ARTICLE III.

TRANSLATION

OF THE

SÛRYA-SIDDHÂNTA,

A TEXT-BOOK OF HINDU ASTRONOMY;

WITH NOTES, AND AN APPENDIX.

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Introductory Note.

Soon after my entrance upon the missionary field, in the Marâtha country of western India, in the year 1839, my attention was directed to the preparation, in the Marâthî language, of an astronomical text-book for schools. I was thus led to a study of the Hindu science of astronomy, as exhibited in the native text-books, and to an examination of what had been written respecting it by European scholars. I at once found myself, on the one hand, highly interested by the subject itself, and, on the other, somewhat embarrassed for want of a satisfactory introduction to it. A comprehensive exhibition of the Hindu system had nowhere been made. The Astronomie Indienne of Bailly, the first extended work upon its subject, had long been acknowledged to be founded upon insufficient data, to contain a greatly exaggerated estimate of the antiquity and value of the Hindu astronomy, and to have been written for the purpose of supporting an untenable theory. The articles in the Asiatic Researches, by Davis, Colebrooke, and Bentley, which were the first, as they still remain the most important, sources of knowledge respecting the matters with which they deal, relate only to particular points in the system, of especial prominence and interest. Bentley's volume on Hindu astronomy is mainly occupied with an endeavor to ascertain the age of the principal astronomical treatises, and the epochs of astronomical discovery and progress, and is, moreover, even in these respects, an exceedingly unsafe guide. The treatment of the subject by Delambre, in his History of Ancient Astronomy, being founded only upon Bailly and the earliest of the essays in the Asiatic Researches, partakes, of course, of the incompleteness of his authorities. Works of value have been published in India also, into which more or less of Hindu astronomy enters, as Warren's Kâla Sankalita, Jervis's Weights Measures and Coins of India, Hoisington's Oriental Astronomer, and the like; but these, too, give, for the most part, hardly more than the practical processes employed in parts of the system, and they are, like many of the authorities already mentioned, only with difficulty accessible. In short, there was nothing in existence which showed the world how much and how little the Hindus know of astronomy, as also their mode of presenting the subject in its totality, the intermixture in their science of old ideas with new, of astronomy with astrology, of observation and mathematical deduction with arbitrary theory, mythology, cosmogony, and pure imagination. It seemed to me that nothing would so well supply the deficiency as the translation and detailed explication of a complete treatise of Hindu astronomy: and this work I accordingly undertook to execute.

Among the different Siddhântas, or text-books of astronomy, existing in India in the Sanskrit language, none appeared better suited to my purpose than the Sûrya-Siddhânta. That it is one of the most highly esteemed, best known, and most frequently employed, of all, must be evident to any one who has noticed how much oftener than any other it is referred to as authority in the various papers on the Hindu astronomy. In fact, the science as practised in modern India is in the greater part founded upon its data and processes. In the lists of Siddhântas given by native authorities it is almost invariably mentioned second, the Brahma-Siddhânta being placed first: the latter enjoys this preminence, perhaps, mainly on account of its name; it is, at any rate, comparatively rare and little known. For completeness, simplicity, and conciseness combined, the Sûrya-Siddhânta is believed not to be surpassed by any other. It is also more easily obtainable. In general, it is difficult, without official influence or exorbitant pay, to gain possession of texts which are rare and held in high esteem. During my stay in India, I was able to procure copies of only three astronomical treatises besides the Sûrya-Siddhânta; the Çâkalya-Sanhitâ of the Brahma-Siddhânta, the Siddhânta-Çiromani of Bhâskara, and the Graha-Lâghava, of which the two latter have also been printed at Calcutta. Of the Sûrya-Siddhânta I obtained three copies, two of them giving the text alone, and the third also the commentary entitled Gûdhârthaprakâçaka, by Ranganâtha, of which the date is unknown to me. The latter manuscript agrees in all respects with the edition of the Sûrya-Siddhânta, accompanied by the same commentary, of which the publication, in the series entitled Bibliotheca Indica, has been commenced in India by an American scholar, and a member of this Society, Prof. Fitz-Edward Hall of Benares; to this I have also had access, although not until my work was nearly completed.

My first rough draft of the translation and notes was made while I was still in India, with the aid of Brahmans who were familiar with the Sanskrit and well versed in Hindu astronomical science. In a few points also I received help from the native Professor of Mathematics in the

Sanskrit College at Pûna. But notwithstanding this, there remained not a few obscure and difficult points, connected with the demonstration and application of the processes taught in the text. In the solution of these, I have received very important assistance from the Committee of Publication of the Society. They have also—the main share of the work falling to Prof. Whitney—enriched the notes with much additional matter of value. My whole collected material, in fact, was placed in their hands for revision, expansion, and reduction to the form best answering to the requirements of modern scholars, my own engrossing occupations, and distance from the place of publication, as well as my confidence in their ability and judgment, leading me to prefer to intrust this work to them rather than to undertake its execution myself.

We have also to express our acknowledgments to Mr. Hubert A. Newton, Professor of Mathematics in Yale College, for valuable aid rendered us in the more difficult demonstrations, and in the comparison of the Hindu and Greek astronomies, as well as for his constant advice and suggestions, which add not a little to the value of the work.

The Sûrya-Siddhânta, like the larger portion of the Sanskrit literature, is written in the verse commonly called the *çloka*, or in stanzas of two lines, each line being composed of two halves, or *pâdas*, of eight syllables each. With its metrical form are connected one or two peculiarities which call for notice. In the first place, for the terms used there are often many synonyms, which are employed according to the exigencies of the verse: thus, the sun has twelve different names, Mars six, the divisions of time two or three each, radius six or eight, and so on. Again, the method of expressing numbers, large or small, is by naming the figures which compose them, beginning with the last and going backward; using for each figure not only its own proper name, but that of any object associated in the Hindu mind with the number it represents. Thus, the number 1,577,917,828 (i. 37) is thus given: Vasu (a class of deities, *eight* in number) -two-eight-mountain (the *seven* mythical chains of mountains) -form-figure (the *nine* digits) -seven-mountain-lunar days (of which there are *fifteen* in the half-month). Once more, the style of expression of the treatise is, in general, excessively concise and elliptical, often to a degree that would make its meaning entirely unintelligible without a commentary, the exposition of a native teacher, or such a knowledge of the subject treated of as should show what the text must be meant to say. Some striking instances are pointed out in the notes. This over-conciseness, however, is not wholly due to the metrical form of the treatise: it is characteristic of much of the Hindu scientific literature, in its various branches; its text-books are wont to be intended as only the text for written comment or oral explication, and hint, rather than fully express, the meaning they contain. In our translation, we have not thought it worth while to indicate, by parentheses or otherwise, the words and phrases introduced by us to make the meaning of the text evident: such a course would occasion the reader much more embarrassment than satisfaction. Our endeavor is, in all cases, to hit the true mean between unintelligibility and diffuseness, altering the phraseology and construction of the original only so far as is necessary. In both the translation and the notes, moreover, we keep steadily in view the interests of the two classes of readers for whose benefit the work is undertaken: those who are orientalists without being astronomers, and those who are astronomers without being orientalists. For the sake of the former, our explanations and demonstrations are made more elementary and full than would be necessary, were we addressing mathematicians only: for the sake of the latter, we cast the whole into a form as occidental as may be, translating every technical term which admits of translation: since to compel all those who may desire to inform themselves respecting the scientific content of the Hindu astronomy to learn the Sanskrit technical language would be highly unreasonable. To furnish no ground of complaint, however, to those who are familiar with and attached to these terms, we insert them liberally in the translation, in connection with their English equivalents. The derivation and literal signification of the greater part of the technical terms employed in the treatise are also given in the notes, since such an explanation of the history of a term is often essential to its full comprehension, and throws valuable light upon the conceptions of those by whom it was originally applied.

We adopt, as the text of our translation, the published edition of the Siddhânta, referred to above, following its readings and its order of arrangement, wherever they differ, as they do in many places, from those of the manuscripts without commentary in our possession. The discordances of the two versions, when they are of sufficient consequence to be worth notice, are mentioned in the notes.

As regards the transcription of Sanskrit words in Roman letters, we need only specify that *c* represents the sound of the English *ch* in "church," Italian *c* before *e* and *i*: that *j* is the English *j*: that *ç* is pronounced like the English *sh*, German *sch*, French *ch*, while *sh* is a sound nearly resembling it, but uttered with the tip of the tongue turned back into the top of the mouth, as are the other lingual letters, *ṭ*, *ḍ*, *ṇ*: finally, that the Sanskrit *r* used as a vowel (which value it has also in some of the Slavonic dialects) is written with a dot underneath, as *ṛ*.

The demonstrations of principles and processes given by the native commentary are made without the help of figures. The figures which we introduce are for the most part our own, although a few of them were suggested by those of a set obtained in India, from native mathematicians.

For the discussion of such general questions relating to this Siddhânta as its age, its authorship, the alterations which it may have undergone before being brought into its present form, the stage which it represents in the progress of Hindu mathematical science, the extent and character of the mathematical and astronomical knowledge displayed in it, and the relation of the same to that of other ancient nations, especially of the Greeks, the reader is referred to the notes upon the text. The form in which our publication is made does not allow us to sum up here, in a preface, the final results of our investigations into these and kindred topics. It may perhaps be found advisable to present such a summary at the end of the article, in connection with the additional notes and other matters to be there given.

SÛRYA-SIDDHÂNTA.

CHAPTER I.

OF THE MEAN MOTIONS OF THE PLANETS.

CONTENTS:—1, homage to the Deity; 2–9, revelation of the present treatise; 10–11, modes of dividing time; 11–12, subdivisions of a day; 12–14, of a year; 14–17, of the Ages; 18–19, of an Æon; 20–21, of Brahma's life; 21–23, part of it already elapsed; 24, time occupied in the work of creation; 25–27, general account of the movements of the planets; 28, subdivisions of the circle; 29–33, number of revolutions of the planets, and of the moon's apsis and node, in an Age; 34–39, number of days and months, of different kinds, in an Age; 40, in an Æon; 41–44, number of revolutions, in an Æon, of the apsides and nodes of the planets; 45–47, time elapsed from the end of creation to that of the Golden Age; 48–51, rule for the reduction to civil days of the whole time since the creation; 51–52, method of finding the lords of the day, the month, and the year; 53–54, rule for finding the mean place of a planet, and of its apsis and node; 55, to find the current year of the cycle of Jupiter; 56, simplification of the above calculations; 57–58, situation of the planets, and of the moon's apsis and node, at the end of the Golden Age; 59–60, dimensions of the earth; 60–61, correction, for difference of longitude, of the mean place of a planet as found; 62, situation of the principal meridian; 63–65, ascertainment of difference of longitude by difference between observed and computed time of a lunar eclipse; 66, difference of time owing to difference of longitude; 67, to find the mean place of a planet for any required hour of the day; 68–70, inclination of the orbits of the planets.

1. To him whose shape is inconceivable and unmanifested, who is unaffected by the qualities, whose nature is quality, whose form is the support of the entire creation—to Brahma be homage!

The usual propitiatory expression of homage to some deity, with which Hindu works are wont to commence.

2. When but little of the Golden Age (*kṛta yuga*) was left, a great demon (*asura*), named Maya,ˣ being desirous to know that mysterious, supreme, pure, and exalted science,

3. That chief auxiliary of the scripture (*vedânga*), in its entirety—the cause, namely, of the motion of the heavenly bodies (*jyotis*), performed, in propitiation of the Sun, very severe religious austerities.

x See JRAS 1917 p. 132 where Mr K. R. V. Raja identifies Asura with the Assyrian god Ahura, or the Assur.

According to this, the Sûrya-Siddhânta was revealed more than 2,164,960 years ago, that amount of time having elapsed, according to Hindu reckoning, since the end of the Golden Age; see below, under verse 48, for the computation of the period. As regards the actual date of the treatise, it is, like all dates in Hindu history and the history of Hindu literature, exceedingly difficult to ascertain. It is the more difficult, because, unlike most, or all, of the astronomical treatises, the Sûrya-Siddhânta attaches itself to the name of no individual as its author, but professes to be a direct revelation from the Sun (*sûrya*). A treatise of this name, however, is confessedly among the earliest text-books of the Indian science. It was one of the five earlier works upon which was founded the Pañca-siddhântika, Compendium of Five Astronomies, of Varâha-mihira, one of the earliest astronomers whose works have been, in part, preserved to us, and who is supposed to have lived about the beginning of the sixth century of our era. A Sûrya-Siddhânta is also referred to by Brahmagupta, who is assigned to the close of the same century and the commencement of the one following. The arguments by which Mr. Bentley (Hindu Astronomy, p. 158, etc.) attempts to prove Varâha-mihira to have lived in the sixteenth century, and his professed works to be forgeries and impositions, are sufficiently refuted by the testimony of al-Bîrûnî (the same person as the Abu-r-Raiḥân, so often quoted in the first article of this volume), who visited India under Mahmûd of Ghazna, and wrote in A.D. 1031 an account of the country: he speaks of Varâha-mihira and of his Pañca-siddhântika, assigning to both nearly the same age as is attributed to them by the modern Hindus (see Reinaud in the Journal Asiatique for Sept.–Oct. 1844, iv[me] Série, iv. 286; and also his Mémoire sur l'Inde). He also speaks of the Sûrya-Siddhânta itself, and ascribes its authorship to Lâta (Mémoire sur l'Inde, pp. 331, 332), whom Weber (Vorlesungen über Indische Literaturgeschichte, p. 229) conjecturally identifies with a Lâḍha who is cited by Brahmagupta. Bentley has endeavored to show by internal evidence that the Sûrya-Siddhânta belongs to the end of the eleventh century: see below, under verses 29–34, where his method and results are explained, and their value estimated.

Of the six Vedângas, "limbs of the Veda," sciences auxiliary to the sacred scriptures, astronomy is claimed to be the first and chief, as representing the eyes; grammar being the mouth, ceremonial the hands, prosody the feet, etc. (see Siddhânta-Çiromaṇi, i. 12–14). The importance of astronomy to the system of religious observance lies in the fact that by it are determined the proper times of sacrifice and the like. There is a special treatise, the Jyotisha of Lagadha, or Lagata, which, attaching itself to the Vedic texts, and representing a more primitive phase of Hindu science, claims to be the astronomical Vedânga; but it is said to be of late date and of small importance.

The word *jyotis*, "heavenly body," literally "light," although the current names for astronomy and astronomers are derived from it, does not elsewhere occur in this treatise.

4. Gratified by these austerities, and rendered propitious, the Sun himself delivered unto that Maya, who besought a boon, the system of the planets.

The blessed Sun spoke:

5. Thine intent is known to me; I am gratified by thine austerities; I will give thee the science upon which time is founded, the grand system of the planets.

6. No one is able to endure my brilliancy; for communication I have no leisure; this person, who is a part of me, shall relate to thee the whole.

The manuscripts without commentary insert here the following verse: "Go therefore to Romaka-city, thine own residence; there, undergoing incarnation as a barbarian, owing to a curse of Brahma, I will impart to thee this science."

If this verse really formed a part of the text, it would be as clear an acknowledgment as the author could well convey indirectly, that the science displayed in his treatise was derived from the Greeks. Romaka-city is Rome, the great metropolis of the West; its situation is given in a following chapter (see xii. 39) as upon the equator, ninety degrees to the west of India. The incarnation of the sun there as a barbarian, for the purpose of revealing astronomy to a demon of the Hindu Pantheon, is but a transparent artifice for referring the foreign science, after all, to a Hindu origin. But the verse is clearly out of place here; it is inconsistent with the other verses among which it occurs, which give a different version of the method of revelation. How comes it here then? It can hardly have been gratuitously devised and introduced. The verse itself is found in many of the manuscripts of this Siddhânta; and the incarnation of the Sun at Romaka-city, among the Yavanas, or Greeks, and his revelation of the science of astronomy there, are variously alluded to in later works; as, for instance, in the Jñâna-bhâskara (see Weber's Catalogue of the Berlin Sanskrit Manuscripts, p. 287, etc.), where he is asserted to have revealed also the Romaka-Siddhânta. Is this verse, then, a fragment of a different, and perhaps more ancient, account of the origin of the treatise, for which, as conveying too ingenuous a confession of the source of the Hindu astronomy, another has been substituted later? Such a supposition, certainly, does not lack plausibility. There is something which looks the same way in the selection of a demon, an Asura, to be the medium of the sun's revelation; as if, while the essential truth and value of the system was acknowledged, it were sought to affix a stigma to the source whence the Hindus derived it. Weber (Ind. Stud. ii. 243; Ind. Lit. p. 225), noticing that the name of the Egyptian sovereign Ptolemaios occurs in Indian inscriptions in the form *Turamaya*, conjectures that Asura Maya is an alteration of that name, and that the demon Maya accordingly represents the author of the Almagest himself; and the conjecture is powerfully supported by the fact that al-Bîrûnî (see Reinaud, as above) ascribes the Pâuliça-Siddhânta, which the later Hindus attribute to a Puliça, to Paulus al-Yûnânî, Paulus the Greek, and that another of the astronomical treatises, alluded to above, is called the Romaka-Siddhânta.

It would be premature to discuss here the relation of the Hindu astronomy to the Greek; we propose to sum up, at the end of this work, the evidence upon the subject which it contains.

7. Thus having spoken, the god disappeared, having given directions unto the part of himself. This latter person thus addressed Maya, as he stood bowed forward, his hands suppliantly joined before him:

8. Listen with concentrated attention to the ancient and exalted science, which has been spoken, in each successive Age, to the Great Sages (*maharshi*), by the Sun himself.

9. This is that very same original text-book which the Sun of old promulgated: only, by reason of the revolution of the Ages, there is here a difference of times.

According to the commentary, the meaning of these last verses is that, in the successive Great Ages, or periods of 4,320,000 years (see below, under vv. 15–17), there are slight differences in the motions of the heavenly bodies, which render necessary a new revelation from time to time on the part of the Sun, suited to the altered conditions of things; and that when, moreover, even during the continuance of the same Age, differences of motion are noticed, owing to a difference of period, it is customary to apply to the data given a correction, which is called *bîja.* All this is very suitable for the commentator to say, but it seems not a little curious to find the Sun's superhuman representative himself insisting that this his revelation is the same one as had formerly been made by the Sun, only with different data. We cannot help suspecting in the ninth verse, rather, a virtual confession on the part of the promulgators of this treatise, that there was another, or that there were others, in existence, claiming to be the sun's revelation, or else that the data presented in this were different from those which had been previously current as revealed by the Sun. We shall have more to say hereafter (see below, under vv. 29–34) of the probable existence of more than one version of the Sûrya-Siddhânta, of the correction called *bîja*, and of its incorporation into the text of the treatise itself. The repeated revelation of the system in each successive Great Age, as stated in verse 8, presents no difficulty. It is the Puranic doctrine (see Wilson's Vishnu Purâṇa, p. 269, etc.) that during the Iron Age the sources of knowledge become either corrupted or lost, so that a new revelation of scripture, law, and science becomes necessary during the Age succeeding.

10. Time is the destroyer of the worlds; another Time has for its nature to bring to pass. This latter, according as it is gross or minute, is called by two names, real (*mûrta*) and unreal (*amûrta*).

There is in this verse a curious mingling together of the poetical, the theoretical, and the practical. To the Hindus, as to us, Time is, in a metaphorical sense, the great destroyer of all things; as such, he is identified with Death, and with Yama, the ruler of the dead. Time, again, in the ordinary acceptation of the word, has both its imaginary, and its appreciable and practically useful divisions: the former are called real (*mûrta*, literally "embodied"), the latter unreal (*amûrta*, literally "unembodied"). The following verse explains these divisions more fully.

The epithet *kalanâtmaka*, applied to actual time in the first half of the verse, is not easy of interpretation. The commentary translates it "is an object of knowledge, is capable of being known," which does not seem satisfactory. It evidently contains a suggested etymology (*kâla*, "time," from *kalana*), and in translating it as above we have seen in it also an antithesis to the epithet bestowed upon Time the divinity. Perhaps it should be rather "has for its office enumeration."

11. That which begins with respirations (*prâṇa*) is called real; that which begins with atoms (*truṭi*) is called unreal. Six respirations make a *vinâḍî*, sixty of these a *nâḍî*;

12. And sixty nâḍîs make a sidereal day and night. . . .

The manuscripts without commentary insert, as the first half of v. 11, the usual definition of the length of a respiration: "the time occupied in pronouncing ten long syllables is called a respiration."

The table of the divisions of sidereal time is then as follows:

10 long syllables (*gurvakshara*)	= 1 respiration (*prâṇa*, period of four seconds);
6 respirations	= 1 vinâḍî (period of twenty-four seconds);
60 vinâḍîs	= 1 nâḍî (period of twenty-four minutes);
60 nâḍîs	= 1 day.

This is the method of division usually adopted in the astronomical text-books: it possesses the convenient property that its lowest subdivision, the respiration, is the same part of the day as the minute is of the circle, so that a respiration of time is equivalent to a minute of revolution of the heavenly bodies about the earth. The respiration is much more frequently called *asu*, in the text both of this and of the other Siddhântas. The vinâḍî is practically of small consequence, and is only two or three times made use of in the treatise: its usual modern name is *pala*, but as this term nowhere occurs in our text, we have not felt justified in substituting it for vinâḍî. For nâḍî also, the more common name is *daṇḍa*, but this, too, the Sûrya-Siddhânta nowhere employs, although it uses instead of nâḍî, and quite as often, *nâḍikâ* and *ghaṭikâ*. We shall uniformly make use in our translation of the terms presented above, since there are no English equivalents which admit of being substituted for them.

The ordinary Puranic division of the day is slightly different from the astronomical, viz:

15 twinklings (*nimesha*)	= 1 bit (*kâshthâ*);
30 bits	= 1 minute (*kalâ*);
30 minutes	= 1 hour (*muhûrta*);
30 hours	= 1 day.

Manu (i. 64) gives the same, excepting that he makes the bit to consist of 18 twinklings. Other authorities assign different values to the lesser measures of time, but all agree in the main fact of the division of the day into thirty hours, which, being perhaps an imitation of the division of the month into thirty days, is unquestionably the ancient and original Hindu method of reckoning time.

The Sûrya-Siddhânta, with commendable moderation, refrains from giving the imaginary subdivisions of the respiration which make up "unreal" time. They are thus stated in Bhâskara's Siddhânta-Çiromaṇi (i. 19, 20), along with the other, the astronomical, table:

100 atoms (*truṭi*)	= 1 speck (*tatpara*);
30 specks	= 1 twinkling (*nimesha*);
18 twinklings	= 1 bit (*kâshṭhâ*);
30 bits	= 1 minute (*kalâ*);
30 minutes	= 1 half-hour (*ghaṭikâ*);
2 half-hours	= 1 hour (*kshaṇa*);
30 hours	= 1 day.

