

THEORY OF CONSONANCE AND STRUCTURE OF SCALE*

B. K. Bose

Theory of Consonance

Relationships of musical notes are of three categories, viz. : Identity (*Samatva*), Consonance (*Samvada*) and Dissonance (*Vivada*). The first kind of relationship is, in popular usage, supposed to exist between one note and another which is eighth in degree above it. This would appear from the long-established custom of calling these two notes by the same name ; though, in point of fact, one is higher in pitch than the other, having double the number of vibrations.

Two notes are said to be Consonant (*Samvadi*) to each other if they have a common upper partial perceptible to the ear. The feeling of consonance is produced by the common partial which being in a sense identical with each of the two notes serves as the connecting link between them and produces an aesthetic satisfaction or pleasure by enabling the sub-conscious mind to connect the two notes with a common bond. There are altogether six relationships of consonance. These, according to the degree of their consonance, with their respective ratios are :—Fifth (*panchama*) — 2 : 3; Fourth (*chaturtha*) — 3 : 4; Major Sixth (*ayata shastha*) — 3 : 5; Major Third (*ayata tritiya*) — 4 : 5; Minor Third (*laghu tritiya*) — 5 : 6 and Minor Sixth (*laghu shastha*) — 5 : 8.

The notes *Sa* and *Pa* constitute one of the several pairs of notes having the relationship of Fifth to each other. Taking *Sa* to represent a note of 10 vibrations, *Pa* would be a note of 15 vibrations. The least common multiple of 10 and 15 is 30. This is the number of vibrations of the common upper partial of the two notes. This note having twice the

*This article is based on material from Chapters 2 & 3 of "The Melodic Types of Hindusthan" by Shri Narendra Kumar Bose, the father of the present writer. Permission for material used or quoted has kindly been granted by M/s. Jaico (1960). Reference can be made to the book for detailed study.

number of vibrations of *Pa* is its second partial, which is *Pá* of the higher octave. It is likewise the third partial of *Sa*, as its vibration number is three times that of the latter note. The ratio numbers of the last of the above-mentioned consonances are 5 and 8; the common upper partials in this case is, therefore the eighth partial of the first and the fifth partial of the second of the two notes having this relationship. The eighth being the last of the partials perceptible to the ear, the Minor Sixth is to be considered the worst of the consonances. The six consonances may be divided into two groups and designated Perfect and Imperfect consonances. The first two in the list are included in the first group and the remaining four in the second. There is a marked difference in the degree of consonance of these two groups. It is probably owing to this difference that in ancient India they were given separate names, viz. : *Samvadi* and *Anuvadi*. It will however be seldom required to observe this distinction in the practical field.

The third kind of relationship, called Dissonance, needs no definition as all relationships, which are neither those of Identity nor those of Consonance belong to this category. Any relationship between two notes, which the human ear cannot connect with each other, may be considered dissonance. It would be natural to conclude from this that dissonances have no place in music. It is, however, a well known fact that the number of dissonant intervals in a scale is almost equal to that of the consonant ones. A note-worthy feature of every scale is that the two notes, which are nearest to a particular note in it, are both dissonant to it. For example, *Sa* and *Ga* are both dissonant to *Ra*. All consecutive notes in a scale are dissonant (*Vivadi*) to each other. Dissonant intervals are as frequent as consonant ones in melodic compositions. The explanation for this apparently paradoxical phenomenon is that the notes of most of these intervals are related to each other indirectly through a common consonant note. Dissonant intervals so related have a special aesthetic value in music and are termed Related Dissonance (*Sambaddha Vivada*) in order to distinguish them from the remaining mass of dissonant intervals, which are useless for musical purposes.

There are four kinds of intervals between consecutive notes, namely, Major Tone, Minor Tone, Major Semitone and Minor Semitone. An interval having the ratio 25 : 24, which is the difference between Major and Minor Thirds or Sixths, is used in some very popular scales found only in India. This interval is called a Small Semitone. Another interval between consecutive notes which may be expressed by the ratio

75 : 64 is also to be found in a few scales and will be called a Large Tone. There are, thus, altogether six kinds of intervals between consecutive notes of a scale. These intervals, with their ratios, are shown below in the order of their size, the largest one being placed first :— Large Tone—75 : 64, Major Tone—9 : 8, Minor Tone—10 : 9, Major Semitone—27 : 25, Minor Semitone—16 : 15 and Small Semitone—25 : 24.

