MATHEMATICS IN INDIAN UNIVERSITIES

Report of the
University Grants Commission
Review Committee

UNIVERSITY GRANTS COMMISSION
NEW DELHI

Contents

Foreword

The University Grants Commission appointed some time ago "Review Committees" in a number of important subjects in Science, Humanities and Social Sciences. The Mathematics Committee consisted of the following:

1. Dr. R. P. Bambah, Professor of Mathematics,
   Panjab University,
   Chandigarh.

2. Dr. P. L. Bhatnagar, Professor of Mathematics,
   Indian Institute of Science,
   Bangalore.

3. Dr. B. N. Prasad, Professor of Mathematics (Rtd.),
   Allahabad University,
   Allahabad.

4. Dr. Ram Behari, Professor of Mathematics,
   Delhi University,
   Delhi.

5. Dr. C. R. Rao,
   Head of Research and Training Division,
   Indian Statistical Institute,
   Calcutta.

6. Dr. Alladi Ramakrishnan,
   Professor of Physics,
   Madras University,
   Madras.

7. Dr. B. R. Seth, Professor of Mathematics,
   Indian Institute of Technology,
   Kharagpur.

8. Dr. B. D. Laroia, —*Member-Secretary*
Development Officer (Science),
University Grants Commission,
New Delhi.

The Committee visited a number of University Departments and held several meetings to discuss problems relating to education and research in Mathematics.

Mathematics is generally recognised as fundamental to the study of all science subjects. One of our important and urgent needs is to raise the level of teaching in this subject, both at the School and the University level. I have no doubt the Report of the Committee and its recommendations will be of real value and benefit in improving the pattern and content of the courses of study (including the reform of syllabus) in Mathematics and in the promotion of research in the Universities.

The Commission is grateful to members of the Committee and all others who helped in making this review possible. At the invitation of the Commission, Professor Marshall H. Stone, Andrew McLeish Distinguished Professor of Mathematics, University of Chicago assisted the Committee, and I would like to express our indebtedness to him.

D. S. Kothari

Chairman
University Grants Commission

1

Introduction

1.1 In 1959 the University Grants Commission appointed 'Reviewing Committees' in a number of important subjects to broadly survey and assess the standard of teaching and research and the facilities available for the purpose, and to recommend steps to be taken (including modifications of syllabuses) in order to raise the general level of academic attainment and research in the universities. Knowledge these days is expanding at such a rapid pace that a serious and sustained effort is required if teaching is to be kept even reasonably up-to-date. The value and necessity of "Reviewing Committees" is apparent, and, if anything, these are all the more important in the context of our development plans for expansion of higher education and research, specially in the fields of science and technology.

1.2 The Review Committee for Mathematics and Statistics was constituted in August, 1959 with the following members:

1. Dr. B. R. Seth, Professor of Mathematics,
   Indian Institute of Technology,
   Kharagpur.

2. Dr. Ram Behari, Professor of Mathematics,
   Delhi University,
   Delhi.

3. Dr. B. N. Prasad, Professor of Mathematics (Rtd.)
   Allahabad University,
   Allahabad.

4. Dr. P. L. Bhatnagar, Professor of Mathematics,
   Indian Institute of Science,
   Bangalore.

5. Dr. R. P. Bambah, Professor of Mathematics,
Panjab University,
Chandigarh.

6. Dr. C. R. Rao,
Head of Research and Training Division,
Indian Statistical Institute,
Calcutta.

7. Dr. Alladi Ramakrishnan,
Professor of Physics,
Madras University,
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Development Officer (Science),
University Grants Commission,
New Delhi.

1.3 The Committee had wide terms of reference and was free to determine its own programme and procedure of work. The Committee could also visit some of the Indian Universities and other institutions. At its first meeting the Committee laid down the following items for study and investigation:

(1) The present state of development in Mathematics and Statistics.

(2) A qualitative and quantitative appraisal of the existing facilities for teaching and research.

(3) Trends of research, its potentialities and steps to be taken for expansion of training and research facilities in various disciplines of the subject.

(4) Syllabi and examination system at different levels of university education; improvement and modernisation of syllabi, preparation of model syllabi.

(5) Ways and means of co-ordination between institutions including university and non-university centres of teaching and research.

(6) Improvement of facilities for students and teaching personnel.

1.4 The Review Committee on Mathematics and Statistics held seven meetings as listed below:

1st meeting: at Delhi on 3rd October, 1959.

2nd meeting: at the Indian Institute of Technology, Kharagpur on 4th and 5th December, 1959.

3rd meeting: at the University of Madras on 28th and 29th January, 1960.

4th meeting: at the Indian Statistical Institute, Calcutta on the 11th and 12th April, 1960.

5th meeting: at Delhi on the 9th and 10th December, 1960.

6th meeting: at the University of Calcutta on the 1st and 2nd April, 1961.

7th meeting: at the Office of the U.G.C. New Delhi on 3rd June, 1961.

1.5 The Committee reviewed the position as it existed at the end of the Second Plan period with regard to the standards of teaching and research in Mathematics and Statistics in the country. The review covered physical facilities such as accommodation, equipment and library and academic problems, syllabi, examination system and staff. The problems of institutional training and inservice training and betterment of teaching personnel were examined. The Committee visited some universities and other institutions and held meetings and had discussions with the teachers and research workers in Mathematics in those universities.

1.6 In order to collect the necessary data regarding the existing facilities for teaching and research in Mathematics, the Committee prepared a questionnaire (Appendix I) and requested all the universities and colleges to supply the information. The information received from different regions was analysed and considered. (Appendices II, III and IV).

1.7 The authorities of the India Wheat Loan Educational Exchange Programme made available to the Committee the services of Prof. Marshall H. Stone, Andrew McLeish Distinguished Service Professor, Department of Mathematics, University of Chicago, Chicago, U.S.A.

Prof. Stone visited several Indian Universities, had discussions with the teaching and research personnel and gave lectures on advanced topics and mathematical education. He attended the 6th meeting of the Committee held at Calcutta on the 1st and 2nd April, 1961, and made valuable suggestions particularly for the proposed changes in the curriculum and examination system.

1.8 At a later date it was decided that the subject of Statistics may be separately dealt with by another committee, and hence this committee has reported mainly on Mathematics.

2

A Brief Historical Sketch of Mathematical Research in India

Ancient Indian mathematicians gave to the world the decimal place value system of numerical notation, the concept of zero and the foundations of Indeterminate Analysis. For various reasons, however, original work of a high level was not continued in India after the 12th century. Original research along modern lines began to spring up in various parts of the country at the beginning of the present century. In 1907, V. Ramaswami Aiyar and a group of enthusiasts—R. P. Paranjpye, M. T. Narayana Aiyangar, and others founded the Indian Mathematical Society; in 1909 Asutosh Mukherjee founded the Calcutta Mathematical Society and in 1919 Ganesh Prasad, the Banaras Mathematical Society. These societies started three journals, viz., the Journal of the Indian Mathematical Society (which has now been divided into two quarterlies—the Journal of the Indian Mathematical Society and the Mathematics Student), Bulletin of the Calcutta Mathematical Society and Proceedings of Banaras Mathematical Society (which is now called Ganita, the name of the Society having changed to Bharat Ganita Parishad). Recently a number of Indian Universities have started mathematical societies of their own. Some of these are also publishing journals.

The history of modern Indian Mathematics is thus a history of the last six decades. India's contribution during this relatively short period has however been quite appreciable. It was within this period that India produced Srinivasa Ramanujan (1887-1920) who did most outstanding work in the fields of Analytical Theory of Numbers, Theory of Partitions, and Elliptic Modular Functions. His first paper entitled 'Some Properties of Bernoulli's Numbers' was published in the journal of the Indian Mathematical Society in 1911. Unfortunately for India and the world this mathematical prodigy passed away at a very young age of 33.

Among those who organised mathematical research in the early stages, may be mentioned the name of Ashutosh Mukherjee who built an active centre of research in Calcutta with which are associated such names as Ganesh Prasad, S. Mukopadhyaya, F. W. Levy, W. H. Young, S. N. Bose, S. S. Pillai, R. C. Bose and N. R. Sen. At Calcutta, N. R. Sen developed the School of Research in Relativity, Fluid Mechanics and Ballistics, while R. N. Sen developed research in Geometry. In 1917, K. Anandarao started research activities on modern lines in Madras and built up a School in Analysis. Three years later in 1920, R. Vaidyanathaswamy initiated work in Abstract Algebra, Algebraic Geometry and Topology which is followed up by V. S. Krishnan and M. Venkataraman at present. Work on Stochastic Processes has been started under Alladi Ramakrishnan. About the same time K. S. K. Aiengar organised research activities in Mysore University. With this School have been associated B. S. Madhava Rao in Algebra and Quantum Mechanics and C. N. Srinivasa Iyengar in Differential Equations and Differential Geometry. A. Narasinga Rao organised work in Geometry at Annamalai University, where Theory of Functions and Functional Analysis are being pursued under V. Ganapaty Aiyar. S. Minakshi Sundaram and V. Ramaswamy have carried on work on Analysis and Number Theory at Andhra University. About the mid-twenties, research in Mathematics started at the Universities of Allahabad, Banaras and Lucknow. At Allahabad A. C. Banerjee initiated work in Astrophysics, P. L. Srivastava in Theory of Functions of a Complex Variable, and B. N. Prasad in Trigonometric Series and theory and applications of Absolute and Strong Summability. At Banaras, work has been done on Theory of Relativity under V. V. Narlikar, at Lucknow on Non-differentiable Functions and history of Hindu Mathematics under A. N. Singh and on Integral Transforms under R. S. Verma ad S. C. Mitra. At the University of Delhi, a School of research in Differential Geometry has been developed by Ram Behari. The work on Partitions and Statistical Mechanics has been started by F. C. Auluck and D. S. Kothari. At the University of Panjab Number Theory has been studied under Pt. Hemraj, S. Chowla, Hansraj Gupta and R. P. Bambah. At the University of Aligarh S. M. Shah worked on the Theory of Functions of a Complex Variable. At the Tata Institute of Fundamental Research, Bombay an active School of research and training in Analysis, Modern Algebra and Topology has grown up under K. Chandrasekharan and K. G. Ramanathan. At the Indian Institute of Technology, Kharagpur, B. R. Seth has developed a centre for research in Elasticity and Hydrodynamics. At the Indian Institute of Science, Bangalore, work is being done on Fluid mechanics, Hydromagnetics and Astrophysics under P. L. Bhatnagar. The Ramanujan Institute of Mathematics, Madras, started its activities under T. Vijayaraghavan who himself made important contributions to Analysis and Number Theory and work is being continued there on Tauberian Theorems, Number Theory and Summability Theory under C. T. Rajagopal. S. S. Pillai made outstanding contributions to Number Theory, especially Waring's problem.

It is not possible in this brief account of the growth of Indian Mathematics to make reference to several isolated workers. Mention may, however, be made of the workers, D. D. Kosambi of Tata Institute of Fundamental Research, Bombay, in Differential Geometry and Statistics, U. N. Singh of Baroda University in Analysis, P. C. Vaidya of Gujarat University in Relativity, R. Mohanty of Utkal University, Cuttack on Summability of Fourier Series, P. Masani on Theory of Operators, G. Bandyopadhya of Kharagpur on Field Theory and Fluid Mechanics, and Saheb Ram Mandan on Geometry, S. Ghosh and B. B. Sen of Calcutta University on Elasticity and Fluid Mechanics, V. R. Thiruvenkatachar of Defence Science Laboratory, Delhi, in Elasticity, Brij Mohan of Banaras University in self Reciprocal Functions, M. Ray of Agra University in Fluid Dynamics, Ram Ballabh of Lucknow University in Fluid Mechanics, T. P. Srinivasan of Panjab University in Measure Theory, T. Pati of Allahabad University in Absolute Summability, R. P. Aggarwal and S. K. Bose of Lucknow University in Theory of Functions of Complex Variables, R. S. Misra of Gorakhpur University on Differential Geometry, S. N. Barua of Defence Science Organisation in Rotating Fluids, M. R. Parameshvaran of Madurai on Summability, J. A. Siddiqui of Aligarh University in Analysis, M. S. Ramanujan in Hausdorff and quasi-Hausdorff Methods of Summability, J. N. Kapur of Indian Institute of Technology, Kanpur on Ballistics, R. P. Kanwal on Compressible Flows, M. K. Jain on High Speed Computation and S. S. Shrikande of Banaras on Combinatorial Analysis.

Besides these, there are several Indian mathematicians of distinction working abroad. S. Chandrasekhar (Chicago) has made outstanding contributions in many branches including Theory of Turbulence, Hydromagnetics, Integral Equations and Astronomy. Important contributions have also been made by S. Chowla in Number Theory, Harish Chandra in theory of Lie groups and S. K. Abhyankar in Algebraic Geometry.

The above is only an indicative but not a comprehensive list of prominent research workers in the country.

During recent years, Chairs have been created with the financial help of the University Grants Commission in several universities where research activities are fast expanding.

Almost all the universities have at present facilities for instruction in Mathematics at the post-graduate level. These facilities may be (a) in the university departments as in the case of nearly 30 universities, (b) in constituent colleges, (c) in colleges affiliated to universities and (d) in non-university institutions such as, Tata Institute of Fundamental Research, Four All-India Institutes of Technology at Bombay, Madras, Kanpur and Kharagpur, the Indian Institute of Science, Bangalore and the Indian Statistical Institute, Calcutta. The only exceptions with no post-graduate departments of Mathematics are perhaps the following universities: Sanskrit University, Varanasi, S.N.D.T. Women's University, Bombay and Visva-Bharati at Santiniketan. Most of the University departments and institutions of research provide facilities for research leading to Ph.D. Degree. Details regarding these institutions and the subjects of study at the post-graduate level are given in Appendix II.

A large number of colleges affiliated to various universities carry on post-graduate teaching work leading to the Master's degree in Mathematics. A list of all such colleges is given in the Appendix II. Very few of them, however, are engaged in research or provide facilities for research work. A list of Doctoral Theses accepted by various universities during the past five years is given in the Appendix III to indicate the major trends of research and branches in which the students are trained in the various departments.

There also exists amongst a few universities a practice wherein post-graduate degree in mathematics is given as M.A., while most universities award M.Sc. degree. The total enrolment of students in Mathematics at post-graduate level including M.A. and M.Sc. is about 3,000. The universitywise enrolment is given in the Appendix IV.

3

Syllabus

It is common knowledge that curricula and courses in sciences have failed to evolve at a pace commensurate with the rapid growth of scientific knowledge. The Committee has examined the existing syllabi for the under-graduate and post-graduate courses and has subsequently prepared the model syllabi indicating the extent of training desired at each level. The syllabli for B.A./B.Sc., and M.A./M.Sc. courses follow. It is desirable that these syllabi are reviewed and revised once in every five years. The syllabi are meant to indicate the broad outlines and the universities are free to modify the same according to their needs and the facilities available. The Committee is not in favour of drawing inflexible and uniform syllabi for all the universities. The syllabi prepared by the Committee are thus illustrative only.

I. B.A./B.Sc. Courses

1. The purpose of the B.A./BSc. syllabus is to bring the knowledge of the student to the level which the committee considers necessary in order that the student may later on take to post-graduate studies or fulfill adequately his responsibilities if he were to pursue other professions, such as teaching etc. While designing the courses the following objectives have been kept in view:

(i) Training in logical thinking and reasoning.

(ii) Preparing the students for higher studies in Mathematics.

(iii) Equipping the students for the teaching profession.

(iv) Providing the basic training for higher studies in other branches such as physical sciences, social sciences, statistics and technology.

(v) Fulfilling the needs of society by providing men who can ably pursue professions like Insurance, Accountancy and Computational work.

