An Analysis of Gamakams of Carnatic Music Using the Computer

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Indian classical music is characterized by movement between and around notes. In particular, the Carnatic music system uses a variety of such movements with the collective name '*gamakam*', each gamakam having its own name. The term 'gamakam' is defined in the *Sangeeta Ratnakara* as a pleasant shaking of a note (*Sangeeta Ratnakara* 3,87). In present-day Carnatic music, ragams can be distinguished by gamakams even when the basic notes are the same. In his work *Sangeeta Sampradaya Pradarshini* (*c.* 1900), Subbarama Deekshitar has described the different gamakams in detail (some on the basis of techniques used in Veena-playing), assigned symbols to them, and used the symbols in his notations for the lyrics given in the book.

Past Work on Frequency Measurements in Indian Music

Use of electronic or mechanical gadgets to analyse the frequencies of Indian classical music has been attempted earlier (see 'Intonation in Present-day North Indian Classical Music', N.A. Jairazbhoy and A.W. Stone, *Journal of the Indian Musicological Society*, Vol. 7, No. 1, March 1976; and *The Grammar of South Indian [Karnatic] Music*, C.S. Ayyar, 1951, pp. 138–140, which describes the experiments made in this connection and refers to Ayyar's papers published in *Current Science*). The study by Jairazbhoy and Stone covered vocal as well as instrumental music, while C. S. Ayyar's study was based on his own violin play. Both these studies used oscillographs to record the music after it was subjected to filtration to remove the higher harmonics. The record shows a graph of the sound amplitude against time. An estimate of the frequency was obtained by counting the number of periods over a certain time interval. The oscillations were recorded on a photographic plate using suitable techniques along with a reference waveform for the time measurement, and the peaks were observed using microscopes. In a paper by C.S. Ayyar, the range of gamakams on certain notes was studied and depicted graphically ('Microtonal Variations in Frequencies in Karnatic Music – II', *Current Science*, Feb. 1953, 22, 39–40). These studies were significant in the sense that an objective analysis of Indian music was attempted using electronic instruments. (The procedure was apparently laborious and expensive as photographic plates were required.) However, no special attention has been paid in the past to the time durations involved in gamakams, although it is essential to measure them in order to delineate a particular gamakam correctly.

The study by Jairazbhoy and Stone showed some interesting results: even steady notes were not held at constant frequencies. There was a leading part which often overshot the note; subsequently, the note settled at some reasonably steady frequency. From the one example where details are given in the paper (for a vocal music recording), it is seen that the overall variation was between 272 cycles and 264 cycles (a range of 52 cents or about half a semitone), and even within the steady part, the variation was between 271 cycles and 266 cycles (a range of 32 cents). For the purpose of frequency estimation, the authors took the part where the note was reasonably steady. The study by C. S. Ayyar also showed variations of frequency in the steady note *pa* on the violin *pa* string ranging from 609 to 612, i.e., a variation of 8.5 cents. The lower variation in this case is attributable to the fact that it is the open string of the instrument, and therefore even the small variation noticed may be due to changes in bowing pressure. The mean of the entire range was taken as the frequency of the note. In the study by Jairazbhoy and Stone, the note Shuddha Gandhar of raga Yaman-Kalyan was found to be held at different frequencies by different artists. The variation was from 382 cents to 429 cents relative to Shadja. In one case, a variation from 382 cents to 403 cents was noticed in the playing of Sitar by the same artist. (For an explanation of the cent system to refer to relative frequencies, see Appendix D, p. 47.)

Personal Computer for Analysing Frequencies

The advent of the personal computer and sound cards which incorporate Analog Digital Converters has made it very easy to record music in digital form and analyse it. (See Appendix B, p. 45, for a note on digital sound files.) The present article describes my attempts to analyse some gamakams of ragam Mayamalavagowla of Carnatic music using an ordinary PC with a sound card, recording the music from CDs or directly with a microphone to the PC (to eliminate possible distortions due to defects in the tape-transport mechanism of a tape recorder). The frequencies were analysed using different techniques for which a computer program was written. The study covered both the frequency aspect and the time durations of the movements.

Pitch, Frequency, and Frequency Estimation

Pitch is the musical sensation felt by the listener, while frequency is a measurable physical quantity. Although it is generally assumed that pitch and frequency are identical, enough evidence exists to show that they are not so. (For information on this subject, see Appendix C, p. 46.) The pitch perception of notes varying in frequency, as notes in gamakam do, is a new dimension of the subject. In the 'Conclusions' to this article, a possible connection between frequencies and pitch perception for notes held with gamakam has been suggested.

Many techniques exist for measuring the frequencies of tones including those containing harmonics. These are described in Appendix A, (p. 43) which also explains how the results in this study were arrived at.

Methodology Used

The music was stored as a PCM wave file in the computer either by capturing it from an audio CD directly or by recording it using a microphone. A sampling rate of 11025 Hz was used, which can faithfully reproduce frequencies up to 5512 Hz, which is adequate for the music recorded and analysed. A computer program was specially written for the purpose of the present study. The program can generate graphs of frequencies against time for the fundamental and the harmonics, and can create log files of frequencies apart from enabling estimation of frequencies by three different techniques: Fast Fourier Transform (FFT), Average Magnitude Difference, and manual measurement (see Appendix A). In practical analysis, when there was any doubt, the values obtained by FFT were checked with the values returned by manual measurement, and the latter values were taken to be the more accurate. However, this technique is laborious, and is useful only for the fundamental frequency. For drawing graphs of frequencies over a time period, only the FFT technique was of practical use because of its speed and facility for automatic generation of a graph to depict the movement of the melody. The graphs and log files produced using the FFT technique were used to locate the peaks or valleys in the soundscape, which were subjected to manual analysis. For estimating the frequency of the tonic (Adhara Shadjam), the mean of a range of values of a steady note in the log file was used.