Notes separated by a Major Tone are indirectly related to each other through the note which is Fourth below the lower note and Fifth below the upper. Notes having a Minor Tone between them may be related either through the note which is Fifth below the lower and Major Sixth below the upper note, or through the note which is Minor Third below the lower and Fourth below the upper note. The other intervals are also similarly related.

Anushruti

It has been shown earlier in the treatise that a cyclic division of an octave into fifty-three (53) degrees is the best of all small cycles both for theoretical and practical use. This cycle gives a scale in almost precisely just intonation. A degree of this cycle being much smaller in magnitude than a *Shruti* (22 degrees), it has been called an *Anu-shruti* (small *shruti*). For its English equivalent, the word 'nonatone' has been coined, as it is one-ninth of a Major Tone. The unit degree of *Anushruti* is almost identical with the Comma (22 cents).

The number of *Anushrutis* representing the sixth consonant intervals and those of the six dissonant intervals mentioned above are given here in the order of their magnitude :—Major Sixth 39, Minor Sixth 36, Fifth 31, Fourth 22, Major Third 17, Minor Third 14, Large Tone 12, Major Tone 9, Minor Tone 8, Major Semitone 6, Minor Semitone 5 and Small Semitone 3.

Structure of Scales

The object of all music being the production of pleasurable feeling in the mind of the listener, any rational explanation of the structure of scales on which music is based must start with the hypothesis that the relationships subsisting amongst the notes constituting a scale are capable of producing such feeling. The science of acoustics and experimental psychology have proved that only the relationships of consonance possess this property. There are indeed some dissonant relationships which play an important part in the musical art. But as

these relationships have no independent value of their own and must be related through consonant relationships in order to be acceptable, we must confine our attention to the consonant relationships only in our attempt to explain the structure of scales. We have seen that the consonant relationships are only six in number. In order that a scale be fit for being used in music, its notes must, therefore, be connected with each other by some one or more of these six relationships.

The most spontaneous and easily appreciated consonant relationship is the descending Fourth, which is the natural leap that the human voice takes in an affirmative sentence. A descending tetrachord may, therefore, be considered as the earliest stage in the evolution of scale. The tetrachord of the *Vedic Saman* chants and that from which the Greek Doric Scale was formed, were both descending tetrachords. The consonant interval which comes next to the descending Fourth as regards spontaneity is the ascending Fifth, which the human voice takes naturally in an interrogative sentence. So, the ascending pentachord may be regarded as the second stage in the development of scales. The ascending tetrachord and the descending pentachord must have come as later developments.

Elementary and Perfect Tetrachords

A full tetrachord ordinarily consists of two tones and a semitone. As notes separated by a tone or semitone are dissonant to each other the lowest note of a tetrachord may have only two notes consonant to it, viz. : the third and the fourth notes above it. Likewise, the highest note of a tetrachord may have only two notes consonant to it, viz. : the third and the fourth notes below it. Now, the third may be either a Major or a Minor Third. As in constructing a scale we would proceed naturally from the starting note to those notes only which are consonant to it, the most elementary tetrachord would be a trisilient one, consisting of three notes with a leap of a Major or Minor Third from the starting note to the next higher or lower note, as the case may be. There would be two such ascending and two descending Elementary Tetrachords according to the direction of the progression. These Elementary Tetrachords are

Ascending —(1) $Sa - Ga \quad Ma$; (2) $Sa - Ga \quad Ma$
 17 5 14 8

Descending—(1) $Ma - Ro \quad Sa$; (2) $Ma - Ra \quad Sa$
 17 5 14 8

If we now combine one of the Elementary Ascending Tetrachords with one of the Elementary Descending Tetrachords we would get four Perfect Tetrachords capable of being used both in ascent and in descent and having no gaps of a Third in them. These Perfect Tetrachords are :—

- (1) *Sa Ra Ga Ma*; (2) *Sa Ra Go Ma*; (3) *Sa Ro Go Ma*; (4) *Sa Ro Ga Ma*
 8 9 5 8 6 8 5 9 8 5 12 5

According to the *Naradiya Shiksha*, the first four of the notes of the *Shadja Grama* were identical with the notes of original tetrachord of the *Saman* Chanters. This is the perfect Tetrachord No. 2 given above, in which the extreme notes have a Minor Third above or below them. The tetrachord of the *Ahvarakas* was the ascending Elementary Tetrachord No. 2; that of the *Taittiriya*s was the Perfect Tetrachord No. 1 in descending order; and that on which the Greek Scales were based, was the Perfect Tetrachord No. 3 also in descending order. The Tetrachord No. 4, which contains two semitones and an interval which is much larger than a tone, is not to be found either in ancient India or in ancient Greece.

