$. **3.** $x^2-2ax+\alpha^2-\beta^2=0$.

4. Latter part of Euc. vi. 20. **5.** Former part of Euc. vi. 20.

7. (i.) See text-book; (ii.) To construct a line of length $2+\sqrt{5}$, produce the hypotenuse of a triangle of sides 2 and 1, making the part produced equal to the side of length 2.

8. $5\frac{5}{8}$ years.

9. At simple interest, £250. 9*s.*; at compound interest, £249. 17*s.*

10. £131. 2*s.* $11\frac{25}{81}$*d.*

VII.

1. $a\left(\dfrac{1\pm2\sqrt{-2}}{3}\right)^6$, or $\dfrac{a}{729}(329\pm230\sqrt{-8})$.

2. $1+\frac{1}{2}\cdot x+\frac{1}{2}\cdot\frac{3}{4}\cdot x^2+\frac{1}{2}\cdot\frac{3}{4}\cdot\frac{5}{6}\cdot x^3+\frac{1}{2}\cdot\frac{3}{4}\cdot\frac{5}{6}\cdot\frac{7}{8}\cdot x^4+$ etc.

3. 3·1622; $5-1$ figures; ·063245553, etc.

4. If two intersecting lines are each parallel to two other intersecting lines, then (by Geometry of Book i.) the contained angles are equal, ∴ the triangles are equiangular, ∴ (by vi. 4) they are similar.

5. Euc. vi. 22. **6** and **7.** See Text-book.

8. ·4579. **9.** (i.) 3456; (ii.) ·447214. **10.** 92.

	Sine.	Cosine.	Tangent.	Cotangent.	Secant.	Cosecant.	Versed Sine.
$\sin\theta =$	$\sin\theta$	$\sqrt{(1-\cos^2\theta)}$	$\frac{\tan\theta}{\sqrt{(1+\tan^2\theta)}}$	$\frac{1}{\sqrt{(1+\cot^2\theta)}}$	$\frac{\sqrt{(\sec^2\theta-1)}}{\sec\theta}$	$\frac{1}{\operatorname{cosec}\theta}$	$\sqrt{(2\operatorname{vers}\theta-\operatorname{vers}^2\theta)}$
$\cos\theta =$	$\sqrt{(1-\sin^2\theta)}$	$\cos\theta$	$\frac{1}{\sqrt{(1+\tan^2\theta)}}$	$\frac{\cot\theta}{\sqrt{(1+\cot^2\theta)}}$	$\frac{1}{\sec\theta}$	$\frac{\sqrt{(\operatorname{cosec}^2\theta-1)}}{\operatorname{cosec}\theta}$	$1-\operatorname{vers}\theta$
$\tan\theta =$	$\frac{\sin\theta}{\sqrt{(1-\sin^2\theta)}}$	$\frac{\sqrt{(1-\cos^2\theta)}}{\cos\theta}$	$\tan\theta$	$\frac{1}{\cot\theta}$	$\sqrt{(\sec^2\theta-1)}$	$\frac{1}{\sqrt{(\operatorname{cosec}^2\theta-1)}}$	$\frac{\sqrt{(2\operatorname{vers}\theta-\operatorname{vers}^2\theta)}}{1-\operatorname{vers}\theta}$
$\cot\theta =$	$\frac{\sqrt{(1-\sin^2\theta)}}{\sin\theta}$	$\frac{\cos\theta}{\sqrt{(1-\cos^2\theta)}}$	$\frac{1}{\tan\theta}$	$\cot\theta$	$\frac{1}{\sqrt{(\sec^2\theta-1)}}$	$\sqrt{(\operatorname{cosec}^2\theta-1)}$	$\frac{1-\operatorname{vers}\theta}{\sqrt{(2\operatorname{vers}\theta-\operatorname{vers}^2\theta)}}$
$\sec\theta =$	$\frac{1}{\sqrt{(1-\sin^2\theta)}}$	$\frac{1}{\cos\theta}$	$\sqrt{(1+\tan^2\theta)}$	$\frac{\sqrt{(1+\cot^2\theta)}}{\cot\theta}$	$\sec\theta$	$\frac{\operatorname{cosec}\theta}{\sqrt{(\operatorname{cosec}^2\theta-1)}}$	$\frac{1}{1-\operatorname{vers}\theta}$
$\operatorname{cosec}\theta =$	$\frac{1}{\sin\theta}$	$\frac{1}{\sqrt{(1-\cos^2\theta)}}$	$\frac{\sqrt{(1+\tan^2\theta)}}{\tan\theta}$	$\sqrt{(1+\cot^2\theta)}$	$\frac{\sec\theta}{\sqrt{(\sec^2\theta-1)}}$	$\operatorname{cosec}\theta$	$\frac{1}{\sqrt{(2\operatorname{vers}\theta-\operatorname{vers}^2\theta)}}$
$\operatorname{vers}\theta =$	$1-\sqrt{(1-\sin^2\theta)}$	$1-\cos\theta$	$1-\frac{1}{\sqrt{(1+\tan^2\theta)}}$	$1-\frac{\cot\theta}{\sqrt{(1+\cot^2\theta)}}$	$1-\frac{1}{\sec\theta}$	$1-\frac{\sqrt{(\operatorname{cosec}^2\theta-1)}}{\operatorname{cosec}\theta}$	$\operatorname{vers}\theta$

VIII.

1. Because (1) it avoids the necessity of evaluating *each* term of the denominator; (2) it is easier to divide by an integer than a long fraction; (3) the quotient is more accurate with less labour.

2. ·08392. 3. $\frac{2}{b}\sqrt{(a^2-b^2)}$. 4. Euc. vi. 25.

5. Ded. vi. 25. 6. $x = 90°$ or $0°$. 7. $\theta = n \,.\, 360° \pm 60°$ or $n \,.\, 180°$.

8. $\frac{a^2-1}{a^2+1}$. 9. 5·6457. 10. $\frac{\sqrt{3}+1}{\sqrt{2}}$ and $\frac{\sqrt{3}-1}{\sqrt{2}}$.

IX.

1. $x = 3$ or 5. 2. $x = \pm\sqrt{-26}$ or ± 5.

3. $\frac{1 \pm \sqrt{5}}{2}$. 4. Euc. vi. 31. 5. Ded. vi. 20 and vi. 2.

6. $x = n\pi + (-1)^n \alpha$. 7. $n \,.\, 180° + (-1)^n 30°$. 8. $n\pi + A$.

9. $4\frac{1}{2}$ per cent. 10. 6300.

X.

1. All numbers may be represented, where $a, b, c, \ldots\ldots l$ are the digits, by $a.10^n + b.10^{n-1} + c.10^{n-2} + \ldots\ldots k.10 + l$ which may be decomposed into

$$a(10^n-1)+b(10^{n-1}-1)+c(10^{n-2}-1)+\ldots\ldots+k(10-1) \text{ and}$$
$$a + b + c + \ldots\ldots + k + l$$

the former expression is clearly divisible by 9 $(=10-1)$ and the latter expression was given divisible by 9
∴ the whole expression is divisible by 9. *Q.E.D.*

2. (i.) 50s.; (ii.) the negative answer -500d. is the solution of 'What would be given per hundred loads to have ashes *removed*, when eight loads less removed for a sovereign would raise the price 1d. per load?'

3. ± 24 and ± 23.

4. (i.) Make the rect ab. On c make the rect cd = rect ab, then $d = \frac{a\ b}{c}$.

(ii.) Hypotenuse of triangle with sides a and b is $\sqrt{a^2+b^2}$, and hypotenuse of triangle with sides $\sqrt{a^2+b^2}$ and c, is $\sqrt{a^2+b^2+c^2}$.

5. Ded. Euc. vi. 2. 6. $\theta = 180n \pm 45^0$.

7. $n\pi \pm \frac{\pi}{3}$ or $n\pi \pm \frac{\pi}{6}$. 8. See Text-book.

9. (i.) $3{\cdot}45\dot{2}0987654\dot{3}$; (ii.) 1·857983. 10. 3303014·4

XI.

1. 2 and 1. 3. $\frac{af - be}{e - f}$

4. A circle by Euc. i. 47, the normal and hypotenuse being constant, the base therefore must be constant.

5. Wilson, Defs. 4, 7, 8 and 9. 6. See Text-book.

7. Tan 15^0=tan $(60^0 - 45^0)$, which by formula for tan $(A - B) = \frac{\sqrt{3}-1}{1+\sqrt{3}} = \frac{\sqrt{3}\ \ 1}{\sqrt{3}+1} \times \frac{\sqrt{3}-1}{\sqrt{3}-1} = \frac{4-2\sqrt{3}}{2} = 2 - \sqrt{3}$.

8. Construct by same rules as ordinary figure; the letters are also the same as usual, but some of the signs different.

9. $\left(\frac{l\,x_1 + k\,x_2}{l+k}\right)\left(\frac{l\,y_1 + k\,y_2}{l+k}\right)$, where A is (x_1, y_1), B (x_2, y_2), and $k : l$ the ratio into which the line is divided.

10. A straight line cutting the axis of x in the point 4
and ,, y ,, —4.

XII.

4. Euc. xi. 4.
5. Euc. xi. 8.
6. See Text-book.
7. Tan θ.
8. $-$tan θ.
9. Zero.
10. $\frac{1}{2}(a^2 - b^2)$.

XIII.

1. (i.) See Text-book; (ii.) Let $A=mB^2$ $B^3=nC^4$, $C^5=pD^6$ and $D^7=qE^4$ then $AB^3C^5D^7=mB^2nC^4pD^6qE^4$ or $\frac{ABCD}{E^4}=mnpq$, a constant. 2. See Text-book.

3. Let $x=my$, then $\frac{x^2+y^2}{x^2-y^2}=\frac{(m^2+1)y^2}{(m^2-1)y^2}=\frac{m^2+1}{m^2-1}$, a constant.

4. Euc. xi. 11. 5. No. Parallel to all four lines.

7. Tan 4θ. 8. See Text-book.

9. (i.) A point at origin; (ii.) Two lines bisecting the right angles between the axes; (iii.) A line making an angle of $\tan^{-1}\frac{1}{3}$ with axis of x.

10. A circle.

XIV.

1. The middle term multiplied by the number of terms.

2. This is easily converted into common text-book formula $S=\frac{n}{2}(2a+\overline{n-1}.b)$.

3. 8 or 6. The 7th term + 8th term = zero.

4. Euc. xi. 15.

5. See Euc. xi. 10 and 15. The second part is equivalent to Euc. xi. 15.

6. See Text-book.

7. $\frac{\cot A\cot B\cot C-\cot C-\cot B-\cot A}{\cot B\cot C+\cot C\cot A+\cot A\cot B-1}$

8. $\frac{a+b+c-abc}{1-ab-ac-bc}$. 9. and 10. See Text-book.

XV.

1. £1. 15*s*. 6*d*.; £377. 10*s*.

2. Substitute m, $m+d$, $m+2d$ . . . for x, then y becomes $am+b$, $am+ad+b$, $am+2ad+b$ . . . a series of which the common difference is ad.

3. 2. 4. Euc. xi. 17. 5. Euc. xi. 18. 6. See Text-book.

7. (i.) See Text-book;

(ii.) $\cos. 36^0 = 1 - 2\sin^2 18^0 = 1 - 2\left(\frac{\sqrt{5}-1}{4}\right)^2 = \frac{\sqrt{5}+1}{4}$.

9. and **10.** See Text-book.

XVI.

1. $a\left(\frac{b}{a}\right)^{\frac{n-1}{9}}$. **2.** (i.) $1\frac{1}{2}$; (ii.) $1\frac{364}{729}$.

3. $\frac{(a+b)^n-(a-b)^n}{2b(a+b)^{n-2}}$.

4. By Euc. xi. 18 the plane containing the two perpendiculars is itself perpendicular to each of the two intersecting planes; therefore, by Euc. xi. 19, it is perpendicular to their intersection

5. Compare Euc. xi. 19.

6. $x=n\pi$.

7. $x=n\pi+(-1)^n\frac{\pi}{10}, \frac{\pi}{2}(4n\pm 1)$ and $n\pi+(-1)^n\left(-\frac{3\pi}{10}\right)$.

8. $\theta=2n\pi$, $n\pi+(-1)^n\frac{\pi}{7}$ and $\frac{\pi}{7}(4n\pm1)$.

9. $23y-13x+64=0$.

10. OP is $ay-bx=0$, OQ is $by-ax=0$, PQ is $y+x=a+b$.

XVII.

1. 3, 12 and 48. **2.** $r=10$. **4.** Euc. xi. 20. **5.** Euc xi. 19. **7.** 13. **8.** 3 and $\frac{9}{13}$. **9.** See Text-book.

10. $29y-43x+71=0$.

XVIII.

1. $\frac{n}{2}[(n+3)a-(n-1)b]$. **2.** $\frac{nx^{n+1}-x^n(n+1)+1}{(1-x)^2}$.

3. $\frac{1}{5}$ **4.** $\sqrt{2}$ and $\frac{\sqrt{2}}{\sqrt{2}}(=1)$.

5. $\frac{2}{3}\sqrt{2}$ and $\sqrt{\frac{2}{3}}$. **8.** $x\cos\alpha+y\sin\alpha-p$.

9. The lines $3y-x=1$ and $3x+y=7$ are perpendicular to one another, the equation of the perpendicular to

$x+7y+11=0$ is $7x-y-13=0$.

10. $11x-11y+38=0$.

XIX.

1. ·8037053. **2.** (i.) 1·1760913 ; (ii.) $\bar{3}$·39794.

3. (i.) log 4·5=·6532125531 ; (ii.) log 6·75=·8293038516 ; (iii.) log 10·125=1·0053951501.

4. Wilson, 29 (ii.). **5.** Wilson, 34.

6 and 7. See Text-book. **8.** $x-y-2=0$ $5x+5y-12=0$.

9. Angle $=90°$; point of intersection $\left\{\frac{c(a-b)}{a^2+b^2}, \frac{c(a+b)}{a^2+b^2}.\right\}$

The two bisectors are $x(a+b)-y(a-b)=0$ and $x(a-b)+y(a+b)=2c$.

XX.

1. $x=\pm\frac{10A}{\sqrt{(A^2+B^2+C^2)}}$, $y=\pm\frac{10B}{\sqrt{A^2+B^2+C^2)}}$, $z=\pm\frac{10C}{\sqrt{(A^2+B^2+C^2)}}$ where all the signs must be taken alike.

2. (i.) $\frac{2^n-1}{3(2^{n-1})}$; (iii.) 13. **3.** Illustration 1, 25 and 49.

4. (i.) The prism is three times that of the pyramid ; (ii.) See Wilson 29, end.

5. (i.) $\frac{1}{6}a^3$; (ii.) $\frac{a^2\sqrt{3}}{2}$. **6 and 7.** See Text-book.