This makes the atom equal to $\frac{1}{2\,916\,000\,000}$th of a day, or $\frac{1}{33\,750}$th of a second. Some of the Purâṇas (see Wilson's Vish. Pur. p. 22) give a different division, which makes the atom about $\frac{1}{2\,110}$th of a second; but they carry the division three steps farther, to the subtilissima (*paramâṇu*), which equals $\frac{1}{3\,280\,500\,000}$th of a day, or very nearly $\frac{1}{38\,000}$th of a second.

We have introduced here a statement of these minute subdivisions, because they form a natural counterpart to the immense periods which we shall soon have to consider, and are, with the latter, curiously illustrative of a fundamental trait of Hindu character: a fantastic imaginativeness, which delights itself with arbitrary theorizings, and is unrestrained by, and careless of, actual realities. Thus, having no instruments by which they could measure even seconds with any tolerable precision, they vied with one another in dividing the second down to the farthest conceivable limit of minuteness; thus, seeking infinity in the other direction also, while they were almost destitute of a chronology or a history, and could hardly fix with accuracy the date of any event beyond the memory of the living generation, they devised, and put forth as actual, a framework of chronology reaching for millions of millions of years back into the past and forward into the future.

12. . . . Of thirty of these sidereal days is composed a month; a civil (*sâvana*) month consists of as many sunrises;

13. A lunar month, of as many lunar days (*tithi*); a solar (*sâura*) month is determined by the entrance of the sun into a sign of the zodiac: twelve months make a year. . . .

We have here described days of three different kinds, and months and years of four; since, according to the commentary, the last clause translated means that twelve months of each denomination make up a year of the same denomination. Of some of these, the practical use and value will be made to appear later; but as others are not elsewhere referred to in this treatise, and as several are merely arbitrary divisions of time, of which, so far as we can discover, no use has ever been made, it may not be amiss briefly to characterize them here.

Of the measures of time referred to in the twelfth verse, the day is evidently the starting-point and standard. The sidereal day is the time of the earth's revolution on its axis; data for determining its length are given below, in v. 34, but it does not enter as an element into the later processes. Nor is a sidereal month of thirty sidereal days, or a sidereal year of three hundred and sixty such days (being less than the true sidereal year by about six and a quarter sidereal days), elsewhere mentioned in this work, or, so far as we know, made account of in any Hindu method of reckoning time. The civil (*sâvana*) day is the natural day: it is counted, in India, from sunrise to sunrise (see below, v. 36), and is accordingly of variable length: it is, of course, an important element in all computations of time. A month of thirty, and a year of three hundred and sixty, such days, are supposed to have formed the basis of the earliest Hindu chronology, an intercalary month being added once in five years. This method is long since out of use, however, and the month and year referred to here in the text, of thirty and three hundred and sixty natural days respectively, without intercalations, are elsewhere assumed and made use of only in determining, for astrological purposes, the lords of the month and year (see below, v. 52).

The standard of the lunar measure of time is the lunar month, the period of the moon's synodical revolution. It is reckoned either from new-moon to new-moon, or from full-moon to full-moon; generally, the former is called *mukhya*, "primary," and the latter *gâuna*, "secondary": but, according to our commentator, either of them may be denominated primary, although in fact, in this treatise, only the first of them is so regarded; and the secondary lunar month is that which is reckoned from any given lunar day to the next of the same name. This natural month, containing about twenty-nine and a half days, mean solar time, is then divided into thirty lunar days (*tithi*), and this division, although of so unnatural and arbitrary a character, the lunar days beginning and ending at any moment of the natural day and night, is, to the Hindu, of the most prominent practical importance, since by it are regulated the performance of many religious ceremonies (see below, xiv. 13), and upon it depend the chief considerations of propitious and unpropitious times, and the like. Of the lunar year of twelve lunar months, however, we know of no use made in India, either formerly or now, except as it has been introduced and employed by the Mohammedans.

Finally, the year last mentioned, the solar year, is that by which time is ordinarily reckoned in India. It is, however, not the tropical solar year, which we employ, but the sidereal, no account being made of the precession of the equinoxes. The solar month is measured by the continuance of the sun in each successive sign, and varies, according to the rapidity of his motion, from about twenty-nine and a third, to a little more than thirty-one and a half, days. There is no day corresponding to this measure of the month and of the year.

In the ordinary reckoning of time, these elements are variously combined. Throughout Southern India (see Warren's Kâla Sankalita, Madras: 1825, p. 4, etc.), the year and month made use of are the solar, and the day the civil; the beginning of each month and year being counted, in practice, from the sunrise nearest to the moment of their actual commencement. In all Northern India the year is luni-solar; the month is lunar, and is divided into both lunar and civil days; the year is composed of a variable number of months, either twelve or thirteen, beginning always with the lunar month of which the commencement next precedes the true commencement of the sidereal year. But, underneath this division, the division of the actual sidereal year into twelve solar months is likewise kept up, and to maintain the concurrence of the civil and lunar days, and the lunar and solar months, is a process of great complexity, into the details of which we need not enter here (see Warren, as above, p. 57, etc.). It will be seen later in this chapter (vv. 48–51) that the Sûrya-Siddhânta reckons time by this latter system, by the combination of civil, lunar, and sidereal elements.

13. . . . This is called a day of the gods.

14. The day and night of the gods and of the demons are mutually opposed to one another. Six times sixty of them are a year of the gods, and likewise of the demons.

"This is called," etc.: that is, as the commentary explains, the year composed of twelve solar months, as being those last mentioned; the sidereal year. It appears to us very questionable whether, in the first instance, anything more was meant by calling the year a day of the gods than to intimate that those beings of a higher order reckoned time upon a grander scale: just as the month was said to be a day of the Fathers, or Manes (xiv. 14), the Patriarchate (v. 18), a day of the Patriarchs (xiv. 21), and the Æon (v. 20), a day of Brahma; all these being familiar Puranic designations. In the astronomical reconstruction of the Puranic system, however, a physical meaning has been given to this day of the gods: the gods are made to reside at the north pole, and the demons at the south; and then, of course, during the half-year when the sun is north of the equator, it is day to the gods and night to the demons; and during the other half-year, the contrary. The subject is dwelt upon at some length in the twelfth chapter (xii. 45, etc.). To make such a division accurate, the year ought to be the tropical, and not the sidereal; but the author of the Sûrya-Siddhânta has not yet begun to take into account the precession. See what is said upon this subject in the third chapter (vv. 9–10).

The year of the gods, or the divine year, is employed only in describing the immense periods of which the statement now follows.

15. Twelve thousand of these divine years are denominated a Quadruple Age (*caturyuga*); of ten thousand times four hundred and thirty-two solar years

16. Is composed that Quadruple Age, with its dawn and twilight. The difference of the Golden and the other Ages, as measured by the difference in the number of the feet of Virtue in each, is as follows:

17. The tenth part of an Age, multiplied successively by four, three, two, and one, gives the length of the Golden and the other Ages, in order: the sixth part of each belongs to its dawn and twilight.

The period of 4,320,000 years is ordinarily styled Great Age (*mahâyuga*), or, as above in two instances, Quadruple Age (*caturyuga*). In the Sûrya-Siddhânta, however, the former term is not once found, and the latter occurs only in these verses; elsewhere, Age (*yuga*) alone is employed to denote it; and always denotes it, unless expressly limited by the name of the Golden (*kṛta*) Age.

The composition of the Age, or Great Age, is then as follows:

	Divine years.		Solar years.	
Dawn,	400		144,000	
Golden Age (*kṛta yuga*),	4000		1,440,000	
Twilight,	400		144,000	
Total duration of the Golden Age,		4,800		1,728,000
Dawn,	300		108,000	
Silver Age (*tretâ yuga*),	3000		1,080,000	
Twilight,	300		108,000	
Total duration of the Silver Age,		3,600		1,296,000
Dawn,	200		72,000	
Brazen Age (*dvâpara yuga*),	2000		720,000	
Twilight,	200		72,000	
Total duration of the Brazen Age,		2,400		864,000
Dawn,	100		36,000	
Iron Age (*kali yuga*),	1000		360,000	
Twilight,	100		36,000	
Total duration of the Iron Age,		1,200		432,000
Total duration of a Great Age,		12,000		4,320,000

Neither of the names of the last three ages is once mentioned in the Sûrya-Siddhânta. The first and last of the four are derived from the game of dice: *kṛta*, "made, won," is the side of the die marked with four dots—the lucky, or winning one; *kali* is the side marked with one dot only—the unfortunate, the losing one. In the other names, of which we do not know the original and proper meaning, the numerals *tri*, "three," and *dvâ*, "two," are plainly recognizable. The relation of the numbers four, three, two, and one, to the length of the several periods, as expressed in divine years, and also as compared with one another, is not less clearly apparent. The character attached to the different Ages by the Hindu mythological and legendary history so closely resembles that which is attributed to the Golden, Silver, Brazen, and Iron Ages, that we have not hesitated to transfer to them the latter appellations. An account of this character is given in Manu i. 81–86. During the Golden Age, Virtue stands firm upon four feet, truth and justice abound, and the life of man is four centuries; in each following Age Virtue loses a foot, and the length of life is reduced by a century, so that in the present, the Iron Age, she has but one left to hobble upon, while the extreme age attained by mortals is but a hundred years. See also Wilson's Vishṇu Purâṇa, p. 622, etc., for a description of the vices of the Iron Age.

This system of periods is not of astronomical origin, although the fixing of the commencement of the Iron Age, the only possibly historical point in it, is, as we shall see hereafter, the result of astronomical computation. Its arbitrary and artificial character is apparent. It is the system of the Purâṇas and of Manu, a part of the received Hindu cosmogony, to which astronomy was compelled to adapt itself.

We ought to remark, however, that in the text itself of Manu (i. 68–71) the duration of the Great Age, called by him Divine Age, is given as twelve thousand years simply, and that it is his commentator who, by asserting these to be divine years, brings Manu's cosmogony to an agreement with that of the Purâṇas. This is a strong indication that the divine year is an afterthought, and that the period of 4,320,000 years is an expansion of an earlier one of 12,000. Vast as this period is, however, it is far from satisfying the Hindu craving after infinity. We are next called upon to construct a new period by multiplying it by a thousand.

18. One and seventy Ages are styled here a Patriarchate (*manvantara*); at its end is said to be a twilight which has the number of years of a Golden Age, and which is a deluge.

19. In an Æon (*kalpa*) are reckoned fourteen such Patriarchs (*manu*) with their respective twilights; at the commencement of the Æon is a fifteenth dawn, having the length of a Golden Age.

The Æon is accordingly thus composed:

	Divine years.		Solar years.	
The introductory dawn,		4,800		1,728,000
Seventy-one Great Ages,	852,000		306,720,000	
A twilight,	4,800		1,728,000	
Duration of one Patriarchate,	856,800		308,448,000	
Fourteen Patriarchates,		11,995,200		4,318,272,000
Total duration of an Æon,		12,000,000		4,320,000,000

Why the factors fourteen and seventy-one were thus used in making up the Æon is not obvious; unless, indeed, in the division by fourteen is to be recognized the influence of the number seven, while at the same time such a division furnished the equal twilights, or intermediate periods of transition, which the Hindu theory demanded. The system, however, is still that of the Purâṇas (see Wilson's Vish. Pur. p. 24, etc.); and Manu (i. 72, 79) presents virtually the same, although he has not the term Æon (*kalpa*), but states simply that a thousand Divine Ages make up a day of Brahma, and seventy-one a Patriarchate. The term *manvantara*, "patriarchate," means literally "another Manu," or, "the interval of a Manu." Manu, a word identical in origin and meaning with our "man," became to the Hindus the name of a being personified as son of the Sun (*Vivasvant*) and progenitor of the human race. In each Patriarchate there arises a new Manu, who becomes for his own period the progenitor of mankind (see Wilson's Vish. Pur. p. 24).

20. The Æon, thus composed of a thousand Ages, and which brings about the destruction of all that exists, is styled a day of Brahma; his night is of the same length.

21. His extreme age is a hundred, according to this valuation of a day and a night....

We have already found indications of an assumed destruction of existing things at the termination of the lesser periods called the Age and the Patriarchate, in the necessity of a new revelation of virtue and knowledge for every Age, and of a new father of the human race for every Patriarchate. These are left, it should seem, to show us how the system of cosmical periods grew to larger and larger dimensions. The full development of it, as exhibited in the Purâṇas and here, admits only two kinds of destruction: the one occurring at the end of each Æon, or day of Brahma, when all creatures, although not the substance of the world, undergo dissolution, and remain buried in chaos during his night, to be created anew when his day begins again; the other taking place at the end of Brahma's life, when all matter even is resolved into its ultimate source.

According to the commentary, the "hundred" in verse 21 means a hundred years, each composed of three hundred and sixty days and nights, and not a hundred days and nights only, as the text might be understood to signify; since, in all statements respecting age, years are necessarily understood to be intended. The length of Brahma's life would be, then, 864,000,000,000 divine years, or 311,040,000,000,000 solar years. This period is also called in the Purâṇas a *para*, "extreme period," and its half a *parârdha* (see Wilson's Vish. Pur. p. 25); although the latter term has obtained also an independent use, as signifying a period still more enormous (ibid. p. 630). It is curious that the commentator does not seem to recognize the affinity with this period of the expression used in the text, *param âyuh*, "extreme age," but gives two different explanations of it, both of which are forced and unnatural.

The author of the work before us is modestly content with the number of years thus placed at his disposal, and attempts nothing farther. So is it also with the Purâṇas in general; although some of them, as the Vishṇu (Wilson, p. 637) assert that two of the greater *parârdhas* constitute only a day of Vishṇu, and others (ibid. p. 25) that Brahma's whole life is but a twinkling of the eye of Kṛshṇa or of Çiva.

21. . . . The half of his life is past; of the remainder, this is the first Æon.

22. And of this Æon, six Patriarchs (*manu*) are past, with their respective twilights; and of the Patriarch Manu son of Vivasvant, twenty-seven Ages are past;

23. Of the present, the twenty-eighth, Age, this Golden Age is past: from this point, reckoning up the time, one should compute together the whole number.

The designation of the part already elapsed of this immense period seems to be altogether arbitrary. It agrees in general with that given in the Purâṇas, and, so far as the Patriarchs and their periods are concerned, with Manu also. The name of the present Æon is *Vârâha*, "that of the boar," because Brahma, in performing anew at its commencement the act of creation, put on the form of that animal (see Wilson's Vish. Pur. p. 27, etc.). The one preceding is called the *Pâdma*, "that of the lotus." This nomenclature, however, is not universally accepted: under the word *kalpa*, in the Lexicon of Böhtlingk and Roth, may be found another system of names for these periods. Manu (i. 61, 62) gives the names of the Patriarchs of the past Patriarchates; the Purâṇas add other particulars respecting them, and also respecting those which are still to come (see Wilson's Vish. Pur. p. 259, etc.).

The end of the Golden Age of the current Great Age is the time at which the Sûrya-Siddhânta claims to have been revealed, and the epoch from which its calculations profess to commence. We will, accordingly, as the Sun directs, compute the number of years which are supposed to have elapsed before that period.

	Divine years.	Solar years.
Dawn of current Æon,	4,800	1,728,000
Six Patriarchates,	5,140,800	1,850,688,000
Twenty-seven Great Ages,	324,000	116,640,000
Total till commencement of present Great Age,	5,469,600	1,969,056,000
Golden Age of present Great Age,	4,800	1,728,000
Total time elapsed of current Æon,	5,474,400	1,970,784,000
Half Brahma's life,	432,000,000,000	155,520,000,000,000
Total time elapsed from beginning of Brahma's life to end of last Golden Age,	432,005,474,400	155,521,970,784,000

As the existing creation dates from the commencement of the current Æon, the second of the above totals is the only one with which the Sûrya-Siddhânta henceforth has any thing to do.

We are next informed that the present order of things virtually began at a period less distant than the commencement of the Æon.

24. One hundred times four hundred and seventy-four divine years passed while the All-wise was employed in creating the animate and inanimate creation, plants, stars, gods, demons, and the rest.

That is to say:

	Divine years.	Solar years.
From the total above given,	5,474,400	1,970,784,000
deduct the time occupied in creation,	47,400	17,064,000
the remainder is	5,427,000	1,953,720,000

This, then, is the time elapsed from the true commencement of the existing order of things to the epoch of this work. The deduction of this period as spent by the Deity in the work of creation is a peculiar feature of the Sûrya-Siddhânta. We shall revert to it later (see below, under vv. 29–34), as its significance cannot be shown until other data are before us.

25. The planets, moving westward with exceeding velocity, but constantly beaten by the asterisms, fall behind, at a rate precisely equal, proceeding each in its own path.

26. Hence they have an eastward motion. From the number of their revolutions is derived their daily motion, which is different according to the size of their orbits; in proportion to this daily motion they pass through the asterisms.

27. One which moves swiftly passes through them in a short time; one which moves slowly, in a long time. By their movement, the revolution is accounted complete at the end of the asterism Revatî.

We have here presented a part of the physical theory of the planetary motions, that which accounts for the mean motions: the theory is supplemented by the explanation given in the next chapter of the disturbing forces which give rise to the irregularities of movement. The earth is a sphere, and sustained immovable in the centre of the universe (xii. 32), while all the heavenly bodies, impelled by winds, or vortices, called provectors (ii. 3), revolve about it from east to west. In this general westward movement, the planets, as the commentary explains it, are, owing to their weight and the weakness of their vortices, beaten by the asterisms (*nakshatra* or *bha*, the groups of stars constituting the lunar mansions [see below, chapter viii], and used here, as in various other places, to designate the whole firmament of fixed stars), and accordingly fall behind (*lambante*=*labuntur, delabuntur*), as if from shame: and this is the explanation of their eastward motion, which is only apparent and relative, although wont to be regarded as real by those who do not understand the true causes of things. But now a new element is introduced into the theory, which does not seem entirely consistent with this view of the merely relative character of the eastward motion. It is asserted that the planets lag behind equally, or that each, moving in its own orbit, loses an equal amount daily, as compared with the asterisms. And we shall find farther on (xii. 73–89) that the dimensions of the planetary orbits are constructed upon this sole principle, of making the mean daily motion of each planet eastward to be the same in amount, namely 11,858.717 *yojanas:* the amount of westward motion being equal, in each case, to the difference between this amount and the whole orbit of the planet. Now if the Hindu idea of the symmetry and harmony of the universe demanded that the movements of the planets should be equal, it was certainly a very awkward and unsatisfactory way of complying with that demand to make the relative motions alone, as compared with the fixed stars, equal, and the real motions so vastly different from one another. We should rather expect that some method would have been devised for making the latter come out alike, and the former unlike, and the result of differences in the weights of the planets and the forces of the impelling currents. It looks as if this principle, and the conformity to it of the dimensions of the orbits, might have come from those who regarded the apparent daily motion as the real motion. But we know that Âryabhaṭṭa held the opinion that the earth revolved upon its axis, causing thereby the apparent westward motion of the heavenly bodies (see Colebrooke's Hindu Algebra, p. xxxviii; Essays, ii. 467), and so, of course, that the planets really moved eastward at an equal rate among the stars; and although the later astronomers are nearly unanimous against him, we cannot help surmising that the theory of the planetary orbits emanated from him or his school, or from some other of like opinion. It is not upon record, so far as we are aware, that any Hindu astronomer, of any period, held, as did some of the Greek philosophers (see Whewell's History of the Inductive Sciences, B. V. ch. i), a heliocentric theory.

The absolute motion eastward of all the planets being equal, their apparent motion is, of course, in the (inverse) ratio of their distance, or of the dimensions of their orbits.

The word translated "revolution" is *bhagaṇa*, literally "troop of asterisms;" the verbal root translated "pass through" is *bhuj*, "enjoy," from which comes also the common term for the daily motion of a planet, *bhukti*, literally "enjoyment." When a planet has "enjoyed the whole troop of asterisms," it has made a complete revolution.

The initial point of the fixed Hindu sphere, from which longitudes are reckoned, and at which the planetary motions are held by all the schools of Hindu astronomy to have commenced at the creation, is the end of the asterism Revatî, or the beginning of Açvinî (see chapter viii. for a full account of the asterisms). Its situation is most nearly marked by that of the principal star of Revatî, which, according to the Sûrya-Siddhânta, is 10′ to the west of it, but according to other authorities exactly coincides with it. That star is by all authorities identified with ζ Piscium, of which the longitude at present, as reckoned by us, from the vernal equinox, is 17° 54′. Making due allowance for the precession, we find that it coincided in position with the vernal equinox not far from the middle of the sixth century, or about A. D. 570. As such coincidence was the occasion of the point being fixed upon as the beginning of the sphere, the time of its occurrence marks approximately the era of the fixation of the sphere, and of the commencement of the history of modern Hindu astronomy. We say approximately only, because, in the first place, as will be shown in connection with the eighth chapter, the accuracy of the Hindu observations is not to be relied upon within a degree; and, in the second place, the limits of the asterisms being already long before fixed, it was necessary to take the beginning of some one of them as that of the sphere, and the Hindus may have regarded that of Açvinî as sufficiently near to the equinox for their purpose, when it was, in fact, two or three degrees, or yet more, remote from it, on either side; and each degree of removal would correspond to a difference in time of about seventy years.

In the most ancient recorded lists of the Hindu asterisms (in the texts of the Black Yajur-Veda and of the Atharva-Veda), Kṛttikâ, now the third, appears as the first. The time when the beginning of that asterism coincided with the vernal equinox would be nearly two thousand years earlier than that given above for the coincidence with it of the first point of Açvinî.

28. Sixty seconds (*vikalâ*) make a minute (*kalâ*); sixty of these, a degree (*bhâga*); of thirty of the latter is composed a sign (*râçi*); twelve of these are a revolution (*bhagaṇa*).

The Hindu divisions of the circle are thus seen to be the same with the Greek and with our own, and we shall accordingly make use, in translating, of our own familiar terms. Of the second (*vikalâ*) very little practical use is made; it is not more than two or three times alluded to in all the rest of the treatise. The minute (*kalâ*) is much more often called *liptâ* (or *liptikâ*); this is not an original Sanskrit word, but was borrowed from the Greek λεπτόν. The degree is called either *bhâga* or *ança*; both words, like the equivalent Greek word μοῖρα, mean a "part, portion." The proper signification of *râçi*, translated "sign," is simply "heap, quantity;" it is doubtless applied to designate a sign as being a certain number, or sum, of degrees, analogous to the use of *gaṇa* in *bhagaṇa* (explained above, in the last note), and of *râçi* itself in *dinarâçi*, "sum of days" (below, v. 53). In the Hindu description of an arc, the sign is as essential an element as the degree, and no arcs of greater length than thirty degrees are reckoned in degrees alone, as we are accustomed to reckon them. The Greek usage was the same. We shall hereafter see that the signs into which any circle of revolution is divided are named Aries, Taurus, etc., beginning from the point which is regarded as the starting point; so that these names are applied simply to indicate the order of succession of the arcs of thirty degrees.

