APPENDIX D Cyclic Cent System for Relative Frequencies

In music, the relation between any two notes (the interval) is their ratio, and not the actual difference in the number of cycles. This makes it difficult to appreciate intervals correctly. For instance, it is difficult to say without further calculation whether 16/15 is higher or lower than 256/243, and the fractions make it difficult to appreciate the pitch difference. The cyclic cent system converts the relationship of ratios into one of arithmetic addition or subtraction by using logarithms.

As the octave is a basic interval and as most musical systems use 12 notes in an octave, the relative frequency ratio of 2 of the octave is equated to 1200 cents. The interval between 2 adjacent notes (a semitone) then becomes 100 cents in the Equally Tempered Scale.

To calculate the cent value of any relative frequency ratio, the following formula is used: cent value = $1200 \times \log (r.f.) / \log (2)$,

where 'r.f.' is the relative frequency ratio.

The cent can be calculated using any scientific calculator which provides for logarithms. Some cent values of common relative frequency ratios are given below:

Relative Frequency	Cent Value
1	0
16/15	111.73
9/8	203.91
6/5	315.64
5/4	386.31
4/3	498.05
3/2	701.96
8/5	813.69
5/3	884.36
16/9	996.09
15/8	1088.27
2	1200

A 1% increase in frequency corresponds to an increase of 17.23 cents.

To obtain a note which is at an interval of 3/2 (sa to pa), we have to add 701.96 in the cent system. For moving up by 4/3 (the sa to ma interval), we have to add 498.05. Thus, a note with r.f. 9/8 (203.91 cents), when increased by 4/3, will give us a note of 203.91 + 498.05 = 701.96 (which is 3/2). The above table shows that the equally tempered fifth note (pa) with a cent value of 700 is very close to the natural pa (701.96 cents). However, the natural r.f. 5/4 (ga) has a cent value of 386.31, which is somewhat lower than the equally tempered third with a cent value of 400.

Acknowledgements

I wish to thank Madurai Sri S. Balasubramanian and Srimathi Vijayalakshmi Rajaram for their cooperation in recording the music. I wish to thank Dr N. Ramanathan, Head of the Department of Music, University of Madras, for reading the article in manuscript and giving me valuable suggestions. — M.S.