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DIFFERENTIAL CALCULUS
FOR BEGINNERS.

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WITH A SELECTION OF EASY EXAMPLES.

BY

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London:
MACMILLAN AND CO.
1884.

GLASGOW:
Printed at the University Press
BY ROBERT MACLEHOSE, 153 WEST NILE STREET.

PREFACE.

It has been found, almost invariably, that students beginning the Calculus meet, at the outset, with a stumbling-block. The Differential Co-efficient is shrouded in a haze. The few pages which follow may help to bring the idea of a Differential Co-efficient more within the grasp of beginners.

ALEXANDER KNOX.

5th June, 1884.

CONTENTS.

ON CERTAIN INFINITESIMALS, LIMITS,

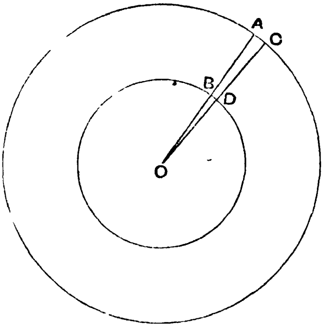
AND

DIFFERENTIAL CO-EFFICIENTS.

I. *Point, Line, and Superficies*

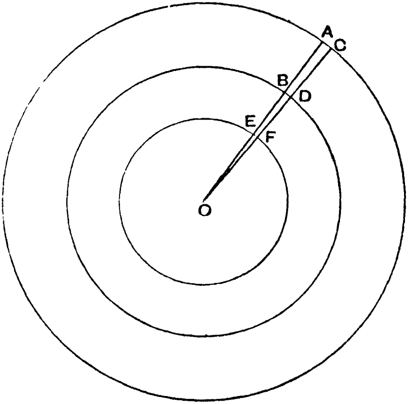
1. A point is defined as "that which has no parts and no magnitude." In order to obtain some more precise comprehension of the meaning of this term *point*, the following considerations may be of assistance

2. If we take two circles, having the same centre, and take any point in the circumference of the outer



circle, and join this point with the centre by drawing a straight line between the two points, it is evident that there will be a corresponding point on the circumference of the inner circle at the point where the straight line cuts this circle. Let O be the common centre, A the point on the circumference of the outer circle, and B the corresponding point on that of the inner circle. Then it is evident that for every such point on the circumference of the outer circle (as A) there will be a corresponding point on that of the inner circle (as B).

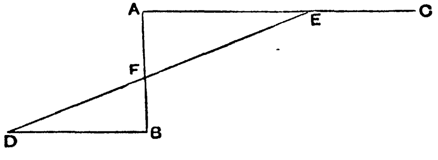
For if another point C be taken very near to A, then the radius CO will cut the inner circumference in some point D, other than B; because if CO were to pass through B, two straight lines AO, CO would have a common segment BO, or two straight lines BO, DO would enclose a space, and both of these are impossible. Therefore there will be a corresponding point (D) on the inner circle, other than B; and this will hold good



when C is as near to A as is conceivable; and it will also be the case, however large the outer circle, and however small the inner circle may be. For if we take any circle smaller than that on which are the points *B* and *D*, the straight lines *AO*, *CO* will cut it in two corresponding points *E* and *F*, and so for any still smaller circle.

Now, evidently, the larger the circle the larger is the circumference; and however long the circumference of the outer circle may be, and however short the circumference of the inner circle, and however great the number of points taken in the circumference of the outer circle, there will still be found a like number of corresponding points on the circumference of the inner circle. If we take the outer circle as described with a radius reaching from here to the fixed stars, and the inner circle as represented by the prick of a needle on this paper, then, for every possible point, which can be conceived, on the circumference of this enormous outer circle, there will be a corresponding point on the circumference of the circular puncture made by the needle.

3. Again, if we take any terminated straight line *AB*, and from the points *A* and *B* draw two parallel straight lines *AC*, *BD* in opposite directions, and take any fixed point *D* in *BD*, and take any other



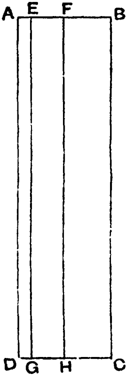
point *E* in *AC*, and join *DE* by a straight line, this will cut *AB* in *F* (say), and, similarly, for any other point taken in *AC*, there will be a corresponding point in *AB*; and remembering, as before, that two straight lines can neither enclose a space nor have a common segment, this will be true however near to E the point be taken. If then the straight line AB remain of fixed length, and D be a fixed point, and AC be supposed to be of unlimited length, extending, say, to one of the fixed stars, then, for every conceivable point in the whole length of AC, there will be a corresponding point in AB; and this is the case however small we may take AB to be. It will thus be seen that, if we take any very long line and take in it any very large number of points, we can always take a shorter line having the same number of corresponding points in it, and a still shorter line having the same number of corresponding points, and so on, until the second line is of inappreciable length, and, yet, in it can be taken the same number of corresponding points as in the longest line which can be conceived. The point, then, under these circumstances can have no parts and no magnitude, as it does not matter how long a circumference or how long a straight line may be taken, or what infinite number of points be taken in either, we can always take the same infinite number of points in the shortest circumference or the shortest straight line, or in one shorter than any assignable circumference or straight line.

4. Hence we may always take more points than any assignable number in any line, however short.

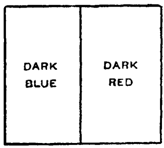
5. Suppose, now, that there be an enormously large surface—for simplicity's sake let it be supposed plane, though the result would be the same of whatever character the surface be; and suppose this surface, again for the same reason, to be bounded by straight lines—say four. Then in one of the bounding lines we can take a larger number of points than any assignable number, and consequently we can draw through these points a number of straight lines, parallel to one of the adjacent sides, larger than any number that can be mentioned.

Suppose the side, in which the points were taken, to become smaller and smaller, we can still take the same number of points and draw the same number of straight lines.

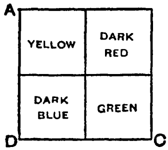
Let the space be now represented by the figure *ABCD*. Then as *AB* becomes shorter we can still draw the same number of straight lines, such as *EG*, *FH*, parallel to *AD*, as we could when *AB* was of enormous length, and when *AB* becomes shorter than any straight line that can be imagined, the same number of parallels can be drawn; that is to say, a line has length *without* breadth, for in the narrowest space, narrower than any that can be mentioned, there can be drawn a number of parallels larger in number than any number that can be assigned.



6. The manner in which we are accustomed to represent points and lines creates a mental prejudice against the acceptation of the definitions, for our points and lines have endless parts, considerable magnitude, and undoubted breadth. A simple method of representing a line, of putting the mind in possession of indubitable evidence of the existence of such a thing as a mathematical line, a length without a breadth, is to take two smoothly-planed square-cut blocks of wood of different colours and place them side by side in a vice, so that the two upper surfaces may be in one plane, and squeeze them tightly together; there will then be represented a *line*, length and no breadth —or, better still, paint a piece of paper, divided into two parts, two different colours, and then again a line will be apparent at the division or separation of the colours, or rather at the junction of the two colours, without any objection, which may



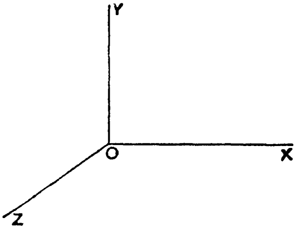
be raised in the first instance, of intervening particles of air.



7. Similarly, a point may be represented by painting such a space as *ABCD* in four colours, making two continuous lines cutting one another, and the intersection of the two lines, or the meeting of the four colours will be a point.

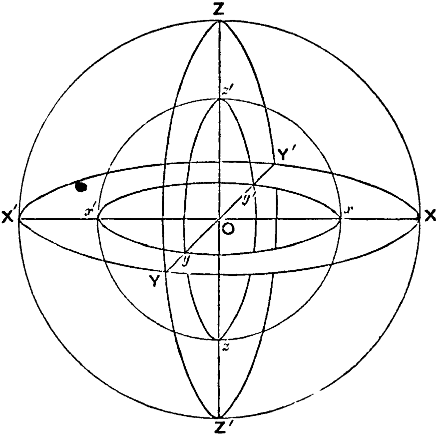
NOTE.—It would be a better method to paint the whole surface one colour first, say yellow; then one half blue over the yellow; and finally half of each of the resultant colour and the yellow, at one wash, with red. This would avoid niceties in laying on the colour.

8. Again, suppose OX, OY to be in the plane of the paper, and OZ perpendicular to that plane; then, employing a similar method of reasoning to that already adopted, we may take in OZ a number of points greater



than any assignable number, and through these points we may draw straight lines parallel to OX and OY, and through each pair we may suppose a plane to pass. Then when OZ is made shorter and shorter we can still have the same number of planes parallel to the plane of the paper, and when OZ is made of less than any assignable length, we can still have the same number of planes, greater than any assigned number, and hence the planes have only length and breadth but no thickness.

9. Again, suppose we have a sphere of radius OX, or OY or OZ; then, since we can take in any one of these

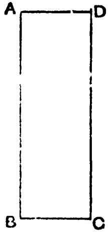


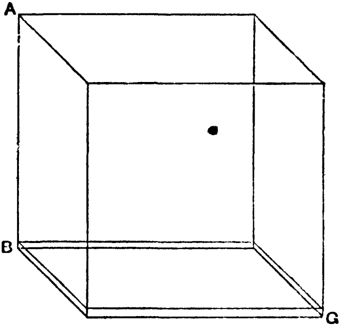
(which are equal) a number of points greater than any assignable number, we can draw spheres passing through each of these points, and we can do this when OZ becomes shorter than any assignable length; that is, when the number of spheres is increased beyond any assignable number, the crust (so to speak) of each sphere has no thickness; and each sphere presents merely a surface with no depth. From these considerations it will be seen what is meant by saying that a surface or superficies has only length and breadth.

10. These three properties of points, lines, and surfaces may be enunciated otherwise thus:—

(1.) If AB be a straight line, and B move up continually towards A, then when, ultimately, B coincides with A, or when AB is indefinitely diminished, the limit, that is the final value, of AB is a point.

(2.) If $ABCD$ be a surface and the line DC move up continuously towards AB, then when the breadth





AD is indefinitely diminished, the limit, or final value of $ABCD$ is a straight line.

(3.) If AG be a cube, then, when AB is indefinitely diminished, the limit of AG is a surface.

II. *The Forms*—$a \times 0$ *and* $\frac{a}{0}$.

11. If one number be multiplied by another, the product becomes less as one of the numbers diminishes. Thus $a \times 10$ is greater than $a \times 9$; and so $a \times \cdot 01$ is smaller than $a \times \cdot 1$; and when the number which is diminishing is very small (say $\cdot 000001$), the product is very small, and ultimately when the diminishing number becomes 0, the product becomes $a \times 0$ or 0. In

other words, the limit of ax when x gradually diminishes and ultimately vanishes is 0; or the limit of the product of two quantities, when one of them ultimately vanishes, is zero.

12. When one quantity is divided by another, the quotient becomes larger as the divisor becomes smaller. Thus $\frac{a}{10}, \frac{a}{9}, \frac{a}{8}$, etc., are in ascending order of magnitude; and again,

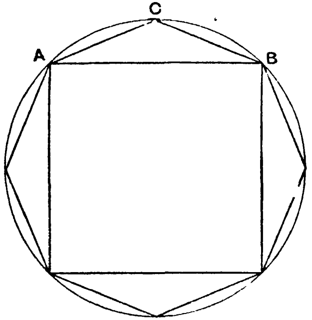
$$\begin{aligned} a \div 1 &= a. \\ a \div \cdot 1 &= 10a. \\ a \div \cdot 01 &= 100a. \\ \text{etc.} &= \text{etc.} \\ a \div \cdot 000001 &= 100000a. \\ \text{etc.} &= \text{etc.} \end{aligned}$$

And a divided by 1 preceded by the decimal point and 100 zeros $= a$ multiplied by 1 followed by 100 zeros; and before the divisor reaches the value 0, the quotient will have reached any value, however great, and ultimately, when the divisor reaches 0, the quotient becomes infinite, or $\frac{a}{x}$, where x is continually diminishing, and ultimately vanishes, is in the limit ∞.

III. *Newton's First Lemma—Recurring Decimals.*

13. Before proceeding farther, it will be advantageous to notice Newton's First Lemma, viz.:—"Quantities and the ratios of quantities which tend constantly to equality, and may be made to approximate to each other by less than any assignable quantity, become ultimately equal." Take any two quantities, and let them tend constantly to become equal; for instance, take a circle and inscribe in it a regular polygon, and let the number of sides be doubled, then the area of this figure is more nearly equal to the area of the circle than was

that of the original figure. Let the first figure be a square, then the eight-sided figure formed by doubling the number of sides is evidently more nearly equal to the area of the circle by four such triangles as ABC.



Again, let the number of sides be doubled, and the area of the new polygon will be still more nearly equal to that of the circle.

It is, then, asserted, that, if this process be indefinitely continued, ultimately, when the number of sides of the inscribed polygon is infinite, the area of the polygon is equal to the area of the circle. For if they are not ultimately equal, let them be ultimately unequal. Then there must be a difference between them. Let us suppose this difference to be D.

Now ultimately, on this supposition, there is a fixed difference between them—that is to say, the two areas cannot approach each other more nearly.

But, by hypothesis, we can make them approximate to each other by less than any assignable quantity, and therefore by less than D.

Therefore ultimately there is not a difference D, and they are not unequal—that is, they are equal.

This is expressed by saying that the limit of the inscribed polygon, when the number of sides is indefinitely increased and their length diminished, is the circle.

And the limit of the circumscribed polygon may be shown to be the same.

14. If we convert $\frac{1}{9}$ into a decimal, by dividing the numerator by the denominator, we obtain ·11111 . . . . , the 1's going on for ever,

or $$\frac{1}{9} = \cdot 11111 \ldots$$

$$= \frac{1}{10} + \frac{1}{100} + \frac{1}{1000} + \text{etc.},$$

to an infinite number of terms.

Now $$\frac{1}{9} - \frac{1}{10} = \frac{10-9}{90} = \frac{1}{90},$$

also $$\frac{1}{9} - \left(\frac{1}{10} + \frac{1}{100}\right) = \frac{100-99}{900} = \frac{1}{900},$$

and $$\frac{1}{9} - \left(\frac{1}{10} + \frac{1}{100} + \frac{1}{1000}\right) = \frac{1}{9000}.$$

Thus, if we take one term of the series

$$\frac{1}{10} + \frac{1}{100} + \frac{1}{1000} + \frac{1}{10000} + \text{etc.},$$

we find that it differs from $\frac{1}{9}$ by $\frac{1}{90}$.

If we take two terms it differs from $\frac{1}{9}$ by $\frac{1}{900}$.

If we take three terms it differs from $\frac{1}{9}$ by $\frac{1}{9000}$;

and, by taking any number of terms, we may make the series differ from $\frac{1}{9}$ by as little as we please,—*i.e.*, we can make it approximate to $\frac{1}{9}$ by less than any assignable difference.

For suppose we wish to make the series differ from $\frac{1}{9}$ by less than $\frac{1}{100000000}$.

Take, as the last term of the series, $\frac{1}{1000000000}$;

then the series differs from $\frac{1}{9}$ by $\frac{1}{9000000000}$,

which is less than $\frac{1}{100000000}$;

and similarly for any other assigned quantity.

Therefore we say that ultimately when the number of terms is indefinitely increased, the series $= \frac{1}{9} = \dot{.1}$.

IV. *The Form* $\frac{0}{0}$.

15. Suppose we have to find the value of the fraction $\frac{a^2 - b^2}{a - b}$ in the limit, when b continually increases, and ultimately becomes equal to a.

If we take the limit of $a^2 - b^2$ when b becomes equal to a, we find this to be 0; and also the limit of $a - b$, when b becomes equal to a, will be 0, and we shall have

$$\frac{a^2 - b^2}{a - b} = \frac{0}{0}.$$

Again, by actual division

$$\frac{a^2 - b^2}{a - b} = a + b$$

$= 2a$, when b becomes ultimately equal to a; and this is the limit required.

Now, it must be borne in mind, that what is meant by the value of a fraction in the limit is not the value obtained by dividing the limit of the numerator by the limit of the denominator; but the value of the quotient, actually obtained by division, in the limit, or the value of the *ratio* of the numerator to the denominator, as the numerator and denominator approach the limit, and ultimately arrive at it.

16. The value of a ratio is not altered if we divide its two terms by the same quantity, or, which is the same thing, the value of a fraction is not altered if we divide both the numerator and denominator by the same quantity. However small the two terms of the ratio may be made, by division by another quantity, they still retain the same ratio, no matter how insignificant they may be in themselves.

17. We must regard the relation existing between two quantities, not as expressed by the difference between them, or how much one is larger than the other, but as how many *times and parts of a time* the one is contained in the other, or what multiple one is of the other. This is, in fact, the manner in which we regard matters of every-day life. We compare them with others of a like nature, and so pronounce them small or great. The quantities may be either great or small in themselves; but it is their relative value which gives us a notion of them as great or small. Thus, if there were 300 men in one assembly and 3000 in another, we should say, as a rule, that there were ten times as many in the latter as there were in the former, and not that there were 2700 more; and, again, the actual number 1000 may vary through any values, from very great to very small—it is all a matter of comparison. If it were stated that 1000 horses started in a race, we should say that it was simply ridiculous, the number was too large; if that 1000 men lived in one hamlet, that it was very large; if that there were 1000 men in one regiment, that it was large or beyond the average; and if that the 1000 men composed an invading army, that it was insignificant. Let us take an improper fraction $\frac{10000}{10}$, this is equal to 1000 or

$$\frac{10000}{10} = 1000.$$

also $$\frac{\cdot 1}{\cdot 00001} = 1000.$$

Similarly $$\frac{\cdot 0000001}{\cdot 00000000001} = 1000,$$

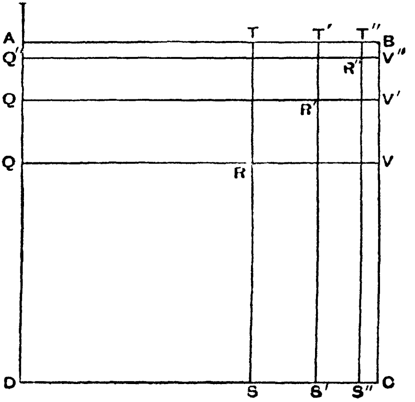
$$\&c. = \&c.,$$

and $\dfrac{\text{the decimal point followed by a million zeros and 1}}{\text{,, ,, a million and 4 zeros and 1}} = 1000.$

Therefore, it follows, that we may make the numerator and denominator differ by less than any assignable quantity, and the *ratio* of the numerator to the denominator still remain equal to 1000.