In order to meet the above objectives the courses have been divided into two categories:

(a) Topics which will acquaint the students with basic mathematical methods.

(b) Topics which develop in the students an idea of rigour and lead to mathematical and logical thinking.

In the teaching of (a) above, although rigorous proofs are not always expected, it is nevertheless essential that the student is given clearly to understand what is assumed, what is proved and which portions have been supported by only a plausible reasoning. In category (b) which deals with concepts of Modern Algebra and Analysis a rigorous treatment is expected.

It is desirable to emphasise fundamental principles rather than indulge in complicated and unrealistic problems, which may not illustrate anything in particular. The instruction in the class room should be necessarily supplemented by tutorials in which the students may be assigned related topics which may be discussed by small groups of students.

It is estimated that in order to reasonably cover the proposed syllabus the student should receive instruction over 9 periods a week, each period of 50 minutes duration, all through the 180 working days. In addition he may be given three tutorials a week.

In formulating the syllabus it has been assumed that the student has undergone training in Arithmetic, Statistics, Algebra, Geometry, Mensuration, Co-ordinate Geometry (standard equations of straight lines and circles) and Plane Trigonometry at the Higher Secondary or equivalent stage.

2. Branches of Study:

(a) Algebra. (b) Trigonometry. (c) Geometry.

(d) Calculus. (e) Differential Equations.

(f) Mechanics. (g) Analysis.

and any one of the following:

(1) Numerical Analysis.

(2) Elements of Modern Algebra.

(3) Set Theory and Foundations of the Number System.

(4) (i) Attractions and Potentials, and

(ii) Elements of Elasticity.

(If necessity arises, other additional subjects may be added to this).

3. DETAILED SYLLABUS FOR B.A./B.Sc. COURSES

All books mentioned in the syllabus are only meant to indicate the spirit of the course. They do not define the course. In most cases they are for the guidance of the teachers only.

(a) *Algebra*

Intuitive ideas of the development of the concept of numbers starting from natural numbers and going on to real and complex numbers without any appeal to Dedekind sections.

Determinants and simultaneous linear equations. Matrices. General properties of polynomial equations. Statement of the Fundamental Theorem of Algebra. Relations between the roots and the coefficients. Symmetric functions of the roots. Nature of the roots of the cubic and biquadratic equations. Cardan's solution of the cubic. Resolution of a quartic expression into quadratic factors. Location of roots. Newton's method. Iterative process.

Summation of finite series. Finite differences. Inequalities (Arithmetic and Geometric Means, Cauchy-Schwartz, Hölder and Minkowski).

Prime numbers. Fundamental Theorem of Arithmetic. Congruences. Fermat's Theorem. Wilson's Theorem.

Scalars and vectors. Vector addition. Scalar and Vector Multiplication. Differentiation of vector functions of a scalar variable.

Introduction to the concepts of Groups, Rings, Fields and Vector spaces. (Number of examples to be given and enough practice to be given for deducing simple properties from given axioms).

(b) *Trigonometry*

Inverse circular functions. De Moivre's theorem for rational indices. Trigonometric methods of solving binomial equations. Summation of finite (Trigonometric) series. Exponential functions. Logarithmic functions. Hyperbolic functions and their inverses.

(c) *Geometry*

(i) *Analytic Geometry of two dimensions*: Statement of the fact that the second degree equation is the object of study and that it represents all varieties of conics and pairs of straight lines. Properties of the circle. Coaxial and orthogonal systems of circles. Properties of the parobola, ellipse, hyperbola. Polar equations of conics. Homogeneous Cartesian co-ordinates. Reduction of the second degree equation.

(ii) *Analytical Geometry of three dimensions*: Direction cosines of straight lines. Planes and straight lines. Equations for the sphere, cone, cylinder, ellipsoid, hyperboloids and paraboloids.

(iii) *Pure Geometry*: Cross-ratio of points and lines. Twenty-four cross-ratios reducible to six. Harmonic points and lines. Invariance of cross-ratio in projection and section. Points at infinity. Principle of duality. Complete quadrangle and complete quadrilateral. Projective generation of conics. Inversion. Reciprocation.

(d) *Calculus*

(i) *Differential Calculus*: Real numbers represented as points on a line. Functions. Their domains and ranges. Intuitive idea of limit.

Algebra of limits. Continuity. Formal differentiation of standard functions including functions of a function. Successive differentiation. Leibnitz rule. Tangents and normals. Indeterminate forms. Simple cases of maxima and minima of functions of one variable. Envelopes. Rolle's theorem (geometrical representation). Mean value theorems. Taylor's and Maclaurin's theorem. Partial differentiation. Asymptotes. Singular points. Curvature. Tracing of curves in Cartesian and polar co-ordinates. Maxima and minima for functions of more than one variable. Change of variables.

(ii) *Integral Calculus*: Integration as inverse of differentiation. Indefinite integrals of standard forms. Methods of substitution, integration by parts, partial fractions, reduction formulae. Definite integral as the limit of a sum. Fundamental theorem of the integral calculus (geometrical demonstration). Algebraic properties of definite integrals. Definition and working knowledge of Beta and Gamma functions (no proof of convergence). Rectification, quadrature, volumes and surfaces of solids of revolution. Numerical integration by Simpson's method. Differentiation under the integral sign (only as a method of integration). Formal double and triple Integration; application in determination of centre of gravity, moment of inertia and centre of pressure.

Note: Examples on various applications may be drawn from Mechanics.

(e) *Differential Equations*

Formation of differential equations. Geometrical approach to the existence theorem for the equation $\frac{dy}{dx} = F(x, y)$ (no proof of existence of solutions).

First order linear and non-linear equations by the method of quadratures. Higher order linear equations with constant coefficients. Cauchy-Euler type equations. Exact differential equations. Second order differential equations occurring in Dynamics.

(f) *Mechanics*

In Mechanics, emphasis should be upon the understanding of principles. Vector methods shall be used.

(i) *Statics*: Forces acting at a point. Parallel forces. Moments. Couples. General conditions of equilibrium, analytical method. Astatic equilibrium. Friction. Centre of gravity. Virtual work in two dimensions. Catenary. Elementary treatment of stability.

(ii) *Dynamics*: Velocity vector. Relative velocity. Acceleration. Angular velocity. Degrees of freedom and constraints. Elementary Dimensional Analysis. Rectilinear motion. Simple harmonic motion. Motion in a plane. Projectiles. Constrained motion. Work and energy. Motion under impulsive forces. Kepler's laws. Orbits under central forces. Eccentric anomaly, true anomaly, mean anomaly. Definition of time. Motion of varying mass. Motion under resistance.

(iii) *Hydrostatics*: Pascal's theorem on the pressure at a point. Resultant thrusts. Centre of pressure. Equilibrium of floating bodies. Pressure equation. Atmospheric pressure. Hydraulic and pneumatic machines.

(g) *Analysis*

(Questions to be set only on theory and direct application): Real numbers introduced as points of the geometrical continuum. Their properties as a complete ordered field explicitly stated. Sets. Denumerable and non-denumerable sets. Bounds. Neighbourhoods. Interior points. Limit points. Open and closed sets. Bolzano-Weierstrass theorem. Sequences. Limits of indeterminacy. Limits. Cauchy's general principle of convergence. Monotonic sequences. Limits of functions. Continuity. Uniform continuity. Properties of continuous functions. Derivatives. Rolle's theorem (Rigorous proof.) Mean Value theorems. L'Hopital rule. Taylor's theorem with remainder: Young's form. Application to maxima and minima.

Infinite series and products: Infinite series with positive terms, Basic comparison tests, root tests, ratio test, Kummer test, condensation test (with examples to show the increase in rapidity of the convergence). Alternating series. Series of arbitrary terms. Absolute convergence. Proof that absolutely convergent series can be deranged and multiplied. Examples of conditionally convergent series. Examples to show that the derangement of a conditionally convergent series may affect its nature. Elementary discussion of infinite products. Infinite products for the sine and cosine functions. Sine, cosine, Gregory, exponential and logarithmic series.

(1) *Numerical Analysis*

Fnite differences, interpolation, extrapolation. Numerical differentiation and numerical integration. Solution of difference equations. Solution of ordinary differential equations. Simultaneous linear equations and their solution. Roots of polynomial equations. Solution of simple problems by Relaxation method. Nomograms.

Books for reference:

A. D. Booth: Numerical Methods

Kunz: Numerical Analysis

Barnard and Child: Higher Algebra

Nielson: Numerical Analysis

(2) *Modern Algebra*

Groups. Subgroups. Normal subgroups. Factor groups, Homomorphisms. Isomorphisms. First Isomorphism Theorem. Conjugate elements. Direct sum (finite).

Rings. Integral domains. Fields. Ideals. Quotient rings. Maximal and prime ideals. Unique factorization domains. Principal ideals domains. Euclidean domains. A Euclidean domain is a principal ideal domain. A principal ideal domain is a unique factorization domain. Ring of polynomials. k(x) is a Euclidean domain if k is a field.

Vector spaces. Linear dependence and independence. Basis of a vector space. Dimension of a vector space. Subspaces. Linear mappings. Dual space.

Matrices: Their algebra. Transpose. Adjoint. Inverse. Linear transformations. Equivalence. Similarity. Reduction. Characteristic roots. Cayley-Hamilton Theorem. Minimal equation of a Matrix. Rank. Linear forms. Bilinear forms. Application to geometry.

Note: Main emphasis of this course will be on matrices. After this course a student should be able to use matrices with ease.

Book for reference:

N. Jacobson: Lectures in Abstract Algebra, Volumes I and II.

3. *Set Theory and Real Numbers*:

(i) *Set Theory*: Sets. Symbols $\in$, $\ni$, $\subset$, $\supset$, $\cup$, $\cap$, Union, intersection, differences and complements of sets. Commutative associative and distributive properties of union and intersection. Finite sets. Infinite sets. Equivalence of sets. Countable and uncountable sets. Cardinal numbers. Bernstein's Theorem. Order in cardinal numbers. Addition and multiplication of cardinals. Commutative, associative and distributive laws. Cardinal α^{β}. Laws of exponentiation. Proofs of $\alpha < 2^{\alpha}$. Statement of the continuum hypothesis.

Books for references:

Joseph Breuer: Introduction to the Theory of Sets (Chapter 1-3).

E. Kamke: Theory of Sets. Chapters 1-2.

Birkoff and Maclane: A Survey of Modern Algebra Chapter XI (11.1-11.3) and Chapter XII.

(ii) *Foundations of the Real Number System*: Peano's axioms for positive integers. Introduction of addition, multiplication and order. Deduction of the basic properties of these operations and order. Subtraction. Division. Extension to rationals. Extension of the algebraic operations and order to rationals. Extension of rationals to reals by the methods of Dedekind and Cantor. Extension of the algebraic operations and order to reals and verification of their properties. Proof of Dedekind's theorem. Theorem on existence of the least upper bound of bounded sets and the theorem of nested intervals. Proof that for integral $n \neq o$ and real $c > o$, $x^n = c$ has a unique solution in positive reals.

Books for reference:

Landau: Foundation of Analysis, supplemented by a standard book on Analysis e.g. Rudin: Principles of Mathematical Analysis.

4. (i) *Attractions and Potentials* and (ii) *Elements of Elasticity.*

(i) *Attractions and Potentials*: Newtonian attraction and potential of a thin rod, circular plate, spherical shell and solid sphere. Change in attraction on crossing a thin attracting surface. Surface integral of normal attraction. Equations of Laplace and Poisson. Equipotential surfaces. Lines and tubes of force. Work done by a self-gravitating system. Distributions for a given potential equivalent layer.

(ii) *Elements of Elasticity*: Deformation. Stress and strain. Shearing stress. Generalised Hooke's Law. Elastic rods. Application to rigid frame works. Shear of cylindrical rod and frustum of a cone. Torsion of uniform cylindrical shaft. Flexure. Bending moment. Beams in tension. Beams subjected to perpendicular forces. Shearing stresses and bending moments of heavy beam and loaded beam. Heavy beam supported at one or both ends. Bending of long columns. Centrifugal whirling shafts. Equation of elastic curve. Potential energy of a bent elastic rod. Equation of equilibrium of plane bent elastic rod.

II. M.A./M.Sc. Course

1. The objectives of a post-graduate course in Mathematics should be:-

   (i) To give training in logical thinking.

(ii) To train students for Advanced Studies in Mathematics leading to research.

(iii) To prepare teachers for schools and colleges.

While preparing the post-graduate syllabus, it is assumed that the student has been trained according to the under-graduate syllabus detailed earlier in Part I. The syllabus has been so framed that by making a suitable choice of the optional group a student can either get introduced to a wide variety of modern topics in Mathematics or can receive intensive training in one or two special fields. This post-graduate course is expected to equip the person adequately for undertaking under-graduate teaching and also for entering into such other professions as require an advanced knowledge of Mathematics. Any person who desires to take to post-graduate teaching should undergo further training and research work.

2. The syllabii proposed consist of both full courses and half courses. Each full course will require 100 hours of lectures and a good number of seminars and discussions. The half course requires about 50 hours. The courses have been further grouped into (i) Compulsory papers and (ii) Optional papers. Each student has to offer (i) and any two full courses or equivalent full and half courses from (ii).

(i) *Compulsory Papers*:

(1) and (2) Theory of functions of a real variable, elementary differential geometry and theory of functions of a complex variable.

(3) Differential Equations and Special Functions.

(4) Potential Theory and Analytical Dynamics.

(5) (a) Mechanics of Deformable Bodies

or

(b) Set Theory and Analytic Topology.

(6) (a) Electromagnetic Theory and Special Theory of Relativity.

or

(b) Modern Algebra and Introduction to Algebraic Geometry.

(ii) *Optional Courses*:

A. *Half Courses*:

(i) Number Theory I, (ii) Number Theory II, (iii) Measure and Integration I, (iv) Measure and Integration II, (v) Functional Analysis I, (vi) Functional Analysis II, (vii) Combinatorial Topology, (viii) Probability Theory, (ix) Stochastic Processes, (x) Information Theory I, (xi) Information Theory II, (xii) Feedback Control and Servomechanisms, (xiii) Mathematical Statistics I, (xiv) Mathematical Statistics II, (xv) Turbulence.

B. *Full Courses*:

(i) Algebraic Number Theory, (ii) Geometry of Numbers, (iii) Differential Geometry, (iv) Riemannian Geometry and General Theory of Relativity, (v) Set Theory and Point set Topology, (vi) Modern Algebra and Introduction to Algebraic Geometry, (vii) Homological Algebra, (viii) Mathematical Logic, (ix) Topological Groups, (x) Linear Operators in Banach and Hilbert spaces, (xi) Integral Equations, (xii) Fourier Series and Allied series with special reference to convergence and Summability aspects, Generalised functions, (xiii) Theory of Summability, (xiv) Projective, Affine and Metric Geometry, (xv) Fourier Series, Integral Transforms and Boundary value and Eigen Value Problems, (xvi) Principles of Statistical Mechanics, (xvii) Principles of Quantum Mechanics, (xviii) Theory of waves and Vibrations, (xix) Non-linear Analysis, (xx) Boundary Layer Theory and Turbulence (xxi) Magneto-fluid Mechanics, (xxii) Elasticity, (xxiii) Plasticity, (xxiv) Compressible Fluid Mechanics, (xxv) Ballistics, (xxvi) Internal constitution of stars, (xxvii) Stellar Atmospheres, (xxviii) Celestial Mechanics, (xxix) Stellar Dynamics.

Note: New optional courses may be introduced according to availability of experts.