9. A straight line.

XXI.

1. 1·930698.

2, 3, and 4. (i.) See answer to 3 in Morning Paper, 1886.

2. (ii.) $\bar{3}$·795880. **3.** (ii.) 1. **4.** (ii.) $\bar{2}$·6197887.

5. (i.) See Text-book ; (ii.) $\overline{18}$·3856065.

6. Vol $=\frac{\pi}{4}$ c. ft.; area $=\frac{3\pi}{2}$ sq. ft. **8.** See Text-book.

9. Centre $\left(-\frac{l}{2}, -\frac{m}{2}\right)$. Radius $=\frac{1}{2}\sqrt{(l^2+m^2-4n)}$.

10. $x^2-2ax+y^2-2by=0$.

XXII.

1. 54·4 years. 2. 3·5265. 3. 58·7.

4. (i.) πra where r is radius of base and a slant side of cone. See Wilson, 33. (ii.) $\frac{1}{\sqrt{2}}$ of height from top.

5. (i.) Vol $=\frac{2}{3}\pi^2 r^3$; (ii.) Area$=\pi r^2\left\{1+\sqrt{4\pi^2+1}\right\}$.

6, 7, and 8. See Text-book.

9. $x^2+y^2-4x-6y-87=0$.

10. Centre $(a \cos \alpha, b \sin \alpha)$. Radius $\sqrt{(a^2+b^2 \sin^2 \alpha)}$.

XXIII.

1. £1858·83125. 2. £47 : 19 : 5¼ approx. 3. £4200.
4. Wilson, 35, and def. 29. 5. 5.
6. (i.) See Text-book.

(ii.) $\text{L} \tan \frac{A}{2}=\frac{1}{2}\{\log(s-b)+\log(s-c)-\log s-\log(s-a)\}+10$.

7 and 8. See Text-book. 9. $y=-x\pm r\sqrt{2}$.

10. $ax+by=0$. Intercepts $2a$ and $2b$.

XXIV.

1. 2·427. 2. £875.

3. (i.) $\frac{100\,A}{r}$ where A is the annuity ; (ii.) £650.

4. Wilson, 37. 5. Wilson, 38.

6. $\tan A=\frac{5}{12}$, $\tan B=\frac{12}{5}$, $\tan C=\tan 90^\circ=\infty$. 7. $\frac{9}{7}$.

8. (i.) $\sin A=\frac{3}{5}$, $\sin B=\frac{4}{5}$, $\sin C=1$;

(ii.) $\sin A=\frac{4}{5}$, $\sin B=\frac{56}{65}$, $\sin C=\frac{12}{13}$;

(iii.) $\sin A=\frac{40}{41}$, $\sin B=\frac{24}{25}$, $\sin C=\frac{496}{1025}$.

9. (ii.) $(a+c.\cos\theta,\ b+c.\sin\theta)$.

10. $a \cos \alpha \cos \theta+b \sin \alpha \sin \theta \pm \sqrt{a^2+b^2 \sin^2 \alpha}$.

XXV.

1. (i.) $\underline{|12}$; (ii.) $\underline{|11}$.

2. (i.) 151200 ; (ii.) $\underline{|9}$ if both directions (clockwise and opposite) be reckoned $\underline{|9} \div 2$ if these are not counted different.

3. 1900. 4. One-fifth of diameter from pole.

5. Wilson, 39 6. $a = \dfrac{c \sin A}{\sin (A+B)}$, $b = \dfrac{c \,.\, \sin B}{\sin (A+B)}$.

7. See Text-book.

8. (ii.) $x+4y=0$; (iii.) Origin and $(8, -2)$.

9. (i.) $x-y+a-b=0$; (ii.) $(a-b)^2=2c$.

10. $x+2y=23$.

XXVI.

1. $\dfrac{\underline{|mn}}{\{\underline{|n}\}^m}$. 2. 172800 ways. 3. See Text-book.

4. (i.) $\dfrac{\pi}{6}$ cubic feet ; (ii.) π feet.

5. Half that of the cone.

6. $c \,.\, \dfrac{\sin A \sin B}{\sin (A+B)}$, or $c \,.\, \dfrac{\tan A \,.\, \tan B}{\tan A + \tan B}$. 7. 120.

8. $PA = \dfrac{bc \sin (\beta+\gamma+A)}{\sqrt{(b^2 \sin^2 \gamma + c^2 \sin^2 \beta + 2bc \sin \beta \sin \gamma \cos (\beta+\gamma+A)}}$
where α, β and γ are the angles at P subtending a, b and c.

9. $x+2y+A+2B \mp \sqrt{5 (A^2+B^2-C)}$.

10. Centre $-(r \cos \phi, r \sin \phi)$. Radius, zero. Length of tangents, r.

XXVII.

1. $\dfrac{a!\, b!\, c!\, d!}{p!\,(a-p)!\, q!\,(b-q)!\, r!\,(c-r)!\, s!\,(d-s)!}$,
where $(a-p)! =$ factorial $a-p$.

2. (i.) 6^5 ; (ii.) 6 ; (iii.) $\underline{|5}$. 3. 165.

4. Ded. on 'the arc between the poles of two great circles measures the angle at which they intersect.'

5. See Wilson on polar triangle just before Theorem 41.

6. $\frac{1}{2}c^2 \frac{\sin A \sin B}{\sin (A+B)}$, or $\frac{1}{2}c^2 \frac{\tan A \cdot \tan B}{\tan A + \tan B}$.

7. See Text-book. 8. (i.) 1470; (ii.) 216.

9. The equation of the radical axis; see Text-book.

10. (i.) $\sqrt{(\alpha^2+\beta^2+a.\alpha+b.\beta+c)}$; (ii.) 15.

XXVIII.

1. £367 4s. $2\frac{79}{1281}$d. 2. $(1+a)^2$, and $(1-a)^2$.

3. $\frac{2}{3}, \frac{4}{9}, \frac{8}{27}$, or $-\frac{5}{3}, \frac{25}{9}$ and $-\frac{125}{27}$.

4. Wilson, 42 Cor. 5. $\frac{1}{360}$.

6. (i.) See Text-book; (ii.) The radius of the circle will be one of the arms of an angle in a semicircle, of which the diameter is the radius of the greater circle, and the other arm the radius of the smaller circle.

7. (i.) See Text-book; (ii.) ·017452. 8. See Text book.

9. (i.) and (ii.) Circles on unit lengths of OY and OX respectively as diameters, and passing through the origin; (iii.) and (iv.) straight lines parallel to axes of OY and OX respectively, and at unit distance from them.

10. Radius $=a$, and coordinates of centre are c, a.

XXIX.

1. £6428$\frac{4}{7}$. 2. 6·9754624.

3. $5 \times 4 \times 17\ (16+1) = 5780$.

4. $\pi r^2 = \pi a(\sqrt{h^2+a^2}+a)$ where r=radius of circle, a= radius of base of cone, and h=height of cone.

5. (i.) Wilson, 39, Cor 1; (ii.) Divide a diameter into the number of equal parts required, and draw planes through the points of division perpendicular to the diameter.

6. (i.) 630; (ii.) 1140. 7. See Text-book.

8. Suppose b and c known, then $\sin A = \frac{2\,\text{Area}}{bc}$, and the triangle may be solved by the usual method for two sides and the included angle. 9. A straight line.

XXX.

1. £46. 6*s*. $5\frac{3}{4}$*d*. **2.** 969 and 909.

4. $a = p + \dfrac{c}{q}$, $b = q + \dfrac{cp}{q}$. **5.** (i.) $\dfrac{|\underline{n}}{|\underline{n-m}}$; (ii.) n^m.

6. $\dfrac{1 + x - (n+2)^2 x^{n+1} + (2n^2 + 6n + 3)\, x^{n+2} - (n+1)^2 x^{n+3}}{1 - x^3}$.

7. 7·643*d*. nearly. **8.** (*a*) $\dfrac{1}{\sqrt{2}}$, (*b*) $\dfrac{1}{2}$. **10.** A circle.

I.—A.

1. (i.) 3 or $-\frac{17}{5}$; (ii.) $\pm 2\sqrt{ab}$ or 0. (iii.) 5 or $-\frac{3}{5}$.
2. ± 21 or ± 8; ± 8 or ± 21.
3. (i.) $x = \pm \dfrac{qr}{p}$, $y = \pm \dfrac{rp}{q}$, $z = \pm \dfrac{pq}{r}$;
(ii.) $x = 2$, $y = 3$, or $x = -\frac{12}{5}$, $y = -\frac{18}{5}$.

4. $\triangle BDE = CDE$ (I., 37). Deduct the common area DEF, $\therefore BFD = CFE$. But $BFD : AFD :: BD : AD$, $CFE : AFE :: CE : AE$, and these ratios are equal; $\therefore BFD : AFD :: CFE : AFE$, $\therefore AFD = AFE$. **5.** Euc. VI., 4.

6. 2·9088 miles. **7.** 135°, 150^g, $\dfrac{3\pi}{4}$ rad.

8. $43\frac{7}{11}$ min. past eight.

9. (i.) ·115625, ·11125; (ii.) 1934, 438·216825. **10.** 2·41068.

II.—A.

1. 63 or 36. **2.** $x + 2$. **3.** $2\frac{2}{3}$ days. **4.** Euc. VI., 12.

5. $BD : DA :: DA : DC$ and $\angle BDA = ADC$, $\therefore$ the triangles BDA, ADC are equiangular (VI., 6);
$\therefore \angle BAC = BAD + DAC = BAD + ABD = ADC =$ right angle.

6. (i.) (ii.) See Text-Book. **7.** See Text-Book.

8. 1. **9.** $\frac{1}{24}$, $\cdot 041\dot{6}$. **10.** $\frac{3}{8}$ or ·375.

III.—A.

1. $\dfrac{14x}{1 - 9x^2}$, $\dfrac{7}{8}$.

2. (i.) $(x^2 - x + 1)(x^2 + x + 1)$; (ii.) $(x - 4)(x + 2)(2x - 1)$;
(iii.) $(x - 1)(x + 1)(x^2 + 1)(x^2 - x + 1)(x^2 + x + 1)(x^4 - x^2 + 1)$.

3. (i.) By guess, 1 and -1 are roots. Divide out and we have a quadratic. Answer is 1, -1, -2, -5. (ii.) $x^2-54x+665=0$.

4. Euclid VI., 19.

5. Let $DEFG$ be the square. The triangles ADG, FEB are similar; $\therefore$ $AD : DG :: EF : EB$;

$$\therefore \text{ rect. } AD \,.\, EB = \text{rect. } DG \,.\, EF = \text{square on } DE.$$

6. See Table, page 74.

7. Take a right angled triangle whose sides AC, CB are 2, 3; then $AB=\sqrt{2^2+3^2}=\sqrt{13}$. Produce BA to D, so that $AD=1$; then $BD=1+\sqrt{13}$. With B centre, BD radius, draw a circle cutting CA produced in E; then $BE=\sqrt{13}+1$, $BC=3$;

$$\therefore \cos EBC = \frac{3}{\sqrt{13}+1}.$$

8. (i.) See Text-Book; (ii.) $\frac{48}{77}$. **9.** £904. 19*s*. $6\frac{126}{221}d$.

10. £786. 17*s*. $8\frac{28}{61}d$.

IV.—A.

1. (i.) x^2+y^2; (ii.) $a+bx-2cx^2$. **2.** 5·916079, ·007052.

3. $\frac{b^2}{2ac}$. **4.** Euclid VI., 23. **5.** Euclid VI., 25.

6. See Text-Book.

7. (i.) $\frac{n\pi}{3}$ or $\frac{1}{7}\left(2n\pi \pm \frac{\pi}{3}\right)$; (ii.) $\frac{n\pi}{2}$ or $2n\pi \pm \frac{2\pi}{3}$.

8. 57263. **9.** $\sqrt{5}+\sqrt{3}$. **10.** 4·6502.

V.—A.

1. (i.) 3 or -9; (ii.) 4. **2.** $3\sqrt{2}+\frac{4}{3}\sqrt{3}-\sqrt{6}$. **3.** 24 or 42.

4. Let AB be any side of the figure. Take a line $LM = 7 \,.\, AB$, and take a mean proportional PQ to AB, LM. Describe on PQ a figure similar to that on AB. It is the one required. For figure on PQ : figure on AB in duplicate ratio of $PQ : AB$, *i.e.*, in ratio equal to $LM : AB$ or 7 : 1. **5.** Euclid VI., 31.

6. (i.) See Text-Book; (ii.) $2n\pi \pm \frac{\pi}{2}$, $2n\pi \pm \frac{\pi}{6}$.

7. See Text-books. **8.** $n\pi$, $n\pi \pm \frac{\pi}{3}$, $n\pi \pm \frac{\pi}{4}$. **9.** £5.

10. (i.) 2·99403533; (ii.) 1·73032.

VI.—A.

1. Put each ratio equal to k, and follow usual method. **2.** $\frac{36}{7}$.

3. (i.) Put $a = bk$, $c = dk$; (ii.) $\frac{\sqrt{x+9}}{\sqrt{x-7}} = \frac{4+1}{4-1} = \frac{5}{3}$, $x = 16$.

4. Wilson, Defs. 2, 9 and 7. **5.** Euc., XI., 6; Wilson, 7 (converse).

6. Draw the figure carefully; the proof is unaltered.

7. (i.) See Text-book; (ii.) $\tan \frac{5A}{2}$.

9. $\frac{10}{11}$, $\frac{27}{11}$. **10.** (i.) 0; (ii.) $2ab$.

VII.—A.

1. (i.) See Text-book. Put $a = pb^2$, $b^4 = qc^3$, $c^4 = rd^5$, $d^7 = se^8$, $e^9 = tf^7$. The expression comes $= pqrst$, a constant. (ii.) 24.

2. $anr + \frac{r(r-1)}{2}b$. **3.** 7 or 9; the eighth and ninth terms cancel each other. **4.** Euclid XI., 11.

5. EC is perpendicular to a plane containing AB, $\therefore$ AB is perpendicular to EC. Similarly, AB is perpendicular to ED, and $\therefore$ to plane ECD. But DF is perpendicular to plane CAB, and $\therefore$ to AB, $\therefore$ DF lies in the plane $CED \perp AB$. $\therefore$ CF is in plane CED, and AB is perpendicular to CF.

6. $\tan 4\theta$. **7.** See Text-books.

8. The locus is that curve on which all points lie whose coordinates x and y satisfy the given equation. (i.) Two straight lines through origin equally inclined to axis of x at angle whose tangent is $\frac{1}{3}$. (ii.) One straight line through origin, inclined to axis of x at angle whose tangent is $\frac{1}{5}$.

9. See Text-book. **10.** See Text-book; $\sqrt{13}$.

VIII.—A.