It will be seen then that it does not matter how small the terms of a ratio are, the value of the ratio remains unaltered.

18. Let us now revert to the limit of $\dfrac{a^2 - b^2}{a - b}$.



Let $ABCD$ be a square whose side is a, and $QRSD$ a square whose side is b.

Produce SR and QR to T and V.

Then $\quad AB=a$ and $QR=b=AT$,

therefore $\quad ABCD=a^2,\ QRSD=b^2$,

and the gnomon $\quad AVS=a^2-b^2$,

and $\quad TB=a-b$.

Now
$$AVS=2AR+TV,$$
$$=2AT.TB+TB^2;$$

therefore
$$\frac{a^2-b^2}{a-b}=\frac{AVS}{TB},$$
$$=\frac{2AT.TB+TB^2}{TB},$$
$$=2AT+TB. \qquad (1)$$

Now suppose b or DQ to become larger and be represented by DQ' or AT'. Then it will be seen that a^2-b^2 becomes smaller, and is now represented by $AV'S'$, and that the rectangle, which was originally AR, has become longer and narrower, and is now represented by AR', and also that $AV'S'$ is more nearly equal to $2AR'$ than AVS was to $2AR$, since the square, which was originally TV, has become $T'V'$.

Suppose, now, that b becomes still larger, and let it be represented by AT''. Then the rectangle will have become still narrower, and the square $T''V''$ very small, and the gnomon $AV''S''$ is more nearly equal to $2AR''$ than $AV'S'$ was to $2AR'$.

By proceeding in this way, it will be seen that, eventually, when T moves up to B, that is, when b becomes equal to a, the rectangle will have become indefinitely narrow, and the square TV will have vanished altogether; that is, the gnomon will be represented by twice the line AB, since it will be represented by AB and BC; or, from (1), the limit of the ratio of a^2-b^2 to $a-b$, when b ultimately becomes a, is equal to $2AT$ or $2a$.

19. This result might have been obtained thus :—

Let $b = a - h.$

Then, when $h = 0,\ b = a.$

Substituting this value for b, we have

$$\frac{a^2 - b^2}{a - b} = \frac{a^2 - (a - h)^2}{a - (a - h)}$$

$$= \frac{a^2 - a^2 + 2ah - h^2}{a - a + h}$$

$$= \frac{2ah - h^2}{h}$$

$$= 2a - h$$

$$= 2a, \text{ when } h = 0 \text{ or } b = a.$$

20. We will illustrate the truth of these remarks by numerical examples.

Let $a = 10$ and $b = 9,$

then $$\frac{a^2 - b^2}{a - b} = \frac{10^2 - 9^2}{10 - 9} = 19,$$

and this differs from $2a$ or 2×10 by $\frac{1}{19}$ part of itself.

Again, let $a = 100$ and $b = 99,$

then $$\frac{a^2 - b^2}{a - b} = \frac{100^2 - 99^2}{100 - 99} = 199;$$

and this differs from $2a$ or 2×100 by $\frac{1}{199}$ part of itself.

Again, let $a = 1000000,$

and $b = 999999.$

then $$\frac{a^2 - b^2}{a - b} = \frac{1000000^2 - 999999^2}{1000000 - 999999}$$

$$= 1999999,$$

which differs from $2a$ or 2×1000000 by $\frac{1}{1999999}$ part of itself.

It is clear, then, the smaller the difference between a and b, the more nearly does $\frac{a^2-b^2}{a-b}$ approximate to $2a$; and, therefore, we say that ultimately, in the limit, when $b=a$, $\frac{a^2-b^2}{a-b}=2a$.

(See also Art. 91.)

V. *Function—Differential Co-efficient—Differential Co-efficient of a Simple Function.*

21. If one quantity depend upon a particular value of another variable quantity, the first quantity is said to be a *Function* of the second; or, if one quantity or expression involve another in any form, it is said to be a *Function* of that quantity. The quantity upon which the other depends is called the *independent variable*, and the function the *dependent variable.*

Thus $3x$, x^2, $\frac{px^n+q}{rx}$, etc., are all functions of x. The independent variable is x, upon whose value the value of the expression, or *function of* x, depends; similarly the *area* of a square is a *function* of its *side*, the *side* being the *independent variable*, upon whose value the value of the area depends; the *volume* of a cube is a *function* of its *edge;* the *circumference* and *area* of a circle are, each of them, *functions* of its *radius;* the *volume* of a spere is a *function* of its radius—the edge of the cube, the radius of the circle, and the radius of the sphere being the *independent* variables in each case, and the *volume* of *cube*, *area*, and *circumference* of the circle, and the *volume* of the sphere, the *dependent* variables.

22. Our object is to find *the ratio of the rate of variation* (*i.e.*, *the rate of increase or decrease*) *of the function to the rate of variation of the independent variable*, as the independent variable undergoes infinitesimally small variations. This ratio is called the *Differential Co-efficient* of the function.

23. If a variable quantity increase uniformly, the function either increases uniformly or accordingly to any variable law.

Let x be a variable quantity, and let it increase uniformly by the quantities 1, 1, 1, etc.

Then the successive values will be

$$x+1,\ x+2,\ x+3, \text{ etc.}$$

Then also any number of times of x will increase uniformly—say $3x$—the values being

$$3x+3,\ 3x+6,\ 3x+9, \text{ etc.},$$

which increase uniformly by 3.

Again, take px, then the successive values are

$$px+p,\ px+2p,\ px+3p, \text{ etc.},$$

which increase uniformly by p.

Further, let x be a variable quantity, and let it increase uniformly by the quantities a, a, a, etc., then it will, at the successive stages, become

$$x+a,\ x+2a,\ x+3a, \text{ etc.},$$

and, as before, any number of times of x will increase uniformly.

First, take $3x$, then the successive values become

$$3x+3a,\ 3x+6a,\ 3x+9a, \text{ etc.},$$

which increase uniformly by $3a$.

Next, take px, then the successive values become

$$px+pa,\ px+2pa,\ px+3pa, \text{ etc.}$$

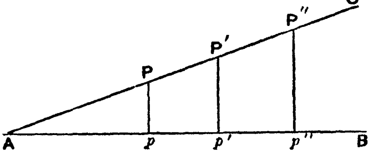
which increase uniformly by pa.

24. It is evident that if a *constant quantity* (i.e., one which does not vary) be connected with the function px by the sign $+$ or $-$, the function will still increase uniformly, for the successive values will be

$$px+pa+C,\ px+2pa+C,\ px+3pa+C, \text{ etc.},$$

25. Again, to illustrate this geometrically, suppose we have a straight line AB, and draw AC, making any acute angle with AB, and let a variable straight line Pp move from A so as to remain always perpendicular to AB, and have one extremity in AB and the other in

AC, and take up, at successive periods, such positions as $P'p'$, $P''p''$, etc.; then it is evident that, as Ap in-



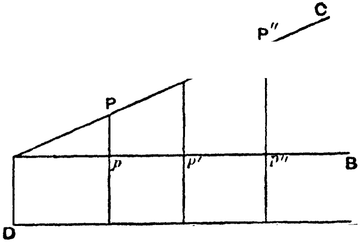
creases uniformly, and becomes Ap', Ap'', etc., Bp increases uniformly, for

$$\frac{AP}{Ap}=\frac{AP'}{Ap'}=\frac{AP''}{Ap''}, \text{ etc.,}$$

and so for any other position of Pp.

Again, let DE be parallel to AB, and let Pq be the new variable line.

Now $Pq=Pp+pq$, and pq is constant; and it is



evident that, as Ap increases uniformly, Pq increases at the same rate as before.

26. Now let x be any given variable quantity, and $3x$ a given function of x, then as x becomes $x+h$, $3x$ becomes $3(x+h)$ or $3x + 3h$, and the ratio of the rate of increase in the function to the rate of increase in the variable $=\frac{3h}{h}=\frac{3}{1}=3$.

Now let h become less and less; this ratio still holds good, and, ultimately, when h is indefinitely diminished, *i.e.*, in the limit, the rate of (increase in this case) variation in the function is to the rate of variation of the independent variable as 3 : 1, *i.e.*, the *differential co-efficient* of $3x$ is 3, and a similar argument will hold if we take nx instead of $3x$. Thus it will be found generally that

the differential co-efficient of nx with respect to x, *i.e.*, where x is the independent variable, $\Big\} = n$.

27. Let us take a quantity $x+C$, where x is the independent variable, and take $n(x+C)$ as a function of this, C being constant; and let x receive a small increment and become $x+h$

$$\text{then } x+C \text{ becomes } (x+h)+C,$$
$$\text{and } n(x+C) \text{ becomes } n(x+h)+nC,$$
$$\text{or } nx+nh+nC,$$

and the ratio of the rate of variation of the function to the rate of variation of the variable $= \dfrac{nh}{h} = n$.

NOTE.—It is obvious that if the rate at which two quantities increase be added together, the sum will be the rate of increase at which the sum of the quantities increases; and the difference, the rate at which the difference increases. Therefore, if we have two functions of the same variable connected by the signs + or −, the differential co-efficient of the whole expression will be the sum or difference of the differential co-efficients of the two parts.

VI. *Differential Co-efficient of* x^2.

28. Let a square have a side of 4 feet, then the area of the square = 16 square feet, that is if

$$x = 4$$
$$x^2 = 16.$$

Now, suppose the side to receive a small increment and become 4·001 feet, then the square becomes 16·008001 square feet.

If we omit ·000001, then the ratio of the increase of the function to the increase of the variable, or of

$$\cdot 008 : \cdot 001 = 8 : 1$$
$$= \text{twice side of square} : 1.$$

Again, suppose the side to receive a still smaller increment and become 4·000001 feet; then the area of the square = 16·000008000001 square feet.

Here by omitting ·000000000001 we commit an almost inappreciable error, and, as before, and still more truly, the ratio of the increase of the function to the increase of the variable is

$$\cdot 000008 : \cdot 000001, \text{ or } 8 : 1,$$

or 2 × side of square : 1.

Therefore we may state that, ultimately, when the increment of the side is indefinitely diminished, or in other words is made indefinitely small, the ratio of the rate of increase in the function (square) to the rate of increase of the variable (side) is $2x : 1$, or the differential co-efficient of x^2 is $2x$.

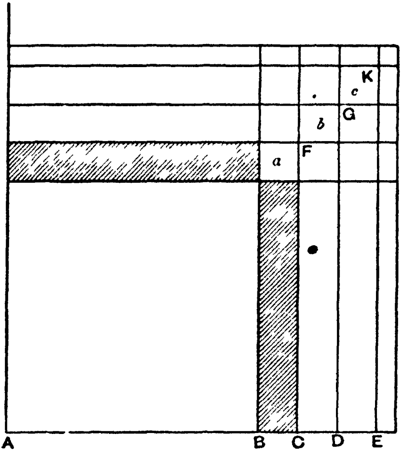
29. Let AB be a straight line, and let a square be described on AB. Then this square is a function of AB. Now let AB receive a small increment BC; the straight line has now become AC, and the square has, in consequence, received an increment of the two shaded rectangles and the small square a.

Let the straight line receive a further increment CD ($=BC$), then the square will have received an increment of four rectangles and four such squares as a.

Now let the straight line receive a further increment DE ($=CD=BC$), then the square will have received an increment of six rectangles, such as the shaded rectangles, and nine such squares as a.

Thus we see that, as the straight line increases uniformly, the square increases, but not uniformly.

30. Again, when the side has an increment BC, the square has an increment of two shaded rectangles (one of whose sides is equal to the side of the original



square) and a small square—*i.e.*, small when compared with the original square.

The second square, whose side AC receives an increment CD, receives an increment of two rectangles such as DF (one of whose sides is equal to the side of the second square) and the square b, which is even smaller, when compared with the square on AC, than a is when compared with the square on AB.

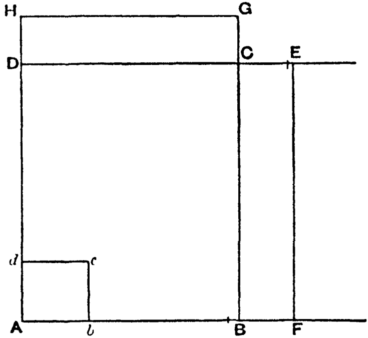
Now let the side AD receive a further increment DE, then, as before, the square receives an increment of two rectangles such as EG (one of whose sides is equal to the side of the square on AD) and the square c, which is *very small* when compared with the square on AD.

Suppose, now, that the side AE receives a very small increment indeed; then the square receives an increment of the two very narrow rectangles (one of whose sides is equal to the side of the square on AE) and the minute square at K.

Finally, when the breadth of the rectangles is indefinitely diminished, or, which is the same thing, when the side receives an infinitesimally small increment, the rectangles become coincident with the sides of the square (see Art. 10), and the small square vanishes, when compared with the square on AE, and the increment in the square corresponding to the infinitesmal increase in the side is made up of two rectangles coincident with the sides—*i.e.*, the ratio of the rate of increase of the square to the rate of increase of the side, when the increment to the side is infinitesimal, is $2 \times$ side : 1, as before.

Now, let us look at this from a different point of view.

Let AC be a square on AB—AB being a variable; and suppose the square to be growing continuously as



AB increased, having originally been Ac, and let AB have arrived at the value x; in consequence of which

$AC = x^2$; and let BF represent the increment which x would receive in the next unit of time.

Now, let the square be checked in its increasing course as soon as it has arrived at the value x^2.

The rate of increase of the square (since it is moving with accelerated motion) will not be represented by the increment which it would receive in the next unit of time, but by the increment it would receive if it *increased* with *uniform* motion at the rate which it had at the instant at which it was stopped.

Therefore, in order that the motion may be uniform, as the sides BC, DC move outwards, they must remain of the same length.

Hence, BF or DH representing the increase in the variable, the corresponding increase in the square will be represented by the two rectangles BE and CH.

i.e., by $2 \times BE$.

But BE = side of square × rate of increase of x,
since BF = rate of increase of x.

∴ rate of increase of square

$$= 2x \times \text{rate of increase of } x,$$

i.e.,
$$\frac{\text{rate of increase of } x^2}{\text{rate of increase of } x} = 2x,$$

or
$$\text{differential coefficients of } x^2 = 2x,$$

VII. *A Falling Body.*

31. Firstly. Suppose a body to fall from rest for $\frac{1}{10}''$, it will have fallen through ·16 feet and have acquired a velocity of 3·2 feet per second. Suppose it then to receive a check which brings it to rest, and then let it, without loss of time, fall, as before, for $\frac{1}{10}''$; it will, as before, fall ·16 feet, and again acquire a velocity of 3·2 feet per second. Let the same process be repeated until, in all, the body has been let fall for 10″, that is 100 times; then the body will have passed

through 16 feet, and the velocity at the end of the time will be 3·2 feet per second.

32. Secondly. Suppose that, after the body has been arrested at the end of $\frac{1}{10}''$, we give it an impulse equal to the velocity it had acquired before it was arrested, viz., a velocity of 3·2 feet per second.

Then at the end of the second $\frac{1}{10}''$ it will have a velocity of 6·4 feet per sec., and the space described will be the original space of ·16 feet

+ that which the body would have described moving uniformly with a velocity of 3·2 feet per sec.

+ the space which it would have described without that impulse

$$=(\text{in feet})\ \cdot16+3\cdot2\times\frac{1}{10}+\frac{32}{2}\times\left(\frac{1}{10}\right)^2$$

$$=\cdot16+\cdot32+\cdot16$$

$$=\cdot64\ \text{feet.}$$

If the body had not been arrested, the space fallen through from rest would have been $\frac{32}{2}\times\left(\frac{2}{10}\right)^2$ feet $=\cdot64$ feet.

Now let the same process be repeated for the third tenth of a second. The starting velocity will be 6·4 feet per second, and the velocity at the end of the third $\frac{1}{10}''$ will be 9·6 feet per second; and the space travelled through will be

that arrived at at the end of the second $\frac{1}{10}''$

+ that which the body would have described moving uniformly with a velocity of 6·4 feet per sec.

+ the space which it would have described without that impulse

$$=(\text{in feet})\ \cdot64+6\cdot4\times\frac{1}{10}+\frac{32}{2}\times\left(\frac{1}{10}\right)^2$$

$$=\cdot64+\cdot64+\cdot16$$

$$=1\cdot44\ \text{feet.}$$

If the body had not been arrested, the space fallen through would have been $\frac{32}{2} \times \left(\frac{3}{10}\right)^2$ feet $= 1 \cdot 44$ feet.

If this process be repeated 100 times, the time of falling will be 10″, and

the velocity acquired will be $= 320$ ft. per sec.
and the space described $= 16 \times 100$ ft.
$= 1600$ ft.

33. In the following table the first column represents the time in seconds during which the body is falling; the second column gives the corresponding spaces through which the body falls (in feet); the third column is obtained from the second by subtracting each number from the one immediately above it, and gives the spaces fallen through in each $\frac{1}{10}$″; the fourth column is obtained from the third in the same manner in which the third is obtained from the second, and gives the difference between the spaces fallen through in the consecutive $\frac{1}{10}$″s seconds, and it will be remarked that these last are all the same.

Sec.	Space fallen through.		
1	16		
		3·04	
·9	16 × ·81 = 12·96		·32
		2·72	
·8	16 × ·64 = 10·24		·32
		2·40	
·7	16 × ·49 = 7·84		·32
		2·08	
·6	16 × ·36 = 5·76		·32
		1·76	
·5	16 × ·25 = 4·00		·32
		1·44	
·4	16 × ·16 = 2·56		·32
		1·12	
·3	16 × ·09 = 1·44		·32
		·80	

Sec.	Space fallen through.			
·2	16 × ·04 =	·64		·32
			·48	
·1	16 × ·01 =	·16		·32
			·16	

Thus we see that the space fallen through in the interval between any two consecutive tenths of seconds is ·32 feet.

This space for 100ths secs. = ·0032 feet,
„ 1000ths secs. = ·000032 feet,
„ 1000000ths secs. = ·000000000032 feet,
etc. = etc.,

and, when the intervals are made infinitesimally small, the space becomes infinitesimally small, but is always a multiple of 32. We may say, then, that when there is a continuous fall, without any interruption, the motion becomes continuous, losing its jerks and impulses (the jerks becoming inappreciable), the space fallen through is increasing, at any instant, by an infinitesimal multiple of 32.

(See also Art. 48, etc.)

VIII. *Differential Co-efficient of* 1^2, 2^2, 3^2, *and* 4^2.

34. (1) Here 1 is supposed to receive small increments of ·01; therefore 1 will be the variable.* The function considered is the square of the variable.