3. DETAILED SYLLABI:

The detailed syllabii for all the compulsory subjects and most of the optional courses have been given below. Where detailed syllabi are not given it is understood that such options will be taught at only such universities where there are specialists in these branches. Suitable syllabii for these options may be framed by the universities concerned.

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PAPERS 1 *and* 2

(i) *Theory of Functions of Real Variables*:

Riemann integration of a function of one variable. Uniform convergence of infinite series and products. Power series. Trigonometric functions. Infinite integrals. Line integrals and rectification. Differential calculus of functions of several variables including Taylor's Theorem and applications to maxima and minima. Theorems on implicit function. Jacobians. Dependence of functions. Double integration. Green's theorem.

Lebesgue measure and integrals (Standard as in Rogosinsiki's Volume and Integrals). Fourier Series. Fejer-Lebesgue Theorem.

(ii) *Differential Geometry*:

Curves: Plane and skew curves. Arc length, tangent. Contact, curvature and torsion. Frenet's formulae. Natural equations and their solution, the fundamental theorem for space curves. Involutes and evolutes.

Theory of Surfaces: The two fundamental forms. Meusnier's Theorem, Euler's Theorem. Dupin's indicatrix. Rodrigues formula. Asymptotic lines and conjugate lines. Lines of curvature. Envelopes. Ruled surfaces. Developable surfaces. Geodesics.

Books for reference:

D. J. Struik: Lectures on Classical Differential Geometry.

Goursat-Hedrick: Mathematical Analysis. Vol. I, Chapters X, XI and XII.

Rudin: Principles of Mathematical Analysis.

De la Vallee Poussin: Cours d'analyse Infinitesimale, Vols. I and II.

(iii) *Functions of a Complex Variable*:

Sequences and functions of a complex variable. Continuity and differentiability. Analytic functions. Cauchy-Riemann equations. Cauchy's theorem and integral formula. Liouville's Theorem. Morera's Theorem. Taylor and Laurent series. Singularities. Riemann Surfaces. Cauchy's Residue Theorem with applications to definite integrals. Cauchy's method of expansions of functions in series and products. Roucke's Theorem. Maximum modulus principle. Integral functions —Weierstrass' theorem on their expansion as infinite products. Meromorphic functions. Mittag-Leffler theorem. Analytic continuation (including the continuation of Γ and ζ functions.)

Conformal transformations (A number of examples). Riemann mapping theorem. Equicontinuous and normal families of functions.

Periodic functions. Weierstrass's $\wp$, ζ and σ functions. Jacobi's sn, cn, dn functions.

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PAPER 3

(i) *Differential Equations and Special Functions*:

Ordinary Differential Equations:

Differential equation of the first order. Singular solutions. Existence theorem for the solution of the equation $\frac{dy}{dx} = f(x, y)$. Riccati's equation. Picard's method for numerical integration of ordinary differential equations. General properties of linear differential equations. Linear independence of solutions. Fundamental sets of solutions of linear differential equation of order n. Exact linear equation of order n. Linear differential equations with constant coefficients. Cauchy-Euler equation. Method of variation of parameters. Particular methods to solve non-linear equations of higher order. Green's function method. Properties of Green's function. Adjoint and self-adjoint equations. Singular points. Solution of ordinary differential equations by the method of power series. Recurrence relations. Integral representations. Asymptotic expansions and zeros of the hypergeometric, Legendre and Bessel functions. Solutions of differential equations by means of a definite integral. Simultaneous equations. Total differential equations.

Books for reference:

Ince: Ordinary Differential Equations.

(ii) *Partial Differential Equations*:

Concepts of geometry of three dimensional space. Envelope. Characteristics of one parameter and two parameter family of surfaces. Formation and classification of partial differential equations. Nature of solution of first order partial differential equations. Classification of integrals of first order partial differential equations. The Cauchy problem. Cauchy's method of characteristics. Charpit's method. Jacobi's method. Higher order partial differential equations with constant coefficients. Euler-Cauchy type. General case of linear partial differential equations of second order. Solution of nonlinear second order partial differential equations by Monge's methods. Classification of second order partial differential equations. Methods of separation of variables and of Fourier series for solving the Laplace, the wave and the diffusion equation with special reference to boundary value problems.

Books for reference:

Sneddon: Elements of Partial Differential Equations.

Webster: Partial Differential Equations of Mathematical Physics.

(iii) *Special Functions*:

Gamma Functions. Legendre Polynomials and generating function. Integral properties of Legendre Polynomials. Recurrence relations between Legendre Polynomials. Associated Legendre polynomials. Addition Theorem for Legendre polynomials. Bessel Functions and generating function. Hankel Functions. Integral properties. Recurrence relations. Addition Theorem. Hermite polynomials and functions. Laguerre polynomials and functions. Generating functions for these polynomials. Expansion of functions in terms of a complete and orthogonal set of functions.

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PAPER 4

Potential Theory and Analytical Dynamics

(i) *Potential Theory*: Newtonian potentials due to volume distribution of mass, at a point of free space and at a point occupied by matter, the Laplace equation and the Poisson equation. Potential due to surface distributions satisfying Poincare's condition. Potential of double distribution. Fundamental properties of Newtonian distributions.

Gravitational and electrical potentials and attractions.

Logarithmic potential, its relation to Newtonian potential and to the theory of analytic functions of a complex variable.

Dirichlet's problem for the sphere, extension of Gauss' mean-value theorem to potentials. Harnack's first and second theorems.

Books for reference:

O.D. Kellogg: Foundations of Potential Theory.

C. Racine: Introduction to Potential Theory.

(ii) *Analytical Dynamics*:

Notes: (i) Emphasis must be laid on the fundamental principles rather than on solving intricate problems of artificial nature, (ii) the vectorial treatment must be adopted, (iii) this syllabus builds upon the B.Sc. syllabus).

(a) *General*: Expressions for velocity and acceleration in various systems of coordinates. Angular velocity. Relative velocity. Moving axes.

Newton's Laws of Motion. Different kinds of forces.

(b) *Particle Dynamics*: Rectilinear motion with varying acceleration with or without dissipation. Motion in a plane. Projectile in a resisting medium. Central orbits. Stability of circular orbits. Kepler's Laws of planetary motions. Disturbed elliptic motion. Principles of conservation of energy, momentum and angular momentum. Impact. Motion in Space. Motion relative to the Earth.

(c) *Rigid Dynamics*: Properties of moments and products of Inertia. Two-dimensional motion of a rigid body under finite and impulsive forces. Compound pendulum. Euler's equations. Elementary treatment of motion of top.

Generalised co-ordinates. Lagrange's equations for holonomic system. Lagrangian function. Ignorable co-ordinates. Hamiltonian equations. Hamilton's principle. Principle of Least Action. (Treatment and standard as in D. E. Rutherford: Classical Mechanics.)

Books for reference:

J. L. Synge and B.A. Griffiths: Principles of Mechanics.

Banach: Mechanics.

E. A. Milne: Vectorial Mechanics.

Goldstein: Classical Mechanics.

Landau: Mechanics.

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PAPER 5 (*a*)

Mechanics of Deformable Bodies: Vector analysis and Cartesian tensors. Kinematics of deformable bodies. Helmholtz's fundamental theorem of kinematics on the independence of translation, rotation and deformation

Statics of Deformable Bodies: Concept of stress. General classification of deformable bodies. Equilibrium of incompressible fluids. Statics of compressible fluids. Stress in an elastic solid. Strain-stress relations. Elastic constants and Elastic potential. Viscous pressures and dissipation.

Dynamics of Deformable Bodies: Analysis of stress. Analysis of strain. Derivation of Navier-Stokes Equations. Continuity and energy equations. Dynamical equations for an elastic body. Existence of waves of dilation and waves of distortion.

Fluid Mechanics: Lagrangian equations of motion. Stream-lines and path-lines. Vorticity and circulation and their constancy in ideal fluid.

Ideal fluid: Bernoulli's theorem and its applications. Potential flow around cylinders and sphere. Blasius' Theorem and its easy applications. Techniques of images and conformal transformations for solution of hydrodynamical problems. Simple properties of vortex motion, uniqueness theorem, motion due to rectilinear vortices.

Viscous Liquid: Flow through straight pipes. Oseen's and Stokes' approximations. Slow streaming past a sphere. Drag and lift.

Elasticity: Bending of beams and plates. Torsion. Torsion and bending of helical springs. Saint-Venant's principle.

Compressible Fluids: Equation of state. First and Second Laws of Thermodynamics. Entropy of a perfect gas. Speed of sound. Enthalpy. Isentropic and isoenergetic flows. Propagation of small disturbances. Characteristics for steady motion. Flow round a corner. Shock-waves, Rankine-Hugoniot relations.

Books for reference:

A. Sommerfeld: Mechanics of Deformable Bodies.

L. M. Milne Thompson: Theoretical Hydrodynamics.

A. M. Kuethe and J. D. Schetzer: Foundations of Aero-dynamics.

I. S. Sokolnikoff: Theory of Elasticity.

PAPER 5 (b)

Set Theory and Analytic Topology

Set Theory: Algebra of sets. Relations. Functions. Equivalence relation, quotient set. Partial ordering. Well ordering. Equivalence of Zorn's Lemma. The Axiom of choice and the Well Ordering Theorem. Transfinite induction. Ordinal Numbers, sets of ordinal Numbers, ordinal arithmetic. The Schroder-Bernstein Theorem. Cardinal arithmetic, cardinal numbers.

Topology: Topological spaces defined through open sets, closed sets and neighbourhood systems. The closure, interior and boundary of a set. Bases and sub-bases. Relative topology. First and second axioms of countability; separability. T_0, T_1 and Hausdorff spaces; regular, completely regular and normal spaces. Tychonoff spaces. Lindelöf spaces. Connected sets, components. Directed sets, nets, convergence of nets. Uniqueness of limit in a Hausdorff space. Theorem on Iterated Limits, subnets, sequences, sub-sequences. Convergence classes.

Continuous Functions, open functions, homeomorphisms. Quotient spaces, quotient topology. Product spaces, product topology.

Urysohn's Lemma, Tietze's Extension Theorem. The Embedding Theorem for a Tychonoff space.

Metric spaces. Urysohn's Metrization Theorem. Smirnov's Theorem on the necessary and sufficient condition for the metrizability of a topological space.

Compactness, various criteria of compactness. Compactness and separation properties. Tychonoff's Theorem. Locally compact spaces. Compactification. One point compactification, Stone-Cech compactification. Para-compactness, various characterisations of para-compactness, a metric space is paracompact. Uniformities, uniform spaces, uniform continuity. Relative uniformity. Product uniformity. Complete spaces, completion of a uniform space.

Existence of uniform structure on a compact space. Necessary and sufficient condition that a topological space be uniformisable.

PAPER 6 (a)

Electromagnetic Theory and Special Theory of Relativity.

(i) *Electromagnetic Theory:*

(a) *Electrostatics and Magnetostatics:* Field due to charges. Flux. Static fields. Conductors and condensers. Dielectrics. Steady currents. Magnetic effects of currents. Steady currents in a magnetic field. Spherical conductor in a uniform field. Dielectric sphere in uniform field. Two-dimensional problems in magnetism. Axially symmetric problems. Induction in stationary and moving circuits. A simple dynamo. Induction in a single circuit. Transformer. Generalised law of induction.

(b) *Electrodynamics:* Displacement current. Maxwell's equations. Conditions to be satisfied at the surface of separation of two media. Electromagnetic potentials. Electromagnetic stresses. Electromagnetic energy. Poynting's theorem. Joule's Heat. Quasi-stationary fields. Alternating currents. Electromagnetic waves in an isotropic dielectric. Plane, circularly and elliptically polarized waves. Reflection and refraction of electromagnetic waves in dielectrics. Waves in conducting media.

Radiation: Motion of electric charges.

(c) *Special Theory of Relativity:* The Galilean principle, Michelson-Morley experiment. The principles of the Theory of Relativity. Space-time co-ordinates and Lorentz transformation. Spatial and temporal world vectors. Geometrical and mechanical consequences of Lorentz transformation. Lorentz invariance of Maxwell's equations. Electrodynamics of a vacuum. Dynamics of the special theory of relativity. Matter and energy.

Notes: (1) Treatment of the section (a) may be as in C.A. Coulson: Electricity. Only problems directly depending on the theory should be asked.

(2) The treatment of section (b) may be as in part III of V.C.A. Ferraro: Electromagnetic Theory.

Books for reference:

Landau and Lipschitz: Classical Theory of Fields.

Ahroni: Special Theory of Relativity.

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PAPER 6 (*b*)

Modern Algebra and Introduction to Algebraic Geometry.

(i) Groups, Sub-groups, Normal subgroups, Factor groups. Homomorphism, Isomorphism. Isomorphism theorems. Conjugate elements. Class equations of a finite group. Normalizer, centre, commutators. Permutation groups. Cayley's Theorem. Sylow's Theorems. Decomposition theorem for finitely generated Abelian groups. Invariants. Normal series, composition series, Jordan-Höder Theorem, soluble groups.

(ii) Rings. Integral Domains. Division Ring. Fields. Ideals. Prime and maximal ideals. Sums and Products of Ideals. Quotient Ring. Isomorphism theorems for rings. The field of quotients of an integral domain. Euclidean domains. Principal ideal domains. Unique factorisation domains. Ring of polynomials over a ring (Commutative). Content. Primitive polynomials, Gauss' Lemma, factorization of polynomials with coefficients from a unique factorization domain. Noetherian rings, Hilbert's Basis Theorem. Primary Ideals. Theorem on decomposition of an ideal into primary ideals in Noetherian rings. The two uniqueness theorems.

(iii) Vector spaces, Basis of a vector space, dimension.

(iv) Field extensions. Degree of an extension and its transitivity. Algebraic extension. Root field of a polynomial. Splitting field of a polynomial. Algebraically closed fields. Algebraic closure of a field. Normal extensions. Separable extension. Simple extension, simplicity of a finite separable extension. Inseparable extension. Reduced degree and degree of inseparability. Galois Theory of finite extensions. Application to solution of equations by radicals. Finite fields.

(v) Ordered fields. Fields with valuation. Completion.

(vi) Algebraic sets. Algebraic varieties. Generic points. Dimension of algebraic variety. Subvariety; Valuations, valuation rings. Places. Extension Theorem. Hilbert's Nullstellensatz.

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A. *HALF COURSES* (Optional)

(i) *Number Theory (I)*

Prime numbers. Unique Factorisation Theorem. Irrational numbers. Farey Series. Continued Fractions. Approximation of irrationals by rationals. Hurwitz Theorem. Congruences. Residues. Quadratic Reciprocity Law. Primitive roots. Arithmetic functions ϕ (n), d(n), σ(n), μ(n) including elementary results on their order and average order (as in Hardy and Wright). Diophantine Equations $x^2 + y^2 = z^2$, $x^4 + y^4 = z^4$, $ax^2 + by^2 + cz^2 = 0$. Representation of a number as a sum of two or four squares. Waring's problem. Elementary results on g(K), G(K).

(ii) *Number Theory (II)*:

Elementary Theory of Binary quadratic forms. Elementary results on the distribution of primes. Brun's theorem on Prime Pairs. Prime Number Theorem. (Landau's proof). Characters. L functions. Dirichlet's theorem on primes in an Arithmetic Progression. Representation of a number as a sum of three squares. The class number of binary quadratic forms.

Books for reference for (i) and (ii):

Landau: Elementary Number Theory.

(iii) *Measure and Integration I*:

Jordan content. Riemann integral in terms of Jordan content and Jordan measurable sets. Families of sets (semirings, σ rings etc.) and set functions. Outer measures and measures. Measure induced by an outer measure. Metric outer measures. Regular Borel measures in locally compact Hausdorff spaces. Extension of measures through outer measures. Applications to Lebesgue measure. Lebesgue–Stieltjes measure and Borel measures from contents.