1. $2(n+1)$. **2.** $a\left(\frac{b}{a}\right)^{\frac{n-1}{r-1}}$, $a\frac{\left\{\left(\frac{b}{a}\right)^{\frac{n}{r-1}}-1\right\}}{\left\{\left(\frac{b}{a}\right)^{\frac{1}{r-1}}-1\right\}}$.

3. $\frac{4}{5}$. **4.** Euclid XI., 17.

5. PQ is perpendicular to plane QRS, $\therefore$ PQ is perpendicular to RS. RS is perpendicular to PQ, QR, and $\therefore$ to plane PQR.

6. See Text-book. $\cos 18° = \frac{1}{4}\sqrt{10 + 2\sqrt{5}}$, $\cos 54° = \frac{1}{4}\sqrt{10 - 2\sqrt{5}}$.

7. (i.) $\frac{1}{7}\left(2n\pi + \frac{\pi}{2}\right)$ or $2n\pi - \frac{\pi}{2}$; (ii.) $2n\pi$ or $\frac{2n\pi}{7}$.

8. See Text-book. **9.** $(\frac{13}{7}, \frac{13}{7})$. **10.** $x + y = 6$.

IX.—A.

1. (i.) $\dfrac{a + b - (a + nb)(a - b)^n}{1 - a + b} + \dfrac{b(a - b)\left\{1 - (a - b)^{n-1}\right\}}{(1 - a + b)^2}$; (ii.) $\frac{25}{114}$.

2. $(\sqrt{a} - \sqrt{b})^2$ is positive, $\therefore \frac{a + b}{2} > \sqrt{ab}$.

3. Take r for common ratio, $\therefore b = ar$, $c = ar^2$, &c. Substitute.

4. Euclid XI., 20. **5.** $\frac{\sqrt{2}}{\sqrt{3}}a$, if a = length of edge; $\frac{\sqrt{2}}{\sqrt{3}}$.

6. $$\tan^2\left(45° + \frac{A}{2}\right) = \left(\frac{1 + \tan\frac{A}{2}}{1 - \tan\frac{A}{2}}\right)^2 = \frac{\left(\cos\frac{A}{2} + \sin\frac{A}{2}\right)^2}{\left(\cos\frac{A}{2} - \sin\frac{A}{2}\right)^2}$$

$$= \frac{1 + \sin A}{1 - \sin A} = \frac{\sec A + \tan A}{\sec A - \tan A}.$$

7. (i.) $\sin 2A + \sin 2B + \sin 2C = 2\sin(A + B)\cos(A - B) + 2\sin C\cos C$
$= 2\sin C\left\{\cos(A + B) - \cos(A + B)\right\} = 4\sin A\sin B\sin C$.

(ii.) $\sin A + \sin B + \sin C = 2\sin\frac{A + B}{2}\cos\frac{A - B}{2} + 2\sin\frac{C}{2}\cos\frac{C}{2}$

$$= 2\cos\frac{C}{2}\left\{\cos\frac{A - B}{2} + \cos\frac{A + B}{2}\right\} = 4\cos\frac{A}{2}\cos\frac{B}{2}\cos\frac{C}{2}.$$

8. $\frac{\pi}{4}, \frac{7}{4}, \frac{19}{22}$. **9** $x - 7y + 5 = 0$.

10. (i.) See Text-book; (ii.) $2x + y = 27$.

X.—A.

1. $\dfrac{a + b - 4c - 4d}{a - b - 4c + 4d} = \dfrac{3a + 3b - 5c - 5d}{3a - 3b - 5c + 5d}$. Add and subtract,

$$\therefore \frac{a - 4c}{b - 4d} = \frac{3a - 5c}{3b - 5d}, \text{ and } \therefore \frac{a - 4c}{3a - 5c} = \frac{b - 4d}{3b - 5d}.$$

Multiply up and cancel, $bc = ad$, $\therefore a : b :: c : d$.

2. $\frac{2}{3}\sqrt{3}$, $n\left\{x^2 + (3n - n^2)x + n^2\right\}$.

3. $\cdot 6488033$, $1\cdot 3222193$, $\bar{3}\cdot 3636120$.

4. Wilson, 29 (ii.), page 36 ; $\frac{a^3}{6\sqrt{2}}$. **5.** $\frac{abc}{6}$.

6. See Text-book ; 3·0370280.

7. (i.) $\frac{\sqrt{5}-1}{4}$, $\frac{\sqrt{5}+1}{4}$, which are the roots of the equation ;

(ii.) $\sin^2 C-(\sin A-\sin B)^2$

$$= 4\sin^2\frac{C}{2}\cos^2\frac{C}{2}-4\sin^2\frac{A-B}{2}\cos^2\frac{A+B}{2}$$

$$= 2\sin^2\frac{C}{2}\left\{2\sin^2\frac{A+B}{2}-2\sin^2\frac{A-B}{2}\right\}$$

$$= 2\sin^2\frac{C}{2}\left\{\cos(A-B)-\cos(A+B)\right\}$$

$$= 4\sin^2\frac{C}{2}\sin A\sin B.$$

8. 90° ; $5x+20y+9=0$ and $20x-5y-21=0$.

9. $\tan^{-1}5$, (2, 1), $\frac{2x+3y-7}{\sqrt{13}} = \pm\frac{x-y-1}{\sqrt{2}}$.

10. Let the lines be $x\cos\alpha+y\sin\alpha=p$, $x\cos\beta+y\sin\beta=q$. Then $m(x\cos\alpha+y\sin\alpha-p)=n(x\cos\beta+y\sin\beta-q)$ is the locus for ratio $n:m$.

XI.—A.

1. See Text-book ; $\bar{3}$·2272439 ; 1·7099794.

2. 22·045 years. **3.** £5. 19*s*. nearly. **4.** 120° ; $3h$

5. $h\left(1-\frac{1}{\sqrt[3]{2}}\right)$. **6.** See Text-book. **7.** $\frac{5}{7}$, $\frac{12}{35}$, $\frac{1}{5}$.

8. See Text-book ; $(a, 0)$ and $2a$. **9.** $x^2+y^2-2x-4y=0$.

10. $(a+b)(x^2+y^2)-(a^2+b^2)(x+y)=0$.

XII.—A.

1. £3380. 4*s*. nearly. **2.** £4000. **3.** 16·4. **4.** Wilson 35 ; 3 inches.

5. $7\frac{1}{5}$ inches. **6.** See Text-book. **7.** 60°, 10, $10\sqrt{3}$.

8. (i.) $\frac{1}{15}\sqrt{105}$, $\frac{12}{143}\sqrt{105}$, $\frac{1}{11}\sqrt{105}$, (ii.) $\frac{\sqrt{7}}{4}$, $\frac{5\sqrt{7}}{16}$, $\frac{3\sqrt{7}}{8}$.

9. See Text-book ; $ax+by=0$.

10. $c=\pm a\sqrt{1+m^2}$; $\left(\mp\frac{am}{\sqrt{1+m^2}},\ \pm\frac{a}{\sqrt{1+m^2}}\right)$.

XIII.—A.

1. $2 \,.\, 8! = 80640$. **2.** $\frac{10!}{4!\,2!\,1!\,3!} = 12600$. **3.** 4.

4. Wilson, 40; radius = that of base of cone.

5. Radius of base of cone = $\sqrt{2}$. radius of sphere.

6. See Text-book. **7.** 65° 29′ 34″ ·6, 30° ·6′ ·25″ ·4.

8. $x - y = 1$, $\left(2 \pm \frac{3}{\sqrt{2}},\ 1 \pm \frac{3}{\sqrt{2}}\right)$.

9. $x + y = \pm\sqrt{2(a^2+b^2+c^2)} + a + b$. **10.** $x + y - 10a = 0$.

XIV.—A.

1. (i.) 3003; (ii.) 1287. **2.** $n^n - n + 1$. **3.** 2.

4. Pass a plane through the points in question; it cuts the sphere in circle required.

5. Wilson, 42, Cor.; $\frac{\pi r^2}{36}$. **6.** (i.) See Text-book; (ii.) $\frac{5a}{2}$, a.

7. (i.) $\sin\theta$ lies between θ and $\theta - \frac{\theta^3}{6}$ (see Text-book), θ being the angle in circular measure. For a small angle, θ is near enough.

(ii.) Circular measure of 20″ = $\frac{20\pi}{180 \times 60 \times 60}$ = ·000096962, whose cube can be neglected.

8. 3. **9.** See Text-book; $2x + 3y = 1$.

10. (i.) A circle passing through the origin, whose diameter = a, and the diameter through the origin is inclined at an angle α to the initial line. (ii.) A straight line, the perpendicular on which from origin is of length a and inclined at angle α to the initial line. (iii.) $r^2 + c^2 - 2rc\cos(\theta - \alpha) = a^2$.

XV.—A.

1. £11466. 13*s.* 4*d.* **2.** ·7506973. **3.** See Text-book.

4. 401 and 441. **5.** Curved surfaces are as $\sqrt{5} : 2$; volumes are equal.

6. (i.) $n\pi - (-1)^n \frac{\pi}{2}$; (ii.) $\theta = n\pi \pm \frac{\pi}{4}$, $\phi = n\pi \pm \frac{\pi}{6}$.

7. $60\sqrt{15}$ feet. **8.** $\frac{(a+d)(b+c)}{2}$.

9. $x^2 + y^2 - \frac{2ka}{1+k}x = 0$; a circle.

10. $x(x' - x'') + y(y' - y'') = 0$.

ANSWERS TO MISCELLANEOUS QUESTIONS.

1. $\pi/12$ cubic feet. **3.** Algebra: H.S. § 408, H. and K. § 348.

4. $x = 2,\ y = 1,\ z = 1$; or, $x = -2,\ y = -1,\ z = -1$.

5. 144000 (see Q. 52). **6.** $\dfrac{2}{(x+1)(x+2)(x+3)}$.

7. $2abc\,(b-c)(c-a)(a-b)$.

8. If c is the fixed side, volume $= \frac{1}{3}\pi ch^3$, surface $= \pi(a+b)h$, where $h = 2\sqrt{\{s(s-a)(s-b)(s-c)\}}/c$. **9.** Tan $\frac{1}{2}A = \frac{1}{5}$, tan $\frac{1}{2}B = \frac{1}{2}$.

11. Conditions are $c^2 - ac + b = 0$, and $c^3 + a'c - b' = 0$.

12. $\dfrac{1}{a-2x}$ **13.** $\dfrac{x^2+x+1}{x^2-x+1}$. **14.** $x = 0$, or $\pm\sqrt{5}$, or $\pm\sqrt{(-2)}$.

16. 4 h. $14\frac{6}{11}$ m.

17. $a = 2,\ b = 3,\ c = 2$; or, $a = -2,\ b = 3,\ c = -2$; or, $a = 2,\ b = 1,\ c = -2$; or, $a = -2,\ b = 1,\ c = 2$.

18. 250 Conservatives, 350 Liberals, 60 Home Rulers—Total, 660.

19. 7 and 17. **21.** £302. 15*s*. $3\frac{1}{2}$*d*.

22. 9 sovereigns 17 shillings 36 pence.

24. Wilson, page 58 (Theor. 40).

25. $x = 0$, or $a + b$, or $(a^2+b^2)/(a+b)$. **26.** 9*s*. (approximately).

27. $d = \dfrac{abc}{2\sqrt{s(s-a)(s-b)(s-c)}}$. **29.** $1 + x + x^2 + x^3 + x^4 + \ldots$.

30. £149. 19*s*. $11\frac{1}{2}$*d*. **32.** H.S. Trig., § 162; L., §§ 221—224.

33. $11x + 37 = 0$. **35.** $x = \frac{167}{229},\ y = \frac{614}{687},\ z = \frac{892}{687}$.

36. 60; the other root 576 being inadmissible.

37. L.C.M. is $(x-1)(x+1)(x+2)(3x-2) = 3x^4 + 4x^3 - 7x^2 - 4x + 4$.

Sum is $= 2x\,(5x^2 + 4x - 6)$.

$$\text{Sum of reciprocals} = \frac{6x}{(x-1)(x+1)(x+2)(3x-2)}.$$

38. $\frac{1330}{909}$ or $1\frac{421}{909}$. **41.** 2 : 1.

42. $(y-y_3)(x_1-x_2) = (x-x_3)(y_1-y_2)$ for parallel;
$(y-y_3)(y_1-y_2) + (x-x_3)(x_1-x_2) = 0$ for perpendicular.

43. $\tan(A+B+C) = \dfrac{\tan A + \tan B + \tan C - \tan A \tan B \tan C}{1 - \tan B \tan C - \tan C \tan A - \tan A \tan B}$;

$$\cot(A+B+C) = \frac{\cot A \cot B \cot C - \cot A - \cot B - \cot C}{\cot B \cot C + \cot C \cot A + \cot A \cot B - 1}.$$

44. $\triangle = 84$ sq. ft. **45.** 0. **47.** $1 + x + x^2 + x^3 + \ldots$. **48.** $x = 4$.

49. $(a+b+c)\,x^2 - 2\,(bc+ca+ab)\,x + 3abc = 0$, whence

$$x = \frac{bc+ca+ab \pm \sqrt{\{b^2c^2 + c^2a^2 + a^2b^2 - abc\,(a+b+c)\}}}{(a+b+c)}.$$

50. 15·012 and 32·425. **51.** 104 ft. **52.** $\frac{m!\,n!\,(p+q)!}{p!\,q!\,(m-p)!\,(n-q)!}$.
(N.B. The same letter is not supposed to occur twice in a word.)

53. $x = b - a$.

54. Present Value $= \frac{A}{r(1+r)^p}\left\{1 - \frac{1}{(1+r)^q}\right\}$, where $r = n/100$.

56. 2·99956 and 3·00043. **58.** ·9071440. **59.** Euc. vi., 5.

60. $x = \pm 3$, $y = 3$ or 5. **61.** 0.

63. 13 terms, last term -9; or, 6 terms, last term 12.

64. £48. 1*s*. 6½*d*.

65. $x = (b^2 - c^2)\frac{(a^2-b^2)(c-a)+(a+b)(c+a)^2-(b+c)(c+a)(a-b)}{(b+c)(c+a)(a+b)-(b-c)(c-a)(a-b)}$,
with similar expressions for y and z.

66. $x = 2$, $y = \frac{1}{2}$. **67.** $x = 16$, $y = 8$; or, $x = -8$, $y = -16$.

68. $b^2 = ac$, $x = -b/a$. **69.** Radius $= c$. **70.** $\frac{\eta - a\xi - b}{\sqrt{a^2+1}}$.

71. C. Smith, § 34 Ex.; Puckle's Conics, § 78. **72.** $x_1 + x_2$, $y_1 + y_2$.

73. Centre $(-1, 3)$, radius $\sqrt{10}$; cuts axis of x at origin and $x = -2$, axis of y at origin and $y = 6$. **74.** $A(x-a) + B(y-b) = 0$.