Commercial CDs have accompaniments which can interfere with analysis. Therefore, only those parts of the music were chosen for analysis where the accompaniment was minimum in volume and the main melody was prominent. In the microphone recordings, there were no accompaniments.

Ragam Mayamalavagowla

The ragam Mayamalavagowla (in which a beginner in Carnatic music learns his or her lessons) uses the notes (in Carnatic terminology) Shadjam (C), Suddha Rishabham (D flat), Anthara Gandharam (E), Suddha Madhyamam (F), Panchamam (G), Suddha Dhaivatam (A flat), Kakali Nishadam (B). Gamakam is used in the four notes *ri*, *ga*, *dha* and *ni*, and occasionally in the note *ma*. In general practice, the notes *ri* and *dha* are oscillated from around the lower notes *sa* and *pa* respectively. In the ascent, while singing *ga*, the note *ma* is reached by an upward movement from below (this gamakam is called '*thirupam*' or '*nokku*'), and then *ga* is held with a '*kampitham*' (simple shake). The note *ni* is held much as *ga*, but is usually anchored on the upper note *sa*. In the descent, the same gamakams are applied, but in reverse order. The purpose of this study is to analyse the manner in which these notes are sung by different artists, and to identify some common parameters such as frequencies involved in the movement, total range of the movement, durations of upward and downward movements, and the total duration of a gamakam compared with the total duration of the note itself.

The music selected for analysis was from (i) a commercial audio CD by D. K. Pattammal with the Muthuswamy Deekshithar Krithi '*Srinathadi*' in raga Mayamalavagowla (referred to here as Voice 1); (ii) the notes *sa ri ga ma pa dha ni sa* sung by Vijayalakshmi Rajaram, an accomplished vocalist (Voice 2); (iii) the same notes sung (Voice 3); and then (iv) played on violin by Madurai S. Balasubramanian, a leading accompanist (referred to as Violin). Two more voices have also been analysed, but for certain notes only. These belong to Sumithra Vasudev (Krithi '*Vidhulaku*' from a Thyagaraja CD-ROM), here referred to as Voice 2a, and Neyveli Santhanagopalan (Krithi '*Mayatheetha*' by Ponnaiah Pillai), referred to here as Voice 2b, from a commercial audio CD.

When a note which sounded steady to the ear showed variations in frequency in the analysis, the weighted mean was taken as the basic pitch of the note (as was done by C. S. Ayyar). The figures were cross-checked using the two other techniques. No significance was attached to change in amplitude — taking amplitude into account, one can attempt a separate study. It was found that notes with gamakams could also be characterized by the mean fequency of the entire note (including long rests at the lower or upper end), or by the mean frequency in the regions of upward and downward movement. In my experience in generating synthetic Carnatic music with a computer ('Synthesizing Carnatic Music with a Computer', *Sangeet Natak* Nos. 133–134, 1999, pp. 16–24), I had found that the mean does carry a sense of the pitch of the whole note.

The results of the analysis are tabulated in the four tables annexed to this article. Instead of showing the actual frequencies, the relative frequencies with reference to the Shadjam of the artist are shown in cyclic cents (Appendix D), which makes it easy to compare the extent of movements. The parameters shown are: (1) the number of oscillations (upward and downward movements of pitch) in the note; (2) the mean relative frequency of the entire note; (3) the maximum range of movements in the gamakam; (4) the relative frequencies of the maximum (peak) points in the oscillation of the gamakams *ri*, *ga*, and *da*, and the minimum (valley) point in the oscillation of the gamakam of *ni*; (5) the mean relative frequency of the gamakam parts, excluding the steady parts; (6) the percentage of time spent in oscillation; (7) the average duration of upward movements; and (8) the average duration of downward movements. In the case of *ni*, where the note is anchored to the higher note *sa* and comes down to the lower point quickly and then goes up again, the minimum is significant (except in the case of the violin sample).

Gamakam on the (Carnatic) Note Suddha Rishabham

The commonly accepted frequency value for (Carnatic) Suddha Rishabham is 16/15 (112 cents), or a comma less than 256/243 (90 cents). The most surprising result of the study is that the peaks touched in the gamakams of the vocalists are consistently higher than the theoretical figures. In the case of two artists, they are almost a major tone. However, in the case of the violinist, the peaks are around 95 cents, i.e., close to the theoretical figure. In the vocal singing of the violinist, the peak frequencies are much lower than the other vocalists, and only in one case does it exceed the theoretical value of 112 cents (r.f. 16/15). However, when we look at the mean of the entire note, the relative frequencies are well below the semitone values. In one case, it is as low as 11 cents, mainly because the artist remains at the lower limit (close to *sa*) for long periods. The measurement of the means of the oscillating parts alone (i.e., from the point where the pitch starts rising, and up to the point it comes down to the steady or lowest value) show lower variations among the artists, the figure being between 60 and 90 cents, except in the case of Voice 1 and Voice 2a, where the figure is higher.