Measurable sets and measurable functions. Measure spaces. Simple functions. Step functions. Elementary integrals for simple functions in a measure space. Product measure of a given measure with the Lebesgue measure in R_1 (Positive-reals,) Fubini's theorem in this case.

Integrable and summable functions. Properties of integrals and integrable functions. L_p spaces as linear spaces. Monotone convergence theorem and Fatou's Lemma for functions in L_p. Dominated convergence theorem. Integration by parts and change of variables in a Lebesgue integral.

(iv) *Measure and Integration II*:

New types of convergence for measurable functions in a measure space like almost uniform convergence, convergence in measure. Egoroff's and Lusin's theorems. L_p norm. Mean convergence. Weak convergence. Relation between convergence in measure and almost everywhere convergence. Completeness theorem. Dominated convergence theorem. L_2 space as a Hilbert space. Indefinite integrals. Radon-Nikodym theorem. Product measures. Fubini's theorem.

Linear functional in Banach spaces. Hahn-Banach theorem. Conjugate spaces of the space of continuous functions and L_p spaces. Random measure in locally compact spaces. Stone-Bourbaki approach.

Integration in groups. Existence and uniqueness of the Haar integral. The measure algebra of regular measures in groups. Convolutions. Fourier transforms.

Books for reference for (iii) and (iv)

Zaanen, A.C.: Linear Analysis, Inter Science, N.Y.

Munroe, M.E.: Introduction to measure and integration, Addison-Wesley, Cambridge.

Halmos, P. R., Measure Theory, Van Nostrand, N. Y.

Weil, A: Integration in Topological Groups.

(v) and (vi) *Functional Analysis I*:

(Syllabus to be framed by the University intending to start these papers).

(vii) *Combinatorial Topology*:

1. Curves and Surfaces. Elementary Curves and 1-Complexes. Connectivity of a curve (1-dimentional Betti-Number). 2-Complexes and Polyhedra. Closed surfaces. Surfaces with Boundary. Triangulations. Subbivisions of Triangulations. Regular Sub-division. Triangulations of Mobius Band. Klein Bottle. Projective Plane. Torus. Orientability of a Surface. Triangulations of Orientable Surfaces. Connectivity of a Surface. Euler Characteristic and its Invariance. Simple Surfaces. Classification of simple Surfaces. Classification of closed surfaces. Genus of a Surface. Normal Surfaces of a given Genus.

(Treatment as in Chapter 3 of Aleksandrov's combinatorial Topology).

2. Simplexes. Complexes (Simplicial, Geometric and Abstract). Skeleton of a complex. Body of a complex-Polyhedron. Star of a complex. Sub-divisions. Barycentric Stars. Connectedness of a complex, decomposition into components. o-dimensional Betti group. Sperner's Lemma. Invariance of the dimension numbers. Invariance of Domain. Fixed point theorem of Brouwer. Betti groups. Betti numbers. Euler—Pomcare formula. Simplicial mapping and approximation. The cone construction. Invariance of Betti groups under Barycentric sub-division and for homeomorphic spaces.

Homotopic mappings. Homotopy groups. Connection with Betti groups. Homotopy groups of standard spaces.

Books for reference:

Aleksandrov: Combinatorial Topology, Graylock, N.Y.

Pontryagin: Combinatorial Topology, Graylock, N.Y.

(viii) *Probability Theory*:

Historical introduction and modern (axiomatic) approach to probability. Concepts of sample space (discrete case only) and events. The algebra of events. The frequency interpretation of probability. Examples of dependent and independent events. Conditional probability. Bayes' theorem. Some classical combinatorial and games of chance problems.

The general definition of a random variable as a function on the sample space. Generating and characteristic functions. Continuous random variables (intuitive approach). Cumulative distribution function. Density function. Stieltjes integral. Marginal and conditional distributions. Conditional expectations. Transformation of random variables.

Chebyecheff's inequality. Convergence in probability and law. Bernoullis' theorem. The law of large numbers. Kolmogorov's inequality and the strong law of large numbers. Properties and uses of characteristic function. Inversion theorem. The first limit theorem and the central limit theorem. Applications of these limit theorems. Some elementary problems of runs and recurrent events.

Probability as measure. Concept of a σ-field and measure. Integration of simple functions and bounded measurable functions. Sub σ-fields and conditional expectation. Product measures and consistency theorem.

Books for reference:

W. Feller: Probability Theory and Applications, Vol. 1, John Wiley and Sons.

M. Loève: Probability Theory. Van Nostrand.

(ix) *Stochastic Processes*:

Sample space, field, measure, extension of measures, product spaces, product measures, consistent family of measures and Kolmogorov's theorem on the construction of measures in the infinite dimensional space. Random variable and convergence notions.

Sequence of independent random variables. Limit theorems for sums of independent random variables.

Strictly stationary sequences of random variables. The individual ergodic theorem. Weakly stationary sequences of random variables. Correlation function. Spectral distributions. The mean ergodic theorem. Problem of prediction. Wiener's prediction formula. Estimation of the spectral distribution.

Martingales, Martingale convergence theorems. Applications to the likelihood ratio etc.

Markov chains. Transition probabilities. Temporally homogeneous Morkov chains. Finite state space. Classification of states and the ergodic theorem. Central limit theorem for ergodic Markov chains.

Processes with continuous parameter. Processes with independent increments, and stationary independent increments. Brownian motion and some of its elementary properties.

Books for reference:

Blanc Lapierre, A., and Fortet, R: Random Functions (Paris).

Doob, J. L.: Stochastic Processes. (John Wiley and Sons).

Kolmogorov, A. N.: Foundations of Probability Theory (Chelsea).

Rosenblatt, M. and U. Grenander: Statistical Analysis of Stationary Time Series. (John Wiley and Sons).

Gnedenko, B. V. and Kolmogorov, A. N.: Limit Theorems for Sums of Independent Random Variables (Addison-Wesley).

Kemeney, J. C. and Snell, J. L.: Finite Markov Chains, Van Nostrand.

Feller, W.: Prob. Theory and its Applications, Vol. I, (John Wiley and Sons).

Loève, M.: Probability Theory. Van Nostrand.

(x) *Information theory I*:

(Pre-requisites: Elements of measure theory. Kolmogorov's consistency theorem may be included in the course at a suitable stage. Elements of Probability theory).

Finite schemes. Entropy of finite schemes. Uniqueness theorem Entropy of Markov chains. Elementary coding problem of Markov chains. Information source, stationary source, entropy of a stationary source. Ergodic theorem and martingable theorem. Macmillan's theorem and the E-property of ergodic sources.

Channels. Stationary channels. Rate of transmission. Ergodic and stationary capacities of a channel without anticipation and with finite memory in the sense of Khinchin and Feinstein. Feinstein's fundamental lemma on the existence of distinguishable set of sequences.

Shannon's theorems on linking a source with a channel by means of block coding. Possibility of transmission at any rate near the capacity.

(xi) *Information theory II*:

Equality of ergodic and stationary capacities. Attainment of capacity. Converse of Shannon's theorems, i.e., on the impossibility of transmission at a rate greater than capacity.

Channels without memory—exponential error bound.

Binary channels. Error detecting and error correcting codes. Slepians group codes. Application of the theory of fractorial designs in the construction of group codes.

Books for reference on Information theory I and II:

A. I. Khinchin: Mathematical Foundations of Information Theory (Dover).

A. Feinstein: Foundations of Information Theory (McGraw-Hill).

C. Shannon and W. Weaver: Mathematical Theory of Communication, Urbana, University of Illinois, 1949.

(xii) *Feedback control and servomechanisms*:

(Pre-requisites: Information theory I, and Elements of Probability Theory).

Fundamentals of system analysis: Circuit theory—Wye Delta Transformation, Thevinin's and Norton's Theorems, Nodal and Mesh analysis. Net work. Use of Laplace Transform in system analysis and synthesis. Open and closed control. Error coefficients.

Stability: Definition, purpose and methods of stability analysis. Routh's and Nyquist's criteria for stability. Root Locus method. Bode Alternation diagram approach.

Relation between transient and frequency response: Feedback system compensation. Noise, random inputs and extraneous signals. Non-linear systems. Sampled data systems and Penodic controllers.

Book for reference:

James, H. M., Nichols and Philips: Theory of Servomechanisms, 1947, McGraw-Hill.

Truxal, J. G.: Automatic Feed-back Control System Synthesis, 1955. McGraw-Hill.

Chestnut, H. and Mayer, R. W.: Servomechanisms and Regulating System Design, Vols. I and II, 1951. John Wiley and Sons.

Grabbe, E. M. and others (Editors): Hand Book of Automation, Computation and Control, Vol. I, Control Foundamentals, John Wiley and Sons.

Wiener, N.: Cybernetics. John Wiley and Sons, 1949.

Davenport, W. B. and Root, W. L.: Introduction to Theory of Random Signals and Noise. McGraw-Hill, 1958.

(xiii) *Mathematical Statistics I*:

(Those who take this half course are advised to take the course on probability concurrently).

Concept of a statistical population. Measures of location and dispersion. Moments, cumulants and generating functions.

Unvariate probability models like the Binomial, Poisson, Hypergeometric, Negative Binomial, Normal, Uniform, Cauchy, Beta, Gamma, Pearson's system of curves, etc., their properties and interrelations.

Concepts of a sample and a statistic. Sampling distributions of mean, variance, Student's t, Fisher's F, etc., in the case of sampling from a Normal distribution.

Bivariate models. Concepts of regression and correlation. Correlation ratio. Sampling distribution of r.

Multivariate models like the Multinomial, Multi-variate Normal and the Hypergeometric. Dispersion matrix. Regression and correlation (total, multiple, partial).

Elements of the theory of sampling with and without replacement from a finite population. Simple random sample. Estimation of mean and of the variance of this estimate.

(xiv) *Mathematical Statistics II*:

(Pre-requisites: course on Probability Theory and Mathematical Statistics I).

General nature of statistical inference. Point estimation. Estimation in large samples, concepts of consistency, efficiency and asymptotic normality. Methods of estimation by moments, least squares, and maximum likelihood. Unbiased minimum variance estimation, Cramer-Rao bounds. Sufficient statistics. Koopman's form, Rao-Blackwell theorem, complete sufficient statistics.

Tests of hypotheses—Null hypothesis, Neyman and Pearson theory. Simple and composite hypotheses. Locally and Uniformly and most powerful tests, unbiased tests, Similar region tests. Tests based on complete sufficient statistics. Tests based on likelihood ratio criterion.

Confidence limits and fiducial probability.

Wald's sequential probability ratio-test and its convergence. The concepts of ASN and OC surves.

Introduction to the general theory of decision functions.

Books for reference for Mathematical Statistics I and II:

H. Cramer: Mathematical Methods of Statistics: Princeton University Press.

Paul G. Hoel: Introduction to Statistics. John Wiley and Sons.

M. G. Kendall and A. Stuart: Advanced Theory of Statistics. Vols. I and II. Griffin and Company.

E. Lehmann: Testing of Hypotheses. John Wiley and Sons.

A. Wald: Sequential Analysis. John Wiley and Sons.

A. Wald: Decision Functions. John Wiley and Sons.

D. A. S. Fraser: Non-parametric Methods. John Wiley and Sons.

C. R. Rao: Advanced Statistics in Biometric Research. John Wiley and Sons.

(xv) *Turbulence*:

Fundamentals of turbulent flow. Mean motion and fluctuations, Additional "apparent" turbulent stresses. Derivation of the stress tensor of apparent turbulent friction from Navier–Stokes equations. Prandtl's mixing length theory and Taylor's vorticity transfer theory. Double correlations between Turbulent-velocity components. Change in double velocity correlation with time and introduction of triple velocity correlations. Double longitudinal and lateral correlations in Eulerian correlations with respect to time. Turbulent diffusion of Homogeneous Turbulence. Macro or integral scale of turbulence. of fluid particles. Taylor's one–dimensional energy spectrum. Energy relations in turbulent flows. Isotropic turbulence. Correlation Tensors. Differential Equation for dynamic behaviour of an isotropic turbulence. Three-dimensional energy spectrum. Dynamic equation for the energy spectrum. Decay of isotropic turbulence. Extension to an isotropic-turbulent scalar field. Pressure fluctuations in isotropic turbulence.

Books for reference:

Agostini and Bass: Statistical Theory of Turbulence.

Hinze: Turbulence.

OPTIONAL FULL COURSES

(i) *Algebraic Number Theory*:

Algebraic numbers. Algebraic number fields. Fundamental system. Degree. Algebraic integers. Divisibility. Units. Basis of an Algebraic number field. Discriminant. Ideals. Prime ideals. Fundamental theorem of Ideal theory (Unique factorization). Infinity of prime ideals. Residue classes modulo an ideal. Norm of an ideal. Function. Degree of a prime ideal. The group of residue classes modulo an ideal. Fractional ideals. Ideal classes. Principal ideal class. Group of ideal classes. Finiteness of the number of ideal classes. Class number. Units. Fundamental units. Dirichlet's theorem. Regulator, Different and Discriminant of an algebraic number field. Relative fields. Relations between ideals in different fields. Relative norm. Relative differents and discriminants. Hilbert's class field. Density of an ideal class. Relation between density and class number. Dedekind's Zeta function. Existence of infinity of primes of the type $mx + 1$. Dirichlet's theorem. Quadratic Number Fields. Relations between ideals in $k(\sqrt{d})$ and binary quadratic forms. Quadratic residue character and Gauss's sums in arbitrary number fields. Theta functions. The quadratic reciprocity law in arbitrary number fields.

Books for reference:

Hecke, E.: Algebraische Zahlen, Chelsa.

(ii) *Geometry of Numbers*:

Hermite's theorem on minima of positive definite quadratic forms. Minkowski's geometrical proof. Convex Bodies. Lattices. Minkowski's fundamental theorm. Its application to linear Forms. Statement of Minkowski-Hajos theorem. Dirichlet's theorem. Minkowski's improvements. Kronecker's theorem. Application of Minkowski's theorem to other convex bodies. Generalisations of Minkowski's theorem due to Blichfeldt, Mordell and Van der corupt. Minkowski's second theorem. Exact minima for binary quadratic forms (first three minima for indefinite forms). Mordell's relation between the minima of positive definite quadratic forms for n and n + 1 variables. Minima for positive definite quadratic forms of three and four variables. Admissible lattices, critical determinant. Critical lattices. Their connection with Diophantine inequalities. Packings and their connection with these problems. Blichfeldt's result on closest packings of n dimensional spheres. Its consequences for minima of positive definite quadratic forms. Critical lattices for two dimensional convex bodies. Application to binary quadratic forms. Minkowski's conjecture about the critical lattices for $|x|^p + |y|^q \leqslant 1$ as amended by Davis.

Covering lattices, covering constants, lattice covering. Their connection with Diophantine inequalities. Lattice coverings for two dimensional convex bodies. Lattice coverings by circles. Statement of known results for converings, lattice as well as non-lattice, in the plance. Definition of θ_n, the density of the best lattice coverings by n-dim spheres. Elementary estimates. Statements of known result for θ_3, θ_4.

Minkowski–Hlawka theorem (any proof). Its analogue for lattice coverings by symmetrical convex bodies. Minimum of products of three linear forms, for the non-real case on the assumption that every critical Lattice has a point on the boundary i.e. prove only that $|L_1(L_2^2+L_3^2)| < \frac{D}{\sqrt{23}} + \epsilon$ Mordell's theorem on the binary cubic form (any proof). Product of n real homogeneous forms (results of Minkowski and Davenport, statement of later results due to Rankin and Rogers).