75. Circle touches axes at distance 1 from origin. The line is a tangent. **76.** $a(x-a) = b(y-b)$. **77.** C. Smith, § 65. Le S., § 54.

78. $7(2x+3y-4) - 9(x+2y-1) = 0$; or, $5x + 3y - 19 = 0$.

79. $(A+Bm)^2 + 4(1+m^2)C - 4Bc - 4c^2 = 0$.

81. $c(x^2+y^2) = 2(ax+by)(y-mx)$.

82. Intercepts on axes are -3, -2. **84.** $x = 0$, and $x\sqrt{3} + y = 0$.

86, 87. C. S., § 65, Le S., § 54; the second represents a "point circle."

88. $(x^2+y^2)/(a^2+b^2) = (x+y)/(a+b)$; lengths of chords are $= (a^2+b^2)/(a+b)$. **89.** C. Smith, § 19; Le S., § 19. **90.** C. S., § 65.

91. Compare C. S., Chap. iv., Ex. 3; Le S., Chap. iii., Ex. 18.

92. $c^2(x^2+y^2) = a^2(y-bx)^2$.

94. $(x-h)/a = (y-k)/b$. **95.** $\tan^{-1}\{(k^2 \sim 1)/2k\}$.

97. Two straight lines bisecting the angles between the axes.

98. $2y + x = 0$; $3/\sqrt{5}$. **99.** $a = y'/2x'$, $b = y'/2$.

100. $x^2 + y^2 - 2ax = 0$.

PART VII.

RECAPITULATION.

Addressed to those Students who have no private Tutor to show them how to use time to the best advantage near the Examination.

After you have gone carefully through Lessons I.—XXX., worked all the Test Papers, and acquired a fair knowledge of all the subjects given in the Regulations, which should be not later than two months before the Examination, you should work through the Course again, paying special attention to the most important parts mentioned in the Course, and the hints given in the following, recapitulating several lessons each week, *e.g.*:

First Week.	Revision of	Lessons	I.—V.
Second „	„	„	VI.—IX.
Third „	„	„	XI.—XV.
Fourth „	„	„	XVI.—XIX.
Fifth „	„	„	XXI.—XXIII.
Sixth „	„	„	XXIV.—XXV.
Seventh „	„	„	XXVI.—XXVIII.
Eighth „	„	„	I.—XXX.

Work one or two questions from each paper, and look up all book-work given.

ALGEBRA.

I. There is generally a Quadratic Equation.

II. Very important.

III. Work the Ex. after Note on page 194, H. Smith, or H.K. § 209.

IV. and V. Revise your solutions to Test Paper Questions on factors.

VI. Rather stiff; fairly important. If you find it difficult, do not spend too much time over it.

VII. Of little importance for this Examination.

VIII. Often required. Revise your solutions. Never mind book-work.

IX. Not very important.

XI. and XII. Not often given. Ex. 2, Test Paper. XI. is the commonest type, and the process is required in establishment of tangent formula in Trigonometry.

XIII. Still down in Syllabus, but no question since '73.

XIV.—XVIII. Of very greatest importance. You must get up all in the small *Algebra* thoroughly, including book-work. Some of the series require more than is given in the ordinary elementary text-books. C. Smith's *Algebra* contains some useful worked examples.

XIX. and XXI. Get up all given in the small *Algebra* thoroughly, including book-work. Revise your solutions of Test Paper XXI., 1 to 5.

XXII. Very important.

XXIII. Of very greatest importance. One given nearly every year.

XXIV. Revise Solutions, Test Papers XXII.—XXIV.

XXV.—XXVII. Very important. Get up book-work set in Test Papers and Miscellaneous Questions.

Work a few examples in the Matriculation subjects not revised, especially Simple and Simultaneous Equations and Fractions.

TRIGONOMETRY.

I. and II. The part relating to Circular Measure is alone fairly important.

III. You will know thoroughly by its constant application.

IV. Remember *all—sin,—tan,—cos* are the positive Trigonometrical Ratios in the 1st,—2nd,—3rd, and 4th quadrants respectively. This important part is not given as often at London as might be expected. Only two questions on it have been set for twenty-five years.

V. and VI. Of fundamental importance.

VII. Comes constantly into practice.

VIII. Work through illustrative Examples of H. Smith, §§ 107, 108; Lock, § 119.

IX. Constantly required, *but very few questions are set at London on* I.—IX.

XI. Most important lesson of all. Get up book-work.

XII. and XIII. Cannot be learnt at the eleventh hour. Fairly important.

XIV. Have up thoroughly.

XV. and XVI. Learn the formulæ at least.

XVII. Master H.S., §§ 133, 134, 137; L., 176, 187.

XVIII. If you have not previously got up well, better leave them alone.

XIX. Same as Algebra, XIX.

XXI. See Twenty-first Week's Work.

XXII. Of fundamental importance, especially H.S., 178, 179, and 210—212; L., 238—240, 261, 262. Work through this lesson. Express sine rule in logs.

XXIII. All the Twenty-third Week's Work is very important. Be able to answer 6, 7, and 8 of Test Paper XXIII. Express the rules in logs., and work examples.

XXIV. Revise your solutions to Test Paper XXIV., and be on the look-out for the common 3, 4, 5, and 5, 12, 13, relations of the sides of a right-angled triangle.

XXV. Have the formulæ at your finger-ends.

XXVI. Work through H. Smith, Chap. XIX.; Lock, Chap. XVIII., and solutions 6 and 7 (not 8) to Test Paper XXVI.

XXVII. Make certain of §§ 219—223 in H. Smith, §§ 273—276 in Lock. There have been eight questions on this in the last twelve papers.

XXVIII. Know H.S., §§ 224—226, and L., §§ 283—287, and omit rest without you are strong in Trigonometry.

GEOMETRY.

The student need not revise Books I.—IV.—questions are never set on them; but, of course, the proofs of

Books VI. and XI. require an acquaintance with the enunciations.

Euc. VI.—Pay special attention to Props. **2**, 4, 10, **12**, 16, **19** (very important), 22 and **25**. If you are pressed for time, omit 7, 20, 21, 26—28, and 29.

Euc. XI.—See Schemes of Work, and lay stress on those marked important. In this book nearly every rider and proposition should be deduced from first principles. Often in examinations, when a proposition is written in language a little varied, very many students treat it as a rider, assume the proposition, and simply go a little roundabout to prove that it is identical with the proposition, a process which gains no marks.

MENSURATION.

Lesson XVIII. is not very important. Work questions in Test Paper XVIII. which have *not* a *direct* bearing on book-work.

XIX. Theorems 29 and 34 are alone important.

XXI. Easy and important. Remember formula for volume.

XXII. Theorem 33 and its first Cor. have provided eight questions in seventeen years.

XXIII. Prop. 35, with introduction and Cors., is of very greatest importance. Work through all your solutions XXI. to XXIII.

XXIV. Fairly important. If, however, you did not fully realise Prop. 38 before, you had better not spend time over it now, as it is rather stiff.

XXV. All very important, especially the Cors. in I. The height refers to the zone, not to a distance from the centre of the sphere.

XXVI. Of very great importance.

XXVII. and XXVIII. The student who has not already mastered these should devote his time to something else. The formula of Prop. 42 Cor. might be got up though. Work XXVIII., 5.

There is one question on the sphere almost every year.

ARITHMETIC.

II. and III., VII. and VIII. are very important. Work through your Arithmetic solutions to these Test Papers, and a few examples in the Matriculation requirements beyond Interest.

CONICS.

XI.—XV. All of very greatest importance. The book-work questions are generally given from these and XXI. Be able to establish the equation of the line in the *tangent* (m), *intercept*, and *perpendicular* (p) forms, and know the meanings of the constants. Be able to answer all Test Papers XI. to XV., also to find the '*distance between two points in terms of their co-ordinates,*' and to investigate formula for the area of a triangle.

XVI. Remember formulæ $\frac{y-y'}{x-x'}=\frac{y''-y'}{x''-x'}=m.$

XVII. The important parts of this lesson are: To find the co-ords. of intersection of two lines, solve their equations simultaneously when the roots give the co-ordinates.

The condition that three lines meet at a point is that the roots of equations of first two lines satisfy the third equation.

XVIII. The condition of perpendicularity $m=-\frac{1}{m'}$ is of fundamental importance. Also the distance of a point from the lines $y=mx+c$, and $x \cos \alpha + y \sin \alpha = p$, is often required.

XIX. See instructions for Nineteenth Week's Work.

XXI. Get up proof of equation of circle in '*central*' form, viz. $(x-d)^2+(y-e)^2=a^2$, thoroughly. It is generally the best to use, and to the general student much the safest. Every solution from XXI.—XXVIII. should be revised in Conics, if you mean to make anything of this subject.

XXII. Important.

XXIII. Remember formulæ. To find the equation of *the tangent* is a very common question.

XXIV. The condition that the line $y=mx+c$ will touch the circle $x^2+y^2=a^2$, viz., $c=a\sqrt{1+m^2}$, is important.

XXV. The equation of the polar is easy and useful. In using it, do not tell the examiner that it is of the same form as the tangent, which is entirely irrelevant.

XXVI. If pressed for time, omit all this.

XXVII. The formula for the length of a tangent should be known, and the *method* of finding equation of the radical axis.

XXVIII. Recently introduced; fairly important.

STRUCTURE OF THE PAPERS.

The *Morning Paper* consists of Arithmetic and Algebra, of late ten questions in all, of which four are Arithmetic, counting Logarithms, Interest, and Annuities under this head. The order of the questions, of course, is not fixed in any way; but it will be convenient here to assume a certain order to show what the paper is likely to be made up of.

1. A fraction, generally decimal. Make sure of the rule for converting repeaters into vulgar fractions.

2. A square root often involving surds in the denominator, which should first be rationalized.

3. On Logarithms. Have up the book-work thoroughly.

4. On annuities, interest, present worth, or discount. A look through your Annuities should refresh all these. At the Examination, if you cannot hit the right method of using the logs. given, you had better leave the question until last.

5. A question involving the Results of Multiplication or Factors.

6. A Progression, sometimes two. An *A.P* is nearly always given. Have up the book-work of this thoroughly.

7. Equations, often simultaneous of the 2nd degree.
8. Generally a Permutation or Combination.

The remaining questions may consist of (*a*) a problem, often quadratic; (*b*) a repetition of 4; or (*c*), a repetition of 7; or, of course, any of the subjects prescribed in the Syllabus.

The *Afternoon Paper* contains questions on Geometry (including Euc., Books vi. and xi., Mensuration, and Conics) and Trigonometry. The Paper is generally constructed thus:

1. A Prop. of Book vi. Formerly more questions were taken from this.
2. A Prop. of Book xi.

3, or 3 and 4. Mensuration and the sphere.

4—7. Trigonometry. Generally a question on the $A \pm B$ formulæ, and on the area of a triangle. Distances now come in for a fair share; Solutions of Triangles and Use of logs now seem likely.

8—10. Conics. Often include a piece of book-work from Lessons XI.—XV., or the establishment of the equation of the circle.

A question on loci is not unfrequently given; but the student who finds this part of the subject difficult had better bestow his time just before the Examination in making himself strong in the other parts of the Paper rather than in following up a hopeless task.

HINTS FOR CANDIDATES.

It is a favourite practice, especially among those who are backward, to devote the evenings during the Examination to an attempt at reviewing the whole of the subject for the next day's Examination, and the plan of working on into the small hours of the morning at such times has still many adherents. We have dissuaded several from doing this; and, when our advice has not been heeded, the result has almost invariably confirmed our views. Cramming up the last thing before the Examination is almost invariably a failure; it is too late to *learn* what you should have known before. Those who have prepared for the Examination in the few spare hours after their professional work is done will probably find it well to revise points which they have beforehand marked weak or of special importance; but those who put in four hours' preparation in trying to review the whole of their work, in addition to the six hours a day spent in the examination room, are generally too fatigued to do such good work as they would otherwise have done had their minds been fresh. Better do no work at all between times than do too much.

With regard to Mathematics, let us suppose you have refreshed your memory by glancing at a few important formulæ on the previous evening, and are now in the examination room with your paper before you. It is not unlikely that you may feel tempted to make a frantic rush at answering the first question. Don't; you will only get flurried, and possibly waste a good deal of time in the long run. It will be wiser to spend several minutes in looking through your paper. You will be almost sure to see some very easy question which you can answer without hesitation. Write the answer out; do not be in too great a hurry, or you may forget some of the essential points. Having answered one question, you will begin to feel greater confidence, and perhaps you will see another question which presents no difficulty. Now answer this, and when you have done any easy questions that there may be on the paper, you will feel better able to grapple with the harder ones. Perhaps, however, what you at first supposed was a simple question may turn out to be more difficult than you imagined. If you see your way clear to finishing it, you will be well repaid; if not, send up your attempts, which may secure you a fair number of marks if you have started the right way; do not try to "beg the question" by *pretending* to get the answer, or you will score less marks than otherwise, and the Examiner will be involuntarily prejudiced at the attempt to humbug him.

If your paper should be a very hard one, you may not be able to solve many of the questions. In that case be more careful than ever to do what you can *thoroughly*, so as to secure the maximum number of marks on it. Be on the look-out for questions resembling book work, but requiring slight modifications in their solution. The Examiners often set such to distinguish those who know their work from others who have merely been cramming. Last, but not least, bear it in mind that to obtain correct results to a number of questions is not the only consideration. The object of the Examiners is to see whether you understand the work, and the result will depend no less on how you have answered the papers than on how much you have done.

Don't be afraid that marks will be deducted for answering the questions out of the order set, but be very careful to prefix the right number to the question.

On no account leave the room before the expiration of the three hours allowed; after you have once finished, do each problem independently of your former working if there is time, employing a method somewhat varied, or an alternative way where a ready one presents itself. Always leave some time at the end for reading over what you have written, especially book work; this will repay you much better than continued attempts at a question of whose solution you are not sure.

Thursday, July 22, 1886. — *Morning*, 10 *to* 1.

ARITHMETIC AND ALGEBRA.

Examiners { Prof. A. G. Greenhill, M.A.
Prof. M. J. M. Hill, M.A.

1. Calculate the value of e to five places of decimals from the formula

$$e = 1 + \frac{1}{1} + \frac{1}{1\,.\,2} + \frac{1}{1\,.\,2\,.\,3} + \frac{1}{1\,.\,2\,.\,3\,.\,4} + \ldots\ldots$$

2. Find the square root of $\dfrac{\sqrt{5}-2}{\sqrt{5}+2}$ to five places of decimals.

3. Explain the nature and general use of logarithms for simplifying numerical calculations.

4. Find the present value of a sum of money, due 10 years hence, at 10 per cent. per annum, Simple Interest. What would be the present value, reckoning bankers' discount?