As the gamakams were anchored at the lower end on *sa*, the range of movements is also roughly equal to the maximum relative frequency of the highest peak. In some cases, the lower end of the movement is slightly higher than *sa*, while in others the voice reaches even below *sa*.

As regards durations, no common point can be noticed except that the duration of the upward movement is generally less than the duration of the downward movement. Also, no common point can be noticed as regards percentage of time spent on the gamakam, except that the shorter the duration of the note, the larger is the percentage.

From the listener's point of view, the gamakams whose peaks are far higher than their theoretical values did not give a sense of a higher pitch. Instead, the vocal Rishabhams gave a sense of greater '*ghana*' compared to the Rishabham of the violinist, which remained mostly within the theoretical frequencies and was felt as subdued or 'soft'.

The figures of 16/15 or 256/243 usually ascribed to these notes do not appear to have any significance when the note is held entirely as a gamakam, and is not prolonged. Only in the case of the violin play are the peaks of the gamakams close to these values. This may be explained by the fact that the production of a note on the violin is by a physical movement of the fingers, and while playing gamakam the artist instinctively moves the fingers to the point where he plays the note when held without gamakam.

Gamakam on (Carnatic) Anthara Gandharam

The relative frequency values commonly assigned to this note are 5/4 (386 cents) and sometimes 81/64 (408 cents, a comma higher)

Unlike in the other three notes — except in the case of the violin — the note was not anchored on either end but oscillated uniformly. In the case of the violin, it was anchored around Gandharam and moved in short bursts upward. However, in the case of one vocal artist, the lower point was below Anthara Gandharam in the descent (where the note ended at *ga*), and in the case of another vocal artist, in the ascent. In the case of one vocal artist, the peaks (up to even 596 cents) were consistently higher than the Suddha Madhyamam (r.f. 498 cents), but in the case of the other artists, they were close to the Madhyamam value. As in the case of Suddha Rishabham, the range of oscillations was much more than a semitone in the case of the vocalists, while it was only 80 cents in the case of the violin. The voice recording of the violinist showed a tendency towards a larger range of oscillations (114 cents), but less than that of regular vocalists. An interesting feature is that in the case of Voice 1, the note went down to an r.f. as low as 274 cents (below Carnatic Sadharana Gandharam or Hindustani Komal Gandhar), but the overall feeling was one of the correct note held with Kampitha Gamakam — presumably because the mean relative frequency of the entire note was 435 cents and the means of the individual oscillations were around 450 cents. The means of individual oscillations showed the greatest consistency — around 450 cents — except in the case of the violin in descent, where it was around 410 cents, and the voice of the violinist in descent (420 cents). Thus (except in these two cases), the mean itself was between Anthara Gandharam and Suddha Madhyamam, generating a feeling of oscillation between these two notes, although the actual limits of frequencies touched were quite different.

Gamakam on (Carnatic) Suddha Dhaivatham

The commonly ascribed frequencies for dhaivatam are 8/5 (814 cents) and 128/81 (792 cents). The results of the analysis for gamakam on this note are similiar to those for Rishabham, to which it is a *samvadi svara*. However, in the case of Voice 1, the note-duration was only half of that for *ri*, and there was only one full oscillation followed by a second incomplete oscillation which led to *ni* without fully coming down to *pa*. The range of movement was 279 cents if only the first oscillation is considered, and 350 cents if the second oscillation is also taken into account. The range was 150 cents (much more than a semitone) for the other vocalist, while the violin remained within the semitone range. The peaks reached by the vocalists were well above the theoretical values of 814 or 792 cents, while the peak of the violin was within this range; the voice of the violinist alone rose above these values. However, the means of relative frequencies within the oscillations fell around 770 cents (except in the case of Voice 1, where the mean was much higher). These are comparable to the figures for Suddha Rishabham if we add 702 cents (the r.f. of *pa*). No common point could be observed as regards durations except in the case of the violin and the vocal tracks of the violinist, which showed uniformly that the upward movement time was much less than the downward movement time. The percentage of time spent in oscillations on the gamakams themselves was around 60; the shorter the note, the larger was the percentage.

Gamakam on Kakali Nishadam

This note is usually held anchored on the upper *sa*, and hence more emphasis was given to measuring the minimum points in the oscillations, which are shown in the table. (It was, however, found that the violinist played this note in the ascent anchoring at the lower end and touching the upper peaks quickly, while in the descent the note was anchored on the upper note.) The theoretical values assigned to the note are 15/8 (1088 cents) and 243/128 (1110 cents). Unlike in the cases of *ri* and *dha*, the movements did not cross the theoretical values significantly except in the case of Voice 1, where it touched 826 cents. (This may be partly attributable to the fact that there was a consonant in the lyric at this point, and the note was held only for half an *aksharam*.) Most of the minima fell within the two theoretical ranges, and the means of each of the oscillations were also around 1150 cents, i.e., 50 cents below the upper *sa* (comparable to 50 cents above *ga* for Anthara Gandharam). As before, no common feature could be found about durations except that in most cases the descent was slower than the ascent.