Independent Variable.	Function (Square).	First Difference.	Second Difference.
1·01	1·0201		
		·0203	
1·02	1·0404		·0002
		·0205	
1·03	1·0609		·0002
		·0207	
1·04	1·0816		

* When *variable* is mentioned *independent variable* is implied.

(2) Here 2 is supposed to receive small increments of ·001 ; therefore 2 will be the variable. The function under consideration in this case is also the square of the independent variable.

Independent Variable.	Function (Square).	First Difference.	Second Difference.
2·001	4·004001		
		·004003	
2·002	4·008004		·000002
		·004005	
2·003	4·012009		·000002
		·004007	
2·004	4·016016		

(3) Here 3 is supposed to receive small increments of ·0001 and the function again is the square.

3·0001	9·00060001		
		·00060003	
3·0002	9·00120004		·00000002
		·00060005	
3·0003	9·00180009		·00000002
		·00060007	
3·0004	9·00240016		

(4) Here 4 is supposed to receive small increments of ·00001, and, as before, the function is the square.

4·00001	16·0000800001		
		·0000800003	
4·00002	16·0001600004		·0000000002
		·0000800005	
4·00003	16·0002400009		·0000000002
		·0000800007	
4·00004	16·0003200016		

The first column in each case represents the independent variable, as it increases uniformly by increments of ·01, ·001, ·0001 and ·00001 respectively.

The numbers in the second columns are the squares of the successive values of the variables.

The numbers in the third columns are the *first differences*, each being the difference between the numbers immediately above and below it in the column to the left.

The numbers in the fourth column are the *second differences*, each being the difference between the two numbers immediately above and below it, in the column to the left.

In each of the cases (1), (2), (3), (4) the *function* is the *square* of the independent variable.

It will be seen that in (1) the first two figures of the first differences are the same, viz., ·02.

When the independent variable is 1·01, the function is 1·0201; when the independent variable has received a further increment, and has become 1·02, the function, in consequence, has become 1·0404—*i.e.*, it has increased by ·02 approximately, if we omit ·0003.

When the variable arrives at the value 1·03, the function has increased from 1·0404 to 1·0609; or, again, by ·02 approximately, if we omit ·0005; and similarly, when the variable assumes the value 1·04, the function again increases approximately by ·02.

Thus, if we omit the ten-thousandths, we may say that, as the independent variable increases by increments of ·01, the function increases by ·02. That is to say, the ratio of the rate of variation (increase in this case) of the function to the rate of variation of the independent variable is ·02 : ·01 or 2 : 1.

This may be stated as follows:—If 1 receive small successive increments, the differential co-efficient of $1^2 = 2 = 2 \times 1$.

35. If we now consider (2), we see that the increments in the independent variable are smaller than in (1); and that, as this variable increases from 2·001, the function increases by increments of ·004, if we omit millionths; and therefore this increment is more

approximately true than was the increment ·02 in the first case, for there we omitted ten-thousandths. In this case, (2), the ratio of the rate of variation of the function to the rate of variation of the variable is ·004 : ·001 or 4 : 1—*i.e.*, if 2 receive small increments successively, the differential co-efficient of $2^2 = 4 = 2 \times 2$.

Similarly, by omitting hundreds-of-millionths in (3), we find that the ratio of the rate of variation of the function to the rate of variation of the variable is ·0006 : ·0001 or 6 : 1—*i.e.*, as before, the differential co-efficient of $3^2 = 6 = 2 \times 3$. And in (4) this ratio, which is still more approximately correct, is ·00008 : ·00001 or 8 : 1.

36. The results of these four cases are

Differential co-efficient of		$1^2 = 2 \times 1$,	(α)
"	"	$2^2 = 2 \times 2$,	(β)
"	"	$3^2 = 2 \times 3$,	(γ)
"	"	$4^2 = 2 \times 4$,	(δ)

and we notice that the differential co-efficient was obtained from the *first difference* by approximation, or by omitting quantities which, when compared with the quantities forming the ratio, were of insignificant value; and we notice also that the smaller the increment the more are the quantities omitted insignificant. And eventually, when the increments are infinitesimal, there is no need of omission at all.

37. Now to refer to (1) again and take into account the more minute quantities, we notice that, as the independent variable increases by small increments, the function also increases, and, if we refer to the second difference we see that the first difference also increases with the increase of the independent variable by increments of ·0002.

Now the ratio of the rate of variation of the *first difference* to (the rate of variation of the independent variable)^2 is called the *second differential co-efficient* of the function, and we see that in

$$(1) \quad \frac{\cdot0002}{.01^2} = \frac{\cdot0002}{\cdot0001} = \frac{2}{1}.$$

$$(2) \quad \frac{\cdot000002}{\cdot001^2} = \frac{\cdot000002}{\cdot000001} = \frac{2}{1}.$$

$$(3) \quad \frac{\cdot00000002}{\cdot0001^2} = \frac{\cdot00000002}{\cdot00000001} = \frac{2}{1}.$$

$$(4) \quad \frac{\cdot0000000002}{\cdot00001^2} = \frac{\cdot0000000002}{\cdot0000000001} = \frac{2}{1}.$$

38. These results may be obtained independently from the first differential co-efficient; for, as we have already seen (Art. 26), the differential co-efficient of nx, where x is the independent variable, $= n$.

Therefore, if in (α) the 1 varies, and in (β) the 2 varies, in (γ) the three varies, and in (δ) the 4 varies, we have

Differential co-efficient of $2 \times 1 = 2$,
" " $2 \times 2 = 2$,
" " $2 \times 3 = 2$,
" " $2 \times 4 = 2$.

So that the second differential co-efficient of a function is the first differential co-efficient of its (function's) first differential co-efficient.

39. We must further notice that, working upwards from the quantity 2 (which is constant for all variations of the variable), this quantity 2 is the origin or germ of the whole system of variable squares, and also of their differences, and that the square is always varying by some function of 2, for since

$$\cdot02 = 2 \times \cdot01,$$
$$\cdot004 = 2 \times \cdot002,$$
$$\cdot0006 = 2 \times \cdot0003,$$
$$\cdot00008 = 2 \times \cdot00004,$$

it follows that ·02, ·004, ·0006, and ·00008 are all of them functions of 2.

IX. *Differential Co-efficient of* 1^3, 2^3, 5^3, *and* 7^4.

	Independent Variable.	Function (Cube).
(1)	1·00001	1·000030000300001
	1·00002	1·000060001200008
	1·00003	1·000090002700027
	1·00004	1·000120004800064
	1·00005	1·000150007500125

First Differences.	Second Differences.	Third Differences.
·000030000900007		
	·000000000600012	
·000030001500019		·000000000000006
	·000000000600018	
·000030002100037		·000000000000006
	·000000000600024	
·000030002700061		

Independent Variable.	Function (Cube).	1st Differences.	2nd Differences.	
(2) 2·01	8·120601			
		·121807		
2·02	8·242408		·001212	
		·123019		·000006
2·03	8·365427		·001218	
		·124237		·000006
2·04	8·489664		·001224	
		·125461		
2·05	8·615125			

40. In (1) the number 1 is supposed to receive small increments of ·00001; and in (2) the number 2 to receive small increments of ·01.

The third differences are obtained from the second differences in the same manner that the second differences were obtained from the first differences, and the first differences from the function in VIII. The function, in each of the cases at present under consideration, is the cube of the independent variable, as the variable in the two cases receives increments of ·00001 and ·01 respectively.

41. Now, considering (1), it will be seen that, if we neglect hundreds-of-thousands-of-millionths, the ratio of the rate of increase of the function to the rate of increase of the variable is

$$\cdot 00003 : \cdot 00001 \text{ or } 3 : 1,$$

i.e., the differential co-efficient of $1^3 = 3 = 3 \times 1^2$.

And the second differential co-efficient of

$$1^3 = 3 \times 2 \times 1 = 6,$$

for the ratio of the rate of variation of the first difference (*which is given by the second difference*) to (the rate of variation of the variable)^2 = second differential co-efficient of 1^3

$$= \frac{\cdot 0000000006}{\cdot 00001^2}$$

$$= \frac{\cdot 0000000006}{\cdot 0000000001}$$

$$= \frac{6}{1}$$

$$= 6 = 3 \times 2 \times 1.$$

Again, the rate of variation of the *second differences* is given by the *third differences*, and the ratio of the rate of variation of the *second differences* to (rate of variation of the variable)^3 is called the *third* differential co-efficient of the function.

Therefore the *third differential co-efficient* of 1^3

$$= \frac{\cdot 000000000000006}{\cdot 00001^3}$$

$$= \frac{\cdot 000000000000006}{000000000000001}$$

$$= \frac{6}{1}$$

$$= 6 = 3 \times 2.$$

42. Now the *second* differential co-efficient might have been obtained from the *first*, for the differential co-efficient of 3×1^2, if the 1 be supposed to vary, is $3 \times 2 \times 1$, which is the same result as was previously obtained.

Similarly, the *third* differential co-efficient may be obtained from the *second* differential co-efficient, for the differential co-efficient of 6×1, if the 1 be supposed to vary, is 6.

43. Precisely similar results will be obtained from (2), but in this case the approximation will not be so far from error, inasmuch as the increments in the variable are not so small. For the first differential co-efficient we shall have to neglect *thousandths*, and for the second differential co-efficient *hundreds-of-thousandths.*

Here *first* differential co-efficient $= \dfrac{\cdot 12}{\cdot 01} = \cdot 12 = 3 \times 2^2$;

second ,, ,, $= \dfrac{\cdot 0012}{\cdot 01^2}$,

$= \dfrac{\cdot 0012}{\cdot 0001}$,

$= \dfrac{12}{1}$,

$= 3 \times 2 \times 2$;

third ,, ,, $= \dfrac{\cdot 000006}{\cdot 01^3}$,

$= \dfrac{\cdot 000006}{\cdot 000001}$,

$= 6$,

$= 3 \times 2$.

44. We shall obtain similar results whatever number we take as the variable: for instance, let us take 5, and let it vary by increments of ·0001.

Variable.	Function (Cube.)	First Difference.
5·0001	125·007500150001	
		·007500450007
5·0002	125.015000600008	
		·007500750019
5·0003	125·022501350027	
		·007501050037
5·0004	125·030002400064	
		·007501350061
5·0005	125·037503750125	

Second Difference.	Third Difference.
·000000300012	
	·000000000006
·000000300018	
	·000000000006
·000000300024	

Here, again, the results will be

first differential co-efficient of $5^3 = \frac{\cdot 0075}{\cdot 0001} = 75 = 3 \times 5^2$;

second " " $= \frac{\cdot 00000030}{\cdot 0001^2}$;

$= \frac{\cdot 00000030}{\cdot 00000001}$,

$= 30 = 3 \times 2 \times 5$;

third " " $= \frac{\cdot 000000000006}{\cdot 0001^3}$

$= \frac{\cdot 000000000006}{\cdot 000000000001}$,

$= 6$,

$= 3 \times 2.$

45. It may be noticed here that, as we found 2 to be the germ or essence of any system of variable *squares*,

so we find 6 to be the germ or essence of any system of variable *cubes*, and their successive differences, for

$$\cdot00003 = 6 \times \frac{1}{2 \times 100000},$$

$$\cdot0012 = 6 \times \frac{2}{10000},$$

$$\text{etc.,} = \text{etc.,}$$

and, therefore, ·00003, ·0012, etc., are all of them functions of 6.

In the systems of squares, we found that the *second* differences received no increment—*i.e.*, were *constant*; so, in the systems of cubes, we find that the *third* differences receive no increment—*i.e.*, are *constant*.

46. If we adopt, for the *fourth* powers of any variable, a method similar to that already used for the squares and cubes, we shall arrive at analogous results; for instance, if 7 be supposed to receive small increments, then

first differential co-efficient of $7^4 = 4 \times 7^3$,
second ,, ,, ,, $= 4 \times 3 \times 7^2$,
third ,, ,, ,, $= 4 \times 3 \times 2 \times 7$,

and, regarding the *fourth* differential co-efficient as the ratio of the rate of variation of the third difference to (the rate of variation of the variable)^4,

fourth differential co-efficient as $7^4 = 4 \times 3 \times 2$
$= 24$;

and 24 will be the essence or germ of any system of fourth powers.

47. Now, we have found that

the germ of *second* power $= 2 = 2 \times 1 = \underline{|2}$,
,, *third* ,, $= 6 = 3 \times 2 \times 1 = \underline{|3}$,
,, *fourth* ,, $= 24 = 4 \times 3 \times 2 \times 1 = \underline{|4}$,

and so it will be found generally, that the germ of the n^{th} power

$$= n.\overline{n-1}\ldots3.2.1 = \underline{|n},$$

and also that *first* differential co-efficient of x^n, where x is the variable, is nx^{n-1}.

NOTE.—Referring to Art. 30, it follows that the ratio of the rate of variation of 3 times the square to the rate of variation of the variable $= 3 \times 2 \times$ side : 1 ; and of n times the square $= n \times 2 \times$ side : 1 ; therefore the differential co-efficient of $ax^2 = 2ax$, and the differential co-efficient of $ax^n = nax^{n-1}$.

X. *Method of Differences applied to the Motion of a Falling Body.*

48. Let us apply this method of differences to the motion of a falling body.

In 1″ a body falls through 16 feet. Now let this 1″ receive increments of ·0001 ; the space fallen through in

Time.	Space.	First Diff.	Second Diff.
1·0001″	=(16 × 1·00020001) ft. =16·00320016 ft.		
		·00320048	
1·0002″	=16·00640064 ft.		·00000032
		·00320080	
1·0003″	=16·00960144 ft.		·00000032
		·00320112	
1·0004″	=16·01280256 ft.		

From this we see that the ratio of the rate of variation of the function (*the space fallen through*) to the rate of variation of the variable (*the time*) $= \frac{\cdot 0032}{\cdot 0001} = 32$, omitting the figures in the seventh and eighth decimal places.

Now the first differences give the space fallen through in each successive interval of ·0001″, and the ratio will be more nearly correct the smaller we make the increments.

But these first differences are themselves receiving increments as the time increases, and the *second differential co-efficient* gives the ratio of their rate of variation to (the rate of variation of the time)^2, viz.:

$$\frac{\cdot 00000032}{\cdot 00000001} = 32,$$

and this ratio has the same value, however small the increments be made.

Therefore, we may say that, at any instant, the space fallen through is increasing by some function of 32, and that that increase is, at that instant, also itself increasing by some function of 32—32 being the germ or essence of the system of spaces fallen through, and also of the differences.

XI. *The Differential Co-efficients of an Inverse Function.*

	Reciprocal of Variable.	Function (Square of Reciprocal).	First Difference.
(1)	$\frac{1}{1 \cdot 01}$	$\frac{1}{1 \cdot 0201} = \cdot 980296$	
			$- \cdot 02$ approx.
	$\frac{1}{1 \cdot 02}$	$\frac{1}{1 \cdot 0404} = \cdot 961168$	
			$- \cdot 02$ "
	$\frac{1}{1 \cdot 03}$	$\frac{1}{1 \cdot 0609} = \cdot 942586$	
			$- \cdot 02$ "
	$\frac{1}{1 \cdot 04}$	$\frac{1}{1 \cdot 0816} = \cdot 924555$	
(2)	$\frac{1}{2 \cdot 001}$	$\frac{1}{4 \cdot 004001} = \cdot 249750187$	
			$- \cdot 000249439$
	$\frac{1}{2 \cdot 002}$	$\frac{1}{4 \cdot 008004} = \cdot 249500748$	
			$- \cdot 000249067$

Reciprocal of Variable.	Function (Square of Reciprocal).	First Difference.
$\frac{1}{2\cdot003}$	$\frac{1}{4\cdot012009} = \cdot249251681$	
		$-\cdot000248689$
$\frac{1}{2\cdot004}$	$\frac{1}{4\cdot016016} = \cdot249002992$	
(3) $\frac{1}{3\cdot0001}$	$\frac{1}{9\cdot00060001} = \cdot11110370$	
		$-\cdot0000074$
$\frac{1}{3\cdot0002}$	$\frac{1}{9\cdot00120004} = \cdot11109629$	
		$-\cdot0000074$
$\frac{1}{3\cdot0003}$	$\frac{1}{9\cdot00180009} = \cdot11108889$	
		$-\cdot0000074$
$\frac{1}{3\cdot0004}$	$\frac{1}{9\cdot00240016} = \cdot11108148$	

49. Now take 1, and let it increase by small increments of ·01, then in the first column of (1) will be found the reciprocals of the successive values of the variable 1; in the second column, the squares of these reciprocals; in the third column, the equivalents of these squares.

It will be seen from the first and third columns that, as the variable 1 increases, the function (viz., the square of the reciprocal) decreases, therefore the differences (the fourth column), which are obtained from the numbers immediately above and below in the column to the left (the third), are negative, and that these differences are approximately in each case ·02.

Therefore the ratio of the rate of variation of the function to the rate of variation of the variable

$$= \frac{-\cdot02}{\cdot01} = -2 = -\frac{2}{1^3};$$

or, the differential co-efficients of $\frac{1}{1^2}$, where the 1 in the denominator is the variable $= -\frac{2}{1^3}$.

50. Now in (2) the number 2 receives successive increments of ·001. The first column, as before, represents the reciprocals of the successive values of the variable, the second column the squares of these reciprocals, etc.; and it will be seen that the first difference in each case is ·00025 approximately; and the ratio of the rate of variation of the function to the rate of variation of the variable

$$= \frac{-\cdot 00025}{\cdot 001} = -\cdot 25 = -\frac{1}{4} = -\frac{2}{8} = -\frac{2}{2^3};$$

or, the differential co-efficient of $\frac{1}{2^2} = -\frac{2}{2^3}$.

51. Similarly from (3) the ratio of the rate of variation of the function to the rate of variation of the variable

$$= \frac{-\cdot 0000074}{\cdot 0001} = -\cdot 074 = -\frac{2}{27} = -\frac{2}{3^3};$$

or, the differential co-efficient of $\frac{1}{3^2} = -\frac{2}{3^3}$;

and, generally, it will be found that the differential co-efficient of $\frac{1}{x^2}$ or x^{-2} is $-\frac{2}{x^{2+1}}$ or $-\frac{2}{x^3}$.

52. Again

Function.	Differences.
$\frac{1}{2\cdot 001} = \cdot 4997501$	
	$-\cdot 0002497$
$\frac{1}{2\cdot 002} = \cdot 4995004$	
	$-\cdot 0002493$
$\frac{1}{2\cdot 003} = \cdot 4992511$	
	$-\cdot 0002492$
$\frac{1}{2\cdot 004} = \cdot 4990019$	

From this it will be seen that the function is the reciprocal of 2, as it receives successive increments of ·001 and the difference in each case is ·00025 approximately.