5. Simplify

$$(a+b+c)^4 - 4(a+b+c)^2(bc+ca+ab) + 2(bc+ca+ab)^2 + 4abc(a+b+c).$$

6. Determine the Greatest Common Measure and the Least Common Multiple, expressed in linear factors, of

$$6x^3 + x^2 - 5x - 2 \quad \text{and} \quad 6x^3 + 5x^2 - 3x - 2.$$

7. Find the sum of an Arithmetical Progression, given the first term a, the last term l, and the number of terms n.

Determine the number of spheres in a pile in which there are 50 spheres in a single line on the top row, and 50 rows.

8. Find x from the equations :

(i.) $(x-b)(x-c)+(x-c)(x-a)+(x-a)(x-b)=0$;

(ii.) $x^2-47x+1=0$.

9. Solve the equations :—

(i.) $\frac{1}{2}x-\frac{1}{5}y=4, \quad \frac{1}{7}x+\frac{1}{15}y=3$;

(ii.) $\frac{1}{4}(y+z)=\frac{1}{5}(z+x)=\frac{1}{2}(x+y), \quad x+y+z=27$.

10. A takes 1 hour longer than B to walk 10 miles; but A calculates that, if he could double his pace, he would take 40 minutes less time than B. Find their rates of walking in miles an hour.

Thursday, July 22, 1886.—*Afternoon*, 3 *to* 6.

GEOMETRY AND TRIGONOMETRY.

Examiners { Prof. A. G. GREENHILL, M.A.
Prof. M. J. M. HILL, M.A.

1. If from the vertical angle of a triangle a straight line be drawn perpendicular to the base, prove that the rectangle contained by the sides of the triangle is equal to the rectangle contained by the perpendicular and the diameter of the circle described about the triangle.

Apply this theorem to express the radius of a circle circumscribing a triangle in terms of a side and the sine of the opposite angle.

2. Through a point O are drawn three straight lines OA, OB, OC, not all in one plane. Prove that any two of the three angles BOC, COA, AOB are together greater than the third.

3. A pyramid stands on a regular hexagon as base. The perpendicular from the vertex of the pyramid on the base passes through the centre of the hexagon, and its length is equal to that of a side of the base. Find the tangent of the angle between the base and any other face of the pyramid, and express the number of units of volume in the pyramid in terms of the number of units of length in a side of the base.

4. Prove that the volume of a sphere is equal to two-thirds of that of the circumscribing cylinder.

5. Calculate the values of (i.) $\sec 30°$, (ii.) $\log_{10} \sec 30°$, (iii.) $L \sec 30°$; having given $\log_{10} 2 = \cdot 30103$ and $\log_{10} 3 = \cdot 4771213$.

6. Assuming the formulæ for $\sin(A-B)$ and $\cos(A-B)$, prove that $$\tan(A-B) = \frac{\tan A - \tan B}{1 + \tan A \tan B}.$$

Prove that $\tan 15° = 2 - \sqrt{3}$.

7. Prove, for a plane triangle, the formula

$$\tan \tfrac{1}{2} (B - C) = \frac{b-c}{b+c} \tan \tfrac{1}{2} (B+C).$$

A, B, and C are three objects in a horizontal plane. If the distance from A to B be $100(\sqrt{3}-1)$ yards, and the distance from A to C be 100 yards, and the angle BAC be 60°, find the distance from B to C.

8. Find the coordinates of the point of intersection of the two straight lines whose equations are

$$2x + 6y + 1 = 0,$$
$$6x - 3y - 4 = 0.$$

Also find the equations of the straight lines, through their point of intersection, which are respectively parallel and perpendicular to the straight line, whose equation is

$$7x - 4y + 3 = 0.$$

9. Find the radius and the polar coordinates of the centre of the circle whose equation is

$$r^2 - 2rc \cos(\theta - \alpha) + c^2 - a^2 = 0.$$

10. The coordinates of the point A are a, 0; of the point B, $-a$, 0. Find the equation of the locus of a point P which moves so that the ratio $\frac{AP}{BP}$ has always the same value m.

Thurs., July 21, 1887.—*Morning,* 10 *to* 1.

ARITHMETIC AND ALGEBRA.

Examiners { Prof. A. G. Greenhill, M.A.
Prof. M. J. M. Hill, M.A.

1. Express as a repeating decimal,

$$\frac{\frac{16}{15}-\frac{7}{11}-\frac{2}{13}}{\frac{11}{13}+\frac{3}{10}-\frac{74}{65}}-35.$$

2. If the square root of ·169 be extracted to five places of decimals, prove that the excess of ·169 over the square of the part of the square root found is ·0000050119.

3. What is the Present Value of £450. 13*s.* 4*d.* due two years hence, reckoning simple interest at 2 per cent. per annum?

4. From 21 consonants and 5 vowels how many words of 5 letters can be formed, each containing 3 consonants and 2 vowels; all the letters in each word formed being different?

5. Calculate to four places of decimals each of the roots of the quadratic equation,

$$16x^2-24x+7=0.$$

6. Solve the simultaneous equations,

$$3(x-y)=2\,;\ 9xy=35.$$

7. Given

$$\log 2 = \cdot 3010300$$
$$\log 3 = \cdot 4771213$$
$$\log 7 = \cdot 8450980$$

calculate the value of $\log \frac{1}{3528}$.

8. Find to the nearest pound how much should be paid now for an annuity of £500, the first instalment of which is paid to the annuitant five years hence, and the last instalment fifteen years hence, reckoning compound interest at 5 per cent.

Given

$$\log 1\cdot 05 = \cdot 0211893$$
$$\log 4\cdot 81017 = \cdot 6821605$$
$$\log 8\cdot 22702 = \cdot 9152428.$$

9. On the supposition that the index law

$$a^m \times a^n = a^{m+n}$$

holds good for all values of m and n, determine the meaning which should be attached to the quantity $a^{\frac{p}{q}}$, where p and q are any two positive integers.

Calculate the value of the expression,

$$(4^{\frac{1}{2}} \times 4^{\frac{2}{3}} \times 4^{\frac{3}{5}}) \div (4^{\frac{4}{15}}).$$

10. There are three places, A., B., C. A. and B. are connected by rail. C. can be reached from B. either by a coach road 15 miles long, or by a footpath 10 miles long. Two travellers, who arrive at C. together at 5 o'clock in the afternoon, find that the first left A. by train at noon, and on reaching B. took the footpath to C.; that the second left A., also by train, at 10 minutes to 1 o'clock, and on reaching B. travelled by coach along the road to C. twice as fast as the first traveller walked; and that the railway journey occupied the same time in each case. Find the time of the railway journey from A. to B., and the rate of the coach.

Thurs., July 21, 1887.—*Afternoon,* 2 *to* 5.

GEOMETRY AND TRIGONOMETRY.

Examiners { Prof. A. G. Greenhill, M.A.
Prof. M. J. M. Hill, M.A.

1. Prove that the areas of similar triangles are to one another in the duplicate ratio of their linear dimensions.

2. Prove that two spheres intersect in a circle, and prove that the length of all tangent lines to the two spheres from any point in the plane of the circle is the same.

3. Determine the volume in cubic feet of a cylindrical gasholder, 140 feet in diameter and 120 feet high.

Determine also, in tons, the quantity of iron plate required in the construction of the gasholder (including the *flat* top), the iron plate weighing 10·2 lbs. the square foot.

4. Define the *sine*, *cosine*, *tangent*, *cotangent*, *secant*, *cosecant*, and *versed sine* of an angle, illustrating their names by reference to a figure.

Construct a table, giving the value of any one in terms of any one of the others.

5. Write down the values of the above trigonometrical ratios for angles of 0°, 30°, 45°, 60°, 90°, 180°.

6. Prove the trigonometrical formulæ:—

(i.) $\cos C - \cos D = -2 \sin \frac{1}{2}(C+D) \sin \frac{1}{2}(C-D)$;

(ii.) $\sqrt{\left(\frac{1-\sin 2A}{1+\sin 2A}\right)} = \tan(45° - A)$.

7. Determine the area and the trigonometrical ratios of the angles of a triangle whose sides are 15, 36, and 39 feet.

8. Prove that, in any triangle,

$$a \operatorname{cosec} A = b \operatorname{cosec} B = c \operatorname{cosec} C$$

= the diameter of the circumscribing circle.

Having measured a base AB, and the angles ABC, BAC, where C is a distant object and these angles very nearly right angles, prove that the distance of C from A or B is approximately

$$AB \operatorname{cosec}(180° - ABC - BAC).$$

9. Prove that in plane co-ordinate geometry an equation of the first degree between x and y,

$$Ax + By + C = 0,$$

represents a straight line; and give the geometrical interpretation of the constants a, b, α, and p in the equations of a straight line in the form

$$\text{(i.)}\ \frac{x}{a} + \frac{y}{b} = 1; \quad \text{(ii.)}\ x \cos \alpha + y \sin \alpha = p.$$

Draw the straight lines $2x+3y=5$ and $3x+5y=8$, and find their point of intersection.

10. Explain the system of polar co-ordinates, and draw the loci whose equations are

$$\text{(i.)}\ r = a \cos(\theta - \alpha); \quad \text{(ii.)}\ r = a \sec(\theta - \alpha);$$

where a and α are constants, and r, θ polar co-ordinates.

Thurs., July 19, 1888.—*Morning*, 10 *to* 1.

ARITHMETIC AND ALGEBRA.

Examiners { JOSEPH LARMOR, Esq., D.Sc., MA.
Prof. M. J. M. HILL, M.A. }

1. Find the remainder left after dividing ·054372 by 26·814 until there are six decimal places in the quotient.

2. Extract to five places of decimals the square root of the sum of $\cdot 742\dot{2}7\dot{0}$ and $\cdot 741\dot{7}2\dot{9}$.

3. Determine the Common Factors of the expressions—

$$12x^3-8x^2-3x+2 \text{ and } 16x^3+12x^2-4x-3.$$

Hence find three values of x which, when substituted in the expression—

$$\left[(12x^3-8x^2-3x+2)+(16x^3+12x^2-4x-3)\right],$$

will give zero for result.

4. Solve the equations—

(i.) $49x^2-154x+121=0$.

(ii.) $x^2+x+1=0$.

(iii.) $3\sqrt{x}+2\sqrt{5-x}=8$.

5. There are four numbers in Arithmetical Progession such that the product of the second and third exceeds the product of the first and fourth by 8; whilst the fourth divided by the third is less than the second divided by the first by $\frac{8}{77}$. Find the numbers.

6. There are five persons A, B, C, D, E; and five chairs numbered 1, 2, 3, 4, 5. Find in how many ways each person can have a chair assigned him, with the restriction that A shall never be seated on a chair having a greater number than either of the chairs on which D and E sit.

7. A man walks a certain distance in a certain time. He calculates that if he had walked a mile per hour slower

than he did, he would have taken 6 hours more than three-fourths of the time he actually took: but if he had walked a mile faster per hour he would have taken 2 hours longer than half the time he actually took. Find the distance walked, and the rate of walking.

8. (i.) Prove that—

$$\log_a x = \log_b x \,.\, \log_a b.$$

(ii.) Given $\log_{10} 2 = {\cdot}30103$, calculate $\log_2 10$ to four places of decimals.

(iii.) Given $\log 3{\cdot}8862 = {\cdot}5895251$,

$$\log 3{\cdot}8863 = {\cdot}5895363,$$

calculate approximately $\log {\cdot}03886245$.

9. (i.) Determine the value of the logarithm of 2401 when the base is the cube root of 7.

(ii.) Calculate the value of $\frac{3}{2} \div 2^{\frac{7}{12}}$, having given $\log 2 = {\cdot}30103$, $\log 1{\cdot}00113 = {\cdot}0004905$, $\log 3 = {\cdot}4771213$.

10. Find how much should be paid for the reversion after ten years of a freehold estate, worth annually £4500, reckoning interest at $2\frac{1}{2}$ per cent.

$$\log 2 = {\cdot}3010300, \quad \log 1{\cdot}025 = {\cdot}0107239,$$
$$\log 3 = {\cdot}4771213, \quad \log 1{\cdot}406156 = {\cdot}1480336.$$

Thurs., July 19, 1888.—*Afternoon,* 2 *to* 5.

GEOMETRY AND TRIGONOMETRY.

Examiners { Prof. M. J. M. Hill, M.A.
Joseph Larmor, Esq., D.Sc., M.A.

1. Prove that a straight line drawn parallel to one side of a triangle divides the other two sides into segments that are proportional to one another.

Show how to draw a line across two of the sides, not parallel to the third side, which will cut off a triangle similar to the original one. When will it be impossible to do this?

2. Prove that, if similar figures are similarly described on the sides of a right-angled triangle, that on the hypotenuse is equal in area to the other two together.

Divide a right-angled triangle into two parts which shall each be similar to the original triangle.

3. Investigate the relations between the trigonometrical ratios of two angles which differ by a right angle.

4. Obtain a formula for $\tan \frac{1}{2}A$ in terms of $\tan A$.

Find to three places of decimals the length of a side of a regular polygon of 12 sides which is circumscribed to a circle of unit radius.

5. Prove the formula for computing the angle A of a triangle whose sides a, b, c have been measured—

$$\tan \tfrac{1}{2}A = \sqrt{\frac{(s-b)(s-c)}{s(s-a)}}$$

where $s = \frac{1}{2}(a+b+c)$; and express it in a form ready for the application of logarithmic tables.

In a triangle the angles at the base are twice and three times the magnitude of the angle at the vertex; calculate the ratios of the sides.

6. The angle α which two objects A and B subtend to an observer at C are measured, and also the angle β they

subtend when he has moved to D, through a measured distance c directly towards the object B; give formulæ by which the distances of A from C and D may be determined from these observations.

7. Prove that the curve of section of a sphere by a plane is a circle.

Through a fixed point any three planes are drawn at right angles to one another so that each intersects a fixed sphere of radius a; prove that the sum of the areas of the three circles of intersection is constant, being equal to $\pi(3a^2 - c^2)$, where c is the distance of the fixed point from the centre of the sphere.

8. A right cylinder is circumscribed to a sphere; prove that the area of the zone of the sphere between two planes drawn perpendicular to the axis of the cylinder is equal to the area of the zone of the cylinder between the same two planes.

Obtain in square miles the areas of the torrid, temperate, and frigid zones of the Earth, which are separated by the circles of latitude $23\frac{1}{2}°$ and $66\frac{1}{2}°$, taking the Earth's radius to be 4,000 miles, $\pi = 3{\cdot}1416$, and $\cos 23\frac{1}{2}° = {\cdot}9170$.