A Short Analysis of Ri and Ga of Saveri

Although this study covers only Mayamalavagowla, a quick analysis of *ri* and *ga* of the ragam Saveri (a *janyam* of Mayamalavagowla omitting *ga* and *dha* in the ascent) was made. The voice of R. K. Srikantan singing the Thyagaraja Krithi '*Ramabana*' was taken from a CD-ROM on Thyagaraja.

The Rishabham of Saveri is generally said to be very low in pitch — it is almost at *sa*. In one sample of *ri* in Madhya Sthayi which was analysed, there were two oscillations, the peaks of which touched 131 cents and 181 cents while the means were 80 cents and 93 cents, which are less than those of Mayamalavagowla for the Voice 1 and Voice 2a examples, but not significantly less than the others; actually, they are more than those of the violin. In the same song, the gamakam on *ri* in Tara Sthayi, with three oscillations, showed peaks at 125 cents, 60 cents, and 106 cents (pitches reduced to middle octave), and the means of the three oscillations were 67 cents, 54 cents, and 64 cents. Though the peaks showed a tendency towards a lower pitch than the results for Mayamalavagowla, the means are lower than the means of only Voice 1 and Voice 2a, and are comparable with the others.

The note *ga* of Saveri is held with extensive gamakam, and is somewhat lower in pitch than that of Mayamalavagowla. The oscillation of this note at the start of the Krithi was from 530 cents, coming down to 322 cents and then rising to 464 cents. The bottom point of the oscillation, i.e., 322 cents, was considerably lower than that of Anthara Gandharam (r.f. 5/4 or 386 cents), while the upper end was near Suddha Madhyamam (498 cents). This may be compared with the gamakam on *ga* in Mayamalavagowla, where the lower end of the oscillation was nearly 380 cents or higher in most cases (except in Voice 1). The range of oscillation, i.e., 208, was also higher than the range in Mayamalavagowla (except for Voice 1). It is thus seen that the *ga* of Saveri is oscillated over a wider range, and is also held at a lower position than that of Mayamalavagowla. The lower pitch is confirmed by the fact that the mean of the entire gamakam was only 378 cents in the Saveri *ga*, while it was around 440 cents in the case of Mayamalavagowla. In fact, some works prescribe Sadharana Gandharam for this ragam. No doubt more samples require to be analysed, especially gamakams in the phrase '*ri ga ri*'.

Conclusions

The study shows that where a note is held by a vocalist with extensive gamakam, and where the music does not linger at one of the ends for any significant time, the actual relative frequencies at that end can vary widely from artist to artist (and even in the case of performances by the same artist in different places). Also, the values obtained in this experiment were higher (for *ri* and *dha*) or lower (for *ni*) than the values usually ascribed to the notes. The ranges of movements of the gamakams too were more than a semitone, and varied from artist to artist. Greater consistency was noticed in the mean relative frequencies for the gamakam parts alone. These were quite close to each other and about 70 cents above the lower note.

This surprising result (that the voice reaches much beyond the theoretical frequencies) has also been confirmed by my analysis of more samples of (Carnatic) Suddha Rishabham and Suddha Dhaivatham sung by other artists (but not documented in this article).

However, where the music stayed for a considerable period at one end of a gamakam, the relative frequencies of the lower end were close to the frequencies of the lower notes *sa* and *pa* respectively in the case of *ri* and *dha*, and the higher note *sa* in the case of *ni*. (The note *ga*, however, did not anchor at either end in most cases.)

Although the end-points of the gamakams were higher or lower than the theoretical values, the listener does not feel they are so. One possible explanation is that the music went beyond the theoretical values only for periods of less than 60 milliseconds. Some empirical studies by me have shown that there is a threshold of duration for proper perception of pitches of separate notes in rapid succession. While this can vary from person to person, the threshold appears to be around 120 milliseconds. Pitch movements in a continuous tone are perceptible where the durations are lower. This also requires further study; the results could vary from person to person and also according to the system of music in which the listener is trained.

The wide variations found in the ranges and peaks of the gamakams indicate that there is considerable flexibility in rendering them, and these perhaps also imply different styles or schools. *The greater uniformity found in the mean values indicates that the overall perception of a note held with gamakam is perhaps based on the mean value of the frequencies in the oscillating part.* This adds one more dimension to the general study of pitch — frequency relationship. Some work has been done on the perception of vibrato, but vibrato and the gamakam of Carnatic music are quite different. The former is faster, covers a smaller range, and consists of more or less uniformly spaced up-and-down movements of the pitch. The vibrato is designed to convey only a sense of the pitch of a steady note, while the gamakam is specifically meant to convey an oscillation in the pitch. The timings of the two movements also vary widely. Studies on vibrato seem to indicate that the central frequency conveys a sense of the pitch (see Appendix C).

As regards durations, it was found that the total duration of the notes (not shown in the tables) had an influence on the number of oscillations, especially when the notes were sung as solfa and not as part of a lyric. Notes of a duration of about 1 second had 2 oscillations, those of 1.5 seconds had 3 oscillations, those of 2 seconds had 4 oscillations, and so on. This factor is well known, and when practising lessons in higher tempos, the standard method is not to fully speed up the gamakams, but to reduce the number of oscillations.

As regards the durations of upward and downward movements, in most cases upward movements were found to be faster than downward movements, but there were many exceptions to this. The upward and downward movement durations in this experiment varied from 50 ms to 200 ms depending on the tempo of the music itself.