Therefore the ratio of the rate of variation of the function to the rate of variation of the variable = $\frac{-\cdot 00025}{\cdot 001} = -\cdot 25 = -\frac{1}{4} = -\frac{1}{2^2}$; and similar results will be found for other numbers, so that

$$\text{differential co-efficient of } \frac{1}{2} = -\frac{1}{2^2},$$
$$\text{,, \quad ,, \quad } \frac{1}{3} = -\frac{1}{3^2},$$
$$\text{,, \quad ,, \quad } \frac{1}{4} = -\frac{1}{4^2},$$
$$\text{etc.,} = \text{etc.},$$

and, generally, this is in accordance with the general form—

$$\text{differential co-efficient of } x^{-1} \text{ or } \frac{1}{x} = -\frac{1}{x^2}.$$

53. Further, let us take a function of the form— $\frac{1}{x^3}$, say $\frac{1}{3^3}$, and let the 3 receive small increments of ·001, then

	Function.	Equivalent.	Difference.
$\frac{1}{3\cdot 001}$	$\frac{1}{27\cdot 027009001}$	$= \cdot 03700002$	
			$-\cdot 00003733$
$\frac{1}{3\cdot 002}$	$\frac{1}{27\cdot 054036008}$	$= \cdot 03696269.$	

Therefore the ratio of the rate of variation of the function to the rate of variation of the variable

$$= -\frac{\cdot 000037}{\cdot 001} \text{ (approximately)}$$
$$= -\cdot 037,$$

but $\frac{3}{81} = \cdot 037$ (approximately);

therefore required ratio $= -\frac{3}{81}$

$$= -\frac{3}{3^4};$$

or, differential co-efficient of $\frac{1}{3^3} = -\frac{3}{3^4}$;

and this is in accordance with the general form—

differential co-efficient of $\frac{1}{x^3}$ or $x^{-3} = -\frac{3}{x^4}$.

54. Tabulating these results, we have

differential co-efficient of x^{-1} or $\frac{1}{x} = -\frac{1}{x^2}$,

" " x^{-2} or $\frac{1}{x^2} = -\frac{2}{x^3}$,

" " x^{-3} or $\frac{1}{x^3} = -\frac{3}{x^4}$,

and these come under the general form—

differential co-efficient of x^{-n} or $\frac{1}{x^n} = -\frac{n}{x^{n+1}}$.

55. Let us now refer again to the function of the form $\frac{1}{x^2}$, and take $\frac{1}{3^2}$ as an example of the function of that form, where 3 is the variable; and let 3 receive small increments, as before, of ·0001. Then, if we take the decimal out to a larger number of places, we shall find that the successive values of the function and the first and second differences become (see Art. 49)

·11110370407		
	− ·00000740631	
·11109629776		·00000000077
	− ·00000740554	
·11108889222		

the second difference being positive, inasmuch as $-\cdot 0000074 0554$ is greater than $-\cdot 00000740631$.

Now the ratio of the rate of variation of the first difference to (the rate of variation of the variable)^2 is the second differential co-efficient of the function; and the rate of variation of the *first* differences is given by the *second* difference.

Therefore, we have, when 3 is the variable,

second differential co-efficient of $\frac{1}{3^2}$ or 3^{-2}

$$= \frac{\cdot 0000000007}{(\cdot 0001)^2} \text{ (approximately)} = \cdot 07.$$

But $$\frac{6}{3^4} = \frac{2}{27} = \cdot 07 \text{ (approximately)},$$

therefore second differential co-efficient of $\frac{1}{3^2} = \frac{6}{3^4} = \frac{2 \times 3}{3^4}$.

56. This result might have been obtained independently from the first differential co-efficient, for

differential co-efficient of $-\frac{2}{3^3} = -2 \times \left(-\frac{3}{3^4}\right) = \frac{2 \times 3}{3^4}$.

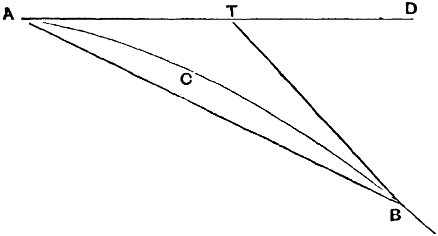
This is of the general form—

second differential co-efficient of $\frac{1}{x^n} = \frac{n(n+1)}{x^{n+2}}$.

XII. *Newton's Lemmas VI. and VII.*

57. "If an arc ACB be subtended by the chord AB, and have the tangent ATD at A; then if the point B move up to A, the angle BAD will diminish indefinitely and ultimately vanish."

Draw the tangent BT at B; then the angle BTD continually diminishes as B approaches A, and ultimately vanishes. Therefore, *a fortiori*, the angle BAT which is less than BTD, continually diminishes and ultimately vanishes—*i.e.*, the ultimate direction of the

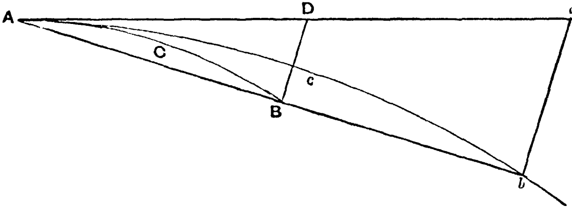


arc, chord, and tangent is the same, and is identical with that of the tangent ATD.

58. *Definition.*—The subtense of an arc is a straight line drawn from one extremity of the arc to meet, at a finite angle, the tangent to the arc at its other extremity.

59. "If BD be a subtense of the arc ACB, and B move up to A, then will the ultimate ratio of the arc ACB, the chord AB, and the tangent AD be a ratio of equality."

Let AD be produced to some fixed point d, and, as B moves up to A, suppose db always drawn through d,



parallel to DB, to meet AB produced in b. Also on Ab suppose an arc Acb to be described, always similar to ACB, and having therefore ADd for its tangent.

Then, by similar figures, we shall always have

$$AB : ACB : AD :: Ab : Acb : Ad;$$

and since this is always true, it is true in the limit, when B moves up to A.

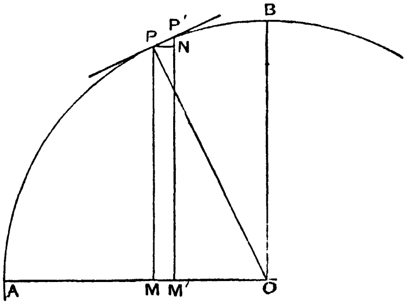
But, when B moves up to A, the angle bAd vanishes, and therefore the point b concides with the point d, and the lines Ab, Ad, and therefore Acb, which lies between them, are equal.

Hence also the arc ACB, the chord AB and the tangent AD, which are always in the same proportion as Acb, Ab, and Ad, are ultimately equal.

Hence, in all reasonings, when the arc is very small indeed, the arc, the chord, and the tangent may be used indifferently for one another.

XIII. *Differential Co-efficient of the Trigonometrical Functions (Geometrically).*

60. Let O be the centre of a circle, whose radius is 1, and in the arc of the quadrant AB take any point



P, and join OP; and from P draw PM perpendicular to AO. Take any other point P' very near to P, on the arc, and draw $P'M'$, PN perpendicular to AO and $P'M$.

Then as P' moves up to P and ultimately coincides with it, the arc PP', the chord PP', and the tangent at P coincide; or, in the immediate neighbourhood of P, may be used indiscriminately, the one for another.

Since the radius of the circle, viz. OP, is 1, it follows that

$$\sin POM = \frac{PM}{OP} = PM;$$

$$\text{and } \cos POM = \frac{MO}{OP} = MO.$$

Now, as the arc AP increases (*i.e.* as the angle POA increases) from AP to AP', it receives a small increment PP' and the *sine* of POM, viz. PM, receives a small increment $P'N$; and the ratio of the rate of variation of the sine (*the function*) to the rate of variation of the arc (the variable) is $\frac{P'N}{PP'}$; and this is true for any position of P', and is therefore true when P' moves up to P; and then PP' becomes a tangent and the angle $OPP' = 90°$.

Therefore, as the angle POM (*i.e.* the arc AP) receives very small increments, the differential coefficient of $\sin POM = \frac{P'N}{PP'}$

$$= \sin P'PN$$
$$= \cos NPO$$
$$= \cos POM.$$

And this is of the general form—

differential co-efficient of $\sin x = \cos x$.

Again, the variation in the *cosine* of POM is represented in magnitude by

$$OM - OM', \text{ or } MM';$$

i.e., as the angle becomes larger the *cosine* gradually becomes smaller, since M moves towards O.

Therefore, the ratio of the rate of variation of cos POM to the rate of variation of the angle POM (*i.e.*, the arc AP) $= -\frac{PN}{PP'}$

$= -\cos P'PN$

$= -\sin NPO$, because $OPP' = 90°$ ultimately,

$= -\sin POM$,

or, the differential co-efficient of cos $POM = -\sin POM$. And this is of the general form—

differential co-efficient of $\cos x = -\sin x$.

61. Versin $POM = 1 - \cos POM$

$= 1 - OM$

$= AM$.

Therefore, using the same method of reasoning as before, and remembering that the small increment or variation in the *versin* is MM, we have the ratio of the rate of variation of versin POM (*i.e.*, the arc AP) $= \frac{MM'}{PP'}$

$= \frac{PN}{PP'}$

$= \cos P'PN$

$= \sin NPO$ (ultimately)

$= \sin POM$,

or, the differential co-efficient of versin $POM = \sin POM$. And this is of the general form—

differential co-efficient of vers $x = \sin x$.

62. Again, $\tan POM = \frac{PM}{MO}$;

now, when the angle POM has received a small increment and become $P'OM'$, the tangent receives a small increment and

$$\tan P'OM' = \frac{P'M'}{M'O}$$
$$= \frac{PM + P'N}{MO - MM'};$$

therefore the rate of variation of the tangent

$$= \frac{PM + P'N}{MO - MM'} - \frac{PM}{MO}$$
$$= \frac{MO(PM + P'N) - PM(MO - MM')}{MO(MO - MM')}$$
$$= \frac{MO.P'N + PM.MM'}{MO.M'O}$$
$$= \frac{P'N \cos POM + MM' \sin POM}{\cos POM . \cos POM},$$

since $M'O = \cos POM$ ultimately.

Therefore the ratio of the rate of variation of the tangent to the rate of variation of the angle (*i.e.*, the arc)

$$= \frac{P'N \cos POM + MM' \sin POM}{\cos^2 POM} : PP'$$
$$= \frac{\frac{P'N}{PP'} . \cos POM + \frac{MM'}{PP'} . \sin POM}{\cos^2 POM}$$
$$= \frac{\cos^2 POM + \sin^2 POM}{\cos^2 POM}$$
$$= \frac{1}{\cos^2 POM}$$
$$= \sec^2 POM.$$

Therefore the differential co-efficient of $\tan POM$, as the angle receives small increments, is $\sec^2 POM$.

And this is of the general form—

differential co-efficient of $\tan x = \sec^2 x$.

63. Also $$\cot POM = \frac{MO}{PM};$$

and $$\cot P'OM' = \frac{M'O}{P'M'} = \frac{MO - MM'}{PM + P'N'}.$$

Therefore the rate of variation of the cotangent

$$= \frac{MO - MM'}{PM + P'N'} - \frac{MO}{PM}$$

$$= \frac{PM(MO - MM') - MO(PM + P'N')}{PM(PM + P'N')}$$

$$= -\frac{PM.MM' + MO.P'N'}{PM(PM + P'N')}$$

$$= -\frac{MM' \sin POM + P'N' \cos POM}{\sin POM. \sin POM},$$

since $P'M' = \sin POM$ ultimately. (See Art. 64.)

Therefore the ratio of the rate of variation of the *cotangent* to the rate of variation of the angle (*i.e.*, the arc)

$$= -\frac{MM' \sin POM + P'N' \cos POM}{\sin^2 POM} : PP'$$

$$= -\frac{\frac{MM'}{PP'} \sin POM + \frac{P'N'}{PP'} \cos POM}{\sin^2 POM}$$

$$= -\frac{\sin^2 POM + \cos^2 POM}{\sin^2 POM}$$

$$= -\frac{1}{\sin^2 POM}$$

$$= -\operatorname{cosec}^2 POM;$$

or the differential co-efficient of $\cot POM$

$$= -\operatorname{cosec}^2 POM.$$

And this is of the general form—

differential co-efficient of $\cot x = -\operatorname{cosec}^2 x$.

64. With reference to the point in the two last preceding arguments (touching the tangent and cotangent),

where the word *ultimately* is used, it will be well to consider the following:—

$$\sin(A+\alpha) = \sin A \cos\alpha + \cos A \sin\alpha,$$
$$\cos(A+\alpha) = \cos A \cos\alpha - \sin A \sin\alpha;$$

now when α, becoming smaller and smaller, ultimately vanishes,

$$\sin\alpha = 0,$$
and
$$\cos\alpha = 1.$$

Therefore, ultimately,

$$\sin(A+\alpha) = \sin A + 0 = \sin A,$$
$$\cos(A+\alpha) = \cos A - 0 = \cos A.$$

Similarly,

$$\sin P'OM' = \sin POM,$$
and
$$\cos P'OM' = \cos POM,$$

when P', moving nearer and nearer to P, ultimately coincides with it.

65. Again,

$$\sec POM = \frac{PO}{MO}$$
$$= \frac{1}{MO}, \text{ since } PO = 1;$$
$$\sec P'OM' = \frac{P'O}{MO - MM'}$$
$$= \frac{1}{MO - MM'}, \text{ since } P'O = 1.$$

Therefore the rate of variation of the *secant*

$$= \frac{1}{MO - MM'} - \frac{1}{MO}$$
$$= \frac{MO - MO + MM'}{MO(MO - MM')}$$
$$= \frac{MM'}{\cos^2 POM} \text{ ultimately.}$$

Therefore the ratio of the rate of variation of the *secant* to the rate of variation of the angle (*i.e.*, the arc)

$$=\frac{MM'}{\cos^2 POM} : PP'$$

$$=\frac{\frac{MM'}{PP'}}{\cos^2 POM}$$

$$=\frac{\sin POM}{\cos^2 POM}$$

$$=\sec POM \tan POM;$$

or, the differential co-efficient of $\sec POM$ is

$$\sec POM . \tan POM.$$

And this is of the general form—

differential co-efficient of $\sec x = \sec x . \tan x$.

66. Similarly,

$$\operatorname{cosec} POM = \frac{PO}{PM} = \frac{1}{PM},$$

and

$$\operatorname{cosec} P'OM' = \frac{PO'}{PM' + P'N}$$

$$= \frac{1}{PM + P'N}.$$

Therefore the rate of variation of the *cosecant*

$$= \frac{1}{PM + P'N} - \frac{1}{PM}$$

$$= \frac{PM - PM - P'N}{PM(PM + P'N)}$$

$$= -\frac{P'N}{\sin^2 POM} \text{ ultimately.}$$

Therefore the ratio of the rate of variation of the *cosecant* to the rate of variation of the angle (*i.e.*, the arc)

$$= -\frac{P'N}{\sin^2 POM} : PP'$$

$$= -\frac{\dfrac{P'N}{PP'}}{\sin^2 POM}$$

$$= -\frac{\cos POM}{\sin^2 POM}$$

$$= -\operatorname{cosec} POM . \cot POM ;$$

or, the differential co-efficient of cosec POM

$$= -\operatorname{cosec} POM . \cot POM.$$

And this is of the general form—

differential co-efficient of $\operatorname{cosec} x = -\operatorname{cosec} x \cot x$.

XIV. *The Differential Co-efficients of the Inverse Trigonometrical Functions.*

67. In a similar manner the differential co-efficients of the inverse trigonometrical functions may be obtained.

$\text{Sin}^{-1}.x$ means the angle whose *sine* is x; let this angle be POM. Then $\sin^{-1}x$ is the function and $\sin x$ the independent variable ; and the ratio of the rate of variation of the *angle* (*i.e.*, the *arc*) to the rate of variation of the *sine* $= PP' : P'N$

$$= \frac{PO}{OM} \text{ ultimately,}$$

$$= \sec POM$$

$$= \frac{1}{\cos POM}$$

$$= \frac{1}{\sqrt{1 - \sin^2 POM}}$$

$$= \frac{1}{\sqrt{1 - x^2}},$$

or, the differential co-efficient of $\sin^{-1} = x = \dfrac{1}{\sqrt{1 - x^2}}$.

68. Now let the function be $tan^{-1}x$. We found before that the ratio of the rate of variation of the tangent to the rate of variation of the angle was $\sec^2 POM$.

Therefore the ratio of the rate of variation of the angle to the rate of variation of the tangent

$$=\frac{1}{\sec^2 POM}$$

$$=\frac{1}{1+\tan^2 POM}$$

$$=\frac{1}{1+x^2},$$

if POM be the angle whose tangent is x;

or, the differential co-efficient of $\tan^{-1}x=\frac{1}{1+x^2}$

69. Again, let the function be $sec^{-1}x$. We found that the ratio of the rate of variation of the secant to the rate of variation of the angle was $sec\,POM.tan\,POM$.

Therefore the ratio of the rate of variation of the angle to the rate of variation of the secant

$$=\frac{1}{\sec POM.\tan POM}$$

$$=\frac{1}{\sec POM\sqrt{\sec^2 POM-1}}$$

$$=\frac{1}{x\sqrt{x^2-1}}$$

or, the differential co-efficient of $\sec^{-1}x=\frac{1}{x\sqrt{x^2-1}}$,

if POM be the angle whose *secant* is x.

Similarly we may find the differential co-efficients of the other inverse trigonometrical ratios.

XV. *The Value of* x^0.

70. It might appear to some that it would be sufficient to say that a quantity which is continually

diminishing may be made as small as we please, without the *proviso* that it may be made smaller than any assignable quantity. But on closer inspection it will be found that, in some cases, quantities may be continually diminishing and yet never become smaller than a certain quantity, which is then the ultimate value, or limit, when the decrease has been carried out to an indefinite extent. For instance, suppose we take the number 100, and take its square root; this will be 10. Now take the square root of 10; this will be 3 followed by a decimal. Take the square root of this, and the result will be 1 followed by a smaller decimal, and so on. However many times we take the square root the 1 will always remain, though the decimal part may be made smaller than any assignable quantity. The limit, then, of any number, when the square root has been taken an infinite number of times, is 1.