9. Find the equation of the straight line joining the points whose rectangular co-ordinates are (1, 2) (−3, 3); and determine its distance from the point (2, 4).

Find in square feet the area of the triangle whose vertices are (1, 1) (−2, 3) (4, −6), the co-ordinates being measured in feet.

10. Write down in rectangular co-ordinates the most general equation that can represent a circle, and find its radius and the co-ordinates of its centre.

Investigate the locus of a point such that the square of the tangent drawn from it to a fixed circle, $x^2 + y^2 = a^2$, is equal to the rectangle contained by its distance from the fixed line $px + qy + r = 0$, and a line of constant length c.

Thursday, July 18th, 1889 — *Morning,* 10 *to* 1.

ARITHMETIC AND ALGEBRA.

Examiners { Prof. M. J. M. Hill, M.A.
J. Larmor, Esq., D.Sc., M.A.

The following logarithms are given :—

$\log 2 = 0 \cdot 3010300$ $\log 32434 = 4 \cdot 511000$
$\log 13 = 1 \cdot 1139434$ $\log 32440 = 4 \cdot 511081.$

---

1. Calculate $\dfrac{10+\sqrt{5}}{(4-\sqrt{5})^2}$ to four places of decimals.

Simplify $10^{\frac{1}{6}} . 2^{\frac{1}{3}} . 4^{\frac{2}{3}} \div 5^{\frac{2}{3}}$.

2. Find the Greatest Common Measure of 556206 and 61180.

Prove that, if n is a positive integer,

$$(y-z)^{2n+1}+(z-x)^{2n+1}+(x-y)^{2n+1}$$

is divisible by $(y-z)(z-x)(x-y)$.

3. A person saves £40 a year out of his income, and invests it at the end of each year, allowing the whole to accumulate at 4 per cent. per annum compound interest. Find the amount, to the nearest shilling, of his savings at the end of 30 years.

4. A. sets his watch by a chronometer. Twenty-four hours afterwards B. sets his watch by the time indicated by A.'s, and twenty-four hours later he finds, on comparison with a chronometer, that his watch is half a minute too fast. He sets it right, and twelve hours later meets A., and finds that his watch is three and a quarter minutes in advance of A.'s. Find the rates at which the watches gain or lose per day.

5. (i.) Find the value of

$$\frac{x-1}{x-4} \div \left(\frac{1}{x}+\frac{2-x}{4-x}\right), \text{ when } x = 3.$$

(ii.) Simplify

$$(x^2+1)(x^2+\sqrt{2}\,x+1)(x^2-1)(x^2-\sqrt{2}\,x+1).$$

Reduce x^6-1 to its simplest real factors.

6. Prove that, if $\dfrac{x}{y}=\dfrac{a}{b}$, then each of these ratios is equal to

$$\frac{\sqrt{(x^2+a^2p^2)}}{\sqrt{(y^2+b^2p^2)}}.$$

7. Solve the equation

$$\frac{1}{x-1}+\frac{1}{x-2}+\frac{1}{x-3}=0.$$

Form the equation whose roots are

$$\tfrac{1}{3}(4+2\sqrt{5}) \text{ and } \tfrac{1}{3}(4-2\sqrt{5}).$$

8. Calculate to six places of decimals

$\log 65$, $\log \cdot 0065$, $\log \frac{4}{65}$, and $\log 324 \cdot 365$.

9. Prove that the number of ways in which n different objects can be arranged in a row is $1 . 2 . 3 \ldots n$.

Prove, also, that if r of the objects are alike, and the remainder unlike, the number of ways is

$$(r+1)(r+2) \ldots n.$$

10. Prove that, if n geometric means are inserted between a and b, their sum is equal to

$$\frac{b-az}{z-1} \text{ where } z^{n+1}=\frac{b}{a}.$$

Prove also that, when n is odd, the sum of the alternate means, beginning with the first, exceeds the sum of the others by $\dfrac{b+az}{z+1}$.

Thursday, July 18*th*, 1889 — *Afternoon*, 2 *to* 5.

GEOMETRY AND TRIGONOMETRY.

Examiners {Prof. M. J. M. Hill, M.A.
J. Larmor, Esq., D.Sc., M.A.

(*All the usual abbreviations may be used in answering this Paper.*)

1. If two triangles ABC, DEF have the angle ABC equal to the angle DEF, and the sides about these angles proportional, viz., $AB : BC :: DE : EF$, prove that the triangles are equiangular, and have those angles equal which are opposite to corresponding sides.

2. Find the area of a regular polygon of n sides, inscribed in a circle whose radius is r.

   Show how to deduce the area of the circle from the result.

3. Show how to draw a straight line perpendicular to a given plane from a given point not in the plane.

4. From a point O are drawn three straight lines OA, OB, OC, in any directions, which are not all in one plane. Prove that any two of the three plane angles BOC, COA, AOB are together greater than the remaining one.

   Hence prove that two sides of a spherical triangle are together greater than the third side.

5. Show how to find the position of the centre of a sphere whose surface passes through four given points in space, all of which are not in one plane.

6. A right circular cone, whose semi-vertical angle is 30° and height is h, has a sphere inscribed in it. Prove that the volume of the part of the cone which is outside of the sphere is $5\pi h^3/81$.

7. Determine in degrees, minutes, and seconds the angle whose sine is ·6.

$$\log 6 = \cdot 7781513$$
$$L \sin 36° \, 52' = 9{\cdot}7781186$$
$$L \sin 36° \, 53' = 9{\cdot}7782870.$$

8. A and B are two stations 531 yards apart. P and Q are two objects in the same horizontal plane as A and B.

The following angles are found by observation :—

$$ABQ = 127° \, 35', \quad BAQ = 36° \, 43', \quad QAP = 73° \, 21',$$
$$ABP = 43° \, 26'.$$

Prove that $AQ = 1555{\cdot}06$ yds., $AP = 818{\cdot}175$ yds., $PQ = 1535{\cdot}744$ yds.

$L \sin 52° \, 25'$	$= 9{\cdot}8989812$	$\log 531$	$= 2{\cdot}7250945$
$L \sin 15° \, 42'$	$= 9{\cdot}4323285$	$\log 1555{\cdot}06$	$= 3{\cdot}1917472$
$L \sin 43° \, 26'$	$= 9{\cdot}8372791$	$\log 818{\cdot}175$	$= 2{\cdot}9128462$
$L \sin 26° \, 30'$	$= 9{\cdot}6495274$	$\log 736{\cdot}885$	$= 2{\cdot}8673998$
$L \cot 36° \, 40' \, 30''$	$= 10{\cdot}1280195$	$\log 2373{\cdot}235$	$= 3{\cdot}3753407$
$L \tan 22° \, 38'$	$= 9{\cdot}6200786$	$\log 1535{\cdot}744$	$= 3{\cdot}1863189$
$L \sin 73° \, 21'$	$= 9{\cdot}9813986$		
$L \sin 30° \, 41' \, 30''$	$= 9{\cdot}7079259.$		

9. Find the equation of the straight line which passes through the point $(2, -3)$ and is perpendicular to the straight line joining the points $(5, 7)$ and $(-6, 3)$.

10. Find the coordinates of the points in which the straight line $y = mx$ cuts the circle

$$x^2 + y^2 - 4x - 6y + 12 = 0.$$

What must the value of m be in order that these two points may coincide?

Find the equations of the tangents to the above circle which pass through the origin of coordinates.

INTERMEDIATE EXAMINATIONS IN ARTS AND IN SCIENCE, 1890.

PASS EXAMINATIONS.

Thursday, July 24.—*Morning*, 10 *to* 1.

ARITHMETIC AND ALGEBRA.

Examiners { Prof. HORACE LAMB, M.A., F.R.S.
JOSEPH LARMOR, Esq., D.Sc., M.A.

1. Find how many feet of copper wire, ·0164 of an inch in diameter, go to the lb, having given that copper is 8·95 times as heavy as water, and that a cubic foot of water weighs 62·3 lbs. [Use by preference contracted methods.]

2. Simplify—

(i.) $\frac{1}{x^2} - \frac{3}{4}\left(\frac{1}{x-1} - \frac{1}{x+1}\right) + \frac{1}{4}\left\{\frac{1}{(x-1)^2} + \frac{1}{(x+1)^2}\right\}$

(ii.) $\frac{(x+y)^5 - x^5 - y^5}{(x+y)^3 - x^3 - y^3}$.

3. Find the Square Roots of

$$8210 \cdot 1721 \quad \text{and} \quad 2(x+y)^4 + 2(x^4+y^4).$$

4. Find an expression of the second degree in x which shall have the values 9, 24, 21, when $x = -2, 1, 4$, respectively. Find also the greatest value of this expression.

5. A man, aged 30, takes out an insurance policy for £1000, payable at death or on his attaining the age of 55, the annual premium being £42. 12*s*. 6*d*, payable at the beginning of each year. If he reaches the above age, what is the gain of the insurance company at the time the transaction is ended, compound interest being reckoned at 4 per cent.?

$$\log 25 = 1 \cdot 39794.$$
$$\log 26 = 1 \cdot 41497.$$
$$\log 2 \cdot 6653 = \cdot 42575.$$

6. Find the condition that the roots of the quadratic,

$$Ax^2 + 2Bx + C = 0,$$

should be equal.

If the roots of $(b-c)x^2+(c-a)x+(a-b)=0$ are equal, then a, b, c are in arithmetic progression.

7. The electric resistance of a wire of given material varies directly as the length and inversely as the square of the diameter. What must be the length and diameter of a wire which is to have double the resistance but only two-thirds the weight of a wire of the same material 100 feet long, and ·018 inch diameter?

8. Simplify—

(i.) $$\frac{\left(\frac{1}{\sqrt{3}}+\frac{1}{\sqrt{5}}\right)(\sqrt{5}-\sqrt{3})}{(\sqrt{5}+\sqrt{3})^2-(\sqrt{5}-\sqrt{3})^2}.$$

(ii.) $(\sqrt{x}+\sqrt{y})(\sqrt[4]{x}+\sqrt[4]{y})(\sqrt[4]{x}-\sqrt[4]{y})$.

9. If three positive quantities are in arithmetic progression, the ratio of the first to the second is less than that of the second to the third.

The differences of the logarithms of successive integers continually diminish: why is this?

10. The number of ways in which $m+n$ things can be divided into two groups containing respectively m and n things is

$$\frac{\underline{|m+n}}{\underline{|m}\,\underline{|n}}.$$

Having five sovereigns and five shillings in my pocket, I am asked for a subscription: between how many different amounts have I choice?

INTERMEDIATE EXAMINATIONS IN ARTS AND IN SCIENCE, 1890.

PASS EXAMINATIONS.

Thursday, July 24.—*Afternoon*, 2 *to* 5.

GEOMETRY AND TRIGONOMETRY.

Examiners { Prof. HORACE LAMB, M.A., F.R.S.
JOSEPH LARMOR, Esq., D.Sc., M.A.

The following logarithms are given:—

$L \tan 24^\circ\ 5' = 9{\cdot}6502809$	$L \sin 22^\circ\ 15' = 9{\cdot}5782$
$L \tan 24^\circ\ 6' = 9{\cdot}6506199$	$L \sin 10^\circ\ 12' = 9{\cdot}2482$
$\log 2 = {\cdot}3010300$	$L \sin 12^\circ\ 3' = 9{\cdot}3197$
$\log 32122 = 4{\cdot}5068$	

1. The triangles ABC, DEF are similar, and on DE, the side homologous to AB, a point K is taken, such that $DE : AB = AB : DK$; prove that the triangles ABC, DKF are equal in area.

2. Three planes BOC, COA, AOB meet in a point O, and lines OA', OB', OC' are drawn perpendicular to them respectively; prove that the lines OA, OB, OC are perpendicular respectively to the planes $B'OC'$, $C'OA'$, $A'OB'$; and draw up a list (without proof) of pairs of angles that are equal.

3. Prove that the sum of two sides of a spherical triangle is greater than the third side.

4. Find the ratio of the volume of a hemisphere to that of a cone with the same base and the same vertex.

Show that the area of the total surface of the cone is approximately ·805 times the area of the total surface of the hemisphere, the base being included in each case.

5. Find to two places of decimals the ratio of the area of an equilateral triangle to that of the circle circumscribed to it.

6. Obtain an equation to determine $\tan \frac{1}{2}\alpha$ in terms of $\tan \alpha$, and apply it to find the value of $\tan 15^\circ$ to four places of decimals.

What is the angle of which the other root of this equation is the tangent?

7. The sides of a triangle are 7, 8, 9; calculate to the nearest second the value of its smallest angle.

If four-figure logarithms instead of seven-figure logarithms were employed in the calculation, within how many seconds would the result be reliable?

8. At a point on a horizontal plain, the elevation above the horizontal of the summit of a mountain is observed to be 22° 15′, and at another point on the plain, a mile further away in a direct line, its elevation is observed to be 10° 12′; calculate the height of the mountain in feet.

9. Plot out on a diagram the points whose co-ordinates referred to rectangular axes are (3, 4), (2, −3), and (−2, $-2\frac{1}{2}$); and find whether the origin lies inside the triangle formed by them.

Find the co-ordinates of the centre of the circle which passes through these points.

10. Construct the line given by the equation $x - \sqrt{3}\, y = 4$; and find (i.) the area of the triangle it cuts off from the axes, (ii.) the length of the perpendicular drawn to it from the point (1, 2), and (iii.) the distance of this perpendicular line from the origin.

SOLUTIONS TO THE PAPER IN ARITHMETIC AND ALGEBRA,

INTERMEDIATE EXAMINATIONS IN ARTS AND IN SCIENCE, 1890.

C. W. C. BARLOW, M.A., Sixth Wrangler, First Div. of First Class in Part II., Math Tripos, Mathematical Honourman at Lond.

T. W. EDMONDSON, B.A. Lond., First in Honours at Matriculation, University Exhibitioner.

1. The radius of the wire

$$= \frac{\cdot 0164}{24} \text{ ft.} = \cdot 00068\dot{3} \text{ ft.} = \frac{6\cdot 833}{10^4} \text{ ft.}$$

Hence the volume of 1 foot of wire in cubic feet

$$= \pi \left(\frac{6\cdot 833}{10^4}\right)^2.$$

Now, since the data are only given to three significant figures, we may take $\pi = \frac{22}{7}$, for this value differs from the true one only in the fourth figure. Hence, performing the calculations to four figures in order to obtain the result correct to three, we find

$$\text{Volume of 1 foot of wire} = \frac{22}{7} \times \frac{46\cdot 69}{10^8} \text{ cub. ft.}$$

$$= \frac{146\cdot 7}{10^8} = \frac{1\cdot 467}{10^6} \text{ cub. ft.}$$

Now the weight of a cubic foot of copper is 8·95 times that of a cubic foot of water, and is therefore

$$8\cdot 95 \times 62\cdot 3 \text{ lbs.} = 557\cdot 6 \text{ lbs.}$$

Hence weight of a foot length of wire

$$= \frac{1\cdot 467}{10^6} \times 557\cdot 6 \text{ lbs.} = \frac{817\cdot 9}{10^6} = \frac{8\cdot 179}{10^4} \text{ lbs.};$$

therefore number of feet in 1 lb. of wire

$$= \frac{10^4}{8\cdot 179} = 1222, \text{ approximately.}$$

The accuracy of the last figure cannot be relied on, as the given data are supposed only approximate. To the degree of approximation they represent, we may take the length of wire as 1220 feet.