A very significant finding is that the violin stuck closely to the theoretical frequencies for the upper limits of the gamakams, and, interestingly, the violinist's voice also showed a smaller range (but still above the theoretical values). The explanation could be that the production of a note on the violin being based on a physical movement of the fingers, the artist instinctively moves only up to the point where he would be playing the note if it was held steady.

Future Studies

Future studies of this kind could cover more complex gamakams. They could also analyse the same lyric sung by different artists belonging to different schools, and played on different instruments. The constraint placed on the gamakam by a lyric set to rhythm, as opposed to free singing as in an *alapana* or *svarams*, could also be studied. Studies too are required in musical perception — for Indian music in general, and Carnatic music, which is more phrase-oriented, in particular.

Tables, graphs, and appendices follow.

Explanation of Tables

In the four tables annexed, all relative frequencies (r.f.) are in cyclic cents (abbreviated as c) relative to the Shadjam (*sa*). (See Appendix D for an explanation of the cent system.) All durations are in milliseconds (abbreviated as ms). The letters A for ascent and D for descent are used to indicate whether the analysis was made for the particular note in the ascent or in the descent; it was not possible to get a suitable point in the music for analysing both ascent and descent in some cases. The mean frequencies have been calculated from the log file created by the program using FFT at predetermined intervals (varying between 10 to 20 milliseconds depending upon the range chosen for analysis), and the mean of the frequencies was calculated using a spreadsheet program. The means of the gamakam parts were similarly calculated from the log file. For actual r.f. values such as the maximum of a peak or the range of oscillation, the values of the log file have been further checked using the average magnitude difference technique and the manual measurement technique.

Table 1
Gamakam on Suddha Rishabham
(Hindustani Komal Rishabh – D flat)

Artist	*Number of oscillations*	*Mean r.f. of whole note*	*Maximum range of movement (r.f.)*	*Maximum r.f. of each oscillation*	*Mean r.f. within the oscillation*	*Percentage of time in gamakam*	*Average time of upward movement*	*Average time of downward movement*
Voice 1	A 2	64 c	228 c	178 c	87 c	36%	78 ms	94 ms
				228 c	114 c			
Voice 2	A 2	11 c	157 c	122 c	53 c	29%	87 ms	110 ms
				148 c	62 c			
	D 3	44 c	165 c	155 c	71 c	24%	59 ms	50 ms
				155 c	88 c			
				155 c	97 c			
Voice 2 a	A 2	76 c	322 c	217 c	100 c	67%	75 ms	150 ms
				286 c	127 c			
	D 2	79 c	198 c	136 c	61 c	30%	66 ms	54 ms
				54 c	69 c			
Voice 2 b	A 2	39 c	193 c	159 c	79 c	59%	100 ms	124 ms
				199 c	117 c			
Violin	A 6	32 c	95 c	85 c	32 c	54%	116 ms	245 ms
				90 c	32 c			
				95 c	54 c			
				95 c	38 c			
				95 c	43 c			
				95 c	54 c			
	D 5	59 c	80 c	95 c	59 c	70%	163 ms	262 ms
				90 c	48 c			
				90 c	54 c			
				85 c	59 c			
				85 c	54 c			
Voice 3 (violinist's voice)	A 4	13 c*	193 c	90 c	71 c	64%	148 ms	220 ms
				103 c	22 c			
				77 c	47 c			
				152 c	36 c			
	D 4	47 c*	174 c	115 c	39 c	100%	100 ms	140 ms
				133 c	70 c			
				103 c	51 c			
				90 c	50 c			

* The voice went below *sa* in the oscillations.

Note: For Suddha Rishabham, the commonly attributed relative frequencies are 16/15 (112 cents) and 256/243 (90 cents).

Table 2
Gamakam on Anthara Gandharam
(Hindustani Shuddha Gandhar – E)

Artist	*Number of oscillations*	*Mean r.f. of whole note*	*Maximum range of movement (r.f.)*	*Maximum r.f. of each upward oscillation*	*Mean r.f. within the oscillation*	*Percentage of time in gamakam*	*Average time of upward movement*	*Average time of downward movement*
Voice 1 (descent stopping at *ga*)	A 3	411 c	232c	561 c 488 c 596 c	480 c 450 c 502 c	100%	116 ms	127 ms
	D 3	435 c	294 c	532 c 525 c 568 c	435 c 450 c 466 c	80%	97 ms	121 ms
Voice 2	A 3	441 c	176 c	516 c 496 c 509 c	459 c 431 c 438 c	100%	155 ms	143 ms
	D 3	452 c	134 c	494 c 494 c 500 c	434 c 431 c 441 c	100%	103 ms	233 ms
Voice 2 b	A 3	453 c	172 c	538 c 529 c 506 c	462 c 469 c 452 c	94%	92 c	175 ms
Violin	A 6	406 c	113 c	482 c 439 c 469 c 482 c 494 c 498 c	410 c 423 c 436 c 440 c 444 c 452 c	50%	96 ms	155 ms
	D 5	406 c	80 c	486 c 465 c 452 c 457 c 457 c	411 c 409 c 407 c 408 c 408 c	50%	80 ms	220 ms
Voice 3	A 4	420 c	114 c	441 c 471 c 471 c 451 c	420 c 420 c 420 c 420 c	100%	112 ms	197 ms

Observations

1. Voice 1 troughs in ascent 380 c, 364 c: in descent 291 c, 274 c, 348 c.
2. Voice 2 troughs in ascent 339 c, 339 c, 376 c: in descent 366 c, 403 c, 410 c.
3. Violin troughs in ascent 384 c, 388 c,410 c, 410 c, 419 c, 410 c: in descent 384 c, 401 c, 397 c, 397 c, 410 c.
4. Voice 3 troughs 378 c, 368 c, 389 c, 357 c.