Minkowski's theorem on the product of two or three non-homogeneous real forms (any proof). Tchebotoroff's theorem on the product of real non-homogeneous forms, with statement of later improvements. Minkowski's conjecture with mention of Dyson's theorem. Non-homogeneous binary cubic forms.

Mahler's existence theorems for Critical lattices of bounded start bodies.

Books for reference:

J. W. S. Cassels: Geometry of numbers, springer verlag, 1959.

H. Minkowski: Diophantische approximationen, Leipzig, 1907.

H. Minkowski: Geometric der Zahlen, Leipzig, 1896.

J. F. Koksma: Diophantische approximationen, 1937, (Berlin), (reprinted by Chelsea).

G. H. Hardy and E. M. Wright: Introduction to the Theory of Numbers, 2nd Edition, Oxford, 1945.

(iii) *Differential Geometry*:

Curves in space. Envelopes. Developable surfaces. Developables associated with a curve. Curvilinear co-ordinates on a surface. Fundamental magnitudes of the first and second order. Curvature of normal section. Lines of curvature. Conjugate systems. Asymptotic lines. Isometric lines. Null lines. The equations of Gauss and of Codazzi. Geodesics and Geodesic parallels. Ruled surfaces. Surface of centres. Parallel surfaces. Inverse surfaces. Conformal representation. Spherical representation. Minimal surfaces. Rectilinear and curvilinear congruences. Triply orthogonal systems of surfaces.

Books for reference:

Weatherburn, C.E.: Differential Geometry of three dimensions, Vol. I.

L. P. Eisenhart: An Introduction to Differential Geometry with aid of the Tensor Calculus.

(iv) *Riemannian Geometry and General Theory of Relativity*:

Riemannian Geometry:

Tensor analysis. Riemannian metric. Christoffel's three-index symbols. Covariant differentiation. Geodesics. Levi-Civita's parallelism of vectors. Congruences and orthogonal ennuples. Riemann-Christoffel Tensor. Curvature of a Riemannian space. Hyper surfaces. Spaces of constant curvature. Subspaces of a Riemannian space.

General Theory of Relativity:

Principle of covariance. Principle of equivalence. Equality of gravitational and inertial mass. The gravitational field in empty space and in the presence of matter and energy. The Schwarzschild line-element. The three crucial tests of general relativity. Relativistic Mechanics. Energy momentum tensor for a perfect fluid. Variational principle for the gravitational field equations. The Lagrangian and the Hamiltonian. Static cosmological models. The Einstein and De Sitter Models.

Books for reference:

Weatherburn: Tensor Calculus and Riemannian Geometry.

Eisenhart: Riemannian Geometry.

Eddington: Mathematical Theory of Relativity.

Möller: The Theory of Relativity.

Bergmann: An introduction to the theory of Relativity.

Tolman: Relativity, Thermodynamics and Cosmology.

Fock: Relativity.

(v) *Set Theory and Analytic Topology*:

[same as Compulsory paper 5(b)]

(vi) *Modern Algebra*:

[same as Compulsory paper 6(b)]

(vii) *Homological Algebra*:

(Syllabus to be framed by the university intending to start the paper).

(viii) *Mathematical Logic*:

(Syllabus to be framed by the university intending to start the paper).

(ix) *Topological Groups*:

Topological Groups. System of Neighbourhoods of the Identity. Open and Closed Subgroups. Normal subgroups. Factor Groups. Isomorphism, Automorphism and Homomorphism. Intersection and Products of Sub-groups. Direct Products. Connected and totally disconnected Groups. Component of the identity. Local Groups, cosets of local Groups. Local Isomorphism. Homogeneous Spaces. General Linear Group and its subgroups. Invariant Integral for Locally Compact Groups, Orthogonality Relations. Representation of Compact Topological Groups. Completeness of the System of Irreducible Representation. Peter-Weyl Theorem. Character Groups of Locally Compact Abelian Groups, Fundamental Relations in characters. Duality Theorem of Pontryagin. Covering Groups. Poincarè Groups. Poincare Group of Special Spaces.

Books for reference:

Pontryagin: Topological Groups.

Chevalley: Lie Groups.

Montgomery and Zippin: Topological Transformation Groups.

A. Weil: Integration in Topological Groups.

(x) *Linear Operators in Banach and Hilbert Spaces*:

Banach and Hilbert Spaces:

Finite dimensional vector spaces. Abstract Banach and Hilbert spaces and their realisations. Linear manifolds. Orthonormal symtems. Schmidt's orthogonalisation process. Bessel's inequality. Parseval's relations.

Linear Transformations:

Linear operators. Symmetric and Self-adjoint operators. Bounded linear operators. Bounded linear functionals. Extensions of linear functionals. The adjoint space. The second adjoint space. Reflexive space. Weak convergence. Closed linear transformations on Banach space. The adjoint transformation. Bounded linear functionals and transformation on direct sums. Annihilators. Projection. Reducibility Resolvents and Spectra.

Transformation in Hilbert Space:

The Hilbert adjoint transformation.

Matrix representation. Unitary and isometric transformations. Bounded self adjoint and unitary transformation Range and null space characteristic values.

Range and null space and spectral properties of bounded linear transformation:

The resolvent and spectrum resolutions of the identity. The resolvent of a closed linear transformation, spectrum of a bounded linear transformation.

Compact linear Transformations:

Compact, Symmetrizable Self-adjoint and Normal Transformations:

The resolvent-representations and reducibility. Representation and spectrum of a Self-adjoint transformation. The unitary equivalence of self-adjoint transformation. Singular values and singular elements of a bounded linear transformation.

Commutativity and Normal Transformations. Symmetric Transformations.

Applications:

Infinite matrices. Integral operators and integral equations. Differential Operators. Operator Calculus. Applications to Quantum and Classical Mechanics.

Books for reference:

Halmos: Finite–dimensional vector space. (Princeton).

J. von Neumann: Functional operators II, (Princeton).

(xi) *Integral Equations*:

Relation between differential and integral equations. General properties of integral equations. Different types of Kernels.

Solution of linear integral equations. Method of Successive approximation. Fredholm's method. The Hilbert-Schmidt theory of integral equations with symmetric Kernels.

Elements of singular integral equations. Equations with Hilbert and Cauchy Kernels.

Applications: Boundary-Value problems and the construction of Green's functions. Integral transforms. Abel's equation. Wiener-Hopf equation. Applications to Quantum Mechanics.

Introduction to operator theory of integral equations.

Book for reference:

Lovitt: Integral Equations.

(xii) *Fourier Series and Allied Series*:

Trigonometric Series:

Orthogonal series, the trigonometric system, Completeness of the Trigonometric system, Bessel's inequality and Parseval's formula, Convex functions, Sets of the first and second categories.

Fourier series and Allied Series:

Fourier series, Formal operations on S(f), differentiation and integration of S(f). Modulus of Continuity. Smooth functions, Order of magnitude of Fourier coefficients. Formula for partial sums of S(f) and $\overline{S}$(f), The Dini test and the principle of localization, Jordan's test, de la Vallée Poussin's test, Relations between the tests of Dini, Jordan and de la Vallée Poussin. Lebesgue's Test, Gibb's phenomenon. Lebesgue's constants, Poisson's summation formula.

Summability of Fourier Series:

Summability (C,1) of S(f) and $\overline{S}$(f). Convergence factors summability (C,α), Abel summability of S(f) and $\overline{S}$(f). Summability of S(dF) and $\overline{S}$(dF), Fourier series at simple discontinuities. Fourier sine series, Gibb's phenomenon for the method (C,α). Approximation of functions by trigonometric polynomials. Existence of the conjugate function.

Absolute Convergence of Trigonometric series:

Theorem of Denjoy and Lusin, Sets N, the absolute convergence of Fourier series. Bernstein's theorem, Theorems of Wiener and Levy. Absolute convergence of lacunary series.

Complex methods in Fourier series:

Existence of conjugate functions, the Fourier character of conjugate series. Mean convergence of S(f) and $\overline{S}$(f), classes H^p, Power series of bounded variation.

Divergence of Fourier series:

Divergence of Fourier Series of continuous functions. Examples of Fourier Series divergent almost everywhere.

Riemann's Theory of Trigonometric series:

The Cantor-Lebesgue theorem. Formal integration of series. Uniqueness of representation by trigonometric series. The principle of localization. Formal multiplication of Trigonometric series. Uniqueness of summable trigonometric series.

The scope of the syllabus is indicated by relevant sections from Chapters I to IX of A. Zygmund's Trigonometric series, Vol. I. (Cambridge, 1959).

Generalized Functions:

The notions of Fourier series and Fourier Transforms (in the classical sense) with elementary properties.

Good functions and fairly good functions, their properties. Generalized Functions, the delta function and its derivatives. Operations with generalized function, Fourier transform of a generalized function. Properties of generalized functions and of their Fourier transforms; Ordinary functions as generalized functions. Equality of a generalized function and an ordinary function in an interval. Even and odd generalized functions, limits of generalized functions. Certain particular generalized functions, their properties and their Fourier transfoms: Non-integral powers, non-integral powers multiplied by logarithms. Integral powers, integral powers multiplied by logarithms. Fourier transform results.

*Asymptotic estimation of Fourier transforms*s

The Riemann-Lebesgue Lemma, generalizations of the Riemann Lebesgue lemma. The asymptotic expression for the Fourier transform of a function with a finite number of singularities.

Fourier series:

Convergence and uniqueness of trigonometric series as series of generalized functions. Determination of the coefficients in a trigonometric series. Existence of Fourier series representation for any periodic generalized function, Poisson's summation formula. Asymptotic behaviour of the coefficients in a Fourier series.

The scope of the syllabus is indicated by:

M. J. Lighthill's: Introduction to Fourier Analysis and Generalised Functions. (Cambridge, 1958).

(xiii) *Theory of Summability*:

General Theorems:

Linear Transformations, Regular transformations, the class T_r (of regular transformations), the classes T_c. and T^*_c. Necessary and sufficient conditions for $T \in T_r$ (Theorems 1, 2 and 3, from Hardy: Divergent series) Positive Transformations (Theorems 9 and 10 from Hardy: Divergent series) Knopp's Kernel Theorem.

Special Methods of Summations:

Nörlund Means, Regularity and consistency of Nörlund Means, inclusion, equivalence, Euler Means, Abelian Means. Complex methods, Lindelöf and Mittag-Leffler's Methods, Methods ineffective for the series $1 - 1 + 1 - 1 + \ldots$. Riesz's Typical Means.

Arithmetic Means:

Hölder's Means, Simple theorems concerning Hölder's summability, Cesàro's means. Means of non-integral orders. Simple theorems concerning Cesàro summability. Equivalence theorem. Mercer's theorem, Cesàro and Abel summability. Cesàro means as Nörlund means. Theorems concerning summable integrals.

Tauberian Theorems for Cesàro summability. Convergence factors.

Tauberian Theorems for Power series.

The Methods of Euler and Borel:

The (E,q) method. Simple properties of (E,q) method, the formal relation between Euler's and Borel's methods, Normal, Absolute and Regular summability. Applications to problems of Analytic Continuation. Tauberian theorems for Borel summability. Examples of series not summable (B).

Multiplications of Series:

Multiplication of summable series, Euler summability, Beorl summability, Dirichlet's multiplication.

Hausdorff Means:

The transformation $\mathfrak{d}$. Expressions of the (E, q) and (C, I) transformations in term of $\mathfrak{d}$. Hausdorff's general transformation. The general Hölder and Cesàro transformations as λ-transformation. Conditions for the regularity of a real Hausdorff transformation.

The scope of the syllabus is indicated by the relevant topics from Chapters III, IV, V, VI, VII, VIII, IX, X and XI of G. H. Hardy: Divergent Series (Oxford, 1949).

(xiv) *Projective, Affine and Metric Geometry*:

Metric and affine transformations. Vector, area, volume. Metric and affine classification of conics and quadrics and their properties. Inversion. Pencils of circles, conics, and quadrics. Confocal systems.

Cross-ratio. Involution. Projective coordinates. Duality. Collineations and correlations. Polarity and system. Generalisation by collineation. Projective classification of conics and quadrics and their properties. Theorems of Desargues, Pappus, Pascal, Brianchon etc.

Groups of transformations and classification of geometries. Sylvester's law of inertia of quadratic forms.

Books for reference:

Graustein: Higher Algebra.

R. N. Sen: A course of geometry.

(xv) *Fourier Series, Integral Transforms and Boundary Value and Eigen Value Problems*:

(i) *Fourier Series*: Othogonal sets of functions. Fourier Series. Fourier integrals. Solution of Boundary Value problems by the use of Fourier Series and Integrals. Uniqueness of solutions. Fourier-Bessel Expansions of functions. Expansion of functions in terms of Legendre polynomials. Applications to Boundary value problems.

(ii) *Integral Transforms*: Transforms of Fourier, Laplace, Hankel, and Mellin. Inversion theorems. Convolution theorems. Applications to boundary value problems.

(*iii*) Eigen functions and eigen values for various types of boundary conditions. Sturm-Liouville problem including degenerate case. Completeness and orthogonality of eigen functions. Series of eigen functions. Factorisation of Śturm-Liouville equation. Eigen functions and the variational principle. Separability of separation constants. Density of eigen values. Continuous distribution of eigen values. Discussion of Schröedinger's wave equation. Laplace and wave equations in cartesian, cylindrical and polar coordinates. Approximate methods of finding eigen values.

Books for reference:

Churchill: Fourier series and Boundary value problems.

Sneddon: Fourier transforms.

Tranter: Integral transforms.

Morse and Feschbach: Methods of theoretical physics, (relevant chapters).

Courant and Hilbert: Methods of Mathematical Physics.

(xvi) *Principles of Statistical Mechanics*:

(a) *Classical Theory*:

(i) *Geometry and Kinematics of phase-space.*

Theorems of Liouville and Birkhoff. Metric indecompasibility. Structure functions.

(ii) *The Ergodic Problems.*

(iii) *Derivation of the Maxwell-Boltzmann Law.*

Reduction to the problem of the theory of probability—application of the central limit theorem.

(iv) Foundations of thermodynamics, statistical interpretation of thermodynamic functions with reference to ideal monotomic gas.

(b) *Quantum Statistics*:

The one dimensional harmonic oscillator in the old theory.

Planck's radiation law. The two and three dimensional harmonic oscillators.

The partition function: transition to classical statistics. The rigid rotator, the hydrogen molecule.

Bose-Einstein and Fermi-Dirac Statistics.

Deviations from Boltzmann statistics. The probability aspects of statistics, the elementary method of statistics. Connection with classical thermodynamics. The perfect Boltzmann gas. The perfect Bose-Einstein gas. The perfect Fermi-Dirac gas.

Books for reference:

Khinchin: Statistical Mechanics (Dover).

D.ter Haar: Statistical Mechanics.

Khinchin: Quantum Statistics.

(xvii) *Principles of Quantum Mechanics*:

Mathematical tools:

The definition of Hilbert Space. The Geometry of Hilbert Space. Closed Linear Manifolds. Operators in Hilbert Space. The Eigen value Problem. Continuation. Initial Considerations concerning the Eigen value Problem. Digression on the Existence and Uniqueness of the Solutions of the Eigen value Problem. Commutative Operators. The Trace.

Original formulation of Quantum Mechanics:

Matrix mechanics of Heisenberg, Jordan and Born. Schrödinger's wave mechanics. Dynamical Variables and operators. Equivalence of the two theories. Transformation theory. Hilbert space.

Probabilistic interpretation of the wave function:

The fundamental basis of this interpretation and its deductive development by Von Neumann. Uncertainty principle.

Perturbation theory of the Schrödinger equation:

Interaction in non-relativistic quantum mechanics. Formal theory of scattering. Transition probabilities. Density of States.