The numerical calculations stand as follows:—

```
4 | ·0164
6 | ·0041
  ---------
   ·0006833

   6·833
   6·833
  ------
  41·00
   5·466
     205
      20
  ------
  46·69
  22
  ------
  933·8
   93·4
7 |1027·2
  ------
  146·7
```

```
     62·3
      8·95
     -----
    498·4
     56·1
      3·1
     -----
    557·6
      1·467
     -----
    557·6
    223·0
     33·4
      3·9
     -----
    817·9

8179 ) 10000 ( 1222
        8179
        ----
        1821
        1636
        ----
         185
         164
         ---
          21
          16
```

2\. (i.) $\frac{1}{x^2}-\frac{3}{4}\left(\frac{1}{x-1}-\frac{1}{x+1}\right)+\frac{1}{4}\left\{\frac{1}{(x-1)^2}+\frac{1}{(x+1)^2}\right\}$

$$=\frac{1}{x^2}-\frac{3}{4}\left(\frac{x+1-x+1}{(x-1)(x+1)}\right)+\frac{1}{4}\left\{\frac{(x+1)^2+(x-1)^2}{(x-1)^2(x+1)^2}\right\}$$

$$=\frac{1}{x^2}-\frac{3}{2}\,\frac{1}{(x-1)(x+1)}+\frac{x^2+1}{2(x-1)^2(x+1)^2}$$

$$=\frac{2(x^2-1)^2-3x^2(x^2-1)+x^2(x^2+1)}{2x^2(x-1)^2(x+1)^2}$$

$$=\frac{2x^4-4x^2+2-3x^4+3x^2+x^4+x^2}{2x^2(x-1)^2(x+1)^2}$$

$$=\frac{2}{2x^2(x-1)^2(x+1)^2}$$

$$=\frac{1}{x^2(x-1)^2(x+1)^2}=\frac{1}{x^2(x^2-1)^2}.$$

(ii.) $$\frac{(x+y)^5-x^5-y^5}{(x+y)^3-x^3-y^3}=\frac{5x^4y+10x^3y^2+10x^2y^3+5xy^4}{3x^2y+3xy^2}$$

$$=\frac{5xy\,(x^3+2x^2y+2xy^2+y^3)}{3xy\,(x+y)}$$

$$=\frac{5\left\{(x^3+y^3)+2xy\,(x+y)\right\}}{3\,(x+y)}$$

$$=\frac{5\,(x+y)\left\{x^2-xy+y^2+2xy\right\}}{3\,(x+y)}$$

$$=\frac{5}{3}\,(x^2+xy+y^2).$$

3. (i.)

$$\begin{array}{r|l}
 & \dot{8}\dot{2}1\dot{0}\cdot1\dot{7}2\dot{1}\ (\ 90\cdot61 \\
 & 81 \\
1806 & 11017 \\
 & 10836 \\
18121 & \quad 18121 \\
 & \quad 18121 \\
 & \quad \cdot\cdot\cdot\cdot\cdot
\end{array}$$

Therefore the square root of 8210·1721 is 90·61.

(ii.) $2\,(x+y)^4+2\,(x^4+y^4)=2\left\{x^4+4x^3y+6x^2y^2+4xy^2+y^4+x^4+y^4\right\}$

$$=4\left\{x^4+2x^3y+3x^2y^2+2xy^3+y^4\right\}.$$

Now find the square root of $x^4+2x^3y+3x^2y^2+2xy^3+y^4$,

$$\begin{array}{l|l}
 & x^4+2x^3y+3x^2y^2+2xy^3+y^4\ (\ x^2+xy+y^2 \\
 & x^4 \\
2x^2+xy & 2x^3y+3x^2y^2 \\
 & 2x^3y+\ \ x^2y^2 \\
2x^2+2xy+y^2 & \quad 2x^2y^2+2xy^3+y^4 \\
 & \quad 2x^2y^2+2xy^3+y^4 \\
 & \quad \cdot\quad\cdot\quad\cdot
\end{array}$$

therefore the square root of $2\,(x+y)^4+2\,(x^4+y^4)$ is

$$2\,(x^2+xy+y^2).$$

Otherwise thus :—Arrange in powers of x^2+y^2 and xy. Thus we have $x^4+2x^3y+3x^2y^2+2xy^3+y^4=(x^2+y^2)^2+2xy\,(x^2+y^2)+x^2y^2$ and the square root of this is evidently x^2+y^2+xy as before.

4. Let the expression of the second degree in x be ax^2+bx+c

Substitute the given values of x; then we have

$$4a-2b+c=9 \quad \text{.......................(1)},$$

$$a+b+c=24 \quad \text{.......................(2)},$$

$$16a+4b+c=21 \quad \text{.......................(3)}.$$

Subtracting (1) from (2), we get $3b-3a=15$,

$$\therefore\ b-a=5 \quad \text{.......................(4)}$$

Subtracting (2) from (3), therefore

$$3b+15a=-3,$$

and therefore $\quad b+5a=-1$ ....(5).

Subtracting (4) from (5), we have $6a=-6$;

$$\therefore\ a=-1;$$

and hence, since $\quad b-a=5,\quad b=4.$

Substituting these values in (2), we get

$$-1+4+c=24;$$

$$\therefore\ c=21;$$

therefore the required expression is

$$-x^2+4x+21.$$

Alternative method.—The expression

$$A\,\frac{(x-b)(x-c)}{(a-b)(a-c)}+B\,\frac{(x-c)(x-a)}{(b-c)(b-a)}+C\,\frac{(x-a)(x-b)}{(c-a)(c-b)}$$

is evidently a quadratic function of x which is equal to A, B, C, respectively, when x is equal to a, b, c. Applying this to the present question, we see that the required expression is

$$9\,\frac{(x-1)(x-4)}{(-2-1)(-2-4)}+24\,\frac{(x+2)(x-4)}{(1+2)(1-4)}+21\,\frac{(x+2)(x-1)}{(4+2)(4-1)},$$

which gives

$$\frac{1}{2}(x-1)(x-4)-\frac{8}{3}(x+2)(x-4)+\frac{7}{6}(x+2)(x-1),$$

or $\quad -x^2+4x+21$, as before.

This expression $\quad =-(x^2-4x+4)+25$

$$=-(x-2)^2+25.$$

Now $(x-2)^2$ cannot be negative, therefore the expression has its greatest value when $x-2=0$, and its value is then 25.

5. At the end of 25 years the first annual premium paid to the Company will have amounted to £42. 12*s*. 6*d*. × $(1{\cdot}04)^{25}$

$$=\text{£}42{\cdot}625\times(1{\cdot}04)^{25};$$

the second premium will have amounted to £42·625 × $(1·04)^{24}$, and so on; and the last or 25th premium, paid at the beginning of the 25th year, will have amounted to £42·625×(1·04) at the end of that year.

Hence the total amount of the premiums paid, together with the compound interest on them, will be

$$£42·625 \left\{(1·04)^{25} + (1·04)^{24} + \ldots + (1·04)^2 + (1·04)\right\}$$
$$= £42·265 \times \frac{1·04 \left\{(1·04)^{25} - 1\right\}}{1·04 - 1}$$
$$= £42·625 \times 26 \times \left\{(1·04)^{25} - 1\right\}.$$

We have now to find $(1·04)^{25}$ with the aid of the logarithms given.

$$\log (1·04)^{25} = 25 \log (1·04) = 25 \log \left(\frac{26}{25}\right)$$
$$= 25 \left\{\log 26 - \log 25\right\} = 25 \left\{1·41497 - 1·39794\right\}$$
$$= 25 \times ·01703 = ·42575 = \log 2·6653;$$
$$\therefore\ (1·04)^{25} = 2·6653;$$

∴ total amount received by the Company

$$= £42·625 \times 26 \times (2·6653 - 1)$$
$$= £1108·25 \times 1·6653$$
$$= £1845·569, \text{ nearly};$$

∴ the gain of the Company, after paying the £1000, is £845·569

= £845. 11*s*. 4½*d*.

6. Proceed to solve the equation $Ax^2 + 2Bx + C = 0$,

$$\therefore\ x^2 + \frac{2B}{A}x = -\frac{C}{A}.$$

Completing the square, we have

$$x^2 + \frac{2B}{A}x + \left(\frac{B}{A}\right)^2 = -\frac{C}{A} + \left(\frac{B}{A}\right)^2 = \frac{B^2 - AC}{A^2};$$
$$\therefore\ x + \frac{B}{A} = \pm\frac{\sqrt{B^2 - AC}}{A};$$
$$\therefore\ x = \frac{-B \pm \sqrt{B^2 - AC}}{A}.$$

If these roots be equal, we must have $\sqrt{B^2 - AC}$ equal to zero, and therefore $B^2 - AC = 0$ is the condition of equality of the two roots of the given quadratic.

Using the above result, we find that, when the roots of

$$(b-c)\,x^2+(c-a)\,x+(a-b)=0 \text{ are equal,}$$

$$\left(\frac{c-a}{2}\right)^2-(b-c)(a-b)=0;$$

$$\therefore\quad (c-a)^2-4\,(b-c)(a-b)=0.$$

This may also be written,

$$\{(a-b)+(b-c)\}^2-4\,(a-b)(b-c)=0;$$

therefore $$\{(a-b)-(b-c)\}^2=0.$$

Hence $$a-b=b-c;$$

therefore a, b, c are in Arithmetical Progression.

7. Let R be the electric resistance of a wire, l its length, and d its diameter. Then $R\propto\dfrac{l}{d^2}$ or $R=k\dfrac{l}{d^2}$, where k is some constant.

When $l=100$ feet, and $d=\cdot018$ inch or $\cdot0015$ feet,

$$R=k\,\frac{100}{(\cdot0015)^2}=k\,.\,\frac{100^5}{15^2}.$$

Let l' be the length of the new wire, and d' its diameter; then

$$2R=k\,\frac{l'}{d'^2},$$

and therefore $$\frac{l'}{d'^2}=2\,.\,\frac{100^5}{15^2}\;\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\;(1).$$

Now, since the weights of the wires are to be in the ratio 2 : 3, and their materials are the same, their volume must also be in the ratio 2 : 3;

$$\therefore\quad \frac{l'\pi\left(\dfrac{d'}{2}\right)^2}{l\pi\left(\dfrac{d}{2}\right)^2}=\frac{2}{3};$$

$$\therefore\quad l'd'^2=\frac{2}{3}\,ld^2=\frac{2}{3}\,100\,.\left(\frac{15}{10000}\right)^2=\frac{2}{3}\,\frac{15^2}{(100)^3}\ldots\ldots.(2).$$

Multiplying (1) and (2), we get $l'^2=\dfrac{4}{3}\,.\,(100)^2$;

$$\therefore\quad l'=\frac{2}{\sqrt{3}}\,100\text{ feet}=\frac{200}{3}\sqrt{3}\text{ feet}=115\cdot47\text{ feet}.$$

Substituting this value of l' in (1), we get

$$d'^2 = \frac{2}{\sqrt{3}} 100 \cdot \frac{15^2}{2.100^5} = \frac{15^2}{\sqrt{3}.100^4};$$

$$\therefore d' = \frac{15}{\sqrt[4]{3} \times 10000} \text{ feet}$$

$$= \frac{5\sqrt[4]{27}}{10000} \text{ feet} = \frac{5 \times 2{\cdot}279}{10000} \text{ feet} = \frac{11{\cdot}395}{10000} \text{ feet}$$

$$= {\cdot}013674 \text{ inch.}$$

8. (i.) $$\frac{\left(\frac{1}{\sqrt{3}} + \frac{1}{\sqrt{5}}\right)(\sqrt{5} - \sqrt{3})}{(\sqrt{5} + \sqrt{3})^2 - (\sqrt{5} - \sqrt{3})^2}$$

$$= \frac{\left(\frac{\sqrt{5} + \sqrt{3}}{\sqrt{15}}\right)(\sqrt{5} - \sqrt{3})}{8 + 2\sqrt{15} - (8 - 2\sqrt{15})}$$

$$= \frac{\frac{(\sqrt{5})^2 - (\sqrt{3})^2}{\sqrt{15}}}{4\sqrt{15}}$$

$$= \frac{2}{60} = \frac{1}{30} = {\cdot}0\dot{3}.$$

(ii.) $$(\sqrt{x} + \sqrt{y})(\sqrt[4]{x} + \sqrt[4]{y})(\sqrt[4]{x} - \sqrt[4]{y})$$

$$= (\sqrt{x} + \sqrt{y})(\sqrt{x} - \sqrt{y})$$

$$= x - y.$$

9. Let the three positive quantities in A. P. be a, b, c.

Then $\frac{a}{b}$ will be less than $\frac{b}{c}$,

if $ac < b^2$,

that is, if $ac < \left(\frac{a+c}{2}\right)^2$,

if $4ac < a^2 + 2ac + c^2$,

if $a^2 - 2ac + c^2 > 0$,

if $(a-c)^2 > 0$.

But $(a-c)^2$, being a square number, is positive and therefore greater than 0, $\therefore$ the ratio $\frac{a}{b}$ is less than the ratio $\frac{b}{c}$.

Suppose $n-1$, n, $n+1$ are three successive integers, and a is the base to which their logarithms are calculated.

Let $x = \log_a(n-1)$, $y = \log_a n$, and $z = \log_a(n+1)$,

then $$y-x = \log_a n - \log_a(n-1) = \log_a \frac{n}{n-1},$$

and $$z-y = \log_a(n+1) - \log_a n = \log_a \frac{n+1}{n}.$$

But $n+1$, n, and $n-1$ are in A. P.,

$$\therefore \quad \frac{n+1}{n} < \frac{n}{n-1},$$

$$\therefore \quad \log_a \frac{n+1}{n} < \log_a \frac{n}{n-1};$$

$$\therefore \quad z-y < y-x.$$

In the same way we can show that the difference between

$$\log_a(n+2) \quad \text{and} \quad \log_a(n+1)$$

is less than the difference between

$$\log_a(n+1) \text{ and } \log_a n.$$

Hence the differences of the logarithms of successive integers continually diminish.

10. Every time we take a group of m things, we leave a group of n things behind.

Therefore the problem before us is obviously the same as that of finding the number of combinations of $m+n$ things taken m at a time.