29. In an Age (*yuga*), the revolutions of the sun, Mercury, and Venus, and of the conjunctions (*çîghra*) of Mars, Saturn, and Jupiter, moving eastward, are four million, three hundred and twenty thousand;

30. Of the moon, fifty-seven million, seven hundred and fifty-three thousand, three hundred and thirty-six; of Mars, two million, two hundred and ninety-six thousand, eight hundred and thirty-two;

31. Of Mercury's conjunction (*çîghra*), seventeen million, nine hundred and thirty-seven thousand, and sixty; of Jupiter, three hundred and sixty-four thousand, two hundred and twenty;

32. Of Venus's conjunction (*çîghra*), seven million, twenty-two thousand, three hundred and seventy-six; of Saturn, one hundred and forty-six thousand, five hundred and sixty-eight;

33. Of the moon's apsis (*ucca*), in an Age, four hundred and eighty-eight thousand, two hundred and three; of its node (*pâta*), in the contrary direction, two hundred and thirty-two thousand, two hundred and thirty-eight;

34. Of the asterisms, one billion, five hundred and eighty-two million, two hundred and thirty-seven thousand, eight hundred and twenty-eight. . . .

These are the fundamental and most important elements upon which is founded the astronomical system of the Sûrya-Siddhânta. We present them below in a tabular form, but must first explain the character of some of them, especially of some of those contained in verse 29, which we have omitted from the table.

The revolutions of the sun, and of Mars, Jupiter, and Saturn, require no remark, save the obvious one that those of the sun are in fact sidereal revolutions of the earth about the sun. To the sidereal revolutions of the moon we add also her synodical revolutions, anticipated from the next following passage (see v. 35). By the moon's "apsis" is to be understood her apogee; *ucca* is literally "height," i. e. "extreme distance:" the commentary explains it by *mandocca*, "apex of slowest motion:" as the same word is used to designate the aphelia of the planets, we were obliged to take in translating it the indifferent term apsis, which applies equally to both geocentric and heliocentric motion. The "node" is the ascending node (see ii. 7); the dual "nodes" is never employed in this work. But the apparent motions of the planets are greatly complicated by the fact, unknown to the Greek and the Hindu, that they are revolving about a centre about which the earth also is revolving. When any planet is on the opposite side of the sun from us, and is accordingly moving in space in a direction contrary to ours, the effect of our change of place is to increase the rate of its apparent change of place; again, when it is upon our side of the sun, and moving in the same direction with us, the effect of our motion is to retard its apparent motion, and even to cause it to seem to retrograde. This explains the "revolutions of the conjunction" of the three superior planets: their "conjunctions" revolve at the same rate with the earth, being always upon the opposite side of the sun from us; and when, by the combination of its own proper motion with that of its conjunction, the planet gets into the latter, its rate of apparent motion is greatest, becoming less in proportion as it removes from that position. The meaning of the word which we have translated "conjunction" is "swift, rapid:" a literal rendering of it would be "swift-point," or "apex of swiftest motion;" but, after much deliberation, and persevering trial of more than one term, we have concluded that "conjunction" was the least exceptionable word by which we could express it. In the case of the inferior planets, the revolution of the conjunction takes the place of the proper motion of the planet itself. By the definition given in verse 27, a planet must, in order to complete a revolution, pass through the whole zodiac; this Mercury and Venus are only able to do as they accompany the sun in his apparent annual revolution about the earth. To the Hindus, too, who had no idea of their proper movement about the sun, the annual motion must have seemed the principal one; and that by virtue of which, in their progress through the zodiac, they moved now faster and now slower, must have appeared only of secondary importance. The term "conjunction," as used in reference to these planets, must be restricted, of course, to the superior conjunction. The physical theories by which the effect of the conjunction (*çighra*) is explained, are given in the next chapter. In the table that follows we have placed opposite each planet its own proper revolutions only.

It is farther to be observed that all the numbers of revolutions, excepting those of the moon's apsis and node, are divisible by four, so that, properly speaking, a quarter of an Age, or 1,080,000 years, rather than a whole Age, is their common period. This is a point of so much importance in the system of the Sûrya-Siddhânta, that we have added, in a second column, the number of revolutions in the lesser period.

In the third column, we add the period of revolution of each planet, as found by dividing by the number of revolutions of each the number of civil days in an Age (which is equal to the number of sidereal days, given in v. 34, diminished by the number of revolutions of the sun; see below, v. 37); they are expressed in days, nâḍîs, vinâḍîs, and respirations; the latter may be converted into sexagesimals of the third order by moving the decimal point one place farther to the right.

In the fourth column are given the mean daily motions.

We shall present later some comparison of these elements with those adopted in other systems of astronomy, ancient and modern.

Mean Motions of the Planets.

Planet.	Number of revolutions in 4,320,000 years.	Number of revolutions in 1,080,000 years.	Length of a revolution in mean solar time.				Mean daily motion.				
			d	n	v	p	°	′	″	‴	⁗
Sun,	4,320,000	1,080,000	365	15	31	3.14		59	8	10	10.4
Mercury,	17,937,060	4,484,265	87	58	10	5.57	4	5	32	20	41.9
Venus,	7,022,376	1,755,594	224	41	54	5.06	1	36	7	43	37.3
Mars,	2,296,832	574,208	686	59	50	5.87		31	26	28	11.1
Jupiter,	364,220	91,055	4,332	19	14	2.09		4	59	8	48.6
Saturn,	146,568	36,642	10,765	46	23	0.41		2	0	22	53.4
Moon:											
sider. rev.	57,753,336	14,438,334	27	19	18	0.16	13	10	34	52	3.8
synod. rev.	53,433,336	13,358,334	29	31	50	0.70	12	11	26	41	53.4
rev. of apsis,	488,203	122,050¾	3,232	5	37	1.36		6	40	58	42.5
" " node,	232,238	58,059½	6,794	23	59	2.35		3	10	44	43.2

The arbitrary and artificial method in which the fundamental elements of the solar system are here presented is not peculiar to the Sûrya-Siddhânta; it is also adopted by all the other text-books, and is to be regarded as a characteristic feature of the general astronomical system of the Hindus. Instead of deducing the rate of motion of each planet from at least two recorded observations of its place, and establishing a genuine epoch, with the ascertained position of each at that time, they start with the assumption that, at the beginning of the present order of things, all the planets, with their apsides and nodes, commenced their movement together at that point in the heavens (near ζ Piscium, as explained above, under verse 27) fixed upon as the initial point of the sidereal sphere, and that they return, at certain fixed intervals, to a universal conjunction at the same point. As regards, however, the time when the motion commenced, the frequency of recurrence of the conjunction, and the date of that which last took place, there is discordance among the different authorities. With the Sûrya-Siddhânta, and the other treatises which adopt the same general method, the determining point of the whole system is the commencement of the current Iron Age (*kali yuga*); at that epoch the planets are assumed to have been in mean conjunction for the last time at the initial point of the sphere, the former conjunctions having taken place at intervals of 1,080,000 years previous. The instant at which the Age is made to commence is midnight on the meridian of Ujjayinî (see below, under v. 62), at the end of the 588,465th and beginning of the 588,466th day (civil reckoning) of the Julian Period, or between the 17th and 18th of February 1612 J.P., or 3102 B. C. (see below, under vv. 45–53, for the computation of the number of days since elapsed). Now, although no such conjunction as that assumed by the Hindu astronomers ever did or ever will take place, the planets were actually, at the time stated, approximating somewhat nearly to a general conjunction in the neighborhood of the initial point of the Hindu sphere; this is shown by the next table, in which we give their actual mean positions with reference to that point (including also those of the moon's apogee and node); they have been obligingly furnished us by Prof. Winlock, Superintendent of the American Ephemeris and Nautical Almanac. The positions of the primary planets are obtained by LeVerrier's times of sidereal revolution, given in the Annales de l'Observatoire, tom. ii (also in Biot's Astronomie, 3me édition, tom. v, 1857), that of the moon by Peirce's tables, and those of its apogee and node by Hansen's Tables de la Lune. The origin of the Hindu sphere is regarded as being 18° 5′ 8″ east of the vernal equinox of Jan. 1, 1860, and 50° 22′ 29″ west of that of Feb. 17, 3102 B. C., the precession in the interval being 68° 27′ 37″. We add, in a second column, the mean longitudes, as reckoned from the vernal equinox of the given date, for the sake of comparison with the similar data given by Bentley (Hind. Ast., p. 125) and by Bailly (Ast. Ind. et Or., pp. 111, 182), which we also subjoin.

Positions of the Planets, midnight, at Ujjayinî, Feb. 17–18, 3102 *B. C.*

Planet.	From beginning of Hindu sphere.			Longitude.			Bentley.			Bailly.		
	°	′	″	°	′	″	°	′	″	°	′	″
Sun,	− 7	51	48	301	45	43	301	1	1	301	5	57
Mercury,	− 41	3	26	268	34	5	267	35	26	261	14	21
Venus,	+ 24	58	59	334	36	30	333	44	37	334	20	18
Mars,	− 19	49	26	289	48	5	288	55	19	288	55	56
Jupiter,	+ 8	38	36	318	16	7	318	3	54	310	22	10
Saturn,	− 28	1	13	281	36	18	280	1	58	293	8	21
Moon,	− 1	33	41	308	3	50	306	53	42	300	51	16
do. apsis,	+ 95	19	21	44	56	42	61	12	26	61	13	33
do. node,	+198	24	45	148	2	16	144	38	32	144	37	41

The want of agreement between the results of the three different investigations illustrates the difficulty and uncertainty even yet attending inquiries into the positions of the heavenly bodies at so remote an epoch. It is very possible that the calculations of the astronomers who were the framers of the Hindu system may have led them to suppose the approach to a conjunction nearer than it actually was; but, however that may be, it seems hardly to admit of a doubt that the epoch was arrived at by astronomical calculation carried backward, and that it was fixed upon as the date of the last general conjunction, and made to determine the commencement of the present Age of the world, because the errors of the assumed positions of the planets at that time would be so small, and the number of years since elapsed so great, as to make the errors in the mean motions into which those positions entered as an element only trifling in amount.

The moon's apsis and node, however, were treated in a different manner. Their distance from the initial point of the sphere, as shown by the table, was too great to be disregarded. They were accordingly ingly exempted from the general law of a conjunction once in 1,080,000 years, and such a number of revolutions was assigned to them as should make their positions at the epoch come out, the one a quadrant, the other a half-revolution, in advance of the initial point of the sphere.

We can now see why the deduction spoken of above (v. 24), for time spent in creation, needed to be made. In order to bring all the planets to a position of mean conjunction at the epoch, the time previously elapsed must be an exact multiple of the lesser period of 1,080,000 years, or the quarter-Age; in order to give its proper position to the moon's apsis, that time must contain a certain number of whole Ages, which are the periods of conjunction of the latter with the planets, together with a remainder of three quarter-Ages; for the moon's node, in like manner, it must contain a certain number of half-Ages, with a remainder of one quarter-Age. Now the whole number of years elapsed between the beginning of the Æon and that of the current Iron Age is equal to 1826 quarter-Ages, with an odd surplus of 864,000 years: from it subtract an amount of time which shall contain this surplus, together with three, seven, eleven, fifteen, or the like (any number exceeding by three a multiple of four), quarter-Ages, and the remainder will fulfil the conditions of the problem. The deduction actually made is of fifteen periods + the surplus.

This deduction is a clear indication that, as remarked above (under v. 17), the astronomical system was compelled to adapt itself to an already established Puranic chronology. It could, indeed, fix the previously undetermined epoch of the commencement of the Iron Age, but it could not alter the arrangement of the preceding periods.

It is evident that, with whatever accuracy the mean positions of the planets may, at a given time, be ascertained by observation by the Hindu astronomers, their false assumption of a conjunction at the epoch of 3102 B. C. must introduce an element of error into their determination of the planetary motions. The annual amount of that error may indeed be small, owing to the remoteness of the epoch, and the great number of years among which the errors of assumed position are divided, yet it must in time grow to an amount not to be ignored or neglected even by observers so inaccurate, and theorists so unscrupulous, as the Hindus. This is actually the case with the elements of the Sûrya-Siddhânta; the positions of the planets, as calculated by them for the present time, are in some cases nearly 9° from the true places. The later astronomers of India, however, have known how to deal with such difficulties without abrogating their ancient text-books. As the Sûrya-Siddhânta is at present employed in astronomical calculations, there are introduced into its planetary elements certain corrections, called *bíja* (more properly *víja*; the word means literally "seed"; we do not know how it arrived at its present significations in the mathematical language). That this was so, was known to Davis (As. Res., ii. 236), but he was unable to state the amount of the corrections, excepting in the case of the moon's apsis and node (ibid., p. 275). Bentley (Hind. Ast., p. 179) gives them in full, and upon his authority we present them in the annexed table. They are in the form, it will be noticed, of additions to, or subtractions from, the number of revolutions given for an Age, and the numbers are all divisible by four, in order not to interfere with the calculation by the lesser period of 1,080,000 years. We have added the corrected number of revolutions, for both the greater and lesser period, the corrected time of revolution, expressed in Hindu divisions of the day, and the corrected amount of mean daily motion.

These corrections were first applied, according to Mr. Bentley (As. Res., viii. 220), about the beginning of the sixteenth century; they are presented by several treatises of that as well as of later date, not having been yet superseded by others intended to secure yet greater correctness.

Mean Motions of the Planets as corrected by the bîja.

Planet.	Correction.	Corrected number of revolutions in 4,320,000 years.	Corrected number of revolutions in 1,080,000 years.	Corrected time of revolution.				Corrected daily motion.				
				d	n	v	p	°	′	″	‴	⁗
Sun,	0	4,320,000	1,080,000	365	15	31	3.14		59	8	10	10.4
Mercury,	− 16	17,937,044	4,484,261	87	58	11	1.26	4	5	32	19	54.5
Venus,	− 12	7,022,364	1,755,591	224	41	56	1.35	1	36	7	43	1.8
Mars,	0	2,296,832	574,208	686	59	50	5.87		31	26	28	11.1
Jupiter,	− 8	364,212	91,053	4,332	24	56	5.56		4	59	8	24.9
Saturn,	+ 12	146,580	36,645	10,764	53	30	1.11		2	0	23	28.9
Moon,	0	57,753,336	14,438,334	27	19	18	0.16	13	10	34	52	3.8
" apsis,	− 4	488,199	122,049¾	3,232	7	12	3.37		6	40	58	30.7
" node,	+ 4	232,242	58,060½	6,794	16	58	0.66		3	10	44	55.0

We need not, however, rely on external testimony alone for information as to the period when this correction was made. If the attempt to modify the elements in such a manner as to make them give the true positions of the planets at the time when they were so modified was in any tolerable degree successful, we ought to be able to discover by calculation the date of the alteration. If we ascertain for any given time the positions of the planets as given by the system, and compare them with the true positions as found by our best modern methods, and if we then divide the differences of position by the differences in the mean motions, we shall discover, in each separate case, when the error was or will be reduced to nothing. The results of such a calculation, made for Jan. 1, 1860, are given below, under v. 67. We see there that, if regard is had only to the absolute errors in the positions of the planets, no conclusion of value can be arrived at; the discrepancies between the dates of no error are altogether too great to allow of their being regarded as indicating any definite epoch of correction. If, on the other hand, we assume the place of the sun to have been the standard by which the positions of the other planets were tested, the dates of no error are seen to point quite distinctly to the first half of the sixteenth century as the time of the correction, their mean being A. D. 1541. Upon this assumption, also, we see why no correction of *bîja* was applied to Mars or to the moon: the former had, at the given time, only just passed his time of complete accordance with the sun, and the motion of the moon was also already so closely adjusted to that of the sun, that the difference between their errors of position is even now less than 10′. Nor is there any other supposition which will explain why the serious error in the position of the sun himself was overlooked at the time of the general correction, and why, by that correction, the absolute errors of position of more than one of the planets are made greater than they would otherwise have been, as is the case. It is, in short, clearly evident that the alteration of the elements of the Sûrya-Siddhânta which was effected early in the sixteenth century, was an adaptation of the errors of position of the other planets to that of the sun, assumed to be correct and regarded as the standard.

Now if it is possible by this method to arrive approximately at the date of a correction applied to the elements of a Siddhânta, it should be possible in like manner to arrive at the date of those elements themselves. For, owing to the false assumption of position at the epoch, there is but one point of time at which any of the periods of revolution will give the true place of its planet: if, then, as is to be presumed, the true places were nearly determined when any treatise was composed, and were made to enter as an element into the construction of its system, the comparison of the dates of no error will point to the epoch of its composition. The method, indeed, as is well known to all those who have made any studies in the history of Hindu astronomy, has already been applied to this purpose, by Mr. Bentley. It was first originated and put forth by him (in vol. vi. of the Asiatic Researches) at a time when the false estimate of the age and value of the Hindu astronomy presented by Bailly was still the prevailing one in Europe; he strenuously defended it against more than one attack (As. Res., viii, and Hind. Ast.), and finally employed it very extensively in his volume on the History of Hindu Astronomy, as a means of determining the age of the different Siddhântas. We present below the table from which, in the latter work (p. 126), he deduces the age of the Sûrya-Siddhânta; the column of approximate dates of no error we have ourselves added.

Bentley's Table of Errors in the Positions of the Planets, as calculated, for successive periods, according to the Sûrya-Siddhânta.

Planet.	Iron Age 0, B. C. 3102.	I. A. 1000, B. C. 2102.	I. A. 2000, B. C. 1102.	I. A. 3000, B. C. 102.	I. A. 3639, A. D. 538.	I. A. 4192, A. D. 1091.	When correct
	° ′ ″	° ′ ″	° ′ ″	° ′ ″	° ′ ″	° ′ ″	A. D.
Mercury,	+33 25 35	+25 9 52	+16 54 9	+8 38 26	+3 21 40	−1 12 28	945
Venus,	−32 43 36	−24 37 31	−16 31 26	−8 25 21	−3 14 45	+1 14 3	939
Mars,	+12 5 42	+ 9 26 32	+ 6 47 22	+4 8 12	+2 26 30	+0 58 29	1458
Jupiter,	−17 2 53	−12 44 16	− 8 25 39	−4 7 2	−1 21 47	+0 41 14	906
Saturn,	+20 59 3	+15 43 20	+10 27 37	+5 11 54	+1 50 10	−1 4 25	887
Moon,	− 5 52 41	− 3 50 48	− 2 9 17	−0 52 33	−0 18 30	−0 0 11	1097
" apsis,	−30 11 25	−23 9 36	−16 7 47	−9 5 58	−4 36 26	−0 43 10	1193
" node,	+23 37 31	+17 59 21	+12 31 11	+7 3 1	+3 33 19	+0 31 50	1188

From an average of the results thus obtained, Bentley draws the conclusion that the Sûrya-Siddhânta dates from the latter part of the eleventh century; or, more exactly, A. D. 1091.

The general soundness of Bentley's method will, we apprehend, be denied at the present time by few, and he is certainly entitled to not a little credit for his ingenuity in devising it, for the persevering industry shown in its application, and for the zeal and boldness with which he propounded and defended it. He succeeded in throwing not a little light upon an obscure and misapprehended subject, and his investigations have contributed very essentially to our present understanding of the Hindu systems of astronomy. But the details of his work are not to be accepted without careful testing, and his general conclusions are often unsound, and require essential modification, or are to be rejected altogether. This we will attempt to show in connection with his treatment of the Sûrya-Siddhânta.

In the first place, Bentley has made a very serious error in that part of his calculations which concerns the planet Mercury. As that planet was, at the epoch, many degrees behind its assumed place, it was necessary, of course, to assign to it a slower than its true rate of motion. But the rate actually given it by the text is not quite enough slower, and, instead of exhausting the original error of position in the tenth century of our era, as stated by Bentley, would not so dispose of it for many hundred years yet to come. Hence the correction of the *bíja*, as reported by Bentley himself, instead of giving to Mercury, as to all the rest, a more correct rate of motion, is made to have the contrary effect, in order the sooner to run out the original error of assumed position, and produce a coincidence between the calculated and the true places of the planet.

In the case of the other planets, the times of no error found by Bentley agree pretty nearly with those which we have ourselves obtained, both by calculating backward from the errors of A. D. 1860, and by calculating downward from those of B. C. 3102, and which are presented in the table given under verse 67. Upon comparing the two tables, however, it will be seen at once that Bentley's conclusions are drawn, not from the sidereal errors of position of the planets, but from the errors of their positions as compared with that of the sun, and that of the sun's own error he makes no account at all. This is a method of procedure which certainly requires a much fuller explanation and justification than he has seen fit anywhere to give of it. The Hindu sphere is a sidereal one, and in no wise bound to the movement of the sun. The sun, like the other planets, was not in the position assumed for him at the epoch of 3102 B. C., and consequently the rate of motion assigned to him by the system is palpably different from the real one: the sidereal year is about three minutes and a half too long. Why then should the sun's error be ignored, and the sidereal motions of the other planets considered only with reference to the incorrect rate of motion established for him? It is evident that Bentley ought to have taken fully into consideration the sun's position also, and to have shown either that it gave a like result with those obtained from the other planets, or, if not, what was the reason of the discrepancy. By failing to do so, he has, in our opinion, omitted the most fundamental datum of the whole calculation, and the one which leads to the most important conclusions. We have seen, in treating of the *bíja*, that it has been the aim of the modern Hindu astronomers, leaving the sun's error untouched, to amend those of the other planets to an accordance with it. Now, as things are wont to be managed in the Hindu literature, it would be no matter for surprise if such corrections were incorporated into the text itself: had not the Sûrya-Siddhânta been, at the beginning of the sixteenth century, so widely distributed, and its data so universally known, and had not the Hindu science outlived already that growing and productive period of its history when a school of astronomy might put forth a corrected text of an ancient authority, and expect to see it make its way to general acceptance, crowding out, and finally causing to disappear, the older version—such a process of alteration might, in our view, have passed upon it, and such a text might have been handed down to our time as Bentley would have pronounced, upon internal evidence, to have been composed early in the sixteenth century; while, nevertheless, the original error of the sun would remain, untouched and increasing, to indicate what was the true state of the case.

But what is the actual position of things with regard to our Siddhânta? We find that it presents us a set of planetary elements, which, when tested by the errors of position, in the manner already explained, do not appear to have been constructed so as to give the true sidereal positions at any assignable epoch, but which, on the other hand, exhibit evidences of an attempt to bring the places of the other planets into an accordance with that of the sun, made sometime in the tenth or eleventh century—the precise time is very doubtful, the discrepancies of the times of no error being far too great to give a certain result. Now it is as certain as anything in the history of Sanskrit literature can be, that there was a Sûrya-Siddhânta in existence long before that date; there is also evidence in the references and citations of other astronomical works (see Colebrooke, Essays, ii. 484; Hind. Alg., p. 1) that there have been more versions than one of a treatise bearing the title; and we have seen above, in verse 9, a not very obscure intimation that the present work does not present precisely the same elements which had been accepted formerly as those of the Sûrya-Siddhânta. What can lie nearer, then, than to suppose that in the tenth or eleventh century a correction of *bîja* was calculated for application to the elements of the Siddhânta, and was then incorporated into the text, by the easy alteration of four or five of its verses; and accordingly, that while the comparative errors of the other planets betray the date of the correction, the absolute error of the sun indicates approximately the true date of the treatise?