Let x be any number; then the square root of x is written $x^{\frac{1}{2}}$, and the square root of this again is $x^{\frac{1}{4}}$, and when we have taken the square root n times the result will be $x^{\frac{1}{2^n}}$; and when we have taken the square root an infinite number of times, *i.e.*, when n has become ∞, the result is $x^{\frac{1}{\infty}}$ or x^0, and therefore $x^0 = 1$.

XVI. *The Differential Co efficients of the Trigonometrical Functions (Arithmetically).*

71. Consider the following :—

Angle.	Arc, or Circular Measure.	Difference.	Natural Sine.	Difference.
70°	1·221730		·9396926	
		·000002		·0000006
70°·0001	1·221732		·9396932	
		·000002		·0000006
70°·0002	1·221734		·9396938	

Here the first column gives the successive values of an angle of 70° as it receives small increments of ·0001 degrees.

The second column gives the corresponding arcs or circular measure of these angles.

The third column the differences of these arcs, or the rate of variation of the arcs.

The fourth column gives the natural sines of the angles, which may be found in any book of logarithmic tables.

The fifth column gives the differences of these.

Here the sine is the function of the arc; and the rate of variation of the function is given by the fifth column.

Therefore the ratio of the rate of variation of the function to the rate of variation of the variable

$$= \frac{\cdot 0000006}{\cdot 000002}$$

$$= \frac{\cdot 6}{2}$$

$$= \cdot 3$$

= *cosine* of an angle whose circular measure is 1·221730 (approximately)

= cos 70°,

or, the differential co-efficient of sin 70° = cos 70°.

And this is of the general form—

differential co-efficient of $\sin x = \cos x$.

The error committed in the above is considerable, because the tables are only carried to 7 places of decimals. Now, the smaller the increments are, the more true is the result, and for very small increments it would be necessary to have tables calculated to a far greater number of decimal places. In the following example the increment is comparatively large—

Angle.	Arc.	Difference.	Sine.	Difference.
30°	·523599		·5000000	
		·001745		·0015114
30°·1	·525344		·5015114	

Therefore, the ratio of the rate of variation of the function to the rate of variation of the variable

$$= \frac{\cdot 0015114}{\cdot 001745}$$
$$= \cdot 866 \text{ etc.}$$
$$= \cos 30^\circ \text{ (approximately)},$$

and, therefore, the differential co-efficient of sin 30° is cos 30°, and this is of the general form—

differential co-efficient of $\sin x = \cos x$.

Similarly, the differential co-efficient of the *cosine* may be shown to be of the general form, from the actual numbers.

72. Now let us take the tangent, and suppose the angle to be 14° and let it receive small increments of ·1°. Then—

Angle.	Arc.	Difference.	Tan.	Difference.
14°	·2443461		·2493280	
		·0017454		·0018546
14°·1	·2460915		·2511826	

Here the ratio of the rate of variation of the tangent (*function*) to the rate of variation of the arc (*variable*)

$$= \frac{\cdot 0018546}{\cdot 0017454}$$
$$= 1 \cdot 0624 \text{ etc.}$$

But $\sec 14^\circ = 1 \cdot 0306136$

therefore $\sec^2 14^\circ = 1 \cdot 0621644$.

Therefore the required ratio $= \sec^2 14^\circ$ (approximately), the error occurring in the fourth decimal place.

Therefore, approximately,

differential co-efficient of $\tan 14^\circ = \sec^2 14^\circ$,

and a similar result may be obtained for any other angle. Further, it will be seen that this result is of the general form—

differential co-efficient of $\tan x = \sec^2 x$.

73. Now take the secant as the function, and let the angle be 84°, and let it receive small increments of ·001°. Then

Angle.	Arc.	Difference.	Secant.	Difference.
84°	1·4660767		9·5667722	
		·0000175		·0001593
84°·001	1·4660942		9·5669315	

From this it will be seen that the ratio of the rate of variation of the function to the rate of variation of the variable

$$= \frac{\cdot0001593}{\cdot0000175}$$

$$= 91\cdot028 \text{ etc.}$$

But $\sec 84° \times \tan 84° = 9\cdot5667722 \times 9\cdot5143645$

$$= 91\cdot0217475.$$

Therefore the required ratio

$$= \sec 84° \times \tan 84° \text{ (approximately)},$$

the error occurring in the third decimal place. And this result is of the general form—

differential co-efficient of $\sec x = \sec x \tan x$.

Similar results may be found for the cosine, co-tangent, and cosecant of an angle.

XVII. *The Differential Co-efficient of a Logarithm.*

74.* Assuming the exponential theorem we may show that—

$$\log\left(1 + \frac{1}{m}\right) = M\left\{\frac{1}{m}\right\} \text{ approximately,}$$

where M is the *modulus* and is found to be ·43429.

For instance, take $\log\left(1 + \frac{1}{345}\right)$.

$$\log\left(1 + \frac{1}{345}\right) = \log 1\cdot0029$$

$$= \cdot0012576.$$

* See Appendix.

Again, since $M = \cdot 43429$

$$\therefore M \times \frac{1}{345} = \cdot 0012576,$$

therefore $\log\left(1 + \frac{1}{345}\right) = M\left(\frac{1}{345}\right)$.

Now the following will be found in any tables of logarithms—

$$\log 41713 = 4 \cdot 6202714,$$
$$\log 41714 = 4 \cdot 6202818,$$
$$\log 41715 = 4 \cdot 6202922.$$

If we take the differences of these, we obtain

$$\cdot 0000104,$$
$$\cdot 0000104.$$

Thus, if 41713 receive small increments of 1, the function receives increments of ·0000104; *i.e.*, the ratio of the rate of variation of the function to the rate of variation of the variable = ·0000104 : 1; or, the differential co-efficient of log 41713 = ·0000104.

Now the general form is—

$$\text{differential co-efficient of } \log x = \frac{1}{x}$$

and the above result does not, at first sight, appear to be of this form. We shall see, presently, that it is.

Converting the above into Naperian logarithms, we have

$$\text{Nap.} \log 41713 = 4 \cdot 6202714 \div \cdot 43429$$
$$= 10 \cdot 6386778 \ldots\ldots\ldots\ldots(1)$$
$$\text{Nap.} \log 41714 = 4 \cdot 6202818 \div \cdot 43429$$
$$= 10 \cdot 6387018 \ldots\ldots\ldots\ldots(2)$$

Taking the difference, as before, between (1) and (2), we obtain

$$\cdot 000024$$
$$= \frac{1}{41713}.$$

Therefore, taking Naperian logarithms, we have, the ratio of the rate of variation of the function to the rate of variation of the variable = 000024 : 1

$$=\frac{1}{41713}$$

or, differential co-efficient of Nap. log 41713 $=\frac{1}{41713}$.

75. We may now show that the result obtained in the previous article is of the same form. (Taking logs to base 10),

$$\log 41713 = 4{\cdot}6202714,$$

that is, $10^{4\cdot6202714} = 41713$;

again, $10^{4\cdot6202818} = 10^{4\cdot6202714 + \cdot0000104}$

$$= 10^{4\cdot6202714} \times 10^{\cdot0000104};$$

therefore $10^{4\cdot6202714} \times 10^{\cdot0000104} = 41714$

$$= 41713 + 1;$$

therefore $10^{\cdot0000104} = \frac{41713+1}{10^{4\cdot6202714}}$

$$= \frac{41713+1}{41713}$$

$$= 1 + \frac{1}{41713},$$

or $\cdot0000104 = \log\left(1 + \frac{1}{41713}\right)$

$$= M\left(\frac{1}{41713}\right).$$

XVIII. *Successive Differentiation.*

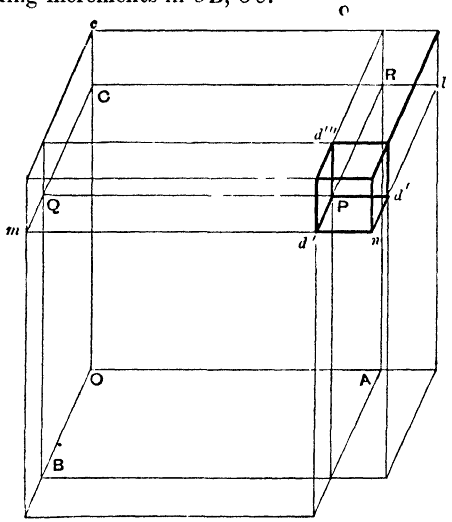
76. The following considerations are of the utmost importance, as they embody the whole principle, not only of differentiation, but also of successive differentiation.

It must be remembered that when a body is moving, not uniformly, but with accelerated motion, its *rate* at any instant is not represented by the space it would pass over in the next unit of time, but by the space it would pass over if it moved *uniformly*, with the velocity it *had* at that instant, for the next unit of time.

Let QA be a cube, which has been growing to its present size,

and let $$OA = x = OB = OC,$$

x being the variable on which size of cube depends, and let OA receive, in the ordinary course of its increase, an increment Aa, and let Bb, Cc be the corresponding increments in OB, OC.



The edge of the cube (x) has, then, received a certain increase—*i.e.*, its rate of increase at the instant it has become x is represented by Aa or Bb or Cc, in the three respective directions.

Our aim is to find the ratio of the rate of increase of the cube, to this rate of increase of x—*i.e.*, the differential co-efficient of x^3; we have therefore to find the corresponding rate of increase of the cube to the increase of its edge (x).

Now if the cube had been stopped suddenly on its increasing course at the instant at which we found it in the form of QA, its rate of increase in the directions of Aa or Pd', Bb or Pd'', Cc or Pd'', corresponding to this rate of increase of x, would be represented by the figures Pa, Pb, and Pc, for the face of the cube would have to remain of the same size as we found it at the instant, in order that we may satisfy the condition of uniformity, already alluded to, in calculating the rate.

Therefore the first rate of increase of cube

$$\begin{aligned}
&= Pa + Pb + Pc \\
&= Pd' \times \text{face of cube} + Pd'' \times \text{face of cube} \\
&\qquad + Pd''' \times \text{face of cube} \\
&= \text{face of cube} \times (Pd' + Pd'' + Pd'') \\
&= \text{face of cube} \times (Aa + Bb + Cc) \\
&= \text{face of cube} \times 3Aa \\
&= \text{face of cube} \times 3\ (\text{rate of increase of } x).
\end{aligned}$$

Therefore

$$\frac{\text{rate of increase of cube}}{\text{rate of increase of } x} = 3 \times (\text{face of cube}),$$

i.e.,

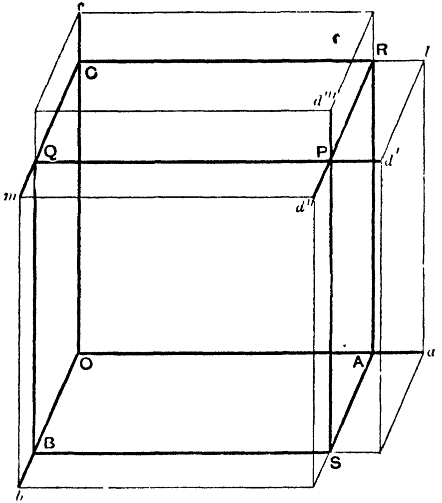
$$\frac{\text{rate of increase of } x^3}{\text{rate of increase of } x} = 3x^2,$$

or differential co-efficient of $x^3 = 3x^2$;

and this is the first differential co-efficient of x^3.

77. Now, a moment ago, we suddenly stopped the cube in its growth. If we had not, it would have increased in size, and, as a necessary and obvious consequence, its three faces would have increased in area. (A cube of course has six faces, but there are only now three under consideration, since the cube is not sup-

posed to increase in the directions AO, BO, CO.) Take the face Pc; Pc would have grown in the direction of Pd' or Rl, and also in the direction of Pd'' or Qm; and would, if not checked in its course, have remained square. But we have stopped the motion, and now inquire, "If the side PR still keeps the same rate as it has now, at the moment of stoppage, where will it be when OA has received an increment Aa?" and we find that it will occupy the position ld'. Similarly, QP will occupy the position md''. And the *rate* of increase of the face PC will be represented by the two rectangles Pl, Pm.



But the face PC of the cube is the base of the solid Pc; and as the base increases the solid tends to increase also; and the rate of increase of the solid.

while the face was increasing by Pl and Pm, would be represented by the solids $d'''l$, $d'''m$.

But each of these solids

$$= x \times (\text{rate of increase of } x)^2,$$

therefore the rate of increase in the cube corresponding to the increase of one of the three faces

$$= 2x \times (\text{rate of increase of } x)^2;$$

and there are three faces which increase, therefore rate of increase of cube

$$= 6x \times (\text{rate of increase of } x)^2;$$

therefore

$$\frac{\text{the rate of this second increase of cube}}{(\text{rate of increase of } x)^2} = 6x,$$

or, second differential co-efficient of $x^3 = 6x$.

78. Again, the solid $d'''m$ would increase in the direction of Pd', and would receive an increment of $d'''n$, which is the cube of Pd' or Aa; and therefore $d'''n$ represents the *rate* of increase of the solid $d'''m$.

Therefore, remembering that, since the solid Pc would have a rate of increase of two such solids as $d'''m$, therefore the whole three solids, such as Pc, would have a rate of increase of six such solids as $d'''m$; and remembering that for each solid, as $d'''m$, there is now a third rate of increase, represented by $d'''n$, we may say that

the rate of the third increase of cube

$$= 6 \times d'''n$$

$$= 6 \times (Pd')^3$$

$$= 6 \times (\text{rate of increase of } x)^3;$$

therefore $\dfrac{\text{rate of third increase of cube}}{(\text{rate of increase of } x)^3} = 6.$

or, third differential co-efficient of $x^3 = 6$.

79. We have already found that

(1) the *first* differential co-efficient of $x^2 = 2x$,

second „ „ $x^2 = 2$;

(2) the *first* „ „ $7^4 = 4 \times 7^3$,

second „ „ $7^4 = 4 \times 3 \times 7^2$,

third differential co-efficient of $7^4 = 4 \times 3 \times 2 \times 7$,
fourth „ „ $7^4 = 4 \times 3 \times 2 \times 1$;
(3) the *first* „ „ $5^3 = 3 \times 5^2$,
second „ „ $5^3 = 3 \times 2 \times 5$,
third „ „ $5^3 = 3 \times 2 \times 1$.

And we have also found that the differential co-efficient of $nx + c = n$.

From the above we gather that

(*a*) If the function be the *first* power of the variable, whether connected with a constant quantity or not, *then* there is only a *first* differential co-efficient; the *second*, *third*, etc., differential co-efficients vanishing, because the *first* is itself a constant quantity and therefore does not vary, and therefore cannot have a differential co-efficient;

(*b*) If the function be the *second* power of the variable, there are both first and *second* differential co-efficients, but no *third*—this being 0—for a similar reason to that in (*a*);

(*c*) If the function be the *third* power, there may be found a first, second, and *third* differential co-efficient, but no *fourth*;

(*d*) If the function be the *fourth* power, we may find first, second, third, and *fourth* differential co-efficients, but no *fifth*;

And so on.

We also notice that—

(*a*) The first differential co-efficient contains the power of the variable, which was contained in the function, decreased by 1;

(*b*) The second differential co-efficient contains the power of the variable, which was contained in the function, decreased by 2;

(*c*) The third decreased by 3;

(*d*) The fourth decreased by 4;

And so on.

Finally, we observe that—

(*a*) The co-efficient of the variable, in the first differential co-efficient, is the power of the variable in the function ;

(*b*) In the second differential co-efficient it is the power of the variable multiplied by (that power decreased by 1) ;

(*c*) In the third it consists of three factors—the first being the power of the variable in the function, the second that power decreased by 1, the third that power decreased by 2 ;

And so on.

Thus, if we have an expression 5^7, which is a function of 5, and the 5 be supposed to undergo small variations,

first differential	co-efficient of 5^7		$= 7\times 5^6$,
second	,,	,,	$=7\times 6\times 5^5$
third	,,	,,	$=7\times 6\times 5\times 5^4$,
fourth	,,	,,	$=7\times 6\times 5\times 4\times 5^3$,
fifth	,,	,,	$=7\times 6\times 5\times 4\times 3\times 5^2$,
sixth	,,	,,	$=7\times 6\times 5\times 4\times 3\times 2\times 5$,
seventh	,,	,,	$=7\times 6\times 5\times 4\times 3\times 2\times 1$,
eighth	,,	,,	$=0$.

Again, suppose the function to be $3x^2 \pm x$:

first differential	co-efficient		$=3\times 2x\pm 1$,
second	,,	,,	$=3\times 2$,
third	,,	,,	$=0$.

Or again, suppose that we have $\frac{5}{4}px^3 \pm bx^2 \pm \frac{1}{2}x$.

first differential co-efficient $=\frac{5}{4}\times p\times 3x^2 \pm b\times 2x \pm \frac{1}{2}$;

second ,, ,, $=\frac{5}{4}\times p\times 3\times 2x \pm 2b$;

third ,, ,, $=\frac{5}{4}\times p\times 3\times 2$;

fourth ,, ,, $=0$.

In order to save time and space let us call the *first* differential co-efficient of any function dc_1 of that function,

the *second*	"	"	dc_2	"
the *third*	"	"	dc_3	"
the p^{th}	"	"	dc_p	"

Now let us take the expression x^n, where x is the variable—

$$
\begin{aligned}
dc_1 &= nx^{n-1},\\
dc_2 &= n(n-1)x^{n-2},\\
dc_3 &= n(n-1)(n-2)x^{n-3},\\
dc_4 &= n(n-1)(n-2)(n-3)x^{n-4},\\
\text{etc.} &= \text{etc.}\\
dc_{n-2} &= n(n-1)\ldots\ldots\{n-(n-3)\}x^{n-(n-2)}\\
&= n(n-1)\ldots\ldots 3x^2,\\
dc_{n-1} &= n(n-1)\ldots\ldots 3\times\{n-(n-2)\}x^{n-(n-1)}\\
&= n(n-1)\ldots\ldots 3\times 2x,\\
dc_n &= n(n-1)\ldots\ldots 3\times 2\times\{n-(n-1)\}x^{n-n}\\
&= n(n-1)\ldots\ldots 3\times 2\times 1\ ;
\end{aligned}
$$

and we notice that these are the co-efficients of the second, third, etc., terms and the last term in the expansion of a binomial—(Binomial theorem).

80. Now by actual multiplication

$$(x+h)^2 = x^2 + 2xh + h^2\ ;$$

and this is a function of $(x+h)$, since it is $(x+h)$ raised to the second power; let us denote this by "function" of $(x+h)$.