Considerations of spin and relativity:

The Dirac equation for the free electron and its solution. Interpretation of negative energy states. Pauli principle. Bosons and Fermions and symmetrization of many-particle wave functions.

Books for reference:

Von Neumann: Mathematical foundations of Quantum Mechanics. (Princeton).

P. A. M. Dirac: Quantum mechanics (Oxford).

L. I. Schiff: Quantum mechanics (McGraw, Hill).

(xviii) *Theory of Waves and Vibrations*:

The Wave equation and its elementary solutions. Plane, spherical and Cylindrical waves. The principle of superposition. The principle of stationary phase.

The application of Lagrange's equations to small oscillations.

Normal modes of vibrating systems. General properties of vibrating systems.

Simple cases of the transverse vibrations of string, membranes and plates. Applications of Fourier's theorem.

Long waves in canals. Gravity waves on the surface of water and at a surface of discontinuity. The phenomenon of dispersion and the concept of group velocity. Cauchy-Poisson wave problem. Waves due to local impulse. The principle of stationary phase.

Plane and spherical sound waves. Solution for a source of simple harmonic waves. General theory.

Elastic waves in an infinite solid. Reflection and refraction of plane simple harmonic waves at a plane interface between two media. Rayleigh waves at the surface of a semi-infinite solid and Love waves at the surface of a solid with a surface layer.

Solution of Lamb's problem.

Simple cases of propagation and diffraction of electromagnetic waves.

Asymptotic approximations with special reference to the method of steepest descent. Application to some problems of wave propagation.

Solution of the initial value problem. Influence and dependence domains.

Huyghen's principle and the method of descent.

Books for reference:

K. O. Friedrich: The theory of wave propagation. (New York, University Institute of Mathematical Sciences, Lecture Notes, 1951-52).

Lord Rayleigh: The Theory of Sound.

Coulson: Waves.

Lamb: Hydrodynamics.

Love: Mathematical Theory of Elasticity.

Kolsky: Stress Waves in Solids.

Baker and Copson: Mathematical Theory of Huyghens' Principle.

Stratton: Electromagnetic Theory.

Schelkunoff: Electromagnetic Waves.

(xix) *Non-Linear Analysis:*

Numerical Methods of integrating differential equation. Graphical methods. Perturbation Method. Reversion Method. Averaging method based on Residuals (a) Gelarkin's method, (b) Ritz Method. Principle of Harmonic balance. Linear Difference Equations. Linear Differential Difference Equation. Non-linear Difference Equation. Non-linear Differential Difference Equation. Second order Equation with varying coefficients. Determination of the particular integral. W K B J (Wentzel, Kramers, Brillonin and Jeffreys) method. Approximate Solutions by variation of parameters. Location of stability boundaries by Perturbation method.

Stability of non-linear Systems.

Structural stability. Dynamical stability.

Test for stability of systems with non-oscillatory steady state. Test for stability of oscillating systems.

Book for reference:

Cunningham: Non-Linear Analysis.

(xx) *Boundary Layer Theory and Turbulence:*

(a) *Laminar Incompressible Boundary Layer Theory:*

Boundary layer equations for two-dimensional flows. General properties of boundary layer equations. Similar solutions. Blasius and Görtler's series method. Comparison of boundary layer solutions with the exact solutions of the Navier-Stokes equations. Stagnation point flow, couette flow, etc.

Approximate methods: Momentum and energy integrals. Karman-Pohlhausen method.

Introduction to thermal boundary layer.

(b) *Turbulence: Phenomenological Theories*:

Definition of Turbulence. Equation of Motion for Turbulent flows. Reynold's Stresses. Momentum transfer theory. Vorticity transfer theory. Diffusion in turbulent motion. Dissipation of energy. Flows in straight and smooth pipes and channels. Prandtl's seventh power law. Turbulent flow past a flat plate. Mixing zone between two parallel flows. Jets. Turbulent wake behind a row of parallel rods. Turbulent wake behind a symmetrical cylinder.

(c) *Elements of Statistical Theory of Turbulence*:

Equation for the conservation of a transferable scalar quantity in a turbulent flow. Double correlation between velocity components. Introduction of triple velocity correlations. Double longitudinal and lateral correlations in a homogeneous turbulence. Isotropic turbulence. Scale of turbulence. Diffusion of fluid particles. Lagrangian correlations. Taylor's one-dimensional energy spectrum. Energy relations in turbulent flows.

Books for reference:

H. Schlichting: Boundary Layer Theory.

S. Goldstein: Modern Developments in Fluid Dynamics.

J. O. Hinze: Turbulence.

H. Görtler: Journal of Mathematics and Mechanics, Vol. 6 (1957), pages 1-63.

L. Howarth: Handbuch der Physik, Vol. 8, 1959.

(xxi) *Magneto-Fluid Dynamics* (Optional):

Electromagnetic Equations for moving material. Equations of motion of an electrically conducting fluid. Equation of magnetic induction and its integral in the case of infinite conductivity. Introduction of dimensionless parameters and simplification of equations of magnetofluid dynamics for cosmical phenomena. Force-free fields and their stability. Alfven waves and their reflection and refraction at the inner face between two liquids. Waves in an electrically conducting viscous fluid with heat conduction. Splitting of hydromagnetic waves into longitudinal and transverse components. Propagation of periodic and aperiodic disturbances. Waves of finite amplitude in an infinitely conducting gas and shock phenomenon. Oscillations of infinitely conducting cylindrical and spherical masses under simple geometry of initial magnetic fields.

General discussion of the theories of production and maintenance of magnetic fields in cosmic bodies.

Extension of criterion of stability of fluid flows to include electromagnetic effects.

Plasmas:

Motion of a charged particle in the presence of magnetic field. Boltzmann's Equation including electromagnetic fields. Deduction of equations determining the motion of charged particles. Study of Plasmas regarding each type of particles forming a fluid and a plasma as a mixture of at least two such fluids. Concept of electrical conductivity in the presence of magnetic field. Oscillation of electron plasma with initial Magnetic field.

Note: Most of the material is available in current journals.

Books for reference:

T. G. Cowling: Magneto-hydrodynamics.

L. Spitzer (Jr.): Physics of Fully Ionised Gases.

F. H. Clauser (Editor): Plasma Dynamics.

(xxii) *Elasticity*:

Analysis of Stress:

Definition and Specification of stress. Body and surface forces. Stress tensor. Principal stress and Mohr's diagram. Differential equation of equilibrium and motion. Boundary conditions in terms of given surface forces. Transformation.

Analysis of Strain:

Specification of strain. Compatibility equations.

Existence of strain energy function. Form of the strain energy function in different cases. Generalised Hooke's Law (Dependence of strain on stress and temperature).

Boundary value problems. Uniqueness of solution in problems of equation and vibration. Variational methods.

Plane Strain and plane stress problems:

Complex representation of biharmonic functions. Complex representation of displacements and stresses. Application of conformal mappnig to two-dimensional problems. Stresses in rotating discs and cylinders. Plates with holes under simple tension or edge thrust.

Torsion and flexure:

Elastic waves—Love and Rayleigh wave.

Vibration of a sphere.

Bending and buckling theory of thin plates. Application of variational principles to solution when effect of transverse shear is taken into account.

Axisymmetric problems:

Boussinesq and Papkovitch functions in general. Three dimensional problem.

Non-linear elasticity:

Finite strain components. Elements of finite strain and finite deformation theory. Large deflection in bending of plates.

Books for Reference:

A. E. H. Love: Mathematical Theory of Elasticity.

I. S. Sokolnikoff: Theory of Elasticity.

N. I. Muskhelihvili: Some Basic Problem of Mathematical Theory of Elasticity.

(xxiii) *Plasticity*:

Invariants of stress tensor. Mohr's representation. Spherical and deviator stress. Octahedral stresses.

Invariants of strain tensor. Spherical and Deviator strain tensors. Octahedral strains.

Theories of failure:

Yield conditions. Yield surfaces.

Plastic stress-strain relations:

Elements of mechanism of flow including strain hardening. Creep. Viscosity. Effective stress. Effective strain.

Work of plastic deformation. Uniqueness of stress distribution under given boundary conditions. Extremum and variational principles.

Plastic flow of ideally plastic materials:

Thick walled spherical shell under external pressure.

Thick walled tube under internal pressure.

Rotating disc and cylinder.

Torsion and bending.

Solutions of Elastic-plastic problems:

Bending under plane strain.

Bending of a prismatic beam.

Plane plastic strain and theory of slip line field:

Plane strain equations referred to slip line. Geometry of slipline field. Discontinuities in the stress. Application to problems of compression of sheet between two parallel rough plates.

Plastic flow of strain-hardening materials:

Torsion of cylindrical bars of solid circular cross section.

Bending of prismatic bars.

Plastic buckling of compressed bars.

Instability of necking of a tensile test specimen.

Books for reference:

R. Hill: Mathematical Theory of Plasticity.

A. Phillips: Introduction of Plasticity.

W. Prager and P. G. Hodge: Theory to Perfectly Plastic Solids.

W. Praget: Introduction to Plasticity.

(xxiv) *Compressible Fluid Mechanics*:

Basic thermodynamics. Ideal gases and gases satisfying the condition $p = k\rho^{\gamma}$.

Eulerian equations of motion. Conservation of energy. General isentropic flow—subsonic and supersonic. Circulation theorem. Bernoulli's law. Limit and critical speeds. Steady and non-steady flows.

Steady isentropic channel flow. Laval Nozzle.

Solution of the equations of steady flow. The method of small perturbation. Linearized flow. Prandtl-Glauert motion. Hodograph methods. Methods of Chaplygin, Karman-Tsien and Karman.

Exact solution for isentropic flow. Prandtl-Meyer flow. Solution of flow equation by method of characteristics. Physical properties of characteristics. Domain of dependence, range of influence. Study of one dimensional flow by method of characteristics. Rarefaction and compression waves.

Normal shocks. Shock equations. Rankine-Hugoniot equation. Determinacy and basic properties of shock transitions. Prandtl relation. Shock strength.

Flow along a wall with a single bend and along wall with continuous bending. Flow between two walls.

Simple discussion of oblique shock waves. Shock polar diagram. Reflection of stationary and non-stationary oblique shocks and combination of shock waves. Mach reflection. Condensation shock. Brief treatment of detonation and deflagration waves.

Speed and pressure measurements.

Elements of the Internal and External Ballistics of Rockets.

Books for Reference:

Liepmann and Roshko: Aerodynamics of a Compressible fluid.

Milne-Thompson: Aerodynamics.

(xxv) *Ballistics*:

Internal Ballistics:

Introduction: The Energy and Dynamical Equations. The form function. The rate of burning. Solution up to all burnt. Maximum pressure. All burnt position. Solution after all burnt. Muzzle velocity. Effects of variations in loading conditions. Introduction to the American System.

External Ballistics:

Motion in vacuum. Form of Resistance Factor. Mach number and

Reynolds number. Motion with air resistance. Ballistic Coefficient. The Siacci solution. Problems in low angle fire. The Primary functions and their applications. Extension to higher angle of projection by the use of vitesse fictive. Small arc methods.

Meteorological factors:

Wind, temperature and humidity.

The dynamics of a rotating shell stability. Yaw. Drift.

Theory of Errors. Gaussian distribution. Probability factor table. Use of 50 per cent zones. Combination of random errors. Circular and elliptical probability zones. Dispersion in space.

(xxvi) *Internal Constitution of Stars*:

Physical Principles:

Study of problem. Perfect gas equations. Mean Molecular weight. fer as applied to Stellar interiors. Flux and Integrated flux of Radiation. Rosseland's correction to opacity. Thermodynamics of a system containing matter and radiation with special reference to specific heats. Polytropic changes. Steller opacity. Bound and Free transitions. Mean absorption coefficient for a Russell mixture. Elementary treatment of Electron scatterings. Thermonuclear reactions. Energy generation law. C-N cycle, P-P reaction.

Fundamental equations of stellar structure

Important Integral theorems. Uniform contraction and Lane's law. Total energy of a star and elementary discussion of its stability. Radiative and convectional modes of transfer of energy.

Models for Stellar Interiors:

Polytropes. Homology theorems. Various types of solutions of the Emden Equation. Composite configurations. Radiative models. Vogt-Russel Theorem. Brief survey of models introduced by Eddington and Milne. Recent work of connective core models.

White Dwarfs:

Degenerate matter. Equation of State. Complete degeneracy. Completely degenerate configurations. Composite configurations.

Books for reference:

S. Chandrasekhar: Stellar Structure.

M. Schwarzschild: Stellar structure and Evolution of stars.

(xxvii) *Stellar Atmospheres*:

Basic concepts of the theory of radiation. The equations of transfer. Radiative equilibrium of the stellar photosphere. The solution of equation of transfer. The theory of radiative equilibrium for an absorption coefficient independent of the frequency. The Coefficient of continuous absorption. The distribution of energy in the continuous spectra of stars for an absorption coefficient depending on frequency.

The structure of stellar photosphere. The application of the laws of thermodynamic equilibrium to photospheres.

The mechanism of formation of absorption lines in atmospheres. The solutions of the equations of transfer for frequencies inside absorption lines. The coefficients of selective absorption. Curves of growth and their interpretation. Non-coherent scattering. Central residual intensities. Methods of studying the Chemical Composition of stars. Spectral sequences.

The physics of the solar envelopes. The structure of solar photosphere. Granulation. Convection. The electro-dynamics of the Sun's atmosphere. Sunspots and faculae. Prominences. The chromosphere. The Corona and radio emission of the sun.

Books for reference:

Ambaratsumyan: Theoretical Astrophysics.

Rosseland: Theoretical Astrophysics.

Aller: The Atmospheres of the sun and stars, Vol. I.

4

Examination System

We strongly feel that the assessment of a student's performance and his calibre is the primary responsibility of his teacher. We have, however, taken note of the existing tradition of a primarily external examination and the administrative difficulties of ensuring uniformity of standards. After a good deal of thought we have come to the conclusion that at least at the M.A./M.Sc. level the following pattern of assessment should be introduced immediately.

1. METHOD OF ASSESSMENT:

(i) The teacher teaching a course should be primarily responsible for the assessment.

(ii) The assessment in any particular course should not be based on one single examination at the end of the course.

A certain percentage of marks (say 25 per cent of the aggregate to start with) should be allotted to periodic tests, consisting of short questions devised to test the student's grasp of the fundamental principles of the course covered during the period. These tests should be given at a short notice.

(iii) At the end of each year the university should conduct an examination on the courses covered during the year. The papers for this examination should be set by internal and external examiners jointly. The role of the external examiner should be to ensure that the standard of teaching, content and examination is maintained.

(iv) At the end of the full period of the courses for the degree there shall be in addition to the test on the courses taught during the final year, a viva voce examination (when a viva is not feasible a written examination may be given) to gauge the student's grasp of the fundamental principles. This test is not intended to test the student's knowledge of details of various courses studied by him.

2. Scheme of marking

(α) The papers should be marked independently by the external and internal examiners. In the exceptional cases of disagreement the average of the awards of the two examiners should be accepted as the final award.

(β) The marking of the papers may be numerical. But the broad classification of the result should be announced and not the numerical awards.

(γ) The students may be classed in the following divisions:

I (with distinction) to be awarded to exceptionally good candidates.

I

II

The pass percentage should be equivalent to the present 45 per cent.

3. Implementation:

At the universities where the M.A./MSc. teaching is centralised at one place and where qualified staff has been appointed, the scheme can be implemented without any qualifications.

At the universities where M.A./M.Sc. teaching is allowed at various places, the scheme may be modified to ensure that the examinations are held under the supervision of the university departments of Mathematics.