In our table, the time of no error of the sun is given as A. D. 250. The correctness of this date, however, is not to be too strongly insisted upon, being dependent upon the correctness with which the sun's place was first determined, and then referred to the point assumed as the origin of the sphere. It was, of course, impossible to observe directly when the sun's centre, by his mean motion, was 10′ east of ζ Piscium, and there are grave errors in the determination by the Hindus of the distances from that point of the other points fixed by them in their zodiac. And a mistake of 1° in the determination of the sun's place would occasion a difference of 425 years in the resulting date of no error. We shall have occasion to recur to this subject in connection with the eighth chapter.

There is also an alternative supposition to that which we have made above, respecting the conclusion from the date of no error of the sun. If the error in the sun's motion were a fundamental feature of the whole Hindu system, appearing alike in all the different text-books of the science, that date would point to the origin rather of the whole system than of any treatise which might exhibit it. But although the different Siddhântas nearly agree with one another respecting the length of the sidereal year, they do not entirely accord, as is made evident by the following statement, in which are included all the authorities to which we have access, either in the original, or as reported by Colebrooke, Bentley, and Warren:

Authority.	Length of sidereal year.	Error.
Sûrya-Siddhânta,	$365^d\ 6^h\ 12^m\ 36^s.56$	$+\ 3^m\ 25^s.81$
Pâuliça-Siddhânta,	365 6 12 36	+ 3 25.25
Pârâçara-Siddhânta,	365 6 12 31.50	+ 3 20.75
Ârya-Siddhânta,	365 6 12 30.84	+ 3 20.09
Laghu-Ârya-Siddhânta,	365 6 12 30	+ 3 19.25
Siddhânta-Çiromaṇi,	365 6 12 9	+ 2 58.25

The first five of these might be regarded as unimportant variations of the same error, but it would seem that the last is an independent determination, and one of later date than the others; while, if all are independent, that of the Sûrya-Siddhânta has the appearance of being the most ancient. Such questions as these, however, are not to be too hastily decided, nor from single indications merely; they demand the most thorough investigation of each different treatise, and the careful collection of all the evidence which can be brought to bear upon them.

Here lies Bentley's chief error. He relied solely upon his method of examining the elements, applying even that, as we have seen, only partially and uncritically, and never allowing his results to be controlled or corrected by evidence of any other character. He had, in fact, no philology, and he was deficient in sound critical judgment. He thoroughly misapprehended the character of the Hindu astronomical literature, thinking it to be, in the main, a mass of forgeries framed for the purpose of deceiving the world respecting the antiquity of the Hindu people. Many of his most confident conclusions have already been overthrown by evidence of which not even he would venture to question the verity, and we are persuaded that but little of his work would stand the test of a thorough examination.

The annexed table presents a comparison of the times of mean sidereal revolution of the planets assumed by the Hindu astronomy, as represented by two of its principal text-books, with those adopted by the great Greek astronomer, and those which modern science has established. The latter are, for the primary planets, from Le Verrier; for the moon, from Nichol (Cyclopedia of the Physical Sciences, London: 1857). Those of Ptolemy are deduced from the mean daily rates of motion in longitude given by him in the Syntaxis, allowing for the movement of the equinox according to the false rate adopted by him, of 36″ yearly.

Comparative Table of the Sidereal Revolutions of the Planets.

Planet.	Sûrya-Siddhânta.	Siddhânta-Çiromani	Ptolemy.	Moderns.
	d h m s	d h m s	d h m s	d h m s
Sun,	365 6 12 36.6	365 6 12 9.0	365 36 9 48.6	365 6 9 10.8
Mercury,	87 23 16 22.3	87 23 16 41.5	87 23 16 42.9	87 23 15 43.9
Venus,	224 16 45 56.2	224 16 45 1.9	224 16 51 56.8	224 16 49 8 0
Mars,	686 23 56 23.5	686 23 57 1.5	686 23 31 56.1	686 23 30 41.4
Jupiter,	4,332 7 41 44.4	4,332 5 45 43.7	4,332 18 9 10.5	4,332 14 2 8.6
Saturn,	10,765 18 33 13.6	10,765 19 33 56.5	10,758 17 48 14.9	10,759 5 16 32.2
Moon:				
sid. rev.	27 7 43 12.6	27 7 43 12.1	27 7 43 12.1	27 7 43 11.4
synod. rev.	29 12 44 2.8	29 12 44 2.3	29 12 44 3.3	29 12 44 2.9
rev. of apsis,	3,232 2 14 53.4	3,232 17 37 6.0	3,232 9 52 13.6	3,232 13 48 29.6
" " node,	6,794 9 35 45.4	6,792 6 5 41.9	6,799 23 18 39.4	6,798 6 41 45.6

In the additional notes at the end of the work, we shall revert to the subject of these data, and of the light thrown by them upon the origin and age of the system.

34. . . . The number of risings of the asterisms, diminished by the number of the revolutions of each planet respectively, gives the number of risings of the planets in an Age.

35. The number of lunar months is the difference between the number of revolutions of the sun and of the moon. If from it the number of solar months be subtracted, the remainder is the number of intercalary months.

36. Take the civil days from the lunar, the remainder is the number of omitted lunar days (*tithikshaya*). From rising to rising of the sun are reckoned terrestrial civil days;

37. Of these there are, in an Age, one billion, five hundred and seventy-seven million, nine hundred and seventeen thousand, eight hundred and twenty-eight; of lunar days, one billion, six hundred and three million, and eighty;

38. Of intercalary months, one million, five hundred and ninety-three thousand, three hundred and thirty-six; of omitted lunar days, twenty-five million, eighty-two thousand, two hundred and fifty-two;

39. Of solar months, fifty-one million, eight hundred and forty thousand. The number of risings of the asterisms, diminished by that of the revolutions of the sun, gives the number of terrestrial days.

40. The intercalary months, the omitted lunar days, the sidereal, lunar, and civil days—these, multiplied by a thousand, are the number of revolutions, etc., in an Æon.

The data here given are combinations of, and deductions from, those contained in the preceding passage (vv. 29–34). For convenience of reference, we present them below in a tabular form.

	In 4,320,000 years.	In 1,080,000 years.
Sidereal days,	1,582,237,828	395,559,457
deduct solar revolutions,	4,320,000	1,080,000
Natural, or civil days,	1,577,917,828	394,479,457
Sidereal solar years,	4,320,000	1,080,000
multiply by no. of solar months in a year,	12	12
Solar months,	51,840,000	12,960,000
Moon's sidereal revolutions,	57,753,336	14,438,334
deduct solar revolutions,	4,320,000	1,080,000
Synodical revolutions, lunar months,	53,433,336	13,358,334
deduct solar months,	51,840,000	12,960,000
Intercalary months,	1,593,336	398,334

Lunar months,	53,433,336	13,358,334
multiply by no. of lunar days in a month,	30	30
Lunar days,	1,603,000,080	400,750,020
deduct civil days,	1,577,917,828	394,479,457
Omitted lunar days,	25,082,252	6,270,563

We add a few explanatory remarks respecting some of the terms employed in this passage, or the divisions of time which they designate.

The natural day, nycthemeron, is, for astronomical purposes, reckoned in the Sûrya-Siddhânta from midnight to midnight, and is of invariable length; for the practical uses of life, the Hindus count it from sunrise to sunrise; which would cause its duration to vary, in a latitude as high as our own, sometimes as much as two or three minutes. As above noticed, the system of Brahmagupta and some others reckon the astronomical day also from sunrise.

For the lunar day, the lunar and solar month, and the general constitution of the year, see above, under verse 13. The lunar month, which is the one practically reckoned by, is named from the solar month in which it commences. An intercalation takes place when two lunar months begin in the same solar month: the former of the two is called an intercalary month (*adhimâsa*, or *adhimâsaka*, "extra month"), of the same name as that which succeeds it.

The term "omitted lunar day" (*tithikshaya*, "loss of a lunar day") is explained by the method adopted in the calendar, and in practice, of naming the days of the month. The civil day receives the name of the lunar day which ends in it; but if two lunar days end in the same solar day, the former of them is reckoned as loss (*kshaya*), and is omitted, the day being named from the other.

41. The revolutions of the sun's apsis (*manda*), moving eastward, in an Æon, are three hundred and eighty-seven; of that of Mars, two hundred and four; of that of Mercury, three hundred and sixty-eight;

42. Of that of Jupiter, nine hundred; of that of Venus, five hundred and thirty-five; of the apsis of Saturn, thirty-nine. Farther, the revolutions of the nodes, retrograde, are:

43. Of that of Mars, two hundred and fourteen; of that of Mercury, four hundred and eighty-eight; of that of Jupiter, one hundred and seventy-four; of that of Venus, nine hundred and three;

44. Of the node of Saturn, the revolutions in an Æon are six hundred and sixty-two: the revolutions of the moon's apsis and node have been given here already.

In illustration of the curious feature of the Hindu system of astronomy presented in this passage, we first give the annexed table; which shows the number of revolutions in the Æon, or period of 4,320,000,000 years, assigned by the text to the apsis and node of each planet, the resulting time of revolution, the number of years which each would require to pass through an arc of one minute, and the position of each, according to the system, in 1850; the latter being reckoned in our method, from the vernal equinox. Farther are added the actual positions for Jan. 1, 1850, as given by Biot (Traité d' Astronomie, tom. v. 529); and finally, the errors of the positions as determined by this Siddhânta.

Table of Revolutions and Present Position of the Apsides and Nodes of the Planets.

Planet.	No. of rev. in an Æon.	Time of revolution, in years.	No. of years to 1′ of motion.	Resulting position, A. D. 1850.	True position, A. D. 1850.	Error of Hindu position.
Apsides:				° ′	° ′	° ′
Sun,	387	11,162,790.7	516.8	95 4	100 22	− 5 16
Mercury,	368	11,739,130.4	543.5	238 15	255 7	− 16 52
Venus,	535	8,074,766.4	373.8	97 39	309 24	−211 45
Mars,	204	21,176,470.6	980.4	147 49	153 18	− 5 29
Jupiter,	900	4,800,000.0	222.2	189 9	191 55	− 2 46
Saturn,	39	110,769,230.8	5128.2	254 24	270 6	− 15 42
Nodes:						
Mercury,	488	8,852,459.0	409.8	38 27	46 33	− 8 6
Venus,	903	4,784,053.2	221.5	77 26	75 19	+ 2 7
Mars,	214	20,186,915.9	934.6	57 49	48 23	+ 9 26
Jupiter,	174	24,827,586.2	1149.4	97 26	98 54	− 1 28
Saturn,	662	6,525,678.2	302.1	118 7	112 22	+ 5 45

A mere inspection of this table is sufficient to show that the Hindu astronomers did not practically recognize any motion of the apsides and nodes of the planets; since, even in the case of those to which they assigned the most rapid motion, two thousand years, at the least, would be required to produce such a change of place as they, with their imperfect means of observation, would be able to detect.

This will, however, be made still more clearly apparent by the next following table, in which we give the positions of the apsides and nodes as determined by four different text-books of the Hindu science, for the commencement of the Iron Age.

Positions of the Apsides and Nodes of the Planets, according to Different Authorities, at the Commencement of the Iron Age, 3102 *B. C.*

Planet.	Sûrya-Siddhânta.	Siddhânta-Çiromaṇi.	Ârya-Siddhânta.	Pârâçara-Siddhânta.
Apsides:	(rev.) s ° ′ ″	(rev.) s ° ′ ″	(rev.) s ° ′ ″	(rev.) s ° ′ ″
Sun,	(175) 2 17 7 48	(219) 2 17 45 36	(210) 2 17 45 36	(219) 2 17 45 36
Mercury,	(166) 7 10 19 12	(151) 7 14 47 2	(154) 7 0 14 24	(162) 7 0 40 19
Venus,	(242) 2 19 39 0	(298) 2 21 2 10	(300) 0 17 16 48	(240) 2 20 42 43
Mars,	(92) 4 9 57 36	(133) 4 8 18 14	(136) 4 3 50 24	(149) 4 2 43 26
Jupiter,	(407) 5 21 0 0	(390) 5 22 15 36	(378) 5 22 48 0	(448) 5 22 35 24
Saturn,	(17) 7 26 36 36	(18) 8 20 53 31	(16) 4 29 45 36	(24) 7 28 14 52
Nodes:				
Mercury,	(221−) 0 20 52 48	(238−) 0 21 20 53	(239−) 0 20 9 36	(296−) [illegible] 1 26
Venus,	(409−) 2 0 1 48	(408−) 2 0 5 2	(432−) 2 0 28 48	(408−) 2 [illegible] 5 2
Mars,	(97−) 1 10 8 24	(122−) 0 21 59 46	(136−) 1 10 19 12	(112−) 1 9 3 36
Jupiter,	(79−) 2 19 44 24	(29−) 2 22 2 38	(44−) 2 20 38 24	(87−) 2 21 43 12
Saturn,	(300−) 3 10 37 12	(267−) 3 13 23 31	(283−) 3 10 48 0	(288−) 3 10 26 24

The data of the Ârya and Pârâçara Siddhântas, from which the positions given in the table are calculated, are derived from Bentley (Hind. Ast. pp. 139, 144). To each position is prefixed the number of completed revolutions; or, in the case of the nodes, of which the motion is retrograde, the number of whole revolutions of which each falls short by the amount expressed by its position.

The almost universal disagreement of these four authorities with respect to the number of whole revolutions accomplished, and their general agreement as to the remainder, which determines the position,* prove that the Hindus had no idea of any motion of the apsides and nodes of the planets as an actual and observable phenomenon; but, knowing that the moon's apsis and node moved, they fancied that the symmetry of the universe required that those of the other planets should move also; and they constructed their systems accordingly. They held, too, as will be seen at the beginning of the second chapter, that the nodes and apsides, as well as the conjunctions (*çighra*), were beings, stationed in the heavens, and exercising a physical influence over their respective planets, and, as the conjunctions revolved, so must these also. In framing their systems, then, they assigned to these points such a number of revolutions in an Æon as should, without attributing to them any motion which admitted of detection, make their positions what they supposed them actually to be. The differences in respect to the number of revolutions were in part rendered necessary by the differences of other features of the systems; thus, while that of the Siddhânta-Çiromani makes the planetary motions commence at the beginning of the Æon, by that of the Sûrya-Siddhânta they commence 17,064,000 years later (see above, v. 24), and by that of the Ârya-Siddhânta, 3,024,000 years later (Bentley, Hind. Ast. p. 139): in part, however, they are merely arbitrary; for, although the Pârâçara-Siddhânta agrees with the Siddhânta-Çiromani as to the time of the beginning of things, its numbers of revolutions correspond only in two instances with those of the latter.

It may be farther remarked, that the close accordance of the different astronomical systems in fixing the position of points which are so difficult of observation and deduction as the nodes and apsides, strongly indicates, either that the Hindus were remarkably accurate observers, and all arrived independently at a near approximation to the truth, or that some one of them was followed as an authority by the others, or that all alike derived their data from a common source, whether native or foreign. We reserve to the end of this work the discussion of these different possibilities, and the presentation of data which may tend to settle the question between them.

45. Now add together the time of the six Patriarchs (*manu*), with their respective twilights, and with the dawn at the commencement of the Æon (*kalpa*); farther, of the Patriarch Manu, son of Vivasvant,

* It is altogether probable that, in the two cases where the Ârya-Siddhânta seems to disagree with the others, its data were either given incorrectly by Bentley's authority, or have been incorrectly reported by him.

46. The twenty-seven Ages (*yuga*) that are past, and likewise the present Golden Age (*kṛta yuga*); from their sum subtract the time of creation, already stated in terms of divine years,

47. In solar years: the result is the time elapsed at the end of the Golden Age; namely, one billion, nine hundred and fifty-three million, seven hundred and twenty thousand solar years.

We have already presented this computation, in full, in the notes to verses 23 and 24.

48. To this, add the number of years of the time since past. . . .

As the Sûrya-Siddhânta professes to have been revealed by the Sun about the end of the Golden Age, it is of course precluded from taking any notice of the divisions of time posterior to that period: there is nowhere in the treatise an allusion to any of the eras which are actually made use of by the inhabitants of India in reckoning time, with the exception of the cycle of sixty years, which, by its nature, is bound to no date or period (see below, v. 55). The astronomical era is the commencement of the Iron Age, the epoch, according to this Siddhânta, of the last general conjunction of the planets; this coincides, as stated above (under vv. 29–34) with Feb. 18, 1612 J. P., or 3102 B. C. From that time will have elapsed, upon the eleventh of April, 1859, the number of 4960 complete sidereal years of the Iron Age. The computation of the whole period, from the beginning of the present order of things, is then as follows:

From end of creation to end of last Golden Age,		1,953,720,000
Silver Age,	1,296,000	
Brazen Age,	864,000	
Of Iron Age,	4,960	2,164,960
Total from end of creation to April, 1859,		1,955,884,960

Since the Sûrya-Siddhânta, as will appear from the following verses, reckons by luni-solar years, it regards as the end of I. A. 4960 not the end of the solar sidereal year of that number, but that of the luni-solar year, which, by Hindu reckoning, is completed upon the third of the same month (see Ward, Kâla Sankalita, Table, p. xxxii).

48. . . . Reduce the sum to months, and add the months expired of the current year, beginning with the light half of Câitra.

49. Set the result down in two places; multiply it by the number of intercalary months, and divide by that of solar months, and add to the last result the number of intercalary months thus found; reduce the sum to days, and add the days expired of the current month;

50. Set the result down in two places; multiply it by the number of omitted lunar days, and divide by that of lunar days; subtract from the last result the number of omitted lunar days

thus obtained: the remainder is, at midnight, on the meridian of Lankâ,

51. The sum of days, in civil reckoning. . . .

In these verses is taught the method of one of the most important and frequently recurring processes in Hindu Astronomy, the finding, namely, of the number of civil or natural days which have elapsed at any given date, reckoning either from the beginning of the present creation, or (see below, v. 56) from any required epoch since that time. In the modern technical language, the result is uniformly styled the *ahargaṇa*, "sum of days;" that precise term, however, does not once occur in the text of the Sûrya-Siddhânta: in the present passage we have *dyugaṇa*, which means the same thing, and in verse 53 *dinarâçi*, "heap or quantity of days."

The process will be best illustrated and explained by an example. Let it be required to find the sum of days to the beginning of Jan. 1, 1860.

It is first necessary to know what date corresponds to this in Hindu reckoning. We have remarked above that the 4960th year of the Iron Age is completed in April, 1859; in order to exhibit the place in the next following year of the date required, and, at the same time, to present the names and succession of the months, which in this treatise are assumed as known, and are nowhere stated, we have constructed the following skeleton of a Hindu calendar for the year 4961 of the Iron Age.

Solar Year.

month.	first day.
(I. A. 4960.)	
12. Câitra,	Mar. 13, 1859.
(I. A. 4961.)	
1. Vâiçâkha,	Apr. 12, do.
2. Jyâishṭha,	May 13, do.
3. Âshâḍha,	June 14, do.
4. Çrâvaṇa,	July 15, do.
5. Bhâdrapada,	Aug. 16, do.
6. Âçvina,	Sept. 16, do.
7. Kârttika,	Oct. 16, do.
8. Mârgaçîrsha,	Nov. 15, do
9. Pâusha,	Dec. 15, do.
10. Mâgha,	Jan. 13, 1860.
11. Phâlguna,	Feb. 11, do.
12. Câitra,	Mar. 12, do.

Luni-solar Year.

month.	first day.
(I. A. 4961.)	
1. Câitra,	Apr. 4, 1859.
2. Vâiçâkha,	May 3, do.
3. Jyâishṭha,	June 2, do.
4. Âshâḍha,	July 1, do.
5. Çrâvaṇa,	July 31, do.
6. Bhâdrapada,	Aug. 29, do.
7. Âçvina,	Sept. 28, do.
8. Kârttika,	Oct. 27, do.
9. Mârgaçîrsha,	Nov. 26, do.
10. Pâusha,	Dec. 25, do.
11. Mâgha,	Jan. 24, 1860.
12. Phâlguna,	Feb. 22, do.
(I. A. 4962.)	
1. Câitra,	Mar. 23, do.

The names of the solar months are derived from the names of the asterisms (see below, chap. viii.) in which, at the time of their being first so designated, the moon was full during their continuance. The same names are transferred to the lunar months. Each lunar month is divided into two parts; the first, called the light half (*çukla paksha*, "bright side"), lasts from new moon to full moon, or while the moon is waxing; the other, called the dark half (*kṛshṇa paksha*, "black side"), lasts from full moon to new moon, or while the moon is waning.

The table shows that Jan. 1, 1860, is the eighth day of the tenth month of the 4961st year of the Iron Age. The time, then, for which we have to find the sum of days, is 1,955,884,960 y., 9 m., 7 d.

Number of complete years elapsed,	1,955,884,960
multiply by number of solar months in a year,	12
Number of months,	23,470,619,520
add months elapsed of current year,	9
Whole number of months elapsed,	23,470,619,529

Now a proportion is made: as the whole number of solar months in an Age is to the number of intercalary months in the same period, so is the number of months above found to that of the corresponding intercalary months: or,

51,840,000 : 1,593,336 : : 23,470,619,529 : 721,384,703 +

Whole number of months, as above,	23,470,619,529
add intercalary months,	721,384,703
Whole number of lunar months,	24,192,004,232
multiply by number of lunar days in a month,	30
Number of lunar days,	725,760,126,960
add lunar days elapsed of current month,	7
Whole number of lunar days elapsed,	725,760,126,967

To reduce, again, the number of lunar days thus found to the corresponding number of solar days, a proportion is made, as before: as the whole number of lunar days in an Age is to the number of omitted lunar days in the same period, so is the number of lunar days in the period for which the sum of days is required to that of the corresponding omitted lunar days: or,

1,603,000,080 : 25,082,252 : : 725,760,126,967 : 11,356,018,395 +

Whole number of lunar days as above,	725,760,126,967
deduct omitted lunar days,	11,356,018,395
Total number of civil days from end of creation to beginning of Jan. 1, 1860,	714,404,108,572

This, then, is the required sum of days, for the beginning of the year A. D. 1860, at midnight, upon the Hindu prime meridian.

The first use which we are instructed to make of the result thus obtained is an astrological one.

51. . . . From this may be found the lords of the day, the month, and the year, counting from the sun. If the number be divided by seven, the remainder marks the lord of the day, beginning with the sun.