Now suppose $h=0$

$$\therefore\ (x+h)^2 = x^2.$$

When we have made this condition that $h=0$, let us denote the function under these circumstances by placing it within brackets; thus

if $\quad$ function $(x+h) = (x+h)^2$

$$(\text{function}) = x^2 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (1)$$

Similarly $\quad dc_1(x+h) = 2x + 2h$

and $\quad (dc_1) = 2x \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (2)$

and $\quad (dc_2) = 2,$

$$\therefore\quad \frac{(dc_2)}{2} = 1 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (3)$$

Now (1) is the first term,
(2) is the co-efficient of h in second term,
(3) „ „ h^2 in third term of the expansion—

$$(x+h)^2 = x^2 + 2xh + h^2 \ldots\ldots\ldots\ldots\ldots (4)$$

Therefore, substituting these values in (4), we have

$$\text{function} = (\text{function}) + (dc_1) \times h + \frac{(dc_2)}{2} h^2.$$

81. Similarly, if we have the "function"

$$(x+h)^3 = x^3 + 3x^2h + 3xh^2 + h^3 \ldots\ldots\ldots\ldots\ldots\ldots\ldots (1)$$

and suppose $h=0$, we obtain

$$(x+h)^3 = x^3,$$

or

$$\left.\begin{aligned} (\text{function}) &= x^3 \\ (dc_1) &= 3x^2 \\ (dc_2) &= 3 \times 2x \\ (dc_3) &= 3.2.1 \end{aligned}\right\}$$

or

$$\left.\begin{aligned} (\text{function}) &= x^3 \\ (dc_1) &= 3x^2 \\ \frac{(dc_2)}{1.2} &= 3x \\ \frac{(dc_3)}{1.2.3} &= 1 \end{aligned}\right\}$$

and these are the first term and the co-efficients of h, h^2, and h^3 in (1); therefore, substituting in (1), we have

$$\text{function} = (\text{function}) + (dc_1)h + \frac{(dc_2)}{1.2} h^2 + \frac{(dc_3)}{1.2.3} h^3.$$

Similarly, if the function were $(x+h)^4$, we should obtain
function

$$= (\text{function}) + (dc_1)h + \frac{(dc_2)}{1.2} h^2 + \frac{(dc_3)}{1.2.3} h^3 + \frac{(dc_4)}{1.2.3.4} h^4,$$

where the co-efficients of h, h^2, h^3, and h^4 may be found by differentiating successively and putting $h=0$.

82. Now suppose we have the function $(x+h)^n$; when $h=0$

$$(x+h)^n = x^n,$$

or $\quad$ $(\text{function}) = x^n,$

$(dc_1) = nx^{n-1},$

$(dc_2) = n(n-1)x^{n-2},$

etc. $=$ etc. (See Art. 79.)

$$\therefore \text{ function} = (\text{function}) + (dc_1)h + \frac{(dc_2)}{2}h^2$$

$$+ \frac{(dc_3)}{1.2.3}h^3 + \text{etc.} + \frac{(dc_n)}{\underline{|n}}h^n \text{ .... (1)}$$

Substituting the values for (dc_1), (dc_2), etc., we have

$$(x+h)^n = x^n + nx^{n-1}h + \frac{n(n-1)}{2}x^{n-2}h^2$$

$$+ \frac{n(n-1)(n-2)}{\underline{|3}}x^{n-3}h^3 + \text{etc.} + h^n,$$

which is the Binomial theorem.

The relation (1) is found to hold good whatever be the function, and is called Maclaurin's theorem.

(See appendix II.)

83. Required the development of $\sqrt{1+x}$.

$\text{function} = (1+x)^{\frac{1}{2}},$

$\therefore$ $\quad$ $(\text{function}) = 1,$

$$dc_1 = \tfrac{1}{2} \times \frac{1}{(1+x)^{\frac{1}{2}}},$$

$$dc_2 = -\tfrac{1}{2} \cdot \tfrac{1}{2}(1+x)^{-\frac{3}{2}},$$

$$= \frac{-\frac{1}{2} \cdot \frac{1}{2}}{(1+x)^{\frac{3}{2}}},$$

$$dc_3 = \frac{\frac{1}{2} \cdot \frac{1}{2} \cdot \frac{3}{2}}{(1+x)^{\frac{5}{2}}},$$

etc. $=$ etc.

Hence $\quad$ $(dc_1) = \frac{1}{2},$

$(dc_2) = -\frac{1}{2} \cdot \frac{1}{2},$

$(dc_3) = \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{3}{2},$

etc. $=$ etc.

But, by Maclaurin's theorem, since h may be any quantity,

$$\text{function} = (\text{function}) + (dc_1)x + \frac{(dc_2)}{\underline{|2}}x^2 + \frac{(dc_3)}{\underline{|3}}x^3 + \text{etc.}$$

Therefore substituting the values we have just found,

$$\sqrt{1+x} = 1 + \tfrac{1}{2}x - \tfrac{1}{2} \cdot \tfrac{1}{2} \cdot \tfrac{1}{2}x^2 + \tfrac{1}{2} \cdot \tfrac{1}{2} \cdot \tfrac{3}{2} \cdot \tfrac{1}{6}x^3 + \text{etc.}$$

$$= 1 + \frac{x}{2} - \frac{x^2}{8} + \frac{x^3}{16} - \text{etc.}$$

By this series we may approximate to the square root to any degree of correctness we please.

84. Expand $\sin x$ to terms of x.

$$\begin{aligned} \text{Here} \quad \text{function} &= \sin x, \\ dc_1 &= \cos x, \\ dc_2 &= -\sin x, \\ dc_3 &= -\cos x, \\ dc_4 &= \sin x, \\ \text{etc.} &= \text{etc.}; \\ \therefore (\text{function}) &= 0, \\ (dc_1) &= 1, \\ (dc_2) &= 0, \\ (dc_3) &= -1, \\ (dc_4) &= 0, \\ \text{etc.} &= \text{etc.} \end{aligned}$$

Substituting these values in Maclaurin's theorem, we have

$$\text{function} = \sin x = x - \frac{x^3}{\underline{|3}} + \frac{x^5}{\underline{|5}} - \text{etc.}$$

85. For small arcs this series will give the sine quite accurately.

In order to apply this, we take the arc of a quadrant —viz., $\frac{\pi}{2}$, the radius being 1. If we divide this $\left(\frac{\pi}{2}\right)$ by (90×60), we obtain the length of one minute of arc, from which any number of minutes or degrees may be calculated. Substituting the value of the arc thus found in the formula, we obtain the length of the natural sine.

86. Again, $(a+h)^2=a^2+2ah+h^2$.
Now if h be very small, we may neglect the h^2 or second power of h, and say—

$$(a+h)^2=a^2+2ah \text{ approximately.}$$

This will be more readily seen if we take a numerical illustration:

$$\begin{aligned}(11+1)^2&=121+2\times 11+1\\&=121+22+1\\&=123+1.\end{aligned}$$

If, therefore, we neglect the 1, which corresponds with h^2, we have

$$(11+1)^2=123 \text{ approximately,}$$

and this is wide of the mark by 1,
since $12^2=144$.

Now let the term which represents h in the binomial expression on the left-hand side be smaller, say ·1, then

$$\begin{aligned}(11{\cdot}9+{\cdot}1)^2&=141{\cdot}61+2{\cdot}38+{\cdot}01\\&=143{\cdot}99+{\cdot}01;\end{aligned}$$

neglecting ·01, which corresponds with h^2, we have

$$(11{\cdot}9+{\cdot}1)^2=143{\cdot}99 \text{ approximately,}$$

and this only differs from 144 by ·01.

Again, let $h={\cdot}01$,
and we have

$$(11{\cdot}99+{\cdot}01)^2=143{\cdot}9999 \text{ approximately.}$$

Therefore we may safely say that, when h is very small,

$$(a+h)^2=a^2+2ah \text{ approximately.}$$

Similarly, if we take

$$(a+h)^3=a^3+3a^2h+3ah^2+h^3,$$

we may neglect the terms involving the second and third powers of h—viz., $3ah^2$ and h^3, when h is *very* small, and then we have

$$(a+h)^3=a^3+3a^2h \text{ approximately.}$$

Let us take an illustration of this also. Let a be represented by 3, and h by 1, then

$$(3+1)^3 = 3^3 + 3\times 3^2\times 1 + 3\times 3\times 1^2 + 1^3$$
$$= \underbrace{27+27}+\underbrace{9+1}$$
$$= \quad 54 \quad + \quad 10.$$

Therefore, in this case where h is comparatively large,

$$(3+1)^3 = 54,$$

and this is wide of the mark by 10, since

$$4^3 = 64.$$

But let h be smaller, say $\cdot 01$,

then $\qquad a = 3\cdot 99$, so that

$a + h = 4$, as before.

Then

$$(3\cdot 99 + \cdot 01)^3 = (3\cdot 99)^3 + 3\times(3\cdot 99)^2\times\cdot 01 + 3\times 3\cdot 99 \times(\cdot 01)^2 + (\cdot 01)^3$$
$$= (3\cdot 99)^3 + 3\times(3\cdot 99)^2\times\cdot 01$$

approximately, omitting the terms involving h^2 and h^3,

$$= 63\cdot 521199 + \cdot 477603$$
$$= 63\cdot 998802.$$

Therefore the error in this case is only $\cdot 001198\ldots$, and the smaller h becomes the more true is the approximation.

87. Now, let us analyse the two cases we have been considering—namely, that when h is very small,

$$(a+h)^2 = a^2 + 2ah \text{ approximately}\ldots\ldots\ldots\ldots(1)$$
$$(a+h)^3 = a^3 + 3a^2h \quad \text{''} \quad \ldots\ldots\ldots\ldots(2)$$

In each case we see that the left-hand member is a function of a; in (1) the right-hand member consists of two terms: the first of these is a^2, which we notice is of the same form as the function—*i.e.*, it is the square of a quantity, and further we notice that it is really (function), for if $h = 0$

$$(a+h)^2 = a^2.$$

Further, the second term of the right-hand member is $h\times 2a$, and $\qquad (dc_1) = 2a$.

Therefore if $(a+h)^2$ be a certain function, we may write (1) as follows:

$$\text{function} = (\text{function}) + (dc_1)h\,;$$

and, similarly, if $(a+h)^3$ be a certain function, we may write (2) thus

$$\text{function} = (\text{function}) + (dc_1)h,$$

and these two results are precisely the same, and it is found that whatever the function be, the result is the same.

88 There appears to be a slight difficulty here which we will not pass over. We have said let $h=0$, and then $(dc_1)=2a$, and immediately afterwards we multiply (dc_1) by h, and one might be led to suppose that this product, viz., $(dc_1)h$, would naturally be 0 also. Not so, however. We only say, what would be the value of dc_1 of the function, supposing h were 0, and we obtain a certain result—a certain quantity. Then, quite apart from that operation, we multiply another quantity (h) by this quantity.

89. We have said that it is found that of whatever form the function be, we always have, as an approximation, when h is small,

$$\text{function} = (\text{function}) + (dc_1)h\ ;$$

we will give a simple example in support of this.

Let the function be

$$\begin{aligned} 3(a+h)^2 + 4(a+h) + 1 & \\ &= 3a^2 + 6ah + 3h^2 + 4a + 4h + 1 \\ &= 3a^2 + 4a + 1 + 6ah + 4h + 3h^2. \end{aligned}$$

Therefore, omitting the term involving h^2, we have

$$\text{function} = (3a^2 + 4a + 1) + h(6a + 4).$$

But since $\text{function} = 3(a+h)^2 + 4(a+h) + 1$

$$(\text{function}) = 3a^2 + 4a + 1$$

and

$$\begin{aligned} (dc_1) &= 2 \times 3a + 4 \\ &= 6a + 4. \end{aligned}$$

Therefore we have again

$$\text{function} = (\text{function}) + (dc_1)h.$$

(This form is a deduction from Taylor's theorem.)

90. We will now show how this result may be practically utilized in approximating to the roots of an equation.

Let the equation be

$$x^3 - 3x + 1 = 0,$$

i.e., function $= 0$,

where $x^3 - 3x + 1$ is the function of x.

Therefore, since

$$\text{function} = (\text{function}) + (dc_1)h,$$

and function $= 0$,

it follows that

$$(\text{function}) + (dc_1)h = 0.$$

$$\therefore \quad h = -\frac{(\text{function})}{(dc_1)}.$$

Now, by trial, 1·5 is found to be near one of the roots. Let h be the difference between 1·5 and the root; that is, let $x = 1·5 + h$, which is of the form $(a + h)$.

Therefore,

$$\begin{aligned}(\text{function}) &= a^3 - 3a + 1\\ &= (1·5)^3 - 3 \times 1·5 + 1\\ &= -·125,\end{aligned}$$

and

$$\begin{aligned}(dc_1) &= 3a^2 - 3\\ &= 3 \times (1·5)^2 - 3\\ &= 6·75 - 3,\\ &= 3·75,\end{aligned}$$

therefore,

$$\begin{aligned}h &= \frac{·125}{3·75}\\ &= ·033,\end{aligned}$$

therefore,

$$x = 1·533.$$

We can now take this as an approximation, as we did 1·5, and so may get a result, by proceeding in this manner, as near to one of the roots as we please.

91. If we wish to find the limit of a fraction, as the variable gradually approaches a certain limit, in the case where the fraction becomes of the form $\frac{0}{0}$, or $\frac{\infty}{\infty}$, we may employ the process of differentiation, and by this means get rid of all artifice in arriving at the correct result.

The method of Bernouilli is to differentiate the numerator and denominator separately, until they do not both vanish, for the value of the limit of the variable.

In No. 2 of "Examples worked out" we found the value of the fraction $\frac{2x+5}{4x+6}$, when x was infinite, by an artifice. We shall get the same result by the method of differentiating.

For $\qquad dc$ of $2x+5=2$

and $\qquad dc$ of $4x+6=4$,

$$\therefore \quad \text{value of fraction} = \frac{2}{4} = \frac{1}{2}.$$

92. Again, find the real value of the fraction

$$\frac{ax^2-2acx+ac^2}{bx^2-2bcx+bc^2} \text{ when } x=c.$$

Here dc_1 of numerator $=2ax-2ac$

$=0$, if $x=c$,

dc_1 of denominator $=2bx-2bc$

$=0$, if $x=c$.

Now let us proceed to the second differential coefficients—

dc_2 of numerator $=2a$,

dc_2 of denominator $=2b$.

Therefore real value of fraction $=\frac{2a}{2b}=\frac{a}{b}$.

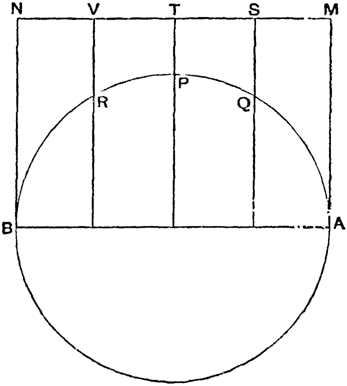
XIX. *Maxima and Minima.*

93. The value of a function is said to be a *maximum* or a *minimum* according as the particular value is greater or less than the values which both immediately precede and immediately succeed it.

94. If, then, a function continually increases or continually decreases, it cannot have a maximum or a minimum.

95. If a function increase at a diminishing rate, like a stone thrown straight up in the air, until at a certain point it ceases to increase and begins to diminish (*i.e.*, in the case of the stone, to diminish its height from the ground), then, at the *turning point*, it has its greatest value, and the values which immediately precede and immediately succeed this value are less than this value, and therefore it is a maximum.

96. Again, if the function decrease until, at a certain point, it ceases to diminish and begins to increase, then the values on either side of it are greater than it, and consequently it is a minimum. Such, for instance, would be the case, if a cork were forced into a vessel filled with water, it would attain its minimum distance from the bottom of the vessel at the *turning point*, when it began to rise. Take the stone thrown straight up into the air as another instance: it decreases in velocity until at the *turning point* it is a minimum, and then begins to increase.



97. Let APB be a circle, and AB its diameter, and

let a straight line move from A so as to be always perpendicular to AB and have its other extremity in the circumference of the circle; it will increase until it reaches the position CP, and then diminish until it reaches B; and in the position CP it will have its maximum value.

Again, a straight line drawn so as to have one extremity in MN, and its other extremity on the circumference, will first have such a position as MA, and will gradually diminish until it reaches the opsition TP, and then it will increase until it reaches the position NB.

Therefore, at the *turning* point, in the position TP it has its minimum value.

98. A function may have more than one maximum or minimum; in fact may have an endless number of both, for a function may increase until it has reached a maximum, and then diminish until it reaches a minimum, and then increase again to a maximum, and so on. From the nature of the case the maximum and minimum values must alternate—that is, there cannot be two maximum values succeeding each other without a minimum value intervening, and *vice versa*. The troughs and crests of the waves of the sea give minima and maxima with regard to a horizontal line. The tide furnishes another example of maxima and minima.

99. The sine of an angle—*i.e.*, the semi-chord—as the angle varies from 0 to 360°, is a minimum at 0 and 180°, and a maximum at 90° and 270°; the values of the sine at any angles on either side of a maximum being smaller, and on either side of a minimum being larger than the maximum and minimum values—viz. (in a circle of radius 1), 1 and 0.

100. Now if, as the variable increases, the function increases, its *rate of variation* must be positive; but if, as the variable increases, the function diminishes, its *rate of variation* must be negative—that is to say, in

the first case the *dc* (remembering the definition) is positive, and in the second case negative.

101. Again, in order that a function may have a maximum or minimum, it is obvious, from what has been said, that the function must first increase and then diminish, or first diminish and then increase; and therefore in either case the *dc* must change its sign.

102. In order that any quantity, which is varying *continuously* may change its sign, it is evident that it must pass through the value 0, from positive to negative, or from negative to positive; and, therefore, in order that there may be a maximum or minimum the *dc* must be equal to 0. In other words, when a function reaches one of its greatest or least values it neither increases nor diminishes, at that instant, and therefore its *rate of variation* is 0, and therefore

$$dc = \frac{\text{rate of variation of function}}{\text{rate of variation of variable}}$$

$$= \frac{0}{\text{rate of variation of variable}} = 0.$$

We have, then, a relation from which we may find the value of the variable which produces this maximum or minimum.