Note: For Anthara Gandharam, the relative frequencies usually attributed are 5/4 (386 cents) and 81/64 (408 cents).

Table 3
Gamakam on Suddha Dhaivatham
(Hindustani Komal Dhaivat – A flat)

Artist	*Number of oscillations*	*Mean r.f. of whole note*	*Maximum range of movement (r.f.)*	*Maximum r.f. of each upward oscillation*	*Mean r.f. within the oscillation*	*Percentage of time in gamakam*	*Average time of upward movement*	*Average time of downward movement*
Voice 1	A 2	826 c[$]	350 c	984 c 1055 c[@]	826 c 922 c[@]	90%	112 ms	74 ms
Voice 2	A 2	730 c	168 c*	855 c 873 c	753 c 770 c	30%	80 ms	82 ms
	D 3	764 c	146 c*	851 c 839 c 834 c	760 c 748 c 771	50%	110 ms	90 ms
Voice 2 a	A 2	800 c	285 c	881 c 964 c[@]	771 c 821 c	100%	88 ms	108 ms
Voice 2 b	A 2	723 c	242 c	887 c 924 c	780 c 787 c	70%	58 ms	63 ms
Violin	A 4	723 c	101 c	807 c 787 c 770 c 774 c	756 c 759 c 752 c 756 c	42%	100 ms	230 ms
	D 4	720 c	88 c	794 c 787 c 780 c 780 c	742 c 749 c 737 c 741 c	58%	118 ms	320 ms
Voice 3	A 4	800 c	135 c	800 c 809 c 841 c 850 c	750 c 758 c 767 c 775 c	62%	115 ms	155 ms
	D 3	792 c	150 c	866 c 850 c 858 c	800 c 783 c 783 c	63%	120 ms	155 c

Observations

1. Voices 1 and 2 a: @, the second oscillation, leads to *ni* without coming down fully to *pa*, which may explain the higher value in each case.
2. The range at $ would be 279 c if the peak of the first oscillation alone was considered; the steady part between the peaks is 705 cents (almost Panchamam).
3. Voice 2: The trough between notes in ascent and descent is 724 c (22 cents above Panchamam): the * range would be 149 c and 127 c if the trough between the two oscillations was taken as the lowest point instead of the steady lead before the peaks.
4. Violin: The trough in ascent is 706 to 709 c.
5. Voice 3: The trough in ascent is 715 c.

Note: For Dhaivatham, the relative frequencies usually attributed are 8/5 (814 cents) and 128/81 (792 cents).

Table 4
Gamakam on Kakali Nishadam
(Hindustani Shuddha Nishad – B)

Artist	*Number of oscillations*	*Mean r.f. of whole note*	*Maximum range of movement (r.f.)*	*Minimum r.f. in each oscillation*	*Mean r.f. within the oscillation*	*Percentage of time in gamakam*	*Average time of upward movement*	*Average time of downward movement*
Voice 1	A 1*	1047 c	403 c	826 c	992 c	72%	140 ms	120 ms
	D 1**	1175 c	138 c	1082 c	1155 c	77%	110 ms	60 ms
Voice 2	A 3	1155 c	112 c	1092 c 1112 c 1117 c	1131 c 1154 c 1154 c	100%	118 c	200 ms
	D 3	1161 c	169 c	1083 c 1093 c 1073 c	1163 c 1140 c 1145 c	100%	115 ms	180 c
Voice 2 b	D 1	1161 c	250 c	1116 c	1148 c	50%	85 ms	75 ms
Violin•	A 5	1135 c	76 c	1102 c 1125 c 1125 c 1128 c 1125 c	1145 c 1162 c 1149 c 1154 c 1157 c	30%	110 ms	105 ms
	D 5	1164 c	101 c	1102 c 1128 c 1133 c 1125 c 1128 c	1157 c 1160 c 1161 c 1164 c 1153 c	60%	168 ms	156 ms
Voice 3	A 4	1187 c 1180 c$	122 c	1112 c 1098 c 1126 c 1126 c	1200 c# 1173 c 1187 c 1180 c	100%	195 ms	108 ms
	D 5	1160 c		1091 c 1112 c 1098 c 1105 c 1119 c	1160 c 1160 c 1160 c 1160 c 1153 c	70% (rest leading steady *sa*)	168 ms	140 ms

* Half note

** Quarter note

• The note was anchored at the lower end; the maximum values of the peaks are 1173 c, 1178 c, 1164 c, 1178 c, and 1175 c.

$ The first oscillation which overshot Tara Shadjam is excluded.

Observation

At #, the mean is equal to Tara Shadjam, possibly because the maximum overshot Tara Shadjam in the lead up.

Note: For Kakali Nishadam, the relative frequencies usually attributed are 15/8 (1088 cents) and 243/128 (1110 cents).