Hence the required number of ways

$$= \frac{\lfloor m+n}{\lfloor m \; \lfloor (m+n)-m} = \frac{\lfloor m+n}{\lfloor m \; \lfloor n}.$$

I may give 5 sovereigns together with any number of shillings from 0 to 5. Hence there are 6 amounts that I can give containing 5 sovereigns. In the same way, there are six amounts each containing 4, 3, 2, or 1 sovereign; and if I give no sovereigns there are 5 amounts to choose from, viz., 1, 2, 3, 4, or 5 shillings.

Hence the total number of amounts to choose from

$$= (5 \times 6) + 5 = 35,$$

the alternative of giving nothing not being included.

SOLUTIONS TO THE PAPER IN GEOMETRY AND TRIGONOMETRY,

INTERMEDIATE EXAMINATIONS IN ARTS AND IN SCIENCE, 1890.

G. H. BRYAN, M.A., Smith's Prizeman, Fellow of St. Peter's College, Cambridge.

1. Since the triangles ABC, DEF are similar, therefore

$$DF : DE = AC : AB,$$

or

$$DF : AC = DE : AB$$
$$= AB : DK, \text{ by construction.}$$

That is, the triangles BAC, KDF have their sides about the angles A and D reciprocally proportional.

But the angles BAC, KDF are equal.

Therefore the triangles are equal in area.

[This is part of the usual proof of Euclid VI., 19.]

2. Since OB' is perpendicular to the plane COA,

$$\therefore\ B'OA \text{ is a right angle.}$$

Since OC' is perpendicular to the plane BOA,

$$\therefore\ C'OA \text{ is a right angle.}$$

Hence OA is perpendicular to the lines OB' and OC', and therefore OA is perpendicular to the plane $B'OC'$.

Similarly, OB is perpendicular to the plane $C'OA'$, and OC is perpendicular to the plane $A'OB'$.

The acute angle between the directions of	OB, OC =	acute dihedral angle between planes	$A'OB'$, $A'OC'$
„	OC, OA =	„	$B'OC'$, $B'OA'$
„	OA, OB -	„	$C'OA'$, $C'OB'$
„	OB', OC' =	„	$A\theta B$, AOC
„	OC', OA' =	„	BOC, BOA
„	OA', OB' =	„	COA, COB.

[If A, B, C, A', B', C' are the intersections of the lines OA, OB, OC, OA', OB', OC' with a sphere whose centre is O, the spherical triangles ABC, $A'B'C'$ are polar triangles, whence these results follow.]

3. If through the centre of the sphere we draw planes meeting the surface in the three sides of the triangle, we have to prove that the sum of any two plane angles (or faces) of the trihedral angle thus formed is greater than the third angle. Euclid XI., 20, or "Wilson's Solid Geometry," page 20.

4. Let a be the radius of the hemisphere. Then the volume of the hemisphere $= \frac{2}{3}\pi a^3$. The area of the base $= \pi a^2$. Hence, since the height of the cone is a, the volume of the cone

$$= \tfrac{1}{3}a \times \pi a^2 = \tfrac{1}{3}\pi a^3.$$

Hence the volume of the hemisphere is double that of the cone.

The curved surface of the hemisphere $= 2\pi a^2$, therefore the total surface including the base $= 3\pi a^2$. The length of a side of the cone is $a\sqrt{2}$. The area of the slant surface $= \frac{1}{2}$ circumference of base $\times$ length of a side

$$= \tfrac{1}{2} . 2\pi a \times a\sqrt{2} = \sqrt{2}\,\pi a^2.$$

Therefore the whole surface is $(1+\sqrt{2})\,\pi a^2$, and is to that of the hemisphere in the ratio $1+\sqrt{2} : 3$.

Now $\sqrt{2} = 1{\cdot}414$. Therefore the required ratio

$$= \tfrac{1}{3}(2{\cdot}414) = {\cdot}805 \text{ nearly}.$$

5. Let ABC be the triangle, O the centre of the circumscribing circle, and let R be its radius. Then

$$\text{area of triangle } BOC = \tfrac{1}{2}R^2 \sin BOC$$

$$= \tfrac{1}{2}R^2 \sin 120^\circ = \tfrac{1}{2}R^2 \times \tfrac{1}{2}\sqrt{3} = \tfrac{1}{4}\sqrt{3}\,R^2;$$

therefore area of whole triangle $ABC = \frac{3}{4}\sqrt{3}\,R^2$.

Also, area of circle $= \pi R^2$;

therefore the required ratio $= \frac{3\sqrt{3}}{4\pi} = \frac{3\sqrt{3}}{4} \times \frac{22}{7}$,

sufficiently approximately for our calculation, since $\pi = \frac{22}{7}$ to two places of decimals. By the usual method,

we find $\sqrt{3} = 1{\cdot}73$ approximately,

and $3 \times 7 \times 1{\cdot}73 \div (4 \times 22) = {\cdot}41$,

which is therefore the required ratio.

6. In the formula

$$\tan 2A = \frac{2 \tan A}{1 - \tan^2 A},$$

write $A = \frac{1}{2}\alpha$; therefore we have

$$\tan \alpha = \frac{2 \tan \frac{1}{2}\alpha}{1 - \tan^2 \frac{1}{2}\alpha},$$

or, reducing this to a quadratic equation in $\tan \frac{1}{2}\alpha$,

$$\tan^2 \tfrac{1}{2}\alpha + 2 \tan \tfrac{1}{2}\alpha / \tan \alpha - 1 = 0.$$

Substituting the known value of $\tan \alpha$ in this equation, and solving the quadratic, we find the values of $\tan \frac{1}{2}\alpha$.

Taking $\frac{1}{2}\alpha = 15°$, we know that $\tan 30° = 1/\sqrt{3}$; therefore the quadratic in $\tan \frac{1}{2}\alpha$ is

$$\tan^2 \tfrac{1}{2}\alpha + 2\sqrt{3} \tan \tfrac{1}{2}\alpha - 1 = 0,$$

$$\therefore\ (\tan \tfrac{1}{2}\alpha + \sqrt{3})^2 = 4,$$

giving $\tan \frac{1}{2}\alpha = -\sqrt{3} \pm 2$.

Now $\tan 15°$ is evidently positive, therefore we must take the upper sign to the radical, and

$$\tan 15° = 2 - \sqrt{3}.$$

By calculation, $\sqrt{3} = 1{\cdot}7320$,

$$\therefore\ \tan 15° = {\cdot}2680.$$

Since $\tan 30° = \tan (180° + 30°)$,

it is evident that we shall obtain the same quadratic for $\tan \frac{1}{2}\alpha$ if we put $\alpha = 210°$.

Therefore the other root is $\tan 105°$.

7. The smallest angle is opposite to the smallest side 7, and s the semi-sum of the sides is

$$\tfrac{1}{2}(7+8+9) = 12.$$

We have therefore by the usual formula

$$\tan^2 \tfrac{1}{2}A = \frac{(s-b)(s-c)}{s(s-a)} = \frac{(12-8)(12-9)}{12(12-7)} = \frac{4 . 3}{12 . 5} = \frac{1}{5}.$$

Therefore $$\tan \tfrac{1}{2}A = \frac{1}{\sqrt{5}} = \frac{\sqrt{2}}{\sqrt{10}},$$

and $$L \tan \tfrac{1}{2}A = 10 + \log \tan \tfrac{1}{2}A = 10 + \tfrac{1}{2}\log 2 - \tfrac{1}{2}\log 10$$
$$= 10 - \cdot 5 + \tfrac{1}{2}(\cdot 3010300) = 9\cdot 5 + \cdot 1505150$$
$$= 9\cdot 6505150.$$

From the given logarithms,

$$L \tan 24^\circ 6' = 9\cdot 6506199$$
$$L \tan 24^\circ 5' = 9\cdot 6502809$$

By subtraction, 3390

is the difference in the L tan corresponding to a difference of $60''$ in the angle.

Again, $$L \tan \tfrac{1}{2}A = 9\cdot 6505150$$
$$L \tan 24^\circ 5' = 9\cdot 6502809$$

$\therefore$ difference for seconds in $\tfrac{1}{2}A =$ $2341.$

Hence the number of seconds in $\tfrac{1}{2}A$ is

$$= 60 \times \frac{2341}{3390} = 41\cdot 4 \text{ nearly.}$$

Therefore $$\tfrac{1}{2}A = 24^\circ\, 5'\, 41''\, 4,$$

giving $$A = 48^\circ\, 11'\, 23'',$$

correct to the nearest second.

If we had used four-figure logarithms, we should have had $$L \tan 24^\circ 6' = 9\cdot 6506,$$

and $$L \tan 24^\circ 5' = 9\cdot 6503.$$

Thus the difference in the L tan corresponding to a difference of 60″ in the angle would be ·0003, and the smallest perceptible difference in the L tan, viz. ·0001, would correspond to a difference in angle of 20″.

The angle $\frac{1}{2}A$ could therefore only be calculated to the nearest multiple of 20″, and thus the value of A would be correct to the nearest multiple of 40″. The result would therefore be reliable to within 20″, more or less, since it would not be more than 20″ greater or less than the *nearest* multiple of 40″.

8. Let P be the top of the mountain, A the nearer and B the further point of observation.* Drop PM perpendicular on the line BA produced.

From the given data

$$\angle PAM = 22^\circ\, 15', \quad \angle PBM = 10^\circ\, 12';$$

$$\therefore\ APB = PAM - PBM = 12^\circ\, 3'.$$

Now $$BP = AB \sin PAB / \sin APB;$$

$$\therefore \text{ height of mountain } MP = BP \sin PBM$$
$$= AB \sin PAM \sin PBM / \sin APB.$$

Let h be the height MP in miles. Since $AB = 1$ mile,

$$\therefore\ h = \sin 22^\circ 15' \sin 10^\circ 12' \div \sin 12^\circ 3';$$

$$\therefore\ \log h = L \sin 22^\circ 15' + L \sin 10^\circ 12' - L \sin 12^\circ 3' - 10$$

(from given data) $= 9{\cdot}5782 + 9{\cdot}2482 - 9{\cdot}3197 - 10$
$= 18{\cdot}8264 - 19{\cdot}3197 = \bar{1}{\cdot}5067.$

Since $\log 32122 = 4{\cdot}5068$, $\therefore \log {\cdot}32122 = \bar{1}{\cdot}5068$;

therefore $h = {\cdot}32122$,

and height in feet $= {\cdot}32122 \times 5280 = 1696{\cdot}04$.

The last figure cannot be relied on. Thus the height is 1696 feet, almost exactly.

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* The reader will have no difficulty in drawing the necessary figure.

9. The points are represented in the accompanying figure, each division marked off on the axes being supposed to represent a unit length.

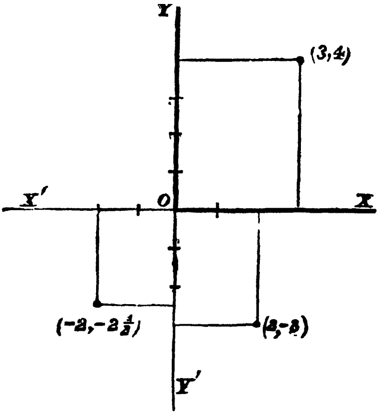

Fig. 1.

From the figure it is evident that the origin will be within the triangle, provided that it lies to the right of (or below) the line joining the points (3, 4) and $(-2, -2\frac{1}{2})$; and it is further evident that this will be the case if the line joining the origin to the point (3, 4) is inclined to the axis of x at a greater angle than the line joining the origin to $(-2, -2\frac{1}{2})$. The former inclination is $\tan^{-1}\frac{4}{3}$; the latter is $\tan^{-1} 2\frac{1}{2}/2$ or $\tan^{-1}\frac{5}{4}$. The first is the greater, thus showing that the origin *is* within the triangle.

[This might be more rigorously proved by finding the equations of the sides of the triangle, and showing that the origin is on the same side of any one of these lines as the corresponding vertex.]

Let (x, y) be the centre of the circle through the three points. Then the squares of the distances of (x, y) from these points are equal. Therefore

$$(x-3)^2+(y-4)^2=(x-2)^2+(y+3)^2=(x+2)^2+(y+2\tfrac{1}{2})^2.$$

Subtracting x^2+y^2 from each member, we have

$$-6x-8y+25=-4x+6y+13=4x+5y+10\tfrac{1}{4}.$$

Solving these simultaneous equations, we find

$$x=\frac{101}{228},\quad y=\frac{181}{228};$$

therefore the centre is the point

$$\left(\frac{101}{228},\ \frac{181}{228}\right).$$

10. Putting in turn $y=0$ and $x=0$ in the equation, we see that the line cuts the axes in the points $(4, 0)$, $(0, -\frac{4}{3}\sqrt{3})$, respectively. The equation may be written

$$y=(x-4)\tan 30^\circ,$$

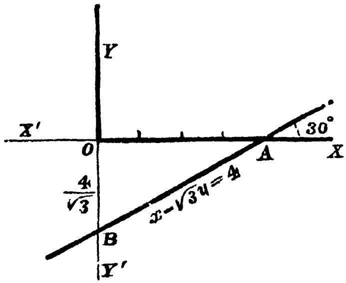

Fig. 2.

which shows that the line makes an angle 30° with the axis of x, as represented in the figure.

(i.) The product of the intercepts on the two axes

$$=-\tfrac{16}{3}\sqrt{3},$$

and the area of the triangle cut off is $=\frac{1}{2}$ of this $=\frac{8}{3}\sqrt{3}$ (neglecting sign).

(ii.) To reduce the line to the "perpendicular form," divide through by 2; thus the equation becomes

$$x\cdot\frac{1}{2}+y\cdot\left(-\frac{\sqrt{3}}{2}\right)=2,$$

or $$x \cos 300^\circ + y \sin 300^\circ = 2.$$

Therefore perpendicular from point (1, 2)

$$= 1 \times \tfrac{1}{2} + 2 \times \left(-\frac{\sqrt{3}}{2}\right) - 2 = -\tfrac{3}{2} - \sqrt{3},$$

or, neglecting the sign, its length is $\frac{3}{2} + \sqrt{3}$.

(iii.) The equation of the perpendicular line through the point (1, 2) is

$$(x-1) \cdot \frac{\sqrt{3}}{2} + (y-2)\,\frac{1}{2} = 0,$$

or $$x \cdot \frac{\sqrt{3}}{2} + y \cdot \frac{1}{2} = \frac{\sqrt{3}}{2} + 1,$$

which is in the form

$$x \cos 30^\circ + y \sin 30^\circ = \tfrac{1}{2}\sqrt{3} + 1.$$

Hence the perpendicular distance of this line from the origin is $\frac{1}{2}\sqrt{3} + 1$.

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