52. Divide the same number by the number of days in a month and in a year, multiply the one quotient by two and the

other by three, add one to each product, and divide by seven; the remainders indicate the lords of the month and of the year.

These verses explain the method of ascertaining, from the sum of days already found, the planet which is accounted to preside over the day, and also those under whose charge are placed the month and year in which that day occurs.

To find the lord of the day is to find the day of the week, since the latter derives its name from the former. The week, with the names and succession of its days, is the same in India as with us, having been derived to both from a common source. The principle upon which the assignment of the days to their respective guardians was made has been handed down by ancient authors (see Ideler, Handbuch d. math. u. tech. Chronologie, i. 178, etc.), and is well known. It depends upon the division of the day into twenty-four hours, and the assignment of each of these in succession to the planets, in their natural order; the day being regarded as under the dominion of that planet to which its first hour belongs. Thus, the planets being set down in the order of their proximity to the earth, as determined by the ancient systems of astronomy (for the Hindu, see below, xii. 84–88), beginning with the remotest, as follows: Saturn, Jupiter, Mars, sun, Venus, Mercury, moon, and the first hour of the twenty-four being assigned to the Sun, as chief of the planets, the second to Venus, etc., it will be found that the twenty-fifth hour, or the first of the second day, belongs to the moon; the forty-ninth, or the first of the third day, to Mars, and so on. Thus is obtained a new arrangement of the planets, and this is the one in which this Siddhânta, when referring to them, always assumes them to stand (see, for instance, below, v. 70; ii. 35–37): it has the convenient property that by it the sun and moon are separated from the other planets, from which they are by so many peculiarities distinguished. Upon this order depend the rules here given for ascertaining also the lords of the month and of the year. The latter, as appears both from the explanation of the commentator, and from the rules themselves, are no actual months and years, but periods of thirty and three hundred and sixty days, following one another in uniform succession, and supposed to be placed, like the day, under the guardianship of the planets to whom belong their first subdivisions: thus the lord of the day is the lord of its first hour; the lord of the month is the lord of its first day (and so of its first hour); the lord of the year is the lord of its first month (and so of its first day and hour). We give below this artificial arrangement of the planets, with the order in which they are found to succeed one another as lords of the periods of one, thirty, and three hundred and sixty days; we add their natural order of succession, as lords of the hours; and we farther prefix the ordinary names of the days, with their English equivalents. Other of the numerous names of the planets, it is to be remarked, may be put before the word *vâra* to form the name of the day: *vâra* itself means literally "successive time," or "turn," and is not used, so far as we are aware, in any other connection, to denote a day.

Name of day.		Presiding Planet.	Succession, as Lord of day,	month,	year,	hour.
Ravivâra,	Sunday,	Sun,	1	1	1	1
Somavâra,	Monday,	Moon,	2	5	6	4
Mangalavâra,	Tuesday,	Mars,	3	2	4	7
Budhavâra,	Wednesday,	Mercury,	4	6	2	3
Guruvâra,	Thursday,	Jupiter,	5	3	7	6
Çukravâra,	Friday,	Venus,	6	7	5	2
Çanivâra,	Saturday,	Saturn,	7	4	3	5

As the first day of the subsistence of the present order of things is supposed to have been a Sunday, it is only necessary to divide the sum of days by seven, and the remainder will be found, in the first column, opposite the name of the planet to which the required day belongs. Thus, taking the sum of days found above, adding to it one, for the first of January itself, and dividing by seven, we have:

7) 714,404,108,573
102,057,729,796 — 1

The first of January, 1860, accordingly, falls on a Sunday by Hindu reckoning, as by our own.

On referring to the table, it will be seen that the lords of the months follow one another at intervals of two places. To find, therefore, by a summary process, the lord of the month in which occurs any given day, first divide the sum of days by thirty; the quotient, rejecting the remainder, is the number of months elapsed; multiply this by two, that each month may push the succession forward two steps, add one for the current month, divide by seven in order to get rid of whole series, and the remainder is, in the column of lords of the day, the number of the regent of the month required. Thus:

30) 714,404,108,572
23,813,470,285 +
2
47,626,940,570
1
7) 47,626,940,571
6,803,848,652 — 7

The regent of the month in question is therefore Saturn.

By a like process is found the lord of the year, saving that, as the lords of the year succeed one another at intervals of three places, the multiplication is by three instead of by two. Upon working out the process, it will be found that the final remainder is five, which designates Jupiter as the lord of the year at the given time.

Excepting here and in the parallel passage xii. 77, 78, no reference is made in the Sûrya-Siddhânta to the week, or to the names of its days. Indeed, it is not correct to speak of the week at all in connection with India, for the Hindus do not seem ever to have regarded it as a division of time, or a period to be reckoned by; they knew only of a certain order of succession, in which the days were placed under the regency of the seven planets. And since, moreover, as remarked above (under vv. 11, 12), they never made that division of the day into twenty-four hours upon which the order of regency depends, it follows that the whole system was of foreign origin, and introduced into India along with other elements of the modern sciences of astronomy and astrology, to which it belonged. Its proper foundation, the lordship of the successive hours, is shown by the other passage (xii. 78) to have been also known to the Hindus; and the name by which the hours are there called (*horâ*=ὥρα) indicates beyond a question the source whence they derived it.

53. Multiply the sum of days (*dinarâçi*) by the number of revolutions of any planet, and divide by the number of civil days; the result is the position of that planet, in virtue of its mean motion, in revolutions and parts of a revolution.

By the number of revolutions and of civil days is meant, of course, their number, as stated above, in an Age. For "position of the planet," etc., the text has, according to its usual succinct mode of expression, simply "is the planet, in revolutions, etc." There is no word for "position" or "place" in the vocabulary of this Siddhânta.

This verse gives the method of finding the mean place of the planets at any given time for which the sum of days has been ascertained, by a simple proportion: as the number of civil days in a period is to the number of revolutions during the same period, so is the sum of days to the number of revolutions and parts of a revolution accomplished down to the given time. Thus, for the sun:

1,577,917,828 : 4,320,000 : : 714,404,108,572 : 1,955,884,960$^{\text{rev}}$ 8$^{\text{s}}$ 17° 48′ 7″

The mean longitude of the sun, therefore, Jan. 1st, 1860, at midnight on the meridian of Ujjayinî, is 257° 48′ 7″. We have calculated in this manner the positions of all the planets, and of the moon's apsis and node—availing ourselves, however, of the permission given below, in verse 56, and reckoning only from the last epoch of conjunction, the beginning of the Iron Age (from which time the sum of days is 1,811,945), and also employing the numbers afforded by the lesser period of 1,080,000 years—and present the results in the following table.

Mean Places of the Planets, Jan. 1*st*, 1860, *midnight, at Ujjayinî.*

Planet.	According to the Sûrya-Siddhânta.					The same corrected by the *bîja*.			
	(rev.)	s	°	′	″	s	°	′	″
Sun,	(4,960)	8	17	48	7	8	17	48	7
Mercury,	(20,597)	4	15	13	8	4	8	36	16
Venus,	(8,063)	10	21	8	59	10	16	11	22
Mars,	(2,637)	5	24	17	36	5	24	17	36
Jupiter,	(418)	2	26	0	7	2	22	41	41
Saturn,	(168)	3	20	11	12	3	25	8	50
Moon,	(66,318)	11	15	23	24	11	15	23	24
" apsis,	(560)	10	9	42	26	10	8	3	13
" node,	(267–)	9	24	26	4	9	22	46	51

The positions are given as deduced both from the numbers of revolutions stated in the text, and from the same as corrected by the *bîja* : prefixed are the numbers of complete revolutions accomplished since the epoch. In the cases of the moon's apsis and node, however, it was necessary to employ the numbers of revolutions given for the whole Age, these not being divisible by four, and also to add to their ascertained amount of movement their longitude at the epoch (see below, under vv. 57, 58).

54. Thus also are ascertained the places of the conjunction (*çîghra*) and apsis (*mandocca*) of each planet, which have been mentioned as moving eastward ; and in like manner of the nodes, which have a retrograde motion, subtracting the result from a whole circle.

The places of the apsides and nodes have already been given above (under vv. 41–44), both for the commencement of the Iron Age, and for A. D. 1850. The place of the conjunctions of the three superior planets is, of course, the mean longitude of the sun. In the case of the inferior planets, the place of the conjunction is, in fact, the mean place of the planet itself in its proper orbit, and it is this which we have given for Mercury and Venus in the preceding table : while to the Hindu apprehension, the mean place of those planets is the same with that of the sun.

55. Multiply by twelve the past revolutions of Jupiter, add the signs of the current revolution, and divide by sixty ; the remainder marks the year of Jupiter's cycle, counting from Vijaya.

This is the rule for finding the current year of the cycle of sixty years, which is in use throughout all India, and which is called the cycle of Jupiter, because the length of its years is measured by the passage of that planet, by its mean motion, through one sign of the zodiac. According to the data given in the text of this Siddhânta, the length of Jupiter's year is $361^{d}\ 0^{h}\ 38^{m}$; the correction of the *bîja* makes it about 12^{m} longer. It was doubtless on account of the near coincidence of this period with the true solar year that it was adopted as a measure of time ; but it has not been satisfactorily ascertained, so far as we are aware, where the cycle originated, or what is its age, or why it was made to consist of sixty years, including five whole revolutions of the planet. There was, indeed, also in use a cycle of twelve of Jupiter's years, or the time of one sidereal revolution : see below, xiv. 17. Davis (As. Res. iii. 209, etc.) and Warren (Kâla Sankalita, p. 197, etc.) have treated at some length of the greater cycle, and of the different modes of reckoning and naming its years usual in the different provinces of India.

In illustration of the rule, let us ascertain the year of the cycle corresponding to the present year, A. D. 1859. It is not necessary to make the calculation from the creation, as the rule contemplates ; for, since the number of Jupiter's revolutions in the period of 1,080,000 years is divisible by five, a certain number of whole cycles, without a remainder, will have elapsed at the beginning of the Iron Age. The revolutions of the planet since that time, as stated in the table last given, are 418, and it is in the 3rd sign of the 419th revolution ; the reduction of the whole amount of movement to signs shows us that the current year is the 5019th since the epoch: divide this by 60, to cast out whole cycles, and the remainder, 39, is the number of the year in the current cycle. This treatise nowhere gives the names of the years of Jupiter, but, as in the case of the months, the signs of the zodiac, and other similar matters, assumes them to be already familiarly known in their succession: we accordingly present them below. We take them from Mr. Davis's paper, alluded to above, not having access at present to any original authority which contains them.

1. Vijaya.
2. Jaya.
3. Manmatha.
4. Durmukha.
5. Hemalamba.
6. Vilamba.
7. Vikârin.
8. Çarvarî.
9. Plava.
10. Çubhakṛt.
11. Çobhana.
12. Krodhin.
13. Viçvâvasu.
14. Parâbhava.
15. Plavanga.
16. Kîlaka.
17. Sâumya.
18. Sâdhâraṇa.
19. Virodhakṛt.
20. Paridhâvin.
21. Pramâdin.
22. Ânanda.
23. Râkshasa.
24. Anala.
25. Pingala.
26. Kâlayukta.
27. Siddhârthin.
28. Râudra.
29. Durmati.
30. Dundubhi.
31. Rudhirodgârin.
32. Raktâksha.
33. Krodhana.
34. Kshaya.
35. Prabhava.
36. Vibhava.
37. Çukla.
38. Pramoda.
39. Prajâpati.
40. Angiras.
41. Çrîmukha.
42. Bhâva.
43. Yuvan.
44. Dhâtar.
45. Îçvara.
46. Bahudhanya.
47. Pramâthin.
48. Vikrama.
49. Bhṛçya.
50. Citrabhânu.
51. Subhânu.
52. Târaṇa.
53. Pârthiva.
54. Vyaya.
55. Sarvajit.
56. Sarvadhârin.
57. Virodhin.
58. Vikṛta.
59. Khara.
60. Nandana.

It appears, then, that the current year of Jupiter's cycle is named Prajâpati: upon dividing by the planet's mean daily motion the part of the current sign already passed over, it will be found that, according to the text, that year commenced on the twenty-third of February, 1859; or, if the correction of the *bîja* be admitted, on the third of April.

Although it is thus evident that the Sûrya-Siddhânta regards both the existing order of things and the Iron Age as having begun with Vijaya, that year is not generally accounted as the first, but as the twenty-seventh, of the cycle, which is thus made to commence with Prabhava. An explanation of this discrepancy might perhaps throw important light upon the origin or history of the cycle.

This method of reckoning time is called (see below, xiv. 1, 2) the *bârhaspatya mâna*, "measure of Jupiter."

56. The processes which have thus been stated in full detail, are practically applied in an abridged form. The calculation of the mean place of the planets may be made from any epoch (*yuga*) that may be fixed upon.

57. Now, at the end of the Golden Age (*kṛta yuga*), all the planets, by their mean motion—excepting, however, their nodes and apsides (*mandocca*)—are in conjunction in the first of Aries.

58. The moon's apsis (*ucca*) is in the first of Capricorn, and its node is in the first of Libra; and the rest, which have been

stated above to have a slow motion—their position cannot be expressed in whole signs.

It is curious to observe how the Sûrya-Siddhânta, lest it should seem to admit a later origin than that which it claims in the second verse of this chapter, is compelled to ignore the real astronomical epoch, the beginning of the Iron Age; and also how it avoids any open recognition of the lesser cycle of 1,080,000 years, by which its calculations are so evidently intended to be made.

The words at the end of verse 56 the commentator interprets to mean: "from the beginning of the current, i. e., the Silver, Age." In this he is only helping to keep up the pretence of the work to immemorial antiquity, even going therein beyond the text itself, which expressly says: "from any desired (*ishtatas*) *yuga*." Possibly, however, we have taken too great a liberty in rendering *yuga* by "epoch," and it should rather be "Age," i. e., "beginning of an Age." The word *yuga* comes from the root *yuj*, "to join" (Latin, *jungo*; Greek, ζεύγνυμι: the word itself is the same with *jugum*, ζυγόν), and seems to have been originally applied to indicate a cycle, or period, by means of which the conjunction or correspondence of discordant modes of reckoning time was kept up; thus it still signifies also the *lustrum*, or cycle of five years, which, with an intercalated month, anciently maintained the correspondence of the year of 360 days with the true solar year. From such uses it was transferred to designate the vaster periods of the Hindu chronology.

As half an Age, or two of the lesser periods, are accounted to have elapsed between the end of the Golden and the beginning of the Iron Age, the planets, at the latter epoch, have again returned to a position of mean conjunction: the moon's node, also, is still in the first of Libra, but her apsis has changed its place half a revolution, to the first of Cancer (see above, under vv. 29–34). The positions of the apsides and nodes of the other planets at the same time have been given already, under verses 41–44.

The Hindu names of the signs correspond in signification with our own, having been brought into India from the West. There is nowhere in this work any allusion to them as constellations, or as having any fixed position of their own in the heavens: they are simply the names of the successive signs (*râçi*, *bha*) into which any circle is divided, and it is left to be determined by the connection, in any case, from what point they shall be counted. Here, of course, it is the initial point of the fixed Hindu sphere (see above, under v. 27). As the signs are, in the sequel, frequently cited by name, we present annexed, for the convenience of reference of those to whose memory they are not familiar in the order of their succession, their names, Latin and Sanskrit, their numbers, and the figures generally used to represent them. Those enclosed in brackets do not chance to occur in our text.

1.	Aries,	♈	*mesha, aja.*	7.	Libra,	♎	*tulâ.*
2.	Taurus,	♉	*vṛshan.*	8.	Scorpio,	♏	[*vṛçcika,*] *âli.*
3.	Gemini,	♊	*mithuna.*	9.	Sagittarius,	♐	*dhanus.*
4.	Cancer,	♋	*karka, karkaṭa.*	10.	Capricornus,	♑	*makara, mṛga.*
5.	Leo,	♌	[*sinha*].	11.	Aquarius,	♒	*kumbha.*
6.	Virgo,	♍	*kanyâ.*	12.	Pisces,	♓	[*mina*].

In the translation given above of the second half of verse 58, not a little violence is done to the natural construction. This would seem to require that it be rendered : "and the rest are in whole signs (have come to a position which is without a remainder of degrees); they, being of slow motion, are not stated here." But the actual condition of things at the epoch renders necessary the former translation, which is that of the commentator also. We cannot avoid conjecturing that the natural rendering was perhaps the original one, and that a subsequent alteration of the elements of the treatise compelled the other and forced interpretation to be put upon the passage.

The commentary gives the positions of the apsides and nodes (those of the nodes, however, in reverse) for the epoch of the end of the Golden Age, but, strangely enough, both in the printed edition and in our manuscript, commits the blunder of giving the position of Saturn's node a second time, for that of his apsis, and also of making the seconds of the position of the node of Mars 12, instead of 24. We therefore add them below, in their correct form.

Motion of the Apsides and Nodes of the Planets, to the End of the last Golden Age.

Planet.	Apsis.					Node.				
	(rev.)	s	°	′	″	(rev.)	s	°	′	″
Sun,	(175)	0	7	28	12					
Mercury,	(166)	5	4	4	48	(220)	8	11	16	48
Venus,	(241)	11	13	21	0	(408)	4	17	25	48*
Mars,	(92)	3	3	14	24	(96)	9	11	20	24
Jupiter,	(407)	0	9	0	0	(78)	8	8	56	24
Saturn,	(17)	7	19	35	24	(299)	4	20	13	12

The method of finding the mean places of the planets for midnight on the prime meridian having been now fully explained, the treatise proceeds to show how they may be found for other places, and for other times of the day. To this the first requisite is to know the dimensions of the earth.

59. Twice eight hundred *yojanas* are the diameter of the earth: the square root of ten times the square of that is the earth's circumference.

60. This, multiplied by the sine of the co-latitude (*lambajyâ*) of any place, and divided by radius (*trijîvâ*), is the corrected (*sphuṭa*) circumference of the earth at that place. . . .

There is the same difficulty in the way of ascertaining the exactness of the Hindu measurement of the earth as of the Greek; the uncertain value, namely, of the unit of measure employed. The *yojana* is ordinarily divided into *kroça*, "cries" (i. e., distances to which a certain cry may be heard); the *kroça* into *dhanus*, "bow-lengths," or *daṇḍa*, "poles;" and these again into *hasta*, "cubits." By its origin, the latter

* The printed edition, by an error of the press, gives 4.

ought not to vary far from eighteen inches; but the higher measures differ greatly in their relation to it. The usual reckoning makes the yojana equal 32,000 cubits, but it is also sometimes regarded as composed of 16,000 cubits; and it is accordingly estimated by different authorities at from four and a half to rather more than ten miles English. This uncertainty is no merely modern condition of things: Hiuen-Thsang, the Chinese monk who visited India in the middle of the seventh century, reports (see Stanislas Julien's Mémoires de Hiouen-Thsang, i. 59, etc.) that in India "according to ancient tradition a yojana equals forty *li;* according to the customary use of the Indian kingdoms, it is thirty *li;* but the yojana mentioned in the sacred books contains only sixteen *li:*" this smallest yojana, according to the value of the *li* given by Williams (Middle Kingdom, ii. 154), being equal to from five to six English miles. At the same time, Hiuen-Thsang states the subdivisions of the yojana in a manner to make it consist of only 16,000 cubits. Such being the condition of things, it is clearly impossible to appreciate the value of the Hindu estimate of the earth's dimensions, or to determine how far the disagreement of the different astronomers on this point may be owing to the difference of their standards of measurement. Âryabhatta (see Colebrooke's Hind. Alg. p. xxxviii; Essays, ii. 468) states the earth's diameter to be 1050 yojanas; Bhâskara (Siddh.-Çir. vii. 1) gives it as 1581: the latter author, in his Lîlâvatî (i. 5, 6), makes the yojana consist of 32,000 cubits.

The ratio of the diameter to the circumference of a circle is here made to be $1 : \sqrt{10}$, or 1 : 3.1623, which is no very near approximation. It is not a little surprising to find this determination in the same treatise with the much more accurate one afforded by the table of sines given in the next chapter (vv. 17–21), of 3438 : 10,800, or 1 : 3.14136; and then farther, to find the former, and not the latter, made use of in calculating the dimensions of the planetary orbits (see below, xii. 83). But the same inconsistency is found also in other astronomical and mathematical authorities. Thus Âryabhatta (see Colebrooke, as above) calculates the earth's circumference from its diameter by the ratio 7 : 22, or 1 : 3.14286, but makes the ratio $1 : \sqrt{10}$ the basis of his table of sines, and Brahmagupta and Çrîdhara also adopt the latter. Bhâskara, in stating the earth's circumference at 4967 yojanas, is very near the truth, since 1581 : 4967 : : 1 : 3.14168: his Lîlâvatî (v. 201) gives 7 : 22, and also, as more exact, 1250 : 3927, or 1 : 3.1416. This subject will be reverted to in connection with the table of sines.

The greatest circumference of the earth, as calculated according to the data and method of the text, is 5059.556 yojanas. The astronomical yojana must be regarded as an independent standard of measurement, by which to estimate the value of the other dimensions of the solar system stated in this treatise. To make the earth's mean diameter correct as determined by the Sûrya-Siddhânta, the yojana should equal 4.94 English miles; to make the circumference correct, it should equal 4.91 miles.

The rule for finding the circumference of the earth upon a parallel of latitude is founded upon a simple proportion, viz., rad. : cos. latitude : : circ. of earth at equator : do. at the given parallel; the cosine of the latitude being, in effect, the radius of the circle of latitude. Radius and cosine of latitude are tabular numbers, derived from the table to be given afterward (see below, ii. 17–21). This treatise is not accustomed to employ cosines directly in its calculations, but has special names for the complements of the different arcs which it has occasion to use. Terrestrial latitude is styled *aksha*, "axle," which term, as appears from xii. 42, is employed elliptically for *akshonnati*, "elevation of the axle," i. e., "of the pole:" *lamba*, co-latitude, which properly signifies "lagging, dependence, falling off," is accordingly the depression of the pole, or its distance from the zenith. Directions for finding the co-latitude are given below (iii. 13, 14).

The latitude of Washington being 38° 54′, the sine of its co-latitude is 2675′; the proportion 3438 : 2675 :: 5059.64 : 3936.75 gives us, then, the earth's circumference at Washington as 3936.75 yojanas.

60. . . . Multiply the daily motion of a planet by the distance in longitude (*deçântara*) of any place, and divide by its corrected circumference;

61. The quotient, in minutes, subtract from the mean position of the planet as found, if the place be east of the prime meridian (*rekhâ*); add, if it be west; the result is the planet's mean position at the given place.

The rules previously stated have ascertained the mean places of the planets at a given midnight upon the prime meridian; this teaches us how to find them for the same midnight upon any other meridian, or, how to correct for difference of longitude the mean places already found. The proportion is: as the circumference of the earth at the latitude of the point of observation is to the part of it intercepted between that point and the prime meridian, so is the whole daily motion of each planet to the amount of its motion during the time between midnight on the one meridian and on the other. The distance in longitude (*deçântara*, literally "difference of region") is estimated, it will be observed, neither in time nor in arc, but in yojanas. How it is ascertained is taught below, in verses 63–65.