Explanation of Graphs

Only some samples of the graphs obtained in the study are included in the following pages.

Figure 1 shows the frequency graph of the first line of '*Srinathadi*' covering the seven notes in the ascending order.

Figure 2 shows the first line of '*Srinathadi*' (Voice 1) starting from the middle of the note *sa*, showing the whole of *ri* with two oscillations, and the beginning of the note *ga*. Six harmonics are seen; the oscillations are better visible in the higher harmonics. The fundamental of *sa* was 174 cycles per second.

Figure 3 shows the note *ga* in the ascent (Voice 2) while singing the svarams of Mayamalavagowla. There are four oscillations. The vertical resolution of the graph was increased to show greater detail, and only the fundamental is shown. The *sa* frequency was 191.5 cycles per second. More prominence is found at the upper end of the gamakam.

Figure 4 shows the note *dha* played on the violin as part of the ascending sequence. There are six oscillations: the last three come in quick succession, while the first three linger considerably at *pa*. The upward durations are less than the downward durations.

Figure 5 shows the note *ni* sung in the descent of the Mayamalavagowla svarams (Voice 2). In this particular case, the oscillations are equal without prominence at either end of the gamakam.

Figure 6 shows the same note as in Figure 4 sung by the violinist. The first three oscillations are anchored at the upper end of the gamakam, which is the upper *sa*.

Figure 7 is the graph of the descent of ragam Saveri from Tara Shadjam to Madhya Shadjam. The entire phrase consists mostly of movements which do not prolong the notes, showing the extent of gamakam used in Carnatic music in the *rakthi-ragam* Saveri.

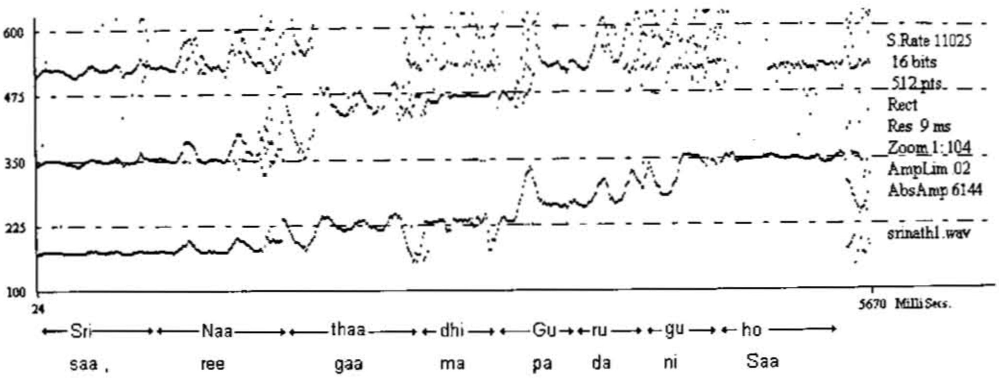

Figure 1. Part of the first line of '*Srinathadi Guruguho*' in Mayamalavagowla showing the extensive gamakam.

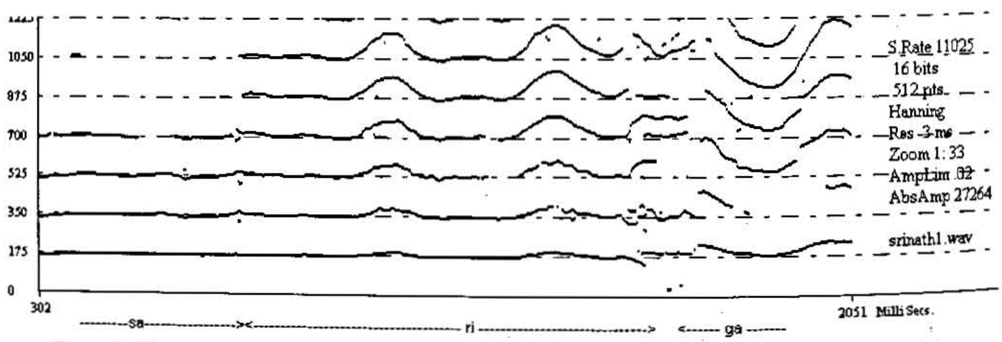

Figure 2. The part '*Srinatha*', covering part of *sa*, the whole of *ri*, and part of *ga* of the first line of the Krithi '*Srinathadi*'. Besides the fundamental, 5 overtones are shown. The X-axis is in milliseconds and the Y-axis shows the frequency. The tonic *sa* frequency is 175 Hz.

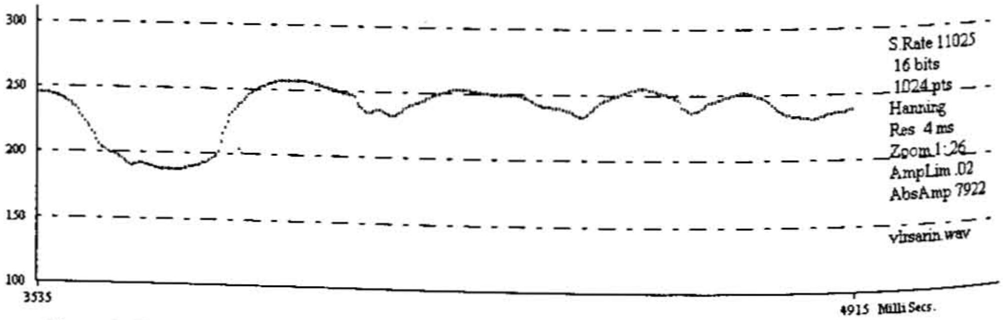

Figure 3. The note *ga* in the Mayamalavagowla svarams (Voice 2) sung in ascent (*sa* frequency 191.5).