103. Suppose we have an expression or function

$$8 + 6x - x^2,$$

and we wish to find for what value of the variable x it will be a maximum or a minimum. We know that dc_1 must be 0, in order that there may be a maximum or minimum.

$$\text{But } dc_1 = 6 - 2x,$$

$$\therefore \quad 6 - 2x = 0,$$

$$\therefore \quad 2x = 6,$$

$$\therefore \quad x = 3;$$

and for this value of x the function

$$8 + 6x - x^2 = 8 + 6 \times 3 - 3^2$$
$$= 8 + 18 - 9$$
$$= 17;$$

and this is, therefore, a maximum or a minimum : we have to determine which. Now, if we substitute in the function values a little larger and a little smaller than 3, we shall see whether the values immediately on either side are both greater or both less than 17.

$$\begin{aligned} \text{If } x&=1, \text{ function}=13 ; \\ x&=2, \text{ function}=16 ; \\ x&=3, \text{ function}=17 ; \\ x&=4, \text{ function}=16 ; \\ x&=5, \text{ function}=13. \end{aligned}$$

From this we see that for the value 3, the function has a value, which is greater than those immediately on either side of it, and therefore this value of the function, namely 17, is a maximum.

104. We might have arrived at this conclusion equally well by substituting these values, 1, 2, 3, etc., in the *dc*; for since the value of the *dc* must change sign—*i.e.*, pass through the value 0—we may see, by substituting these values, whether it is passing from positive to negative, in which case the function must be a maximum ; or from negative to positive, in which case the function must have attained a minimum value.

We found $\qquad dc=6-2x,$

$$\begin{aligned} \therefore \quad \text{if } x&=1, \ dc=+4 ; \\ x&=2, \ dc=+2 ; \\ x&=3, \ dc=0 ; \\ x&=4, \ dc=-2 ; \\ x&=5, \ pc=-4. \end{aligned}$$

From this we see that the *dc* has passed from positive to negative, and therefore the value of the function given by the value 3 of the variable is a maximum.

105. Let us take another example.

Suppose the function to be

$$x^3-9x^2+24x-7 ;$$

we wish to find what value of the variable makes this a maximum or a minimum, and what is the value of that maximum or minimum.

$$dc = 3x^2 - 18x + 24,$$

and this must be equal to 0 ;

$$\therefore \quad 3x^2 - 18x + 24 = 0,$$

or

$$x^2 - 6x + 8 = 0,$$

$$\therefore \quad x^2 - 6x = -8,$$

and

$$x^2 - 6x + 3^2 = -8 + 9$$
$$= 1,$$
$$\therefore \quad x - 3 = \pm 1,$$

therefore, $x = 4$ and $x = 2$ are the two solutions.

Now let us substitute, as before, in the function, numbers immediately larger and immediately smaller than these, and also in the *dc*.

If $x = 1$, function $= 9$, and $dc = +3$;
$d = 2$, function $= 13$, and $dc = 0$;
$x = 3$, function $= 11$, and $dc = -1$;
$x = 4$, function $= 9$, and $dc = 0$;
$x = 5$, function $= 13$, and $dc = +3$;
$x = 6$, function $= 29$, and $dc = +24$.

From this we see that when $x = 2$ the function is a maximum ; since, firstly, the *dc* passes from positive to negative ; and, secondly, from the values of the function which immediately precede and immediately succeed the value of the function when $x = 2$.

Similarly we see that when $x = 4$, the function is a minimum.

106. Now dc_2 is the *dc* of dc_1 (see Art. 37), that is, it gives the rate of variation of dc_1 ; and, when the function is a maximum, it has been increasing and is about to decrease, and the rate of its variation, which is given by dc_1, has been decreasing until the function arrives at the maximum, and then $dc_1 = 0$. Therefore the dc_1 must itself have been receiving negative increments, and therefore dc_2, which gives its rate of variation, must be negative.

Similarly for a minimum, dc_2 must be a positive.

So that we have a third method of testing whether the function be a maximum or minimum, for the particular value of the variable, provided it has a dc_2 which does not vanish.

In the first case which we considered

$$\text{function} = 8 + 6x - x^2,$$
$$dc_1 = 6 - 2x,$$
$$\therefore \quad dc_2 = -2;$$

which shows that the function has a maximum value.

In the second case which we considered

$$\text{function} = x^3 - 9x^2 + 24x - 7,$$
$$dc_1 = 3x^2 - 18x + 24,$$
$$dc_2 = 6x - 18.$$

Substituting in dc_2 the value $x = 2$, we have

$$dc_2 = 12 - 18 = -6;$$

and therefore, as before, for the value 2 of the variable the function has a maximum value.

Again, substituting the value $x = 4$, we have

$$dc_2 = 24 - 18 = +6;$$

and therefore, as before, for the value 4 of the variable the function is a minimum.

107. We will conclude this part of the subject with one more example.

"Divide a straight line into two parts, so that the rectangle contained by the parts may be a maximum."

Let a be the straight line, and x one of the parts,

$$\therefore \quad a - x = \text{other part},$$

and $\quad$ the rectangle $= (a - x)^x$

$$= ax - x^2,$$

or $\quad$ function $= ax - x^2,$

$$dc_1 = a - 2x,$$
$$dc_2 = -2.$$

From the sign of dc_2 we see that there is a maximum.

Putting $\quad dc_1 = 0$, we have

$$a - 2x = 0,$$

or

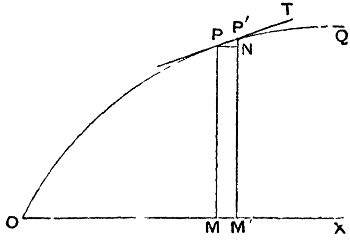
$$x = \frac{a}{2},$$

i.e., the line must be bisected.

XX. *The Tangent to a Curve.*

108. Let OPQ be any curve, and P a point on it. Then the ratio of $PM : OM$ will give the position of the point P, and likewise the tangent of the angle which the chord OP makes with OX.

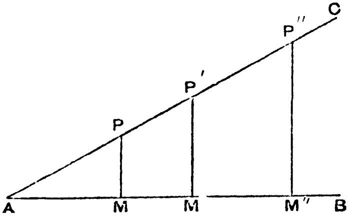
Suppose a point to be taken in the curve near to P, viz., P'; then, if $P'M'$ be drawn parallel to PM (which is perpendicular to OX), and PN be drawn parallel to OX, the ratio of $P'M' : OM'$ gives the position of P'; and if OPQ, instead of being a curve, were a straight line, the ratio $P'M' : OM'$ would be equal to $PM : OM$, *i.e.*, the straight line PP' would pass through O, if produced.



Now if we know the position of P, the position of P' and the direction of the chord PP' are determined by the ratio of $P'N : PN$, which is also the tangent of the angle which the chord PP' makes with PN or OX.

Let P' move up gradually towards P, and eventually coincide with it, then the angle $P'PT$ ultimately vanishes (see Art. 57) and the directions of the arc, chord, and tangent are the same, and are identical with that of the tangent; and the tangent of the angle which the tangent at P makes with PN or OX is represented by the ratio of the very small increase in PM to the very small increase in OM, *i.e.*, by $\frac{P'N}{PN}$, when $P'N$ and PN are indefinitely diminished, or by $\frac{dy}{dx}$, if OM be called x and PM be called y, and $\frac{dy}{dx}$ the ratio of the rate of variation of PM to the rate of variation of OM, when PM and OM receive infinitesimally small increments. This is what is meant by saying that a point which moves in a curve has, *at every instant*, the direction of motion which is represented by the tangent of that curve. It must be remembered that it is not asserted in what direction the point is *actually* moving *at any instant* of its motion, but what fictitious line of *uniform* direction (*i.e.*, what straight line) best represents, at that instant, the line of variable motion (*i.e.*, the curve) on which it is moving; and it has been shown that the direction of this line is given by $\frac{dy}{dx}$, which represents the tangent of the angle, which, at any point, the tangent at that point makes with a certain fixed straight line.

108. Let us look at this from another point of view.



Suppose a point to move along the *straight* line AC; then, for any point on this straight line, if PM, $P'M$, $P''M''$, etc., be all perpendicular to AB, we have

$$\frac{PM}{AM} = \frac{P'M'}{AM'} = \frac{P''M''}{AM''} = \text{etc.}$$

and conversely, if

$$\frac{PM}{AM} = \frac{P'M'}{AM'} = \frac{P''M''}{AM''} = \text{etc.},$$

then the path of the point is a straight line.

But if, as AM increases uniformly, PM have a varying rate of change, then the path of the point will be a curve.

If, at any instant, the varying rate of change of PM were to become uniform, the path of the point would be determined by the constant ratio of $\frac{PM}{AM}$, as before, and therefore would be a straight line.

Let us bear in mind that the position of the point at any instant, whether on the curve or a straight line, is determined by the relative values of PM and AM, that is by the ratio of $\frac{PM}{AM}$.

Now let us adopt a similar method to that employed in Art. 76; and supposing the point to be moving in a curve, such that AM has uniform increases, but PM a varying rate of change, let us suddenly check the point in its path and inquire what its motion would have been, if it had continued in the *direction* which it *had at that instant* for a unit of time; not the direction it would have taken in *its path* in the next unit of time, but the direction it would take if the increments in PM and AM continued *uniformly* at the rate they had at the instant of stoppage. Since PM and AM increase uniformly, the path is a straight line and its direction is given by the ratio of $\frac{PM}{AM}$ which is the tangent of the angle which the straight line makes with AM.

A good example of the idea of a tangent is found in the stone leaving the sling, which has been swung ınd in a curve. The instant the stone leaves the ıg it proceeds (for a short time) in the direction ıich it had at that instant.

A pellet of mud leaving a carriage wheel gives ɔther familiar example.

APPENDIX I.

Assuming the exponential theorem,

$$a^x = 1 + \frac{Ax}{1} + \frac{A^2x^2}{\underline{|2}} + \frac{A^3x^3}{\underline{|3}} + \text{etc.} \ldots\ldots\ldots\ldots\ldots (1),$$

where $A = (a-1) - \frac{1}{2}(a-1)^2 + \frac{1}{3}(a-1)^3 - \text{etc.}$

In (1), put $x = 1$, then

$$a = 1 + \frac{A}{1} + \frac{A^2}{\underline{|2}} + \frac{A^3}{\underline{|3}} + \text{etc.} \ldots\ldots\ldots\ldots\ldots (2).$$

Again, in (2), put $A = 1$, then the series becomes

$$1 + \frac{1}{1} + \frac{1}{\underline{|2}} + \frac{1}{\underline{|3}} + \text{etc.} = 2{\cdot}71828 \ldots\ldots\ldots\ldots\ldots (3),$$

and this is called e, and is the base of the Naperian system of logarithms.

Again, in (1), put $A = 1$, and then e represents the value of a, and

$$e^x = 1 + x + \frac{x^2}{\underline{|2}} + \frac{x^3}{3} + \text{etc.};$$

and in this make x equal to A;
therefore

$$e^A = 1 + A + \frac{A^2}{\underline{|2}} + \frac{A^3}{\underline{|3}} + \text{etc.};$$

and this is identical with (2),
therefore $a = e^A \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (4)$
when, as before,

$$A = (a-1) - \tfrac{1}{2}(a-1^2) + \tfrac{1}{3}(a-1)^3 - \text{etc.}$$

But, Nap. $\log a = A$, from (4),
therefore

$$\text{Nap.} \log a = (a-1) - \tfrac{1}{2}(a-1)^2 + \tfrac{1}{3}(a-1)^3 - \text{etc.};$$

or, reducing this to logs with base 10,

$$\log a = \log e\{(a-1) - \tfrac{1}{2}(a-1)^2 + \tfrac{1}{3}(a-1)^3 - \text{etc.}\};$$

or $$\log a = M\{(a-1) - \tfrac{1}{2}(a-1)^2 + \tfrac{1}{3}(a-1)^3 - \text{etc.}\};$$

in this put $a = 1 + n$, and therefore $a - 1 = n$,

then $$\log\{1+n\} = M\{n - \frac{n^2}{2} + \frac{n^3}{3} - \text{etc.}\};$$

Again, let $n = \frac{1}{m}$, then

$$\log\left(1 + \frac{1}{m}\right) = M\left(\frac{1}{m}\right), \text{ approximately,}$$

and M is found to be ·4342940.

APPENDIX II.

Maclaurin's Theorem.

Assuming the ordinary working of Indeterminate Co-efficients.

Let there be any function of x, and suppose that this function may be expanded in ascending powers of x and constants which do not contain x, but which have to be determined; and let these constants be A, B, C, etc., then the

$$\text{function} = A + Bx + Cx^2 + Dx^3 + \text{etc.}, \quad \text{(1)}$$

$$\therefore \quad dc_1 = B + 2Cx + 3Dx^2 + \text{etc.}, \quad \text{(2)}$$

$$dc_2 = 2C + 3 \times 2Dx + \text{etc.}, \quad \text{(3)}$$

$$dc_3 = 3 \times 2D + \text{etc}, \quad \text{(4)}$$

$$\text{etc.} = \text{etc.}$$

Now let x, being the variable, continuously diminish and ultimately become 0, then

$$\begin{aligned}(\text{function}) &= A,\\ (dc_1) &= B,\\ (dc_2) &= 2C,\\ (dc_3) &= 3\times 2D;\end{aligned}$$

and therefore

$$\begin{aligned}A &= (\text{function}),\\ B &= (dc_1),\\ C &= \tfrac{1}{2}(dc_2),\\ D &= \frac{1}{2\times 3}(dc_3),\\ \text{etc.} &= \text{etc.}\end{aligned}$$

Substituting these values in (1) we have

$$\text{function} = (\text{function}) + (dc_1)x + \tfrac{1}{2}(dc_2)x^2 + \frac{1}{2\times 3}(dc_3)x^3 + \text{ etc.};$$

which is Maclaurin's theorem.

EXAMPLES WORKED OUT.

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1. If the side of a square increases uniformly at the rate of 5 feet per second, at what rate is the area increasing when the side becomes 10 feet.

If the side of square $= x$,

then area of square $= x^2$,

and differential co-efficient of $x^2 = 2x$.

Now when the side becomes 5 feet—

differential co-efficient of $x^2 = 2 \times 5$ feet,

$= 10$ feet,

or

$$\frac{\text{rate of variation of area}}{\text{,, \quad ,, \quad side}} = 10 \text{ feet.}$$

Therefore, when the side becomes 10 feet—

rate of variation of area $= 10 \times 10$ square feet,

$= 100$ square feet.

2. What is the value of the fraction $\frac{2x+5}{4x+6}$, when x becomes infinite?

Divide both numerator and denominator by x, and the fraction becomes

$$\frac{2+\frac{5}{x}}{4+\frac{6}{x}}$$

Now, when x becomes infinite, each of the fractions $\frac{5}{x}$ and $\frac{6}{x}$ becomes nothing.

$\therefore$ the limit of $\frac{2x+5}{4x+6}$, when x becomes infinite

$$= \tfrac{2}{4}$$
$$= \tfrac{1}{2}.$$

3. Find that angle which increases twice as fast as its sine.

Let x be the angle

then $\sin x =$ the function

$\therefore \quad dc = \cos x.$

But $\frac{\text{rate of variation of function}}{\text{rate of variation of angle}} = \frac{1}{2}$ (by question)

$\therefore \quad dc_1 = \frac{1}{2},$

$\therefore \quad \cos x = \frac{1}{2},$

$\therefore \quad \text{angle} = 60°.$

4. Divide a straight line into two parts, so that the rectangle contained by the parts may be the greatest possible.

Let $a =$ the line,

$x =$ one of the parts,

$\therefore \quad a - x =$ other part,

then $\text{rectangle} = (a - x)x$

$$= ax - x^2,$$

$$\therefore \quad dc_1 = a - 2x,$$

$$dc_2 = -2,$$

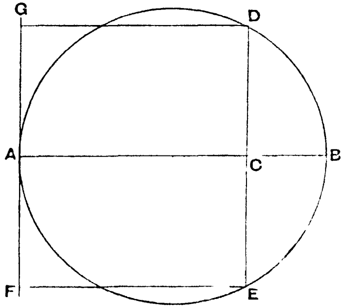
therefore there is a maximum, since the dc_2 is negative, and this maximum is given by equating dc_1 to 0.

$$\therefore \quad a - 2x = 0,$$

or, $$x = \frac{a}{2},$$

that is to say, the line must be bisected.

5. Let AB be the diameter of a given circle, it is required to find a point C in the diameter, so that the rectangle formed by the chord DE, which is perpendicular to AB, and the part AC may be the greatest possible.



Let $AB = a,$

$$AC = x,$$

$$\therefore \quad CB = a - x,$$

then $CD^2 = (a - x)x,$

$$CD = \sqrt{(a - x)x},$$

$$\therefore \quad DE = 2\sqrt{ax - x^2},$$

and $\text{rectangle } EG = x \times 2\sqrt{ax - x^2},$

and this is to be a maximum ; if it is, its square will also be, viz.,

$$4x^2(ax-x^2),$$

or $$4ax^3-4x^4\ ;$$

$$\therefore \quad dc_1=12ax^2-16x^3,$$

and $$dc_2=24ax-48x^2,$$

but $$dc_1=0\ ;$$

$$\therefore \quad 12ax^2-16x^3=0,$$

or $$16x^3=12ax^2,$$

i.e., $$16x=12a,$$

$$x=\frac{12a}{16}$$

$$=\frac{3}{4}a.$$

Substitute this value in dc_2 and we have

$$24a\times\frac{3}{4}a-48\times\frac{9}{16}a^2$$

$$=18a^2-27a^2$$

$$=-9a^2.$$

Therefore there is a maximum, and it is given by the value $\frac{3}{4}a$—*i.e.*, we must take $\frac{3}{4}$ of a to find C.

6. To approximate to the roots of an equation.

Let the equation be

$$x^3-3x+1=0,$$

x^3-3x+1 being a function of x.

But $$\text{function}=(\text{function})+(dc_1)\times h,$$

and $$\text{function}=0,$$

since the function is the left-hand side of the equation.

$$\therefore \quad (\text{function})+(dc_1)\times h=0,$$

$$\therefore \quad h=-\frac{(\text{function})}{(dc_1)}.$$

Now, by trial, 1·5 is found to be near one of the roots. Let h be the difference between 1·5 and the root, so that $x=1\cdot5+h$, which is of the form $(a+h)$.

$$\therefore \quad (\text{function})=a^3-3a+1$$

$$=(1\cdot5)^3-3\times1\cdot5+1$$

$$=-1\cdot25,$$

$$
\begin{aligned}
\text{and} \qquad (dc_1) &= 3a^2 - 3 \\
&= 3 \times (1{\cdot}5)^2 - 3 \\
&= 6{\cdot}75 - 3 \\
&= 3{\cdot}75, \\
\therefore \qquad h &= \frac{{\cdot}125}{3{\cdot}75} \\
&= {\cdot}033 \\
\therefore \qquad x &= 1{\cdot}5 + {\cdot}003 \\
&= 1{\cdot}533.
\end{aligned}
$$

EXERCISES.