The geographical position of the prime meridian (*rekhâ*, literally "line") is next stated.

62. Situated upon the line which passes through the haunt of the demons (*râkshasa*) and the mountain which is the seat of the gods, are Rohîtaka and Avantî, as also the adjacent lake.

The "haunt of the demons" is Lankâ, the fabled seat of Râvana, the chief of the Râkshasas, the abduction by whom of Râma's wife, with the expedition to Lankâ of her heroic husband for her rescue, its accomplishment, and the destruction of Râvana and his people, form the subject of the epic poem called the Râmâyana. In that poem, and to the general apprehension of the Hindus, Lankâ is the island Ceylon; in the astronomical geography, however (see below, xii. 39), it is a city, situated upon the equator. How far those who established the meridian may have regarded the actual position of Ceylon as identical with that assigned to Lankâ might not be easy to determine. The "seat of the gods" is Mount Meru, situated at the north pole (see below, xii. 34, etc.). The meridian is usually styled that of Lankâ, and "at Lankâ" is the ordinary phrase made use of in this treatise (as, for instance, above, v. 50; below, iii. 43) to designate a situation either of no longitude or of no latitude.

But the circumstance which actually fixes the position of the prime meridian is the situation of the city of Ujjayinî, the Ὀζηνη of the Greeks, the modern Ojein. It is called in the text by one of its ancient names, Avantî. It is the capital of the rich and populous province of Mâlava, occupying the plateau of the Vindhya mountains just north of the principal ridge and of the river Narmadâ (Nerbudda), and from old time a chief seat of Hindu literature, science, and arts. Of all the centres of Hindu culture, it lay nearest to the great ocean-route by which, during the first three centuries of our era, so important a commerce was carried on between Alexandria, as the mart of Rome, and India and the countries lying still farther east. That the prime meridian was made to pass through this city proves it to have been the cradle of the Hindu science of astronomy, or its principal seat during its early history. Its actual situation is stated by Warren (Kâla Sankalita, p. 9) as lat. 23° 11′ 30″ N., long. 75° 53′ E. from Greenwich: a later authority, Thornton's Gazetteer of India (London: 1857), makes it to be in lat. 23° 10′ N., long. 75° 47′ E.; in our farther calculations, we shall assume the latter position to be the correct one.

The situation of Rohîtaka is not so clear; we have not succeeded in finding such a place mentioned in any work on the ancient geography of India to which we have access, nor is it to be traced upon Lassen's map of ancient India. A city called Rohtuk, however, is mentioned by Thornton (Gazetteer, p. 836), as the chief place of a modern British district of the same name, and its situation, a little to the north-west of Delhi, in the midst of the ancient Kurukshetra, leads us to regard it as identical with the Rohîtaka of the text. That the meridian of Lankâ was expressly recognized as passing over the Kurukshetra, the memorable site of the great battle described by the Mahâbhârata, seems clear. Bhâskara (Siddh.-Çir., Gan., vii. 2) describes it as follows: "the line which, passing above Lankâ and Ujjayinî, and touching the region of the Kurukshetra, etc., goes through Meru—that line is by the wise regarded as the central meridian (*madhyarekhâ*) of the earth." Our own commentary also explains *sannihitam sarah*, which we have translated "adjacent lake," as signifying Kurukshetra. Warren (as above) takes the same expression to be the name of a city, which seems to us highly improbable; nor do we see that the word *saras* can properly be applied to a tract of country: we have therefore thought it safest to translate literally the words of the text, confessing that we do not know to what they refer.

If Rohîtaka and Rohtuk signify the same place, we have here a measure of the accuracy of the Hindu determinations of longitude; Thornton gives its longitude as 76° 38′, or 51′ to the east of Ujjayinî.

The method by which an observer is to determine his distance from the prime meridian is next explained.

63. When, in a total eclipse of the moon, the emergence (*unmîlana*) takes place after the calculated time for its occurrence, then the place of the observer is to the east of the central meridian;

64. When it takes place before the calculated time, his place is to the west: the same thing may be ascertained likewise from the immersion (*nimîlana*). Multiply by the difference of the two times in nâḍîs the corrected circumference of the earth at the place of observation,

65. And divide by sixty: the result, in yojanas, indicates the distance of the observer from the meridian, to the east or to the west, upon his own parallel; and by means of that is made the correction for difference of longitude.

Choice is made, of course, of a lunar eclipse, and not of a solar, for the purpose of the determination of longitude, because its phenomena, being unaffected by parallax, are seen everywhere at the same instant of absolute time; and the moments of total disappearance and first reappearance of the moon in a total eclipse are farther selected, because the precise instant of their occurrence is observable with more accuracy than that of the first and last contact of the moon with the shadow. For the explanation of the terms here used see the chapters upon eclipses (below, iv–vi).

The interval between the computed and observed time being ascertained, the distance in longitude (*deçântara*) is found by the simple proportion: as the whole number of nâḍîs in a day (sixty) is to the interval of time in nâḍîs, so is the circumference of the earth at the latitude of the point of observation to the distance of that point from the prime meridian, measured on the parallel. Thus, for instance, the distance of Ujjayinî from Greenwich, in time, being $5^h\ 3^m\ 8^s$, and that of Washington from Greenwich $5^h\ 8^m\ 11^s$ (Am. Naut. Almanac), that of Ujjayinî from Washington is $10^h\ 11^m\ 19^s$, or, in Hindu time, $25^n\ 28^v\ 1^p.8$, or $25^n.4718$: and by the proportion 60 : 25.4718 : : 3936.75 : 1671.28, we obtain 1671.28 yojanas as the distance in longitude (*deçântara*) of Washington from the Hindu meridian, the constant quantity to be employed in finding the mean places of the planets at Washington.

We might have expected that calculators so expert as the Hindus would employ the interval of time directly in making the correction for difference of longitude, instead of reducing it first to its value in yojanas. That they did not measure longitude in our manner, in degrees, etc., is owing to the fact that they seem never to have thought of applying to the globe of the earth the system of measurement by circles and divisions of circles which they used for the sphere of the heavens, but, even when dividing the earth into zones (see below, xii. 59–66) reduced all their distances laboriously to yojanas.

66. The succession of the week-day (*vâra*) takes place, to the east of the meridian, at a time after midnight equal to the difference of longitude in nâḍîs; to the west of the meridian, at a corresponding time before midnight.

This verse appears to us to be an astrological precept, asserting the regency of the sun and the other planets, in their order, over the successive portions of time assigned to each, to begin everywhere at the same instant of absolute time, that of their true commencement upon the prime meridian; so that, for instance, at Washington, Sunday, as the day placed under the guardianship of the sun, would really begin at eleven minutes before two on Saturday afternoon, by local time. The commentator, however, sees in it merely an intimation of what moment of local time, in places east and west of the meridian, corresponds to the true beginning of the day upon the prime meridian, and he is at much pains to defend the verse from the charge of being superfluous and unnecessary, to which it is indeed liable, if that be its only meaning.

The rules thus far given have directed us only how to find the mean places of the planets at a given midnight. The following verse teaches the method of ascertaining their position at any required hour of the day.

67. Multiply the mean daily motion of a planet by the number of nâḍîs of the time fixed upon, and divide by sixty: subtract the quotient from the place of the planet, if the time be before midnight; add, if it be after: the result is its place at the given time.

The proportion is as follows: as the number of nâḍîs in a day (sixty) is to those in the interval between midnight and the time for which the mean place of the planet is sought, so is the whole daily motion of the planet to its motion during the interval; and the result is additive or subtractive, of course, according as the time fixed upon is after or before midnight.

In order to furnish a practical test of the accuracy of this text-book of astronomy, and of its ability to yield correct results at the present time, we have calculated, by the rule given in this verse, the mean longitudes of the planets for a time after midnight of the first of January, 1860, on the meridian of Ujjayinî, which is equal to the distance in time of the meridian of Washington, viz. $25^{n}\ 28^{v}\ 1^{p}.8$, or $0^{d}.42453$; and we present the results in the annexed table. The longitudes are given as reckoned from the vernal equinox of that date, which we make to be distant 18° 5′ 8″.25 from the point established by the Sûrya-Siddhânta as the beginning of the Hindu sidereal sphere; this is (see below, chap. viii) 10′ east of ζ Piscium. We have ascertained the mean places both as determined by the text of our Siddhânta, and by the same with the correction of the *bîja.* Added are the actual mean places at the time designated: those of the primary planets have been found from Le Verrier's elements, presented in Biot's treatise, as cited above;* those of the moon, and of her apsis and node, were kindly furnished us from the office of the American Nautical Almanac, at Cambridge.

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* We would warn our readers, however, of a serious error of the press in the table as given by Biot; as the yearly motion of the earth, read 1,295,977.38, instead of . . . 972.38.

Mean Longitudes of the Planets, Jan. 1*st*, 1860, *midnight, at Washington.*

Planet.	According to Sûrya-Siddhânta: text.	According to Sûrya-Siddhânta: with *bîja*.	According to moderns.
	° ′ ″	° ′ ″	° ′ ″
Sun,	96 18 21	96 18 21	100 5 6
Mercury,	155 2 30	148 25 39	151 28 20
Venus,	339 54 55	334 57 18	336 13 36
Mars,	192 36 5	192 36 5	197 26 32
Jupiter,	104 7 22	100 48 56	103 35 17
Saturn,	128 17 11	133 14 49	137 10 10
Moon,	9 4 9	9 4 9	12 41 23
" apsis,	327 50 24	326 11 11	326 47 35
" node,	312 29 51	310 50 38	312 48 10

In the next following table is farther given a view of the errors of the Hindu determinations—both the absolute errors, as compared with the actual mean place of each planet, and the relative, as compared with the place of the sun, to which it is the aim of the Hindu astronomical systems to adapt the elements of the other planets. Annexed to each error is the approximate date at which it was nothing, or at which it will hereafter disappear, ascertained by dividing the amount of present error by the present yearly loss or gain, absolute or relative, of each planet; excepting in the case of the moon, where we have made allowance, according to the formula used by the American Nautical Almanac, for the acceleration of her motion.

Errors of the Mean Longitudes of the Planets, as calculated according to the Sûrya-Siddhânta.

Planet.	Errors according to text: absolute.	when correct.	rel. to sun.	when correct.	The same, with *bîja*: absolute.	when correct.	rel. to sun.	when correct.
	° ′ ″	A. D.	° ′ ″	A. D.	° ′ ″	A. D.	° ′ ″	A. D.
Sun,	−3 46 45	250	0 0 0	....	−3 46 45	250	0 0 0	....
Mercury,	+3 34 10	2332	+7 20 55	3271	−3 2 41	1517	+0 44 5	1970
Venus,	+3 41 19	1222	+7 28 4	941	−1 16 18	2126	+2 30 27	1509
Mars,	−4 50 27	886	−1 3 42	1455	−4 50 27	886	−1 3 42	1455
Jupiter,	+0 32 5	1571	+4 18 50	832	−2 46 21	4203	+1 0 24	1575
Saturn,	−8 52 59	666	−5 6 14	857	−3 55 21	1250	−0 8 36	1825
Moon,	−3 37 14	115	+0 9 31	1067	−3 37 14	115	+0 9 31	1067
" apsis,	+1 2 49	1679	+4 49 34	1252	−0 36 24	1969	+3 10 21	1459
" node,	−0 18 19	1976	+3 28 26	1162	−1 57 32	2714	+1 49 13	1468

To complete the view of the planetary motions, and the statement of the elements requisite for ascertaining their position in the sky, it only remains to give the movement in latitude of each, its deviation from the general planetary path of the ecliptic. This is done in the concluding verses of the chapter.

68. The moon is, by its node, caused to deviate from the limit of its declination (*krânti*), northward and southward, to a distance, when greatest, of an eightieth part of the minutes of a circle;

69. Jupiter, to the ninth part of that multiplied by two; Mars, to the same amount multiplied by three; Mercury, Venus, and Saturn are by their nodes caused to deviate to the same amount multiplied by four.

70. So also, twenty-seven, nine, twelve, six, twelve, and twelve, multiplied respectively by ten, give the number of minutes of mean latitude (*vikshepa*) of the moon and the rest, in their order.

The deviation of the planets from the plane of the ecliptic is here stated in two different ways, which give, however, the same results; thus:

Moon,	$\frac{21600'}{80} = 270'$	or	$27' \times 10 = 270' = 4^\circ\ 30'$
Mars,	$\frac{270'}{9} \times 3 = 90'$	or	$9' \times 10 = 90' = 1^\circ\ 30'$
Mercury,	$\frac{270'}{9} \times 4 = 120'$	or	$12' \times 10 = 120' = 2^\circ$
Jupiter,	$\frac{270'}{9} \times 2 = 60'$	or	$6' \times 10 = 60' = 1^\circ$
Venus,	$\frac{270'}{9} \times 4 = 120'$	or	$12' \times 10 = 120' = 2^\circ$
Saturn,	$\frac{270'}{9} \times 4 = 120'$	or	$12' \times 10 = 120' = 2^\circ$

The subject of the latitude of the planets is completed in verses 6–8, and verse 57, of the following chapter; the former passage describes the manner, and indicates the direction, in which the node produces its disturbing effect; the latter gives the rule for calculating the apparent latitude of a planet at any point in its revolution.

There is a little discrepancy between the two specifications presented in these verses, as regards the description of the quantities specified: the one states them to be the amounts of greatest (*parama*) deviation from the ecliptic; the other, of mean (*madhya*) deviation. Both descriptions are also somewhat inaccurate. The first is correct only with reference to the moon, and the two terms require to be combined, in order to be made applicable to the other planets. The moon has its greatest latitude at 90° from its node, and this latitude is obviously equal to the inclination of its orbit to the ecliptic; for although its absolute distance from the ecliptic at this point of its course varies, as does its distance from the earth, on account of the eccentricity of its orbit, and the varying relation of the line of its apsides to that of its nodes, its angular distance remains unchanged. So, to an observer stationed at the sun, the greatest latitude of any one of the primary planets would be the same in its successive revolutions from node to node, and equal to the inclination of its orbit. But its greatest latitude as seen from the earth is very different in different revolutions, both on account of the difference of its absolute distance from the ecliptic when at the point of greatest removal from it in the two halves of its orbit, and, much more, on account of its varying distance from the earth. The former of these two causes of variation was not recognized by the

Hindus: in this treatise, at least, the distance of the node from the apsis (*mandocca*) is not introduced as an element into the process for determining a planet's latitude. The other cause of variation is duly allowed for (see below, ii. 57). Its effect, in the case of the three superior planets, is to make their greatest latitude sometimes greater, and sometimes less, than the inclination of their orbits, according as the planet is nearer to us than to the sun, or the contrary; hence the values given in the text for Mars, Jupiter, and Saturn, as they represent the mean apparent values, as latitude, of the greatest distance of each planet from the ecliptic, should nearly equal the inclination. In the case of Mercury and Venus, also, the quantities stated are the mean of the different apparent values of the greatest heliocentric latitude, but this mean is of course less, and for Mercury very much less, than the inclination. Ptolemy, in the elaborate discussion of the theory of the latitude contained in the thirteenth book of his Syntaxis, has deduced the actual inclination of the orbits of the two inferior planets: this the Hindus do not seem to have attempted.

We present below a comparative table of the inclinations of the orbits of the planets as determined by Ptolemy and by modern astronomers, with those of the Hindus, so far as given directly by the Sûrya-Siddhânta.

Inclination of the Orbits of the Planets, according to Different Authorities.

Planet.	Sûrya-Siddhânta.		Ptolemy.		Moderns.		
	°	′	°	′	°	′	″
Mercury,	. . . . .		7		7	0	8
Venus,	. . . . .		3	30	3	23	31
Mars,	1	30	1		1	51	5
Jupiter,	1		1	30	1	18	40
Saturn,	2		2	30	2	29	28
Moon,	4	30	5		5	8	40

The verb in verses 68 and 69, which we have translated "caused to deviate," is *vi kshipyate*, literally "is hurled away," *disjicitur*; from it is derived the term used in this treatise to signify celestial latitude, *vikshepa*, "disjection." The Hindus measure the latitude, however, as we shall have occasion to notice more particularly hereafter, upon a circle of declination, and not upon a secondary to the ecliptic. In the words chosen to designate it is seen the influence of the theory of the node's action, as stated in the first verses of the next chapter. The forcible removal is from the point of declination (*krânti*, "gait," or *apakrama*, "withdrawal," i. e., from the celestial equator) which the planet ought at the time to occupy.

The title given to this first chapter (*adhikâra*, "subject, heading") is *madhyamâdhikâra*, which we have represented in the title by "mean motions of the planets," although it would be more accurately rendered by "mean places of the planets;" that is to say, the data and methods requisite for ascertaining their mean places. Now follows the *spashtâdhikâra*, "chapter of the true, or corrected, places of the planets."

CHAPTER II.

OF THE TRUE PLACES OF THE PLANETS.

CONTENTS:—1–3, causes of the irregularities of the planetary motions; 4–5, disturbing influence of the apsis and conjunction; 6–8, of the node; 9–11, different degree of irregularity of the motion of the different planets; 12–13, different kinds of planetary motion; 14, purpose of this chapter; 15–16, rule for constructing the table of sines; 17–22, table of sines; 22–27, table of versed sines; 28, inclination of the ecliptic, and rule for finding the declination of any point in it; 29–30, to find the sine and cosine of the anomaly; 31–32, to find, by interpolation, the sine or versed sine corresponding to any given arc; 33, to find, in like manner, the arc corresponding to a given sine or versed sine; 34–37, dimensions of the epicycles of the planets; 38, to find the true dimensions of the epicycle at any point in the orbit; 39, to find the equation of the apsis, or of the centre; 40–42, to find the equation of the conjunction, or the annual equation; 43–45, application of these equations in finding the true places of the different planets; 46, correction of the place of a planet for difference between mean and apparent solar time; 47–49, how to correct the daily motion of the planets for the effect of the apsis; 50–51, the same for that of the conjunction; 51–55, retrogradation of the lesser planets; 56, correction of the place of the node; 57–58, to find the celestial latitude of a planet, and its declination as affected by latitude; 59, to find the length of the day of any planet; 60, to find the radius of the diurnal circle; 61–63, to find the day-sine, and the respective length of the day and night; 64, to find the number of asterisms traversed by a planet, and of days elapsed, since the commencement of the current revolution; 65, to find the *yoga;* 66, to find the current lunar day, and the time in it of a given instant; 67–69, of the divisions of the lunar month called *karaṇa.*

1. Forms of Time, of invisible shape, stationed in the zodiac (*bhagaṇa*), called the conjunction (*çîghrocca*), apsis (*mandocca*), and node (*pâta*), are causes of the motion of the planets.

2. The planets, attached to these beings by cords of air, are drawn away by them, with the right and left hand, forward or backward, according to nearness, toward their own place.

3. A wind, moreover, called *provector* (*pravaha*) impels them toward their own apices (*ucca*); being drawn away forward and backward, they proceed by a varying motion.

4. The so-called apex (*ucca*), when in the half-orbit in front of the planet, draws the planet forward; in like manner, when in the half-orbit behind the planet, it draws it backward.

5. When the planets, drawn away by their apices (*ucca*), move forward in their orbits, the amount of the motion so caused is called their excess (*dhana*); when they move backward, it is called their deficiency (*ṛṇa*).

In these verses is laid before us the Hindu theory of the general nature of the forces which produce the irregularities of the apparent motions, regarded as being the real motions, of the planets. The world-wide difference between the spirit of the Hindu astronomy and that of the Greek is not less apparent here than in the manner of presentation of the elements in the last chapter: the one is purely scientific, devising methods for representing and calculating the observed motions, and attempting nothing farther; the other is not content without fabricating a fantastic and absurd theory respecting the superhuman powers which occasion the movements with which it is dealing. The Hindu method has this convenient peculiarity, that it absolves from all necessity of adapting the disturbing forces to one another, and making them form one consistent system, capable of geometrical representation and mathematical demonstration; it regards the planets as actually moving in circular orbits, and the whole apparatus of epicycles, given later in the chapter, as only a device for estimating the amount of the force, and of its resulting motion, exerted at any given point by the disturbing cause.

The commentator gives two different explanations of the provector wind, spoken of in the third verse: one, that it is the general current, mentioned below, in xii. 73, as impelling the whole firmament of stars, and which, though itself moving westward, drives the planets, in some unexplained way, towards its own apex of motion, in the east; the other, that a separate vortex for each planet, called provector on account of its analogy with that general current, although not moving in the same direction, carries them around in their orbits from west to east, leaving only the irregularities of their motion to be produced by the disturbing forces. This latter we regard as the proper meaning of the text: neither is very consistent with the theory of the lagging behind of the planets, given above, in i. 25, 26, as the explanation of their apparent eastward motion. The commentary also states more explicitly the method of production of the disturbance: a cord of air, equal in length to the orbit of each planet less the disk of the latter itself, is attached to the extremities of its diameter, and passes through the two hands of the being stationed at the point of disturbance; and he always draws it toward himself by the shorter of the two parts of the cord. The term *ucca*, which we have translated "apex," applies both to the apsis (*manda*, *mandocca*, "apex of slowest motion"—the apogee in the case of the sun and moon, the aphelion, though not recognized as such, in the case of the other planets), and to the conjunction (*çighra*, *çighrocca*, "apex of swiftest motion"). The statement made of the like effect of the two upon the motion of the planet is liable to cause difficulty, if it be not distinctly kept in mind that the Hindus understand by the influence of the disturbing cause, not its acceleration and retardation of the rate of the planet's motion, but its effect in giving to the planet a position in advance of, or behind, its mean place. It may be well, for the sake of aiding some of our readers to form a clearer apprehension of the Hindu view of the planetary motions, to expand and illustrate a little this statement of the effect upon them of the two principal disturbing forces.

First, as regards the apsis. This is the remoter extremity of the major axis of the planet's proper orbit, and the point of its slowest motion.