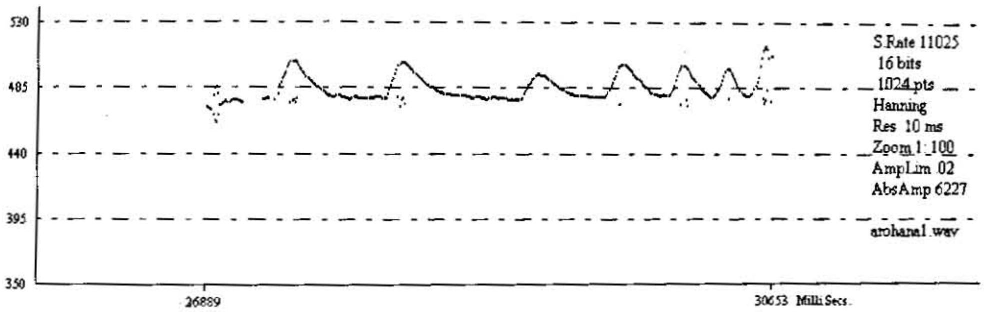

Figure 4. The note *dha* in the Mayamalavagowla svarams played on the violin (in ascent). The tonic *sa* frequency is 318.

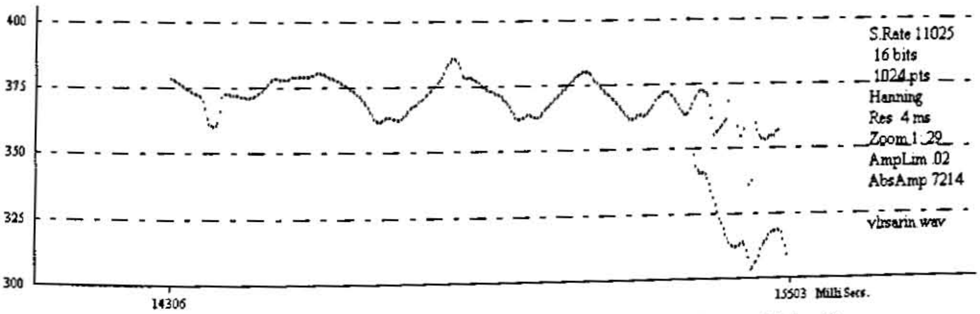

Figure 5. The note *ni* in the Mayamalavagowla svarams sung in descent (Voice 2). The upper *sa* frequency is 392.4 Hz.

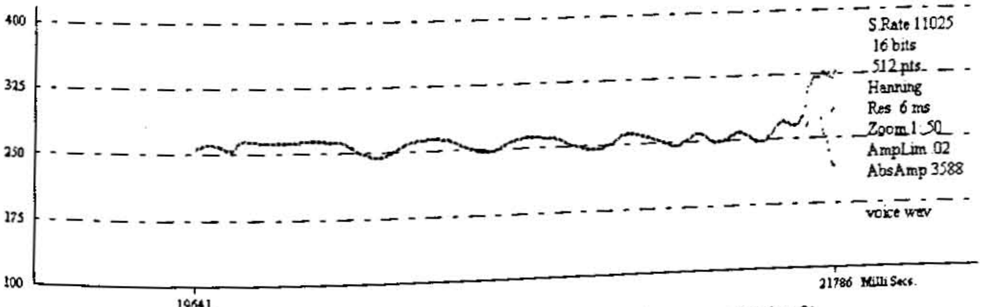

Figure 6. The note *ni* in the descent of Mayamalavagowla svarams (Voice 3). The upper *sa* frequency is 263 Hz.

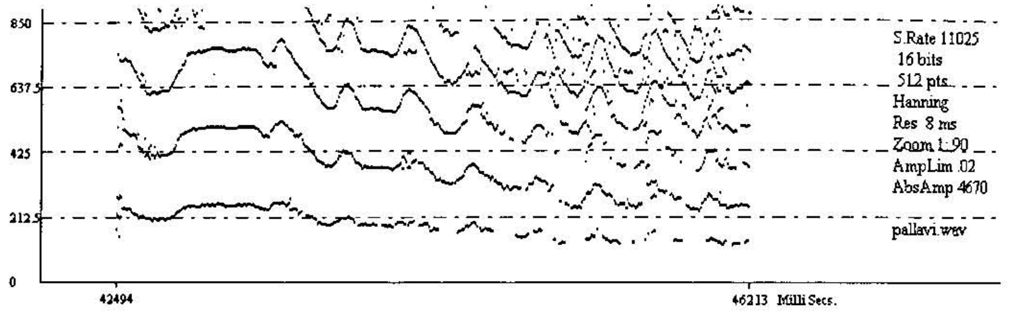

Figure 7. The descent from Tara Sa to Madhya Sa in the Saveri Krithi '*Ramabana*'.
This part is from the Pallavi lyric '[*mana*]*sa*'.