The Universities should be asked to allow M.A./M.Sc. teaching only at centres which fulfil the following minimum conditions:

(1) The centre has at least two teachers with the qualifications and emoluments of university readers and three with those of university lecturers.

(2) The centre has a library equipped with at least Rs. 10,000/- worth of books of post graduate level and spends at least Rs. 1,000/- p.a. on new books and journals.

(3) The centre has adequate buildings for staff, students and library.

सत्यमेव जयते

5

Future Developments

For more than half a century several Indians have made significant individual contributions to Mathematics. But generally speaking our progress in the subject has unfortunately lagged behind rapid developments in the rest of the world. A survey of the status of Mathematics in the Universities and Colleges, as indicated by the replies received to the questionnaire issued, shows that the developments have been both inadequate and lop-sided. A few places have produced good work on selected topics. Recently, a few centres of research have also come into existence, but still much remains to be achieved. The work done even in the same place has not been co-ordinated and most of it has been centred around single individuals. The results of this indifference on the part of the academic profession have been almost tragic. In a large number of places the teaching methods and the subjects are almost quarter of a century old, and no appreciable attempt on a nation wide scale has been made to train young men on modern lines.

It may not be very easy to give all the reasons for this unsatisfactory state of affairs. A number of factors, both political and social, have played an inhibitive part in all branches of learning. A general apathy on the part of the society to recognise creative work and its readiness to give undue importance to political and administrative activities have weaned away some of the best potential mathematicians into non-productive projects. The want of an early recognition and the failure to search for new talent are also responsible. This has resulted in a marked lack of academic fellowship amongst mathematicians. A sort of isolation, with workers both in their own fields and in allied fields, has crept in. The country has not explored the non-teaching opportunities for qualified mathematicians. It has been almost forgotten that the development of the subject is the responsibility of the community as a whole.

All these factors have contributed towards the migration of some of our best workers to other countries and, if it gives any consolation to think, they have made a mark wherever they have gone.

The present system of only one professorship and one or two readerships at most of the universities has also discouraged the growth of centres of research.

Interaction between groups interested in allied subjects has not been encouraged by the universities. Interchange of visiting teachers has been very infrequent. Teachers, therefore, have continued to work on the same problems year after year without getting any chance to take up problems of new type.

It is very encouraging to note that the University Grants Commission and other central bodies have now taken effective steps to foster advanced teaching and research. A large number of new professorships, readerships, fellowships, and scholarships have been created at various universities and colleges. It can now almost be said that no young man who is serious about his research will be left unsupported. The individual salary scales for lecturers, readers and professors have been suitably upgraded and they now compare favourably with those in competitive services. On retirement a number of incentives for renowned workers are now available in the form of national professorships, honoraria, etc. Perhaps sufficient publicity has not been given to the fact that a creative worker can rise to a national professorship which in honour and otherwise compares with any high office in the country.

Centres of Research

But the fact remains that no overall picture of coordinated and collaborative work has evolved. With this aim in view it is suggested that selected centres of research should be recognised and supported in different parts of the country. This is also necessitated by the fact that a large number of suitable posts and persons, especially senior ones, are not available. The University Grants Commission, wherever necessary, should assume full financial responsibility for the development of these centres and thus give a great impetus to the rapid development of the subject. When creating these centres the following points have to be kept in view:

(1) The University Grants Commission should recognise a selected number of centres in prescribed fields,

(2) The centres should form an integral part of the University or Institute of higher learning,

(3) Each centre should be formed around one or more distinguished workers in the field,

(4) The nucleus should consist of eight or ten active workers,

(5) The University Grants Commission should give grants on a 100 per cent basis to these centres for any additional expenditure on account of staff, library, equipment, buildings, publications, symposia, visiting professorships, casual visiting lecturerships, visits of members of other centres or the universities, conferences, etc.

(6) The division of the posts at the centres into various cadres should not be too rigid. If a member of the staff is deemed *at any time* by an expert committee to be fit for promotion to a higher rank, his post should be upgraded. Thus, the number of profesorships, readerships, and fellowships may vary within reasonable limits at the centre. The recruitment of all staff connected with the centre should be done by selection committees with which the University Grants Commission is actively associated.

(7) Admission to the centres should be on an All India basis and on merit alone.

(8) An adequate number of fellowships should be available at each centre. Each centre should have an intensive programme of study and research. It should also arrange symposia, summer schools, refresher courses for both college and school teachers.

(9) The centres may submit plans to invite visiting professors from other universities and countries. If approved, the University Grants Commission should provide all expenses thus incurred. Integrated plans extending over a number of years should be drawn up for visiting mathematicians. The visitors during

the period of one plan should be so chosen that their indivi-daul contributions supplement those of other visitors and members of the centre.

(10) The programme of every visiting professor should be drawn up in consultation with all other centes interested in the subject of specialisation so that maximum benefit may be taken from his visit.

It is obvious that these centres must cooperate with each other and have inter-centre plans. In this connection the following suggestions are made:

Inter-centre plans

(1) Besides its own field of work each centre should have a plan of study of allied fields for at least one term in each year. The seminars and courses in the allied fields should be conducted by experts invited from other centres or universities.

(2) Selected members including research fellows from each centre may go to other centres, their expenses being borne by the University Grants Commission.

(3) Visitors' boarding and lodging arrangements should be made by the host centre, and visitors should be paid some allowances in addition to their regular salaries from their parent institutions.

(4) A number of research workers are sometimes placed in such positions that even if they should very much like to, they cannot get away from their routine work, to do some extensive creative work in their fields. It is, therefore, desirable to set up on the basis of inter-university collaboration an Institute where workers can from time to time spend six months to a year without any formal duties. This will definitely give great impetus to creative work.

Teachers

All the recommendations made above will prove of no avail if we do not continue to get a stream of efficient teachers all over the country. The method of selection of teachers should, therefore, be improved and greater stress laid on research potential and achievement, particularly for the posts of readers and professors. In fixing the initial salary of a teacher the period spent by him in research should be given due recognition.

It is a fact that hundreds of teachers who have no contact with modern developments have to be retained in their present positions. They should, therefore, be required to undertake advanced training and to do creative work at these centres. Any further promotion should depend on the progress made by them.

Younger teachers should be able to get study leave early in their academic career. Senior teachers should be eligible for leave for one year once in every five years for academical work with bnefits which may be found necessary in each case.

Adequate library facilities and working rooms should be provided to the teachers and they should be given some leisure to think and not be loaded with extensive teaching work, as is the case in many places at present.

In short, the University Grants Commission should assume greater responsibility for sponsoring any projects which advance the cause of the subject by providing greater facilities to staff and students all over the country.

Centres of research in Mathematics

It is, therefore, recommended that the University Grants Commission, as a first step, should recognise the following centres of research in the country:

Group (a) — Non-university Instituitons: These centres will not need much financial help from the University Grants Commission. But their recognition will help to attract young men interested in higher training and research.

(i) Tata Institute of Fundamental Research, Bombay—Pure Mathematics.

(ii) Department of Mathematics, Indian Institute of Technology Kharagpur—Applied Mathematics.

Group (b) — The following centres will need substantial financial help to strengthen them:

(i) Department of Mathematics, Punjab University—Pure Mathematics.

(ii) Department of Mathematics, Delhi University—Pure Mathematics.

(iii) University of Madras—Pure Mathematics and Mathematical Physics.

(iv) Indian Institute of Science, Bangalore—Applied Mathematics.

(v) The Universities of Calcutta and Allahabad may be considered for recognition as centres when the vacant professorships are filled up.

6

Summary of Recommendations

1. (a) *Model syllabi* for under-graduate and post-graduate courses are recommended in order to indicate the standard of training desired in Mathematics and to prepare a student for service and research. The syllabi need to be reviewed and revised once in every five years.

(b) The extent of formal lectures should be reduced and reduction compensated by introduction of tutorials and seminars to develop the reasoning capacity and the habit of independent reading in the student.

2. Due recognition should be given to the sessional work in order to reduce the entire dependence on the annual examination only at the end of the course. In order to do this, a beginning may be made by alloting 25 per cent of the marks for sessional work and gradually increase it to 40 per cent or even 50 per cent.

3. There should be no Third Division awarded at the Master's degree level.

4. (a) The University Grants Commission should recognise a selected number of centres in prescribed fields. These centres should form an integral part of a university or an institute of higher learning.

(b) Each centre should be formed around a group of distinguished workers in the field. The nucleus should consist of eight or ten active workers.

(c) The University Grants Commission should give grants on a 100 per cent basis to these centres for any additional expenditure on account of staff, library, equipment, buildings, publications, symposia, visiting professorships, casual visiting lectureships, visits of members of other centres or universities, conferences, etc.

(d) The division of the posts at the centres into various cadres should not be too rigid. If a member of the staff is deemed *at any time* by an expert committee to be fit for promotion to a higher rank, his post should be upgraded. Thus the number of professorships, readerships, and fellowships may vary within reasonable limits at each centre. The recruitment of all staff connected with the centre should be done by selection committees with which the University Grants Commission is actively associated.

(e) Admission to the centres should be on an All-India basis and on merit alone. An adequate number of fellowships should be available at each centre. Each centre should have an intensive programme of study and research. It should also arrange symposia, summer schools, refresher courses for both college and school teachers.

(f) The centres may submit plans to invite visiting professors from other universities and countries. If approved, the University Grants Commission should provide all expenses thus incurred. Integrated plans extending over a number of years should be drawn up for visiting mathematicians. The visitors during the period of one plan should be so chosen that their invidiual contributions supplement those of other visitors and members of the centre. The programme of every visiting professor should be up in consultation with all other centres interested in the subject of specialisation so that maximum benefit may be taken from his visit.

(g) Centres must cooperate with each other and have inter-centre plans.

5. Creative workers should be encouraged and facilities should be given to them to get away from their routine work of teaching to do some research. For this purpose it is desirable to set up on the basis of inter-university collaboration an institute where the workers can come at their will to spend 6 months to a year without any formal duties.

6. Steps should be taken to recognise academic attainments and research abilities of persons. Steps may be evolved to get continuous supply of efficient teachers by proper methods of selection and give them necessary amenities. Stress may be laid on research

potential and achievements particularly for the senior posts. While fixing the initial salary of the teachers, the period spent by him in research should be given due reognition.

7. Such of the teachers who have for some reason or other detached themselves from research and modern developments should be required to undergo advanced training.

8. Younger teachers should be able to take study leave early in their career and all senior teachers should be eligible to get one year's leave once in five or six years for acadmical purposes.

9. (a) A university teacher engaged in research should not have a teaching load in excess of 9 hours per week. The head of the department, who also has to do the administrative work and guide research students, should not have more than 6 hours of teaching load per week. No teacher should be called upon to do more than 15 hours of teaching work including tutorials. Out of these 15 hours not more than 9 hours should be for post-graduate teaching.

(b) Suitable working rooms and library facilities should be provided to all workers.

10. Exchange of professors among the Indian Universities and visits by distinguished foreign scholars should be encouraged.

11. (a) The Univerity Grants Commission should give special importance to the organisation and holding of summer institutes, summer schools, symposia, etc., for university and college teachers and should make special provisions for this purpose in their budget.

(b) *Symposia*: Topics selected for the symposia must be highly specialised and the invitees are to be choosen on the basis of their current status in the subject. The duration of symposia can be 3 to 6 days and the proceedings may be published.

(c) *Summer Schools*: The purpose of summer schools could be to organise extensive lectures on few selected topics by invited lecturers and admission would be by application. Regarding the summer schools it may not be possible to organise a single summer school for the whole country. It is, therefore, suggested that regional summer schools be held and the venues for each region can be selected by rotation. The summer school should preferably be organised during long vacation to extend from a period to 4 to 6 weeks.

(d) *Summer Institutes*: The primary aim of summer institutes will be to bring together research workers for extensive discussions both formal and informal. The duration should be for 4 to 6 weeks. The participation should be by invitation.

12. Suitable publication grants may be given by the University Grants Commission to encourage publication of research material in recognised journals by individual research workers. An *ad hoc* grant may be given to each university to encourage original publication as well as for purchase of reprints of each of such publication for purposes of general distribution.

APPENDIX I

Proforma Circulated to Universities for Collection of Information

Information for the use of Mathematics Review Committee, 1959,

University of ——————————, ———————————————College.

Department of ——————————.

I. No. of *Admissions in the subject.*

Undergraduate			*Post graduate*		*Research*	
B.A.	B.Sc.	B.Sc. Hons.	M.A.	M.Sc.	Ph.D.	D.Sc.

II. Any facilities offered for optional/special papers at M.A./M.Sc. level.

III. Any field in which the department is specialising at research level.

IV. Total number of students taking Mathematics (Please exclude I.A. and I.Sc. enrolment).

V. (a) Teaching staff:

No. of Professors ——————————

No. of Readers ——————————

No. of Lecturers ——————————

Total ——————————

(b) Details of Teaching staff.

Name	*Designation*	*Pay scale*	*Academic qualifications*	*Teaching experience*	*Publications during past 5 years (1955 onwards) giving title, name of journal, etc.)*

Note: Please attach a separate sheet, if necessary)

VI. Library facilities in Mathematics, number of books and journals, Amount of annual grant available for the Department.

VII. Any suggestions for development of teaching and research in Mathematics in Indian Universities.

Signature ——————

Designation ——————

APPENDIX II

List of Universities and Colleges Providing Instruction in Mathematics at Post-Graduate Level with Subjects in Which Facilities For Research are Provided

Sl. No.	*Name of University*	*University Department*	*Teaching Centres*	*Subject of Specialization if any*
1.	Agra		1. Agra College, Agra.	
			2. B. R. College, Agra.	
			3. Bareilly College, Bareilly.	
			4. Christ Church College, Kanpur.	
			5. D.A.V. College, Dehra Dun.	
			6. D.A.V. College, Kanpur.	
			7. D.A.V. College, Muzaffarnagar.	
			8. Dharam Samaj College, Aligarh.	
			9. Hindu College, Moradabad.	
			10. K. N. Government College, Gyanpur.	

Sl. No.	*Name of University*	*University Department*	*Teaching Centres*	*Subject of Specialization if any*
			11. Meerut College, Meerut. 12. N.R.E.C. College, Khurja. 13. S.M. College, Chandausi. 14. St. John's College, Agra. 15. D.S.B. Govt. Degree College, Nainital.	
2.	Aligarh Muslim University	Department of Mathematics and Statistics		Theory of functions of a complex variable, Fourier Series
4.	Allahabad University	Department of Mathematics.		Summability of Trigonometrical Series
4.	Andhra University	University College of Arts and Commerce, Waltair.		Theory of functions, Pure Geometry, Eigenvalue Problems.
5.	Annamalai University	Department of Mathematics.		Theory of Infinite Series, Analytical Functions, Functional Analysis.
6.	Banaras Hindu University	University Department.		Integral Transforms.

Sl. No.	Name of University	University Department	Teaching Centres	Subject of Specialization if any
7.	Baroda University	Department of Mathematics.		
8.	Bihar University	Department of Mathematics.		
9.	Bombay University		1. Elphinstone College, Fort, Bombay. 2. Institute of Science, Fort, Bombay. 3. Ismail Yusuf College, Jogeshwari, Bombay 42. 4. Jai Hind College and Basant Singh Institute of Science, 23-24 Backbay Reclamation 'A' Road, Fort, Bombay. 5. Khalsa College, Matunga, Bombay. 6. Megji Mathradas Arts College, Chowpatty and Narrondass Manordass Institute of Science, Nav Gujrat, Andheri, Bombay 41. 7. Ram Narain Ruia College, Matunga, Bombay.	