Upon passing this point, the planet begins to fall behind its mean place, but at the same time to gain velocity, so that at the quadrature it is farthest behind, but is moving at its mean rate; during the next quadrant it gains both in rate of motion and in place, until at the perigee, or perihelion, it is moving most rapidly, and has made up what it before lost, so that the mean and true places coincide. Upon passing that point again, it gains upon its mean place during the first quadrant, and loses what it thus gained during the second, until mean and true place again coincide at the apsis. Thus the equation of motion is greatest at the apsides, and nothing at the quadratures, while the equation of place is greatest at the quadratures, and nothing at the apsides; and thus the planet is always behind its mean place while passing from the higher to the lower apsis, and always in advance of it while passing from the lower to the higher; that is, it is constantly drawn away from its mean place toward the higher apsis, *mandocca.*

In treating of the effect of the conjunction, the *çîghrocca,* we have to distinguish two kinds of cases. With Mercury and Venus (see above, i. 29, 31, 32), the revolution of the conjunction takes the place, in the Hindu system as in the Greek, of that of the planet itself, the conjunction being regarded as making the circuit of the zodiac in the same time, and in the same direction, as the planet really revolves about the sun; while the mean place of these planets is always that of the sun itself. While, therefore, the conjunction is making the half-tour of the heavens eastward from the sun, the planet is making its eastward elongation and returning to the sun again, being all the time in advance of its mean place, the sun; when the conjunction reaches a point in the heavens opposite to the sun, the planet is in its inferior conjunction, or at its mean place; during the other half of the revolution of the conjunction, when it is nearest the planet upon the western side, the latter is making and losing its western elongation, or is behind its mean place. Accordingly, as stated in the text, the planet is constantly drawn away from its mean place, the sun, toward that side of the heavens in which the conjunction is.

Once more, as concerns the superior planets. The revolutions assigned to these by the Hindus are their true revolutions; their mean places are their mean heliocentric longitudes; and the place of the conjunction (*çîghrocca*) of each is the mean place of the sun. Since they move but slowly, as compared with the sun, it is their conjunction which approaches, overtakes, and passes them, and not they the conjunction. Their time of slowest motion is when in opposition with the sun; of swiftest, when in conjunction with him: from opposition on to conjunction, therefore, or while the sun is approaching them from behind, they are, with constantly increasing velocity of motion, all the while behind their mean places, or drawn away from them in the direction of the sun; but no sooner has the sun overtaken and passed them, than they, leaving with their most rapid motion the point of coincidence between mean and true place, are at once in advance, and continue to be so until opposition is reached again; that is to say, they are still drawn away from their mean place in the direction of the conjunction.

The words used in verse 5 for "excess" and "deficiency," or for additive and subtractive equation, mean literally "wealth" (*dhana*) and "debt" (*ṛṇa*).

6. In like manner, also, the node, Râhu, by its proper force, causes the deviation in latitude (*vikshepa*) of the moon and the other planets, northward and southward, from their point of declination (*apakrama*).

7. When in the half-orbit behind the planet, the node causes it to deviate northward; when in the half-orbit in front, it draws it away southward.

8. In the case of Mercury and Venus, however, when the node is thus situated with regard to the conjunction (*çîghra*), these two planets are caused to deviate in latitude, in the manner stated, by the attraction exercised by the node upon the conjunction.

The name Râhu, by which the ascending node is here designated, is properly mythological, and belongs to the monster in the heavens, which, by the ancient Hindus, as by more than one other people, was believed to occasion the eclipses of the sun and moon by attempting to devour them. The word which we have translated "force" is *ranhas*, more properly "rapidity, violent motion:" in employing it here, the text evidently intends to suggest an etymology for *râhu*, as coming from the root *rah* or *ranh*, "to rush on": with this same root Weber (Ind. Stud. i. 272) has connected the group of words in which *râhu* seems to belong. For the Hindu fable respecting Râhu, see Wilson's Vishṇu Purâṇa, p. 78. The moon's descending node was also personified in a similar way, under the name of Ketu, but to this no reference is made in the present treatise.

The description of the effect of the node upon the movement of the planet is to be understood, in a manner analogous with that of the effect of the apices in the next preceding passage, as referring to the direction in which the planet is made to deviate from the ecliptic, and not to that in which it is moving with reference to the ecliptic. From the ascending node around to the descending, of course, or while the node is nearest to the planet from behind, the latitude is northern; in the other half of the revolution it is southern.

For an explanation of some of the terms used here, see the note to the last passage of the preceding chapter.

As, in the case of Mercury and Venus, the revolution of the conjunction takes the place of that of the planet itself in its orbit, it is necessary, in order to give the node its proper effect, that it be made to exercise its influence upon the planet through the conjunction. The commentator gives himself here not a little trouble, in the attempt to show why Mercury and Venus should in this respect constitute an exception to the general rule, but without being able to make out a very plausible case.

9. Owing to the greatness of its orb, the sun is drawn away only a very little; the moon, by reason of the smallness of its orb, is drawn away much more;

10. Mars and the rest, on account of their small size, are, by the supernatural beings (*dâivata*) called conjunction (*çîghrocca*) and apsis (*mandocca*), drawn away very far, being caused to vacillate exceedingly.

11. Hence the excess (*dhana*) and deficiency (*ṛṇa*) of these latter is very great, according to their rate of motion. Thus do the planets, attracted by those beings, move in the firmament, carried on by the wind.

The dimensions of the sun and moon are stated below, in iv. 1; those of the other planets, in vii. 13.

We have ventured to translate *ativegita*, at the end of the tenth verse, as it is given above, because that translation seemed so much better to suit the requirements of the sense than the better-supported rendering "caused to move with exceeding velocity." In so doing, we have assumed that the noun *vega*, of which the word in question is a denominative, retains something of the proper meaning of the root *vij*, "to tremble," from which it comes.

12. The motion of the planets is of eight kinds: retrograde (*vakra*), somewhat retrograde (*anuvakra*), transverse (*kuṭila*), slow (*manda*), very slow (*mandatara*), even (*sama*); also, very swift (*çîghratara*), and swift (*çîghra*).

13. Of these, the very swift (*atiçîghra*), that called swift, the slow, the very slow, the even—all these five are forms of the motion called direct (*ṛju*); the somewhat retrograde is retrograde.

This minute classification of the phases of a planet's motion is quite gratuitous, so far as this Siddhânta is concerned, for the terms here given do not once occur afterward in the text, with the single exception of *vakra*, which, with its derivatives, is in not infrequent use to designate retrogradation. Nor does the commentary take the trouble to explain the precise differences of the kinds of motion specified. According to Mr. Hoisington (Oriental Astronomer [Tamil and English], Jaffna: 1848, p. 133), *anuvakra* is applied to the motion of a planet, when, in retrograding, it passes into a preceding sign. From the classification given in the second of the two verses it will be noticed that *kuṭila* is omitted: according to the commentator, it is meant to be included among the forms of retrograde motion; we have conjectured, however, that it might possibly be used to designate the motion of a planet when, being for the moment stationary in respect to longitude, and accordingly neither advancing nor retrograding, it is changing its latitude; and we have translated the word accordingly.

14. By reason of this and that rate of motion, from day to day, the planets thus come to an accordance with their observed places (*dṛç*)—this, their correction (*sphuṭîkaraṇa*), I shall carefully explain.

Having now disposed of matters of general theory and preliminary explanation, the proper subject of this chapter, the calculation of the true (*sphuṭa*) from the mean places of the different planets, is ready to be taken up. And the first thing in order is the table of sines, by means of which all the after calculations are performed.

15. The eighth part of the minutes of a sign is called the first sine (*jyârdha*); that, increased by the remainder left after subtracting from it the quotient arising from dividing it by itself, is the second sine.

16. Thus, dividing the tabular sines in succession by the first, and adding to them, in each case, what is left after subtracting the quotients from the first, the result is twenty-four tabular sines (*jyârdhapiṇḍa*), in order, as follows:

17. Two hundred and twenty-five; four hundred and forty-nine; six hundred and seventy-one; eight hundred and ninety; eleven hundred and five; thirteen hundred and fifteen;

18. Fifteen hundred and twenty; seventeen hundred and nineteen; nineteen hundred and ten; two thousand and ninety-three;

19. Two thousand two hundred and sixty-seven; two thousand four hundred and thirty-one; two thousand five hundred and eighty-five; two thousand seven hundred and twenty-eight;

20. Two thousand eight hundred and fifty-nine; two thousand nine hundred and seventy-eight; three thousand and eighty-four; three thousand one hundred and seventy-seven;

21. Three thousand two hundred and fifty-six; three thousand three hundred and twenty-one; three thousand three hundred and seventy-two; three thousand four hundred and nine;

22. Three thousand four hundred and thirty-one; three thousand four hundred and thirty-eight. Subtracting these, in reversed order, from the half-diameter, gives the tabular versed-sines (*utkramajyârdhapiṇḍaka*):

23. Seven; twenty-nine; sixty-six; one hundred and seventeen; one hundred and eighty-two; two hundred and sixty-one; three hundred and fifty-four;

24. Four hundred and sixty; five hundred and seventy-nine; seven hundred and ten; eight hundred and fifty-three; one thousand and seven; eleven hundred and seventy-one;

25. Thirteen hundred and forty-five; fifteen hundred and twenty-eight; seventeen hundred and nineteen; nineteen hundred and eighteen;

26. Two thousand one hundred and twenty-three; two thousand three hundred and thirty-three; two thousand five hundred and forty-eight; two thousand seven hundred and sixty-seven;

27. Two thousand nine hundred and eighty-nine; three thousand two hundred and thirteen; three thousand four hundred and thirty-eight: these are the versed sines.

We first present, in the following table, in a form convenient for reference and use, the Hindu sines and versed sines, with the arcs to which they belong, the latter expressed both in minutes and in degrees and minutes. To facilitate the practical use of the table in making calculations after the Hindu method, we have added a column of the differences of the sines, and have farther turned the sines themselves into decimal parts of the radius. For the purpose of illustrating the accuracy of the table, we have also annexed the true values of the sines, in minutes, as found by our modern tables. Comparison may also be made of the decimal column with the corresponding values given in our ordinary tables of natural sines.

Table of Sines and Versed Sines.

No.	Arcs, in ° ′	Arcs, in ′	Hindu Sines, in ′	Hindu Sines, Diff.	Hindu Sines, in parts of rad.	True Sines, in ′	Versed Sines, in ′
1	3° 45′	225′	225′	224′	.065445	224′.84	7′
2	7° 30′	450′	449′	222′	.130599	448′.72	29′
3	11° 15′	675′	671′	219′	.195172	670′.67	66′
4	15°	900′	890′	215′	.258871	889′.76	117′
5	18° 45′	1125′	1105′	210′	.321408	1105′.03	182′
6	22° 30′	1350′	1315′	205′	.382489	1315′.57+	261′
7	26° 15′	1575′	1520′	199′	.442117	1520′.48	354′
8	30°	1800′	1719′	191′	.500000	1718′.88	460′
9	33° 45′	2025′	1910′	183′	.555555	1909′.91	579′
10	37° 30′	2250′	2093′	174′	.608784	2092′.77	710′
11	41° 15′	2475′	2267′	164′	.659395	2266′.67	853′
12	45°	2700′	2431′	154′	.707097	2430′.86	1007′
13	48° 45′	2925′	2585′	143′	.751894	2584′.64	1171′
14	52° 30′	3150′	2728′	131′	.793484	2727′.35−	1345′
15	56° 15′	3375′	2859′	119′	.831588	2858′.38−	1528′
16	60°	3600′	2978′	106′	.866201	2977′.18−	1719′
17	63° 45′	3825′	3084′	93′	.897033	3083′.22−	1918′
18	67° 30′	4050′	3177′	79′	.924084	3176′.07−	2123′
19	71° 15′	4275′	3256′	65′	.947062	3255′.31−	2333′
20	75°	4500′	3321′	51′	.965969	3320′.61	2548′
21	78° 45′	4725′	3372′	37′	.980803	3371′.70	2767′
22	82° 30′	4950′	3409′	22′	.991565	3408′.34−	2989′
23	86° 15′	5175′	3431′	7′	.997964	3430′.39−	3213′
24	90°	5400′	3438′		1.000000	3437′.75	3438′

The rule by which the sines are, in the text, directed to be found, may be illustrated as follows. Let s, s', s'', s''', s'''', etc., represent the successive sines. The first of the series, s, is assumed to be equal to its arc, or 225′, from which quantity, as is shown in the table above, it differs only by an amount much smaller than the table takes any account of. Then

$$s' = s + s - \frac{s}{s}$$

$$s'' = s' + s - \frac{s}{s} - \frac{s'}{s}$$

$$s''' = s'' + s - \frac{s}{s} - \frac{s'}{s} - \frac{s''}{s}$$

$$s'''' = s''' + s - \frac{s}{s} - \frac{s'}{s} - \frac{s''}{s} - \frac{s'''}{s}$$

and so on, through the whole series, any fraction larger than a half being counted as one, and a smaller fraction being rejected. In the majority of cases, as is made evident by the table, this process yields correct results: we have marked in the column of "true sines" with a plus or minus sign such modern values of the sines as differ by more than half a minute from those assigned by the Hindu table.

It is not to be supposed, however, that the Hindu sines were originally obtained by the process described in the text. That process was, in all probability, suggested by observing the successive differences in the values of the sines as already determined by other methods. Nor is it difficult to discover what were those methods; they are indicated by the limitation of the table to arcs differing from one another by 3° 45′, and by what we know in general of the trigonometrical methods of the Hindus. The two main principles, by the aid of which the greater portion of all the Hindu calculations are made, are, on the one hand, the equality of the square of the hypothenuse in a right-angled triangle to the sum of the squares of the other two sides, and, on the other hand, the proportional relation of the corresponding parts of similar triangles. The first of these principles gave the Hindus the sine of the complement of any arc of which the sine was already known, it being equal to the square root of the difference between the squares of radius and of the given sine. This led farther to the rule for finding the versed sine, which is given above in the text: it was plainly equal to the difference between the sine complement and radius. Again, the comparison of similar triangles showed that the chord of an arc was a mean proportional between its versed sine and the diameter; and this led to a method of finding the sine of half any arc of which the sine was known: it was equal to half the square root of the product of the diameter into the versed sine. That the Hindus had deduced this last rule does not directly appear from the text of this Siddhânta, nor from the commentary of Ranganâtha, which is the one given by our manuscript and by the published edition; but it is distinctly stated in the commentary which Davis had in his hands (As. Res. ii. 247); and it might be confidently assumed to be known upon the evidence of the table itself; for the principles and rules which we have here stated would give a table just such as the one here constructed. The sine of 90° was obviously equal to radius, and the sine of 30° to half radius: from the first could be found the sines of 45°, 22° 30′, and 11° 15′; from the latter, those of 15°, 7° 30′, and 3° 45′. The sines thus obtained would give those of the complementary arcs, or of 86° 15′, 82° 30′, 78° 45′, 75°, etc.; and the sine of 75°, again, would give those of 37° 30′ and 18° 45′. By continuing the same processes, the table of sines would soon be made complete for the twenty-four divisions of the quadrant; but these processes could yield nothing farther, unless by introducing fractions of minutes; which was undesirable, because the symmetry of the table would thus be destroyed, and no corresponding advantage gained; the table was already sufficiently extended to furnish, by interpolation, the sines intermediate between those given, with all the accuracy which the Hindu calculations required.

If, now, an attempt were made to ascertain a law of progression for the series, and to devise an empirical rule by which its members might be developed, the one from the other, in order, nothing could be more natural than to take the differences of the successive sines, and the differences of those differences, as we have given them under the headings Δ' and Δ'' in the annexed table.

Hindu Sines, with their First and Second Differences.

No.	Sine.	Δ'	Δ''	No.	Sine.	Δ'	Δ''
0	000	225		12	2431	154	10
1	225	224	1	13	2585	143	11
2	449	222	2	14	2728	131	12
3	671	219	3	15	2859	119	12
4	890	215	4	16	2978	106	13
5	1105	210	5	17	3084	93	13
6	1315	205	5	18	3177	79	14
7	1520	199	6	19	3256	65	14
8	1719	191	8	20	3321	51	14
9	1910	183	8	21	3372	37	14
10	2093	174	9	22	3409	22	15
11	2267	164	10	23	3431	7	15
12	2431		10	24	3438		

With these differences before him, an acute observer could hardly fail to notice the remarkable fact that the differences of the second order increase as the sines; and that each, in fact, is about the $\frac{1}{225}$th part of the corresponding sine. Now let the successive sines be represented by 0, s, s', s'', s''', s'''', and so on; and let q equal $\frac{1}{225}$, or $\frac{1}{s}$; let the first differences be $d=s-0$, $d'=s'-s$, $d''=s''-s'$, $d'''=s'''-s''$, etc. The second differences will be: $-sq=d'-d$, $-s'q=d''-d'$, $-s''q=d'''-d''$, etc. These last expressions give

$$d' = d - sq = s - sq$$
$$d'' = d' - s'q = s - sq - s'q$$
$$d''' = d'' - s''q = s - sq - s'q - s''q, \text{ etc.}$$

Hence, also,

$$s' = s + d' = s + s - sq$$
$$s'' = s' + d'' = s' + s - sq - s'q$$
$$s''' = s'' + d''' = s'' + s - sq - s'q - s''q,$$

and so on, according to the rule given in the text.

That the second differences in the values of the sines were proportional to the sines themselves, was probably known to the Hindus only by observation. Had their trigonometry sufficed to demonstrate it, they might easily have constructed a much more complete and accurate table of sines. We add the demonstration given by Delambre (Histoire de l'Astronomie Ancienne, i. 458), from whom the views here expressed have been substantially taken.

Let a be any arc in the series, and put $3° 45' = n$. Then $\sin(a-n)$, $\sin a$, $\sin(a+n)$, will be three successive terms in the series: $\sin a - \sin(a-n)$, and $\sin(a+n) - \sin a$, will be differences of the first order; and their difference, $\sin(a+n) + \sin(a-n) - 2\sin a$, will be a difference of the second order. But this last expression, by virtue of the formula $R \sin(a \pm n) = \sin a \cos n \pm \cos a \sin n$, reduces to $2 \sin a \cos n \div R - 2 \sin a$, or $2\left(\frac{\cos n}{R} - 1\right) \sin a$. That is to say, the second difference is equal to the product of the sine of the arc a into a certain constant quantity, or it varies as the sine. When n equals $3° 45'$, as in the Hindu table, it is easy to show, upon working out the last expression by means of the tables, that the constant factor is, as stated by Delambre, $\frac{1}{233.5}$, instead of being $\frac{1}{225}$, as empirically determined by the Hindus.

It deserves to be noticed, that the commentary of Ranganâtha recognizes the dependence of the rule given in the text upon the value of the second differences. According to him, however, it is by describing a circle upon the ground, laying off the arcs, drawing the sines, and determining their relations by inspection, that the method is obtained. The differences of the sines, he says, will be observed to decrease, while the differences of those differences increase; and it will be noticed that the last second difference is $15' \, 16'' \, 48'''$. A proportion is then made: if at the radius the second difference is of this value, what will it be at any sine? or, taking the first sine as an example, $3438' : 15' \, 16'' \, 48''' :: 225 : 1$. Nothing can be clearer, however, than that this pretended result of inspection is one of calculation merely. It would be utterly impossible to estimate by the eye the value of a difference with such accuracy, and, were it possible, that difference would be found very considerably removed from the one here given, being actually only about $14' \, 45''$. The value $15' \, 16'' \, 48'''$ is assumed only in order to make its ratio to the radius exactly $\frac{1}{225}$.

The earliest substitution of the sines, in calculation, for the chords, which were employed by the Greeks, is generally attributed (see Whewell's History of the Inductive Sciences, B. III. ch. iv. 8) to the Arab astronomer Albategnius (al-Battânî), who flourished in the latter part of the ninth century of our era. It can hardly admit of question, however, that sines had already at that time been long employed by the Hindus. And considering the derivation by the Arabs from India of their system of notation, and of so many of the elements of their mathematical science, it would seem not unlikely that the first hint of this so convenient and practical improvement of the methods of calculation may also have come to them from that country. This cannot be asserted, however, with much confidence, because the substitution of the sines for the chords seems so natural and easy, that it may well enough have been hit upon independently by the Arabs; it is a matter for astonishment, as remarked by Delambre (Histoire de l'Astronomie du Moyen Age, p. 12), that Ptolemy himself, who came so near it, should have failed of it. If Albategnius got the suggestion from India, he, at any rate, got no more than that. His table of sines, much more complete than that of the Hindus, was made from Ptolemy's table of chords, by simply halving them. The method, too, which in India remained comparatively barren, led to valuable developments in the hands of the Arab mathematicians, who went on by degrees to form also tables of tangents and co-tangents, secants and co-secants; while the Hindus do not seem to have distinctly appreciated the significance even of the cosine.

In this passage, the sine is called *jyârdha*, "half-chord;" hereafter, however, that term does not once occur, but *jyâ* "chord" (literally "bow-string") is itself employed, as are also its synonyms *jîvâ*, *mâurvikâ*, to denote the sine. The usage of Albategnius is the same. The sines of the table are called *pinda*, or *jyâpinda*, "the quantity corresponding to the sine." The term used for versed sine, *utkramajyâ*, means "inverse-order sine," the column of versed sines being found by subtracting that of sines in inverse order from radius.

The ratio of the diameter to the circumference involved in the expression of the value of radius by 3438′ is, as remarked above (under i. 59, 60), 1 : 3.14136. The commentator asserts that value to come from the ratio 1250 : 3927, or 1 : 3.1416, and it is, in fact, the nearest whole number to the result given by that ratio. If the ratio were adopted which has been stated above (in i. 59), of $1 : \sqrt{10}$, the value of radius would be only 3415′. It is to be observed with regard to this latter ratio, that it could not possibly be the direct result of any actual process adopted for ascertaining the value of the diameter from that of the circumference, or the contrary. It was probably fixed upon by the Hindus because it looked and sounded well, and was at the same time a sufficiently near approximation to the truth to be applied in cases where exactness was neither attainable by their methods, nor of much practical consequence; as in fixing the dimensions of the earth, and of the planetary orbits. The nature of the system of notation of the Hindus, and their constantly recurring extraction of square roots in their trigonometrical processes, would cause the suggestion to them, much more naturally than to the Greeks, of this artificial ratio, as not far from the truth; and their science was just of that character to choose for some uses a relation expressed in a manner so simple, and of an aspect so systematical, even though known to be inaccurate. We do not regard the ratio in question, although so generally adopted among the Hindu astronomers, as having any higher value and significance than this.

28. The sine of greatest declination is thirteen hundred and ninety-seven; by this multiply any sine, and divide by radius; the arc corresponding to the result is said to be the declination.

The greatest declination, that is to say, the inclination of the plane of the ecliptic, is here stated to be 24°, 1397′ being the sine of that angle. The true inclination in the year 300 of our era, which we may assume to have been not far from the time when the Hindu astronomy was established, was a little less than 23° 40′, so that the error of the Hindu determination was then more than 20′: at present, it is 32′ 34″. The value assigned by Ptolemy (Syntaxis, i) to the inclination was between 23° 50′ and 23° 52′ 30″; an error, as compared with its true value in the time of Hipparchus, of only about 7′.