Elementary and Perfect Pentachords

Adding the fifth note *Pa* to the Elementary Ascending Tetrachords we get the following two Elementary Ascending Pentachords :—

- Ascending —(1) *Sa — Ga Ma Pa*; (2) *Sa — Go Ma Pa*
 17 5 9 14 8 9

If we descend from *Pa* as the starting note preserving the above relationship in the inverse order, we get the following two Elementary Descending Pentachords :—

- Descending—(1) *Pa — Go Ra Sa*; (2) *Pa — Ga Ra Sa*
 17 5 9 14 8 9

Combining No. 1 of the one of these groups with No. 2 of the other, we get the following two Perfect Pentachords :—

- (1) *Sa Ra Ga Ma Pa*; (2) *Sa Ra Go Ma Pa*
 9 8 5 9 9 5 8 9

No example of the use of these pentachordal scales has been found out in the Vedic Chants.

Elementary and Perfect Hexachords

If either the Major Sixth *Da* or the Minor Sixth *Do* be added to the Elementary Ascending Pentachords, we get the following four Elementary Ascending Hexachords :—

- (1) *Sa — Ga Ma Pa Da* ; (2) *Sa — Go Ma Pa Da* ;
 17 5 9 8 14 8 9 8
- (3) *Sa — Ga Ma Pa Do* ; (4) *Sa — Go Ma Pa Do*
 17 5 9 5 14 8 9 5

We may, similarly, have four Elementary Descending Hexachords :—

- (1) *Da — Ma Ga Ra Sa* ; (2) *Da — Ma Ga Ra Sa* ;
 17 5 9 8 14 8 9 8
- (3) *Do — Mo Go Ro Sa* ; (4) *Do — Ma Go Ro Sa*
 17 5 9 5 14 8 9 5

Now the Elementary Ascending Hexachord No. 1 has four notes in common with the Elementary Descending Hexachord No. 1 viz. : *So, Ga, Ma* and *Da*. So, if we combine the two, the note *Pa* of the former will fill up the gap of the latter, and the note *Ra* of the latter will fill up the gap of the former. We shall thus have a Perfect Hexachordal Scale capable of being used both in ascent and in descent.

Similarly, the Elementary Ascending Hexachord No. 4 and the Elementary Descending Hexachord No. 4, which have four notes in common, can be combined to form another Perfect Hexachords Scale.

The other Hexachords cannot be so combined for want of common notes. We thus get the following two Perfect Hexachords :—

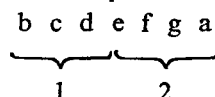
- (1) *Sa Ra Ga Ma Pa Da* ; (2) *Sa Ro Go Ma Pa Do*
 8 9 5 9 8 5 9 8 9 5

The first of these hexachords, known in Europe as the Hexachord of Guido of Arezzo, was the normal scale for singers of that continent throughout the middle ages.

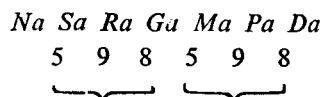
The Greek Heptachord

We have now reached the limit in the size of Perfect Scales of unitary character. As the seventh is dissonant to the starting note, we

cannot get it directly from that note. A perfect heptachordal scale of unitary character is, therefore, not possible. A heptachordal scale of composite character can, however, be had by combining two tetrachords. The ancient Greek Heptachordal Scale was constructed in this method. The Heptachord was constructed by collocating two similar tetrachords by conjunction, that is to say, by so placing one tetrachord after another that the highest note of the lower one was made the lowest note of the higher one. The structure of the Greek Heptachordal Scale of the diatonic genus may be shown thus :—



The two tetrachords which are shown by braces and marked 1 and 2 are exactly similar in character, the semitone being placed between the first and the second notes and also between the fourth and fifth notes. In order to understand the relationships amongst the notes of the above scale, it may be put in the following form :



In this scale the starting note *Ma* has no Perfect Fifth above it and the last note *Da* no Perfect Fifth below it. Every note of the lower tetrachord, has however, a Perfect Fourth above it. This is a particular (tetrachordal) aspect of our Scale of Origin.