Sl. No.	Name of University	University Department	Teaching Centres	Subject of Specialization if any
			8. Siddharath College of Arts and Science, Anand Bhavan, Fort, Dadabhai Nowroji Road, Fort, Bombay. 9. St. Xavier's College, Cruickshank Road, Bombay. 10. The D.G. Ruparel College, Tulsi Pipe Road, Mahim, Bombay. 11. Wilson College, Chowpatty, Bombay.	
10.	University of Calcutta.	University College of Science.		Theory of Elasticity, Fluid Mechanics.
11.	Delhi University.	Department of Mathematics and Statistics.		Differential Geometry
			1. Delhi College, Delhi. 2. Hans Raj College, Delhi. 3. Hindu College, Delhi. 4. Kirori Mal College, Delhi. 5. Miranda House, Delhi.	

Sl. No.	Name of University	University Department	Teaching Centres	Subject of Specialization if any
			6. Ramjas College, Delhi. 7. St. Stephen's College, Delhi.	
12.	Gauhati University	Department of Mathematics.		
13.	Gorakhpur University	Department of Mathematics		Differential Geometry.
14.	Gujarat University		1. Gujarat College, Ahmedabad. 2. M.T.B. College, Surat. 3. Samaldas College and Sir P.P. Institute of Science, Bhavanagar.	Theory of Relativity
15.	Jabalpur University		Mahakoshal Mahavidyalaya, Jabalpur.	
16.	Jadavpur University	University Department.		Theory of Elasticity
17.	Jammu and Kashmir University	Department of Mathematics.		
18.	Karnatak University	Department of Mathematics.		Differential Geometry.

Sl. No.	Name of University	University Department	Teaching Centres	Subject of Specialization if any
19.	Kerala University		1. H.H. The Maharaja's University College, Trivandrum. 2. Maharaja's College, Ernakulam.	
20.	Kuruksetra University	Department of Mathematics		Elastic Waves Homological Algebra
21.	Lucknow University	Department of Mathematics and Astronomy		Theory of Functions, Hydrodynamics.
22.	Madras University	Department of Mathematics	1. Loyola College, Madras. 2. Pachaiyapa's College, Kilpauk, Madras. 3. Presidency College, Chepauk, Madras. 4. Vivekananda College, Mylapore, Madras. 5. Madras Christian College, Tambaram. 6. Madura College, Madurai.	Algebra and Topology

Sl. No.	Name of University	University Department	Teaching Centres	Subject of Specialization if any
			7. St. Joseph's College, Tiruchirapalli.	
23.	Marathwada University	University Department.		
24.	Mysore University	University Department.	Central College, Bangalore.	
25.	Nagpur University		1. College of Science, Nagpur. 2. Vidarbha Mahavidyalaya, Amravati.	
26.	Osmania University	1. University College of Arts and Commerce, Hyderabad. 2. University College of Science, Hyderabad.		
27.	Panjab University	Department of Mathematics and Computation.	1. Mahendra College, Patiala. 2. D.A.V. College, Jullundur. 3. Government College, Ludhiana.	Number Theory, Geometry of numbers, Functional Analysis, Modern Algebra, Measure Theory, Probability

Sl. No.	Name of University	University Department	Teaching Centres	Subject of Specialization if any
28.	Patna University	Department of Mathematics		
29.	Poona University	Department of Mathematics and Statistics.	1. Rajaram College, Kolhapur. 2. Willingdon College, Sangli.	Mathematical Theory of Statistics
30.	Rajasthan University	Department of Mathematics, Jaipur.	1. Birla College of Science, Commerce and Pharmacy, Pilani. 2. Dungar College, Bikaner. 3. Government College, Ajmer. 4. Jaswant College. Jodhpur. 5. Maharaja's College, Jaipur. 6. M. B. College, Udaipur.	
31.	Roorkee University	Department of Applied Mathematics		
32.	Sardar Vallabhai Vidyapeeth.	Department of Mathematics.	Vithalbhai Patel Mahavidyalaya, Vallabh Vidyanagar.	

Sl. No.	Name of University	University Department	Teaching Centres	Subject of Specialization if any
33.	Saugar University	Department of Mathematics.	1. College of Science, Raipur. 2. Maharaja College, Chhatarpur. 3. T.R.S. College, Rewa.	
34.	S.N.D.T. Women's University			
35.	Sri Venkateswara University	Department of Mathematics.		Integral Transforms
36.	Utkal University		Ravenshaw College, Cuttack.	
37.	Vikram University		1. Government Hamidia College, Bhopal. 2. Holkar College, Indore. 3. Madhav College, Ujjain. 4. Maharani Laxmibai College, Gwalior.	
38.	Visva-Bharati University			
39.	Ranchi University			

Sl. No.	Name of University	University Department	Teaching Centres	Subject of Specialization if any
INSTITUTES				
1.	Tata Institute of Fundamental Research, Appolo Pier Road, Bombay.	School of Mathematics.		Summability Theory, Modern Algebra, Topology, Number Theory. Function Theory.
2.	Indian Institute of Technology, Kharagpur.	Department of Applied Mathematics.		Elasticity. Fluid Mechanics, High Speed Computations.
3.	Indian Institute of Science, Bangalore.	Department of Applied Mathematics		Fluid Mechanics, Magnetogasdynamics, Astrophysics, Theory of Functions.
4.	Indian Institute of Technology, Kanpur.	Department of Applied Mathematics.		Ballistics, Fluid Mechanics.

APPENDIX III

Titles of Doctoral Theses in Mathematics Accepted in Indian Universities During the Year 1955-1960

(The information available is included below. The names of those universities from where the information was not available or was nil, have not been included.)

University (1)	*Title* (2)	*Submitted by* (3)	*Year of Award* (4)	*Degrees awarded* (5)
1. *Agra*	i) Hankel Transform and self reciprocal functions.	Shri Ram Kumar	1956	Ph.D.
	ii) Some Problems in Hydro-dynamics.	Shri Jyoti Prasad Agarwal	1957	Ph.D.
	iii) Properties of divergence between a number of Multi-nomial (Multivariate) distributions.	Shri Prem Chand Cousal	1957	Ph.D.
	iv) Integral and Meromorphic Functions.	Shri Sankar Hari Dwivedi	1958	Ph.D.
	v) Some Problems in Operational Calculus.	Shri Suraj Chandra Mittal	1959	Ph.D.

(1)	(2)	(3)	(4)	(5)
2. *Aligarh*	i) Entire Functions.	Shri Mansoor Ahmad.	1955-56	Ph.D.
	ii) Entire Functions of Finite and Infinite Order.	Shri Q. I. Rahman	1956-57	Ph.D.
	iii) Finite Matrices and their characteristic roots.	Shri Nisar Ahmad Khan.	1957-58	Ph.D.
	iv) Quasi-Idempotent Matrices, U-Matrices and Characteristic Values and Moduli Matrices	Shri Marathe, C.R.	1958-59	Ph.D.
3. *Allahabad*	i) Contributions to the Study of Absolute Summability of Series.	Dr. Tribikram Pati	1956	D.Sc.
	ii) Strong Rieszian Summability of Infinite Series.	Dr. (Km.) Pramila Srivastava	1956	Ph.D.
	iii) Certain problems in Summability of Fourier Series.	Dr. Shri Niwas Bhatt	1957	Ph.D.
	iv) The Absolute Summability of Fourier Series and its conjugate Series.	Dr. Sri Rama Sinha	1958	Ph.D.
	v) Flow of Viscous Fluids Past Obstacles.	Dr. Hiral Lal Agarwal	1958	Ph.D.
	vi) Integral Functions	Dr. Kailash Nath Srivastava	1958	Ph.D.

(1)	(2)	(3)	(4)	(5)
	vii) The Convergence and Summability of Ultra-Spherical Series.	Dr. Dharam Prakash Gupta	1959	Ph.D.
4. *Annamalai*	i) Metric methods of Summability with special reference to Hausdorff and Quasi-Hausdorff methods.	Shri M. S. Ramanujan	1958	D.Ss.
	ii) Contribution to a study of some topics in Mathematical Statistics. Sequences of Independent Random variable distribution of quadratic forms and tests of hypotheses.	Shri G. Sankaranarayan	1959	Ph.D.
5. *Banaras*	i) An exploratory study in Riemannian geometry and general relativity.	Shri K. P. Singh	1955	Ph.D.
	ii) The equations of motion as deduced from the relativistic field equation.	Shri B. R. Rao	1956	Ph.D.
6. *Bombay*	i) The Lattice Point Problem	Shri V. Venugopal Rao	1957	Ph.D.

(1)	(2)	(3)	(4)	(5)
	for indefinite Quadratic forms with rational co-efficients.			
	ii) Some applications of complex analysis— Generalized multiplicative Meromorphic Functions on a Complex Analytic Manifold.	Shri C. S. Seshadri	1958	Ph.D
7. Delhi	i) Investigations in the Theory of Statistics.	Shri S. P. Agarwal	1956	Ph.D.
	ii) Differential geometry of sub-spaces.	Miss K. D. Singh	1956	Ph.D.
	iii) Differential Geometry of sub-spaces.	Shri M. K. Saighal	1957	Ph.D.
	iv) Differential geometry of ruled surface and rectilinear Congruences.	Shri Gorwara, K. P.	1957	Ph.D.
	v) Thesis on differential geometry, analytic manifold.	Miss G. Haldar	1957	Ph.D.
	vi) On Magneto-hydrodynamical Stability.	Shri R. K. Jaggi	1959-60	Ph.D.

(1)	(2)	(3)	(4)	(5)
	vii) Integral Ballistics, modurated charges.	Shri J. N. Kapur	1957	Ph.D.
	viii) Extensols.	Shri S. K. Kaul	1959	Ph.D.
	ix) Differential geometry of generalised Riemann spaces and Kachler Manifolds.	Shri S. C. Saxena	1958-59	Ph.D.
	x) Efficiency of Certain Sampling Designs.	Daroga Singh	1958-59	Ph.D.
	xi) Internal Ballistics of Guns.	Vinod Behari	1958-59	Ph.D.
	xii) Some Aspects of the Statistical Theory of Homogeneous Turbulence.	Padam Chand Jain	1958-59	Ph.D.
	xiii) Relativity and Unified Field Theory	Syed Izhar Hussain	1958-59	Ph.D.
8. *Gujarat*	A mass particle in a homogeneous cosmological model and its complete conversion into radiation in the scheme of the theory of general relativity.	Shri Shah Kanaiyalal Bhogilal	1960	Ph.D.
9. *Lucknow*	i) Study of Whittakar Transform, the Laplace transform	Shri S. K. Bose	1955	D.Sc.

(1)	(2)	(3)	(4)	(5)
	and the meromorphic and integral functions			
	ii) Study of certain generalisations of some special functions and polynomials	Shri A. M. Chak	1955	Ph.D.
	iii) Analytic functions of a complex variable	Shri Dinesh Chandra	1955	Ph.D.
	iv) Analytic functions of a complex variable	Shri M. K. Jain	1955	Ph.D.
	v) Application of Fractional Integration and differentiation to certain transforms and integrals	Krishna Ji Srivastava	1957	Ph.D.
	vi) A study of a generalised Laplace transform	Shri Roop Narain	1957	Ph.D
	vii) Study of certain Hypergeometric functions of three variables	Shri Shanti Saran	1955	Ph.D.
	viii) Astronomy in the seventh century in India: Bhaskara I and his works	Shri K. S. Shukla	1955	D.Litt.

(1)	(2)	(3)	(4)	(5)
	ix) Singular functions and symmetry of derivatives	Shri U. K. Shukla	1955	Ph.D.
	x) A study of certain Hypergeometric Identities	Shri Vishwa Nath Singh	1957	Ph.D.
	xi) A study of certain transformations of generalised bilateral hypergeometric series	Shri Hari Shankar Shukla	1959	Ph.D.
	xii) A study of the derivatives of Integral Functions and Meromorphic functions.	Shri Ram Prasad Srivastava	1958	Ph.D.
	xiii) Self reciprocal functions involving infinite series	Shri Ved Prakash Mainra	1958	Ph.D.
	xiv) Some aspects of superposability in Hydrodynamics	Shri Chandra Dutt Ghidyal	1958	Ph.D.
	xv) A study of Meijer Transform of one and two variables	Shri Amar Nath Mehra	1959	Ph.D.

(1)	(2)	(3)	(4)	(5)
	xvi) Theory of Transforms	Shri Vishnu Kant Verma	1959	Ph.D.
	xvii) On Interpolation and Mechanical Quadrature	Shri Rajendra Bahadur Saxena	1959	Ph.D.
10. *Madras*	Studies in the convergence of Topological and Uniform spaces	Mr. S. Swaminathan	1957	Ph.D.
11. *Nagpur*	i) Relative Growth of parts in some Indian Species of the Fresh-water Prawn of the Genus Plaemon	Shri R. K. Mishra	1959	Ph.D.
	ii) Some properties of special function of spherical Harmonics and Mathematical Physics	Shri B. R. Bhonsale	1959	Ph.D.
12. *Osmania*	i) Finite Deformation of Aelotropic and Isotropic hollow and Composite bodies	Mr. J. Rama Kanth	1956	Ph.D.
	ii) Some problems on Finite Deformation of Elastic Solids.	Mr. Mahfooz Ali Siddiqi	1956	Ph.D.
	iii) Studies in the Theory of Non-linear differential Equations	Mr. V. Lakshmikantam	1959	Ph.D.

(1)	(2)	(3)	(4)	(5)
	iv) Some Problems in the Mathematical Theory of Elasticity	Mr. G. Lakshminarayana	1960	Ph.D.
13. *Poona*	Contributions to the theory of Statistical Estimation	Mr. B. Raja Rao.	14.10.'59	Ph.D.
14. *Rajasthan*	Contributions to the study of Ballistics	Shri Gopichand Patni	1959	Ph.D.
15. *Saugar*	Strong Summability of Fourier Series and allied topics.	Shri Basudeo Singh	1958	Ph.D.

APPENDIX IV

Post-Graduate Enrolment at the M.Sc. and M.A. Levels in Mathematics in Indian Universities

(All figures given below refer to the year 1960-61)

University	*Enrolment in M.Sc. (Maths.) both Previous and Final*
1. Agra	452
2. Aligarh	22
3. Allahabad	196
4. Andhra	21
5. Annamalai	36
6. Banaras	58
7. Bihar	25
8. Baroda	16
9. Bombay	128
10. Calcutta	164
11. Delhi	15
12. Gauhati	52
13. Gorakhpur	57
14. Gujarat	32
15. Jabalpur	20

University		Enrolment in M.Sc. (Maths.) both Previous and Final
16. Jadavpur	..	63
17. Jammu and Kashmir	..	15
18. Karnatak	..	26
19. Kerala	..	86
20. Kurukshetra	..	—
21. Lucknow	..	100
22. Madras	..	246
23. Marathwada	..	—
24. Mysore	..	25
25. Nagpur	..	95
26. Osmania	..	24
27. Panjab	..	53
28. Poona	..	127
29. Patna	..	83
30. Rajasthan	..	88
31. Roorkee	..	—
32. Saugar	..	37
33. Ranchi	..	—
34. S. V. V. Peeth	..	10
35. S. N. D. T.	..	—
36. Sri Venkateswara	..	44
37. Utkal	..	20
38. Vikram	..	20
39. Visva Bharti	..	—
	Total	2,463

Post-Graduate Enrolment at the M.A. Level in Mathematics in Indian Universities (Affiliated Colleges Etc.)

University	*Enrolment in M.A. (Maths.) both Previous and Final*
1. Delhi	234
2. Marathwada	20
3. Agra	159
4. Gujarat	4
5. Kerala	37
6. Panjab	196
7. Saugar	—
8. Bombay	29
9. Nagpur	25
Total	704