The Roman numbers refer to the sections in the body of the book.

II.

1. What is the value of $\frac{1}{a-x}$, when $x = a$?

2. What does the fraction $\frac{a}{a-x}$ become, when $x = a$?

3. Place $\frac{a-b}{{\cdot}01}$, $a-b$, and $\frac{a-b}{{\cdot}001}$ in ascending order of magnitude, and state the value of each when $b = a$.

4. Develop into a series, by actual division, the fraction $\frac{1}{1-x^2}$, and show, by this means, that its value is infinite when $x = 1$.

III.

5. In the series which is equivalent to $\frac{1}{9}$, if we take

10 terms, by how much does their sum differ from $\frac{1}{9}$?

6. How many terms of the series must be taken in order that their sum may differ from $\frac{1}{9}$ by less than $\frac{1}{10000000}$?

7. How many terms must be taken that their sum may differ from $\frac{1}{9}$ by less than $\frac{43}{18562}$?

IV.

8. Find the value of $\frac{a^3 - b^3}{a - b}$, when $b = a$.

9. Show that $\frac{a^5 - b^5}{a^3 - b^3} = \frac{5}{3}a^2$, when $b = a$.

10. Find the value of the fraction $\frac{x^2 - 1}{x - 1}$, when $x = 1$.

11. What is the value of the fraction $\frac{5x + 7}{15x + 17}$, when x becomes infinite?

12. Find the value of the fraction in (8) by substituting $a - h$ for b.

13. Show how in (8) the value of the fraction becomes more and more nearly the value of the limit, as b approaches a, by means of numerical illustrations.

14. Find the value of $\frac{6x - 5}{2x + 7}$, when x becomes infinite.

15. What is the limit of the ratio $\frac{xh + h^2}{h}$, when $h = 0$?

16. What is the limit to which the ratio of

$$h^2 : 3x^2h^2 + 3xh^3 + h^4$$

approaches, as h diminishes and ultimately vanishes?

V.

17. Define *Differential Co-efficient.*

18. State what you mean by a *function;* and give 5 examples of a function of x, 5 of a function of y, and 5 of a function of z.

19. If y be a variable quantity and receive small increments of ·1, show that the corresponding values of ·01 × y increase uniformly.

20. If $px - C$ be a function of x, show that it increases uniformly as the variable receives successive increments of $a + b$.

21. Find the differential co-efficient of $5x$.

22. Give the differential co-efficients of

(1) ax,
(2) $3bx$,
(3) $(a - b)x$,
(4) $(a^2 - b^2)x$,
(5) $ax + b$,
(6) $2a + 5x$,
(7) $m + nx$,
(8) $(a^2 - b^2)x - (a^2 - b^2)$,
(9) $\frac{px}{q} + r$,
(10) $\frac{4p^2x}{3q^2} - \frac{l^3}{m^3}$.

VI.

23. If the side of a square increase uniformly at the rate of 3 feet per second, at what rate is the area of the square increasing when the side becomes 10 feet?

24. If x increase uniformly at the rate of 2 per unit of time, at what rate does ax^2 increase when $a = 4$, and $x = 10$?

25. If x increase uniformly at the rate of 1 per unit of time, at what rate does the value of the function $a + 2x^2$ increase when $a = 4$, and $x = 6$.

26. If x increase uniformly at the rate of ·1 per second, at what rate does $\frac{x^2}{a}$ increase when x becomes 4, the constant a being equal to 10?

27. The radius of a circular plate of metal is 12

inches; find the increase in the area, when the radius is increased by ·001 inch.

[Area of circle of radius $r = \pi r^2$
and $\pi = 3{\cdot}1416$.]

VII.

28. Show, by constructing a table of spaces fallen through in hundredths of seconds (·1, ·09, ·08 . . . . ·01 sec.) and then taking differences, that the space fallen through in the interval between any two consecutive hundredths of a second is ·0032 ft.

29. If the interval were between two consecutive $\frac{1}{100000000}$ths of seconds, what would the space fallen through be?

30. If the intervals were seconds what would the spaces be?

VIII.

31. Show, by forming a table, that if 2 be a variable and receive small successive increments of ·001, the differential co-efficient of $2^2 = 2 \times 2$.

32. If 5 be a variable and receive small increments of ·0001, show, by forming a table, that the differential co-efficient of $5^2 = 2 \times 5$.

33. Find, by constructing a table, the second differential co-efficient of 3^2, supposing 3 to receive small increments of ·001.

34. Supposing 25 to be a variable and to receive small increments of ·0000001, what is the first differential co-efficient of 25^2? What is the second differential co-efficient?

35. If the numbers, whose squares are the functions, be supposed to vary, give the first and second differential coefficients of 19^2, 37^2, 1001^2.

IX.

36. If 2 be supposed to vary, and to receive small increments of ·0001, find, by constructing a table, the first, second, and third differential co-efficients of 2^3.

37. If 2 be supposed to vary and to receive small increments of ·0001, find, by forming a table, the first, second, third, and fourth differential co-efficients of 2^4.

38. What is the germ or essence of the 7th power?

39. What is the germ or essence of the $(n-1)$th power?

40. What is the germ or essence of the $(p-q)$th power? Prove the truth of your answer by substituting 225 for p and 220 for q.

41. Give the first differential co-efficients of

(1) x^2,	(4) x^{17},
(2) x^3,	(5) x^{45},
(3) x^4,	(6) x^{100}.

42. Find the second, third, fourth, fifth, ninth, and twentieth differential coefficients of x^{20}.

43. Give the differential coefficients of (see Arts. 27 and 46)

(1) $x+x^2$,	(6) $x^{\frac{1}{2}}$,
(2) ax^2+c,	(8) $2\sqrt{x}$,
(3) $4ax^3+b$,	(9) $(a+b)x^{p+q}+k$,
(4) $c-2x^3$,	(10) ax^{n+1}.
(5) $\dfrac{3x^{\frac{2}{3}}-a^2}{b}$,	

44. A cube of metal, whose edge is 12 inches, has this edge increased by ·001 inch. Find the cubical expansion.

XI.

45. Show, by forming a table, that, if 3 be a variable and receive small increments of ·0001, the differential co-efficient of $\frac{1}{3} = -\frac{1}{3^2}$.

46. Find, by forming a table, the differential co-efficient of $\frac{1}{5^2}$ as 5 varies and receives small increments of ·001.

47. Find the differential co-efficient of $\frac{1}{4^3}$, 4 being a variable and receiving small increments of ·00001.

48. Find the differential co-efficients of

(1) $\frac{1}{x^2}$,

(2) $\frac{a}{x^3}$,

(3) $\frac{3a}{x^4}+b$,

(4) $\frac{a^2-b^2}{x}$.

49. By constructing a table, find the second differential co-efficient of $\frac{1}{2^3}$ when 2 is the variable, and receives small increments of ·001.

50. What is the second differential co-efficient of $\frac{1}{3^4}$, when 3 receives small increments?

51. Find the second differential co-efficients of

(1) $\frac{1}{x^{99}}$,

(2) $\frac{a}{x^5}$,

(3) $\frac{a^2-b^2}{x^{10}}+c$,

(4) $\frac{x^{-21}}{a}$

XIII.

52. Find the differential co-efficient of the sine of an angle, which lies between 180° and 270° (geometrically).

53. Find the differential co-efficient of the cosine of an angle, which lies between 90° and 180° (geometrically).

54. Find the differential co-efficient of the tangent of an angle, which lies between 270° and 360° (geometrically).

55. Find the differential co-efficient of the cotangent of an angle, which lies between 90° and 180° (geomettrically).

56. Find the differential co-efficient of the secant of an angle, which lies between 180° and 270° (geometrically).

57. Find the differential co-efficient of the cosecant of an angle, which lies between 270° and 360° (geometrically).

58. Find that angle which increases twice as fast as its cosine.

XIV.

59. Find the differential co-efficient of $\cos^{-1}x$.

60. Find the differential co-efficient of $\cot^{-1}x$.

61. Find the differential co-efficient of $\operatorname{cosec}^{-1}x$.

XV.

62. Establish, by taking successive cube roots of 1000, the principle laid down in Section XV.

63. What is the value of $95a^{n-x}$ when $x=n$?

64. What is the value of $1000^\circ - 1^\circ$?

XVI.

65. Having given—

$$\text{arc} = \frac{\text{angle}}{180^\circ} \times 3{\cdot}1416\,;$$

natural cosine of 30° = ·8660254, and difference for 1′ = 1454;

and natural sine of 30° = ·5000000;

let 30° receive small increments of ·001 and show, by constructing a table, that the differential co-efficient of $\cos x = -\sin x$ approximately.

66. Having given—

$$\text{arc} = \frac{\text{angle}}{180^\circ} \times 3{\cdot}1416\,;$$

natural cot 14° = 4·0107809, difference for
1′ = −49644 ;
and natural cosec 14° = 4·1335655 ;
let 14° receive small increments of ·1 and show, by constructing a table, that the differential co-efficient of cot 14° = −cosec²14°.

67. Find the angle which increases at the rate of $\sqrt{2}$ times the rate of its sine.

XVII.

68. Given

Common log 62300 = 4·7944880,
„ log 62301 = 4·7945578,
„ log 62302 = 4·7946276.

Convert these into Naperian logarithms, and show that

$$\text{differential co-efficient of Nap. log } 62300 = \frac{1}{62300}.$$

69. Given

Common log 33·863 = 1·5297254,
„ log 33·864 = 1·5297383,
„ log 33·865 = 1·5297512.

Convert these into Naperian logarithms, and show that

$$\text{differential co-efficient of Nap. log } 33{\cdot}863 = \frac{1}{33{\cdot}863}.$$

XVIII.

70. Show by successive differentiating that the fourth differential co-efficient of $x^4+x^3+x^2+x+1 = 2\times 3\times 4$.

71. Find the fourth differential co-efficient of $\frac{1}{x}$.

72. Find the eighth differential co-efficient of x^n.

73. Find the third differential co-efficient of

$$x^3+ax^2+bx+c.$$

74. Find the second differential co-efficient of

$$x^{\frac{5}{2}}+\frac{a}{x}+b.$$

75. Find the fifth differential co-efficient of $x^4 - x^{-4}$.

76. Required the seventh and eighth differential co-efficients of $\cos x$.

77. Expand $\cos x$, by Maclaurin's theorem, in terms of x.

78. Differentiate the series in (77), and show that the result is the expression for $\sin x$.

79. Approximate to the roots of the equation

$$x^3 - 12x - 28 = 0.$$

80. Approximate to the roots of the equation

$$x^4 + x - 3 = 0.$$

XIX.

81. Find when $16x - x^2$ will be a maximum or a minimum.

82. Find when the function

$$2x^3 - 9ax^2 + 12a^2x - 4a^3$$

will be a maximum or minimum, and give the value of the function which is a maximum or minimum.

83. When is the function

$$x^3 - 3ax^2 + 4a^3$$

a maximum, and when a minimum?

84. Find when

$$6x^2 - 30x + 24$$

is a maximum and a minimum.

85. Give the maximum and minimum values of the function

$$4x^3 - x^2 - 2x + 1.$$

86. Give the maximum and minimum values of

$$x^3 - 7x^2 + 8x + 32.$$

87. Find the fraction which exceeds its second power by the greatest possible quantity.

88. Divide the quantity a into two such parts that their product shall be the greatest possible.

89. Divide a given line AB into two parts so that the sum of the areas of the squares described on the parts shall be the least possible.

90. A gentleman has a plot of ground in the form of

a triangle, the base of which is 400 feet and the perpendicular 300 feet, in which he wishes to make the greatest rectangular garden possible, one of the sides of which is in the base. It is required to find how many feet from the vertex the other side must be drawn.

MISCELLANEOUS EXERCISES.

91. Upon AB describe a semi-circle, draw a chord AP; draw PN perpendicular to AB; then prove that $AP=PN$ ultimately—*i.e.*, at the moment when the arc AP vanishes.

Note.—If $AN=x$,

$AB=2a$,

$AP=\sqrt{2ax}$ and $PN=\sqrt{2ax-a^2}$.

92. Develop into a series, by Maclaurin's theorem, $\sqrt{a+x}$.

93. In (92) put $a=1$, then $\sqrt{a+x}=\sqrt{1+x}$. Now, by putting $x=1$, find the value of $\sqrt{2}$, correct to three decimal places.

94. Expand into a series, by Maclaurin's theorem, $\sqrt[3]{1+x}$; and, by substituting 8 for x, give the series for the calculation of $\sqrt[3]{9}$.

95. Find the differential co-efficient of

$$(1+2x^2)(1+4x^3).$$

96. Find the real value of the fraction

$$\frac{x^3+2x^2-x-2}{x^3-1}, \text{ when } x=1.$$

97. Find the value, when $x=2$, of the fraction

$$\frac{x^3-x^2-8x+12}{x^4-9x^2+4x+12}.$$

98. If x increase uniformly at the rate of 1 per second, at what rate is the expression $\frac{4x^3+a}{b}$ increasing when x becomes 10, a being equal to 4 and b to 6?

99. Find the n^{th} dc of $\frac{1}{x}$.

100. Divide a number into two such parts, that their product multiplied by the difference of their squares shall be a maximum.

ANSWERS TO THE EXERCISES.

II.

1. ∞.
2. ∞.
3. $a-b$, $\frac{a-b}{\cdot 01}$, $\frac{a-b}{\cdot 001}$; 0.

III.

5. $\frac{1}{900000000000}$.
6. 8.
7. 2.

IV.

8. $3a^2$.
10. 2.
11. $\frac{1}{3}$.
14. 3.
15. $x : 1$.
16. $1 : 3x^2$.

V.

21. 5.
22. (1) a
(2) $3b$.
(3) $a-b$.
(4) a^2-b^2.
(5) a.
(6) 5.
(22. 7) n.
(8) a^2-b^2.
(9) $\frac{p}{q}$.
(10) $\frac{4p^2}{q^2}$.

VI.

23. 60 square feet per second.
24. 160.
25. At the rate of 24.
26. At the rate of ·08 per second.
27. ·0753984 square inches.

VII.

29. ·000000000000000032.
30. 32 ft.

VIII.

33. 2.
34. 50 ; 2.
35. First differential co-efficients are 38, 74 and 2002; the second differential co-efficients are 2, 2, 2.

IX.

36. 12 ; 12 ; 6 ; or 3×2^2 ; $3 \times 2 \times 2$; $3 \times 2 \times 1$
37. 4×2^3 ; $4 \times 3 \times 2^2$; $4 \times 3 \times 2 \times 2$; $4 \times 3 \times 2 \times 1$.
38. $7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1 = \underline{|7}$.
39. $\underline{|n-1}$.
40. $\underline{|p-q}$.
41. (1) $2x$.
(2) $3x^2$.
(3) $4x^3$.
(4) $17x^{16}$.
(5) $45x^{44}$.
(6) $100x^{99}$.
42. (1) $20 \times 19x^{18}$.
(2) $20 \times 19 \times 18x^{17}$.
(3) $20 \times 19 \times 18 \times 17x^{16}$.
(4) $20 \times 19 \times 18 \times 17 \times 16x^{15}$.
(5) $20 \times 19 \times 18 \times \ldots \times 12x^{11}$
(6) $\underline{|20}$.
43. (1) $1 + 2x$.
(2) $2ax$.
(3) $12ax^2$.
(4) $-6x^2$.
(5) $\frac{9x^2}{b}$.
(6) $\frac{1}{2}x^{-\frac{1}{2}}$.
(7) $x^{-\frac{1}{2}}$ or $\frac{1}{\sqrt{x}}$.

(8) $(a+b)(p+q)x^{p+q-1}$.
(9) $a(n+1)x^n$.

44. ·432 cubic inches.

XI.

46. $-\dfrac{2}{5^3}$.

47. $-\dfrac{3}{4^4}$.

48. (1) $-\dfrac{2}{x}$.
(2) $-\dfrac{3a}{x^2}$.
(3) $-\dfrac{12a}{x^5}$.
(4) $-\dfrac{a^2-b^2}{x^2}$.

49. $\dfrac{3\times 4}{2^5}$.

50. $\dfrac{4\times 5}{3^6}$.

51. (1) $\dfrac{99\times 100}{x^{101}}$.
(2) $\dfrac{30a}{x^7}$.
(3) $\dfrac{110(a^2-b^2)}{x^{12}}$.
(4) $\dfrac{462}{ax^{23}}$.

XIII.

58. 210°.

XV.

63. 95.

64. 0.

XVI.

67. 45°.

XVIII.

71. $\dfrac{2.3.4}{x^5}$.

72. $n(n-1)(n-2)'n-3)(n-4)(n-5)(n-6)\times (n-7)x^{n-8}$.

73. $1\times 2\times 3$.

74. $\dfrac{15}{4}x^{\frac{1}{4}}+2ax^{-3}$.

75. $4\times 5\times 6\times 7\times 8x^{-9}$.

76. $\sin x$ and $\cos x$.

79. 4·302.

80. 1.165.

XIX.

81. Maximum when $x=8$.
82. Maximum when $x=a$, function $=a^3$.
Minimum when $x=2a$, function $=0$.
83. $x=0$ gives a maximum.
$x=2a$ gives a minimum.
84. $x=1$ gives a maximum.
$x=4$ gives a minimum.
85 $\frac{38}{27}$; $\frac{1}{4}$.
86. $34\frac{14}{27}$ when $x=4$. 16 when $x=\frac{2}{3}$.
87. $\frac{1}{8}$.
88. The parts must be equal.
89. The line must be bisected.
90. The perpendicular must be bisected.

MISCELLANEOUS EXERCISES.

92. $a^{\frac{1}{2}}+\frac{1}{2}\frac{x}{a^{\frac{1}{2}}}-\frac{1}{8}\frac{x^2}{a^{\frac{3}{2}}}+\frac{1}{16}\frac{x^3}{a^{\frac{5}{2}}}-\text{etc.}$
93. 1·414.
94. $1+\frac{1}{3}\times 8-\frac{1}{9}\times 8^2+\frac{5}{81}\times 8^3-\text{etc.}$
95. $4x+12x^2+40x^4$.
96. 2.
97. $\frac{1}{3}$.
98. At rate of 200 per second.
99. $\frac{1, 2, 3 \ldots\ldots n}{x^{n+1}}$, being + or − according as n is even or odd.
100. If $2a=$ the number,
$a+x=$ one part,
$a-x=$ the other,
then $x=\frac{a}{\sqrt{3}}$.

GLASGOW: ROBERT MACLEHOSE, PRINTER TO THE UNIVERSITY

January, 1885.

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