Combination of two Unitary Scales

The Octachord Scale of the Greeks was composed of two disjunct similar tetrachords. It may be shown as follows — *e f g a b c d e*. The gap *a—b* between the tetrachords in the above scale, which has descending in character, is a dissonant interval of a tone, which cannot be easily bridged. The easier and more rational method of constructing the complete scale would be to extend the upper tetrachord by a Fifth below its initial note *e* and then starting the lower tetrachord from that note. The upper tetrachord is thus converted into a descending pentachord. The complete scale should, therefore, be considered as composed not of two similar tetrachords but of a pentachord and a tetrachord. Most of the scales are constructed on this principle. There are, however, a few scales in use which must be considered as composed of two similar tetrachords, the most conspicuous amongst them being the scale used in *Raga Ramakali*.

Composite character of Scales

We have seen above that the scale of Origin may be conceived and as a Heptachord Scale composed of two conjunct similar tetrachords. If it is started from *Ga*, it would also appear as an Octachord Scale composed of two disjunct similar tetrachords. Thus : *Ga Ma Pa Ra Na Sa Ra Ga*. The same scale may also be considered as composed of a pentachord and a tetrachord. Thus : *Sa Ra Ga Ma Pa Da Ma Sa*

9 8 5 9 8 9 5

The portion of the Scale from *Sa* to *Pa* is Perfect Pentachord No. 1 and the portion from *Pa* to *Sa* is Perfect Tetrachord No. 1.

A complete scale must, therefore, be considered not as a single unitary scale, but as a composite scale made up of a pentachordal and a tetrachordal unitary scale. Some purposes it may also be considered as a combination of two tetrachords, either conjunct or disjunct.

Bi-centric character of Scales.

Every Unitary Scale is, as shown before, composed of two scales of elementary character, one of which is ascending, and the other descending. An Elementary Scale is constructed by starting from a certain note and placing either above or below it two or more notes, all of which are consonant to it. These notes may be dissonant to each other, but they are bound together by the common bond of the starting note, which is consonant to all of them. The starting note is, therefore the centre round which the other notes cluster and by which they are together. The concluding note of an Ascending Elementary Scale can be made the starting note of a Descending Elementary Scale and the concluding note of the latter can be made the starting note of the former. A Perfect Unitary Scale, which can be used both in ascent and in descent has therefore one centre at the time of ascent and another at the time of descent. It must, accordingly, be considered as bi-centric in its character, one of the centres being placed at each extremity.

Now, as the first five notes of the scale of Origin constitute a pentachord and the last four a tetrachord, and as a note is identical with its octave, the lower centre (*Sa*) of the pentachord is the same as the upper centre (*Sa*¹) of the tetrachord and the upper centre (*Pa*) of the former is the lower centre of the latter. So, the same two notes which are the centres of the pentachord are also the centres of the tetrachord. The full scale is therefore also bi-centric in character.

The two centres are similar in function : In modern European music that centre of a scale which stands fifth below or fourth above the other is considered as the “key-note” or the “tonic”. Let us therefore examine whether there is any superiority in the position of this centre over the other. If we analyse the scale of Origin, which is identical with the European Major Scale, we find the following relation of *Sa* and *Pa* :

Firstly, they are perfectly consonant to each other.

Secondly, there is only one note in the scale,

viz : *Ga*, which is related to both of them.

Thirdly, each is related to two notes which are unrelated to the other. *Ma* and *Da* are related to *Sa*, but unrelated to *Pa* *Ra* and *Ma* are related to *Pa*, but unrelated to *Sa*.

The relations of the two centres may be shown thus :—

Notes related to *Sa* — *Ga Ma Pa Da*
17 22 31 39

Notes related to *Pa* — *Na Sa¹ Ra¹ Ga¹*
17 22 31 39

The figures given below a note indicates the number of nonatones separating it from the centre concerned. It will be observed that not only the number but also the kind of the relationships of both the centres are quite identical. These relationships are : Major Third, Fourth, Fifth and Sixth. The two notes unrelated to each centre are those situated next and below it in the scale. It follows from these facts regarding the two centres that so far as the notes related to one of them are concerned it stands on the same footing as the other and may serve the same functional purpose. Each of them may, therefore, be taken as the starting note or used as the final.

The two centres which have been shown to be similar in every respect should, therefore, have a common name. In ancient India they were called *Amsas*. We shall call them both “Tonic” in English. Taking the original form of the scale of Origin we shall call its initial note the Lower Tonic, in Sanskrit—*Adhara Amsa* and fifth note the Upper Tonic, in Sanskrit—*Uttara Amsa*. These names will be used irrespective of the actual positions which the notes occupy in the plagal or other forms of the scale. They are both to be considered as the “key-notes” of the scale.

Continuously Consonant Character of Scales

In its third aspect, a scale is a chain of consonant notes. This feature follows naturally from the fundamental principle that a scale must be based on the relationship of consonance. The bi-centric character of a scale, shown above also points to this fact ; for all the notes of the scale are consonant to one or other of its two centres, which are themselves consonant to each other. The notable feature of a scale is that it is a continuous chain of consonances of a uniform nature.

Of the six consonant relationships Fourth, Minor Sixth and Major Sixth being the inversions of Fifth, Major Third and Minor Third respectively, the latter three may be taken as the primary consonant relationships. In order, therefore, to make the scale a chain of uniform structure, all the links in it must be either Fifths or Thirds.

To ascertain whether a scale is a chain of Fifths or not we may consider the following chain of seven notes starting with *Ma*, and placed a Fifth apart from each other :—

$$\begin{array}{ccccccc} Ma_1 & Sa & Pa & Ra^1 & D\hat{a}^1 & G\hat{a}^2 & N\hat{a}^2 \\ 31 & 31 & 31 & 31 & 31 & 31 & 31 \end{array}$$

Let us divide this chain into two parts, bring the three notes of the second part two octaves lower thus making them $D\hat{a}_1$, $G\hat{a}$ and $N\hat{a}$. Placing $D\hat{a}_1$ between Ma_1 and Sa , $G\hat{a}$ between Sa and Pa , and $N\hat{a}$ between Pa and Ra^1 , we get the following series :—

$$\begin{array}{ccccccc} Ma_1 & D\hat{a}_1 & Sa & G\hat{a} & Pa & N\hat{a} & Ra^1 \\ 18 & 13 & 18 & 13 & 18 & 13 & \end{array}$$

Any two consecutive notes of this series are dissonant to each other. None of the relationships of Major Third (17) and Minor Third (14) or the Major and Minor Sixths, which are inversions of the Thirds occur in the scale based on this series. Such a scale would be colourless and insipid and almost useless for the composition of any good melody.

A developed musical faculty cannot do without correct Thirds and Sixths. The above series can be improved by a very simple process. We have only to lower the notes *Da Ga* and *Na* by one nonatone each and thus convert them to *Da*, *Ga* and *Na*. With these changes the above series will appear thus :—

$$\begin{array}{ccccccc} Ma_1 & Da_1 & Sa & Ga & Pa & Na & Ra^1 \\ 17 & 14 & 17 & 14 & 17 & 14 & \end{array}$$

In this series correct Major and Minor Thirds are placed alternately. From the arrangement of the notes it will be clear that this series contains the largest possible number of consonant relationships. It is the basis of one of the best of scales, viz. : the scale of Origin. We have only to place Ma_1 and Da_1 an Octave higher and Ra^1 an Octave lower in order to have that scale. Instead of a continuous chain of fifths, we now have one of Thirds. Continuity of constant relationships essential for a scale is, therefore, maintained through Thirds and not through Fifths.

The Scale of Origin is thus, a chain of consonant Thirds so placed as to form also two chains of Fifths. The three notes Da , Ga and Na of one of these chains are variable and by their variations give rise to various scales. The four notes Ma_1 , Sa , Pa and Ra^1 of the other chain are, however, fixed and unalterable and constitute, as it were, the steel-frame of all scales. This chain of Thirds is very important as it shows at a glance the inter-relationship of the notes of a scale. We shall call it the Scale-Heptad *Gramā*—(*Saptaswarya*).

Break in the Chain

It will be seen from the Scale-Heptad that the Major and Minor Thirds are placed alternately, the lowest being a major and the highest a Minor one. If we now extend this chain upwards by placing a Minor Third above Ra , we get a note which is sharper than Ma by one nonatone and is, therefore, to be called Ma . This note is foreign to the scale. The note Ma which is the third note above Ra is dissonant to it, being separated from it by 13 nonatones. It is thus evident that there is a break in the chain of Thirds in a scale.

This feature of scales plays a very important part in the formation of what are known as transilient scales and also in the structure of the melodic types (*Ragas*) of India.