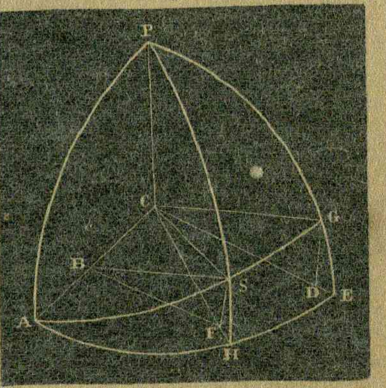
The second half of the verse gives, in the usual vague and elliptical language of the treatise, the rule for finding the declination of any given point in the ecliptic. We have not in this case supplied the ellipses in our translation, because it could not be done succinctly, or without introducing an element, that of the precession, which possibly was not taken into account when the rule was made. See what is said upon this subject under verses 9 and 10 of the next chapter. The "sine" employed is, of course, the sine of the distance from the vernal equinox, or of the longitude as corrected by the precession.

The annexed figure will explain the rule, and the method of its demonstration.

Let A C E represent a quadrant of the plane of the equatorial, and A C G a quadrant of that of the ecliptic, A C being the line of their intersection: then A P is the equinoctial colure, P E the solstitial, G E, or the angle G C E, the inclination of the ecliptic, or the greatest declination (*paramâpakrama*, or *paramakrânti*), and G D its sine (*paramakrântijyâ*). Let S be the position of the sun, and draw the circle of declination P H; S H, or the angle S C H, is the declination of the sun at that point, and S F the sine of declination (*krântijyâ*). From S and F draw S B and F B at right angles to A C; then S B is the sine of the arc A S, or of the sun's longitude. But G C D and S B F are similar right-angled triangles, having their angles at C and B each equal to the inclination. Therefore C G : G D :: S B : S F; and $SF = \frac{GD \times SB}{CG}$;

that is, $\text{sin decl.} = \frac{\text{sin incl.} \times \text{sin long.}}{R}$.

Fig. 1.



The same result is, by our modern methods, obtained directly from the formula in right-angled spherical trigonometry: $\sin c = \sin a \sin C$; or, in the triangle A S H, right-angled at H, sin S H = sin S A sin S A H.

29. Subtract the longitude of a planet from that of its apsis (*mandocca*); so also, subtract it from that of its conjunction (*çîghra*); the remainder is its anomaly (*kendra*); from that is found the quadrant (*pada*); from this, the base-sine (*bhujajyâ*), and likewise that of the perpendicular (*koṭi*).

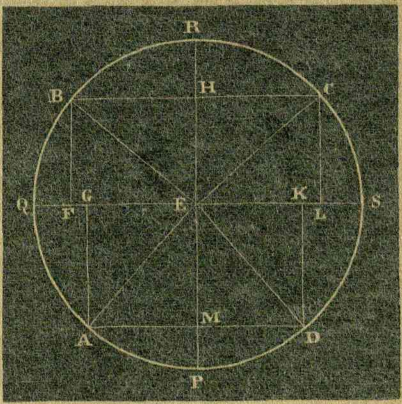
30. In an odd (*vishama*) quadrant, the base-sine is taken from the part past, the perpendicular from that to come; but in an even (*yugma*) quadrant, the base-sine (*bâhujyâ*) is taken from the part to come, and the perpendicular-sine from that past.

The distance of a planet from either of its two apices of motion, or centres of disturbance, is called its *kendra*; according to the commentary, its distance from the apsis (*mandocca*) is called *mandakendra*, and that from the conjunction (*çîghrocca*) is called *çîghrakendra*: the Sûrya-Siddhânta, however, nowhere has occasion to employ these terms. The former of the two corresponds to what in modern astronomy is called the anomaly, the latter to what is known as the commutation. The word *kendra* is not of Sanskrit origin, but is the Greek κέντρον; it is a circumstance no less significant to meet with a Greek word thus at the very foundation of the method of calculating the true place of a planet by means of a system of epicycles, than to find one, as noticed above (under i. 52), at the base of the theory of planetary regency upon which depend the names and succession of the days of the week. Both anomaly and commutation, it will be noticed, are, according to this treatise, to be reckoned always forward from the planet to its apsis and conjunction respectively; excepting that, in the case of Mercury and Venus, owing to the exchange with regard to those planets of the place of the planet itself with that of its conjunction, the commutation is really reckoned the other way. The functions of any arc being the same with those of its negative, it makes no difference, of course, whether the distance is measured from the planet to the apex (*ucca*), or from the apex to the planet.

The quantities actually made use of in the calculations which are to follow are the sine and cosine of the anomaly, or of the commutation. The terms employed in the text require a little explanation. *Bhuja* means "arm;" it is constantly applied, as are its synonyms *bâhu* and *dos*, to designate the base of a right-angled triangle; *koṭi* is properly "a recurved extremity," and, as used to signify the perpendicular in such a tri..ngle, is conceived of as being the end of the *bhuja*, or base, bent up to an upright position: *bhujajyâ* and *koṭijyâ*, then, are literally the values, as sines, of the base and perpendicular of a right-angled triangle of which the hypothenuse is made radius: owing to the relation to one another of the oblique angles of such a triangle, they are respectively as sine and cosine. We have not been willing to employ these latter terms in translating them, because, as before remarked, the Hindus do not seem to have conceived of the cosine, the sine of the complement, of an arc, as being a function of the arc itself.

To find the sine and cosine of the planet's distance from either of its apices (*ucca*) is accordingly the object of the directions given in verse 30 and the latter part of the preceding verse. The rule itself is only the awkward Hindu method of stating the familiar truth that the sine and cosine of an arc and of its supplement are equal. The accompanying figure will, it is believed, illustrate the Hindu manner of looking at the subject. Let P be the place of a planet, and divide its orbit into the four quadrants P Q, Q R, R S, and S P; the first and third of these are called the odd (*vishama*) quadrants; the second and fourth, the even (*yugma*) quadrants. Let A, B, C, and D, be four positions of the apsis (or of the conjunction); then the arcs P A, P Q B, P Q R C, P Q R S D will be the values of the anomaly in each case. A M, the base-sine, or sine of anomaly, when the apsis is in the first quadrant, is

Fig. 2.



determined by the arc A P, the arc passed over in reckoning the anomaly, while A G or E M, the perpendicular-sine, or cosine, is taken from the arc A Q, the remaining part of the quadrant. The same is true in the other odd quadrant, R S; the sine C H, or E L, comes from R C, the part of the quadrant between the planet and the apsis; the cosine C L is from its complement. But in the even quadrants, Q R and S P, the case is reversed; the sines, B H, or E F, and D M, are determined by the arcs B R and D P, the parts of the quadrant not included in the anomaly, and the cosines, B F and K D, or E M, correspond to the other portions of each quadrant respectively.

This process, of finding what portion of any arc greater than a quadrant is to be employed in determining its sine, is ordinarily called in Hindu calculations "taking the *bhuja* of an arc."

31. Divide the minutes contained in any arc by two hundred and twenty-five; the quotient is the number of the preceding tabular sine (*jyâpindaka*). Multiply the remainder by the difference of the preceding and following tabular sines, and divide by two hundred and twenty-five;

32. The quotient thus obtained add to the tabular sine called the preceding; the result is the required sine. The same method is prescribed also with respect to the versed sines.

33. Subtract from any given sine the next less tabular sine; multiply the remainder by two hundred and twenty-five, and divide by the difference between the next less and next greater tabular sines; add the quotient to the product of the serial number of the next less sine into two hundred and twenty-five: the result is the required arc.

The table of sines and versed sines gives only those belonging to arcs which are multiples of 3° 45′; the first two verses of this passage state the method of finding, by simple interpolation, the sine or versed sine of any intermediate arc; while the third verse gives the rule for the contrary process, for converting any given sine or versed sine in the same manner into the corresponding arc.

In illustration of the first rule, let us ascertain the sine corresponding to an arc of 24°, or 1440′. Upon dividing the latter number by 225, we obtain the quotient 6, and the remainder 90′. This preliminary step is necessary, because the Hindu table is not regarded as containing any designation of the arcs to which the sines belong, but as composed simply of the sines themselves in their order. The sine corresponding to the quotient obtained, or the sixth, is 1315′: the difference between it and the next following sine is 205′. Now a proportion is made: if, at this point in the quadrant, an addition of 225′ to the arc causes an increase in the sine of 205′, what increase will be caused by an addition to the arc of 90′: that is to say, 225 : 205 : : 90 : 82. Upon adding the result, 82′, to the sixth sine, the amount, 1397′, is the sine of the given arc, as stated in verse 28. The actual value, it may be remarked, of the sine of 24°, is 1398′.26.

The other rule is the reverse of this, and does not require illustration.

The extreme conciseness aimed at in the phraseology of the text, and not unfrequently carried by it beyond the limit of distinctness, or even of intelligibility, is well illustrated by verse 33, which, literally translated, reads thus: "having subtracted the sine, the remainder, multiplied by 225, divided by its difference, having added to the product of the number and 225, it is called the arc." In verse 31, also, the important word "remainder" is not found in the text.

The proper place for this passage would seem to be immediately after the table of sines and versed sines: it is not easy to see why verses 28–30 should have been inserted between, or indeed, why the subject of the inclination of the ecliptic is introduced at all in this part of the chapter, as no use is made of it for a long time to come.

34. The degrees of the sun's epicycle of the apsis (*manda-paridhi*) are fourteen, of that of the moon, thirty-two, at the end of the even quadrants; and at the end of the odd quadrants, they are twenty minutes less for both.

35. At the end of the even quadrants, they are seventy-five, thirty, thirty-three, twelve, forty-nine; at the odd (*oja*) they are seventy-two, twenty-eight, thirty-two, eleven, forty-eight,

36. For Mars and the rest; farther, the degrees of the epicycle of the conjunction (*çighra*) are, at the end of the even quadrants, two hundred and thirty-five, one hundred and thirty-three, seventy, two hundred and sixty-two, thirty-nine;

37. At the end of the odd quadrants, they are stated to be two hundred and thirty-two, one hundred and thirty-two, seventy-two, two hundred and sixty, and forty, as made use of in the calculation for the conjunction (*çighrakarman*).

38. Multiply the base-sine (*bhujajyâ*) by the difference of the epicycles at the odd and even quadrants, and divide by radius (*trijyâ*); the result, applied to the even epicycle (*vṛtta*), and additive (*dhana*) or subtractive (*ṛṇa*), according as this is less or greater than the odd, gives the corrected (*sphuṭa*) epicycle.

The corrections of the mean longitudes of the planets for the disturbing effect of the apsis (*mandocca*) and conjunction (*çighrocca*) of each—that is to say, for the effect of the ellipticity of their orbits, and for that of the annual parallax, or of the motion of the earth in its orbit—are made in Hindu astronomy by the Ptolemaic method of epicycles, or secondary circles, upon the circumference of which the planet is regarded as moving, while the centre of the epicycle revolves about the general centre of motion. The details of the method, as applied by the Hindus, will be made clear by the figures and processes to be presented a little later; in this passage we have only the dimensions of the epicycles assumed for each planet. For convenience of calculation, they are measured in degrees of the orbits of the planets to which they severally belong; hence only their relative dimensions, as compared with the orbits, are given us. The data of the text belong to the planets in the order in which these succeed one another as regents of the days of the week, viz., Mars, Mercury, Jupiter, Venus, and Saturn (see above, under i. 51, 52). The annexed table gives the dimensions of the epicycles, both their circumferences, which are presented directly by the text, and their radii, which we have calculated after the method of this Siddhânta, assuming the radius of the orbit to be 3438′.

Dimensions of the Epicycles of the Planets.

Planet.	Epicycle of the apsis: at even quadrant, circ.	rad.	at odd quadrant, circ.	rad.	Epicycle of the conjunction: at even quadrant, circ.	rad.	at odd quadrant, circ.	rad.
Sun,	14°	133′.70	13° 40′	130′.52	....	....	....	....
Moon,	32°	305′.60	31° 40′	302′.42	....	....	....	....
Mercury,	30°	286′.50	28°	267′.40	133°	1270′.15	132°	1260′.60
Venus,	12°	114′.60	11°	105′.05	262°	2502′.10	260°	2483′.00
Mars,	75°	716′.25	72°	687′.60	235°	2244′.25	232°	2215′.60
Jupiter,	33°	315′.15	32°	305′.60	70°	668′ 50	72°	687′.60
Saturn,	49°	467′.95	48°	458′.40	39°	372′.45	40°	382′.00

A remarkable peculiarity of the Hindu system is that the epicycles are supposed to contract their dimensions as they leave the apsis or the conjunction respectively (excepting in the case of the epicycles of the conjunction of Jupiter and Saturn, which expand instead of contracting), becoming smallest at the quadrature, then again expanding till the lower apsis, or opposition, is reached, and decreasing and increasing in like manner in the other half of the orbit; the rate of increase and diminution being as the sine of the distance from the apsis, or conjunction. Hence the rule in verse 38, for finding the true dimensions of the epicycle at any point in the orbit. It is founded upon the simple proportion: as radius, the sine of the distance at which the diminution (or increase) is greatest, is to the amount of diminution (or of increase) at that point, so is the sine of the given distance to the corresponding diminution (or increase); the application of the correction thus obtained to the dimensions of the epicycle at the apsis, or conjunction, gives the true epicycle.

We shall revert farther on to the subject of this change in the dimensions of the epicycle.

The term employed to denote the epicycle, *paridhi*, means simply "circumference," or "circle;" it is the same which is used elsewhere in this treatise for the circumference of the earth, etc. In a single instance, in verse 38, we have *vṛtta* instead of *paridhi*; its signification is the same, and its other uses are closely analogous to those of the more usual term.

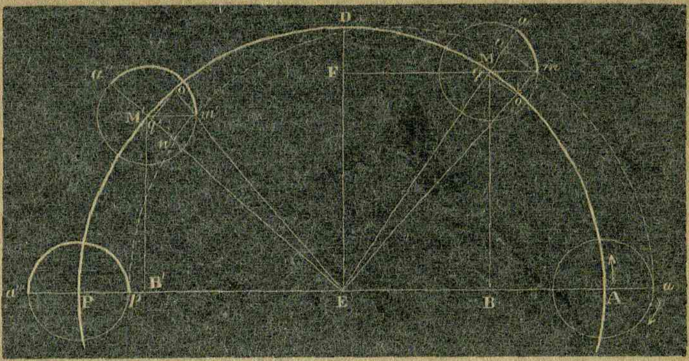
39. By the corrected epicycle multiply the base-sine (*bhujajyâ*) and perpendicular-sine (*koṭijyâ*) respectively, and divide by the number of degrees in a circle: then, the arc corresponding to the result from the base-sine (*bhujajyâphala*) is the equation of the apsis (*mânda phala*), in minutes, etc.

All the preliminary operations having been already performed, this is the final process by which is ascertained the equation of the apsis, or the amount by which a planet is, at any point in its revolution, drawn away from its mean place by the disturbing influence of the apsis. In modern phraseology, it is called the first inequality, due to the ellipticity of the orbit; or, the equation of the centre.

Figure 3, upon the next page, will serve to illustrate the method of the process.

Let A M M′ P represent a part of the orbit of any planet, which is supposed to be a true circle, having E, the earth, for its centre. Along this orbit the planet would move, in the direction indicated by the arrow, from A through M and M′ to P, and so on, with an equable motion, were it not for the attraction of the beings situated at the apsis (*mandocca*) and conjunction (*çîghrocca*) respectively. The general mode of action of these beings has been explained above, under verses 1–5 of this chapter: we have now to ascertain the amount of the disturbance produced by them at any given point in the planet's revolution. The method devised is that of an epicycle, upon the circumference of which the planet revolves with an equable motion, while the centre of the epicycle traverses the orbit with a velocity equal to that of the planet's mean motion, having always a position coincident with the mean place of the planet. At present, we have to do only with the epicycle which represents the disturbing effect of the apsis (*mandocca*). The period of the planet's revolution about the centre of the epicycle is the time which it takes the latter to make the circuit of the orbit from the apsis around to the apsis again, or the period of its anomalistic revolution. This is almost precisely equal to the period of sidereal revolution in the case of all the planets excepting the moon, since their apsides are regarded by the Hindus as stationary (see above, under i. 41–44): the moon's apsis, however, has a forward motion of more than 40° in a year; hence the moon's anomalistic revolution is very perceptibly longer than its sidereal, being $27^{d}\ 13^{h}\ 18^{m}$. The arc of the epicycle traversed by the planet at any mean point in its revolution is accordingly always equal to the arc of the orbit intercepted between that point and the apsis, or to the mean anomaly, when the latter is reckoned, in the usual manner, from the apsis forward to the planet. Thus, in the figure, suppose A to be the place of the apsis (*mandocca*, the apogee of the sun and moon, the aphelion of the other planets), and P that of the opposite point (perigee, or perihelion; it has in this treatise no distinctive name); and let M and M′ be two mean positions of the planet, or actual positions of the centre of the epicycle; the lesser circles drawn about these four points represent the epicycle: this is made, in the figure, of twice the size of that assumed for the moon, or a little smaller than that of Mars. Then, when the centre of the epicycle is at A, the planet's place in the epicycle is at a; as the centre advances to M, M′, and P, the planet moves in the opposite direction, to m, m', and p, the arc $a'm$ being equal to A M, $a''m'$ to A M′, and $a'''p$ to A P. It is as if, while the axis Ea revolves about E, the part of it Aa remained constant in direction, parallel to E A, assuming the positions Mm, M′m', and Pp successively. The effect of this combination of motions is to make the planet virtually traverse the orbit indicated in the figure by the broken line, which is a circle of equal radius with the true orbit, but having its centre removed from E, toward A, by a distance equal to

Fig. 3.



A *a*, the radius of the epicycle. This identity of the virtual orbit with an eccentric circle, of which the eccentricity is equal to the radius of the epicycle, was doubtless known to the Hindus, as to Ptolemy: the latter, in the third book of his Syntaxis, demonstrates the equivalence of the suppositions of an epicycle and an eccentric, and chooses the latter to represent the first inequality: the Hindus have preferred the other supposition, as better suited to their methods of calculation, and as admitting a general similarity in the processes for the apsis and the conjunction. The Hindu theory, however, as remarked above (under vv. 1–5 of this chapter), rejects the idea of the actual motion of the planet in the epicycle, or on the eccentric circle: the method is but a device for ascertaining the effect of the attractive force of the being at the apsis. Thus the planet really moves in the circle A M M′ P, and if the lines E *m*, E *m′* be drawn, meeting the orbit in *o* and *o′*, its actual place is at *o* and *o′*, when its mean place is at M and M′ respectively. To ascertain the value of the arcs *o* M and *o′* M′, which are the amount of removal from the mean place, or the equation, is the object of the process prescribed by the text.

Suppose the planet's mean place to be M, its mean distance from the apsis being A M: it has traversed, as above explained, an equal arc, *a′ m*, in the epicycle. From M draw M B and M F, and from *m* draw *m n*, at right angles to the lines upon which they respectively fall: then M B is the base-sine (*bhujajyâ*), or the sine of mean anomaly, and M F, or its equal E B, is the perpendicular-sine (*koṭijyâ*), or cosine, and *m n* and *n* M are corresponding sine and cosine in the epicycle. But as the relation of the circumference of the orbit to that of the epicycle is known, and as all corresponding parts of two circles are to one another as their respective circumferences, the values of *m n* and *n* M are found by a proportion, as follows: as 360° is to the number of degrees in the circumference of the epicycle at M, so is M B to *m n*, and E B to *n* M. Hence *m n* is called the "result from the base-sine" (*bhujajyâphala*, or, more briefly, *bhujaphala*, or *bâhuphala*), and *n* M the "result from the perpendicular-sine" (*koṭijyâphala*, or *koṭiphala*): the latter of the two, however, is not employed in the process for calculating the equation of the apsis. Now, as the dimensions of the epicycle of the apsis are in all cases small, *m n* may without any considerable error be assumed to be equal to *o q*, which is the sine of the arc *o* M, the equation: this assumption is accordingly made, and the conversion of *m n*, as sine, into its corresponding arc, gives the equation required.

The same explanation applies to the position of the planet at M′: *a″ m′*, the equivalent of A M M′, is here the arc of the epicycle traversed; *m′ n′*, its sine, is calculated from M′B′, as before, and is assumed to equal *o′ q′*, the sine of the equation *o′* M′.

To give a farther and practical illustration of the process, we will proceed to calculate the equation of the apsis for the moon, at the time for which her mean place has been found in the notes to the last chapter, viz., the 1st of January, 1860, midnight, at Washington.

Moon's mean longitude, midnight, at Ujjayinî (i. 53),	$11^s\ 15°\ 23'\ 24''$
add the equation for difference of meridian (*deçântaraphala*), or for her motion between midnight at Ujj. and Wash. (i. 60, 61),	5 35 37
Moon's mean longitude at required time,	11 20 59 1
Longitude of moon's apsis, midnight, at Ujjayinî (i. 53),	10 9 42 26
add for difference of meridian, as above,	2 50
Longitude of moon's apsis at required time,	10 9 45 16
deduct moon's mean longitude (ii. 29),	11 20 59 1
Moon's mean anomaly (*mandakendra*),	10 18 46 15

The anomaly being reckoned forward on the orbit from the planet, the position thus found for the moon relative to the apsis is, nearly enough for purposes of illustration, represented by M in the figure. By the rule given above, in verse 30, the base-sine (*bhujajyâ*)—since the anomaly is in the fourth, an even, quadrant—is to be taken from the part of the quadrant not included in the anomaly, or A M; the perpendicular-sine (*koṭijyâ*) is that corresponding to its complement, or M D. That is to say:

From the anomaly,	$10^s\ 18°\ 46'\ 15''$
deduct three quadrants,	9
remains the arc M D,	1 18 46 15
take this from a quadrant,	3
remains the arc A M,	1 11 13 45

And by the method already illustrated under verses 31, 32, the sine corresponding to the latter arc, which is the base-sine (*bhujajyâ*), or the sine of mean anomaly, M B, is found to be 2266′; that from M D, which is M F, or E B, the perpendicular-sine (*koṭijyâ*), or cosine of mean anomaly, is 2585′.

The next point is to find the true size of the epicycle at M. By verse 34, the contraction of its circumference amounts at D to 20′; hence, according to the rule in verse 38, we make the proportion, sin A D : 20′ : : sin A M : diminution at M; or,

3438 : 20 : : 2266 : 13

Deducting from 32°, the circumference of the epicycle at A, the amount of diminution thus ascertained, we have 31° 47′ as its dimensions at M.

Once more, by verse 39, we make the proportion, circ. of orbit : circ. of epicycle : : MB : *mn*; or,

360° : 31° 47′ : : 2266 : 200

The value, then, of *mn*, the result from the base-sine (*bhujajyàphala*), is 200′; which, as *mn* is assumed to equal *oq*, is the sine of the equation. Being less than 225′, its arc (see the table of sines, above) is of the same value: 3° 20′, accordingly, is the moon's equation of the apsis (*mânda phala*) at the given time: the figure shows it to be subtractive (*ṛṇa*), as the rule in verse 45 also declares it. Hence, from the

Moon's mean longitude,	11ˢ 20° 59′
deduct the equation,	3 20
Moon's true longitude,	11 17 39

We present below, in a briefer form, the results of a similar calculation made for the sun, at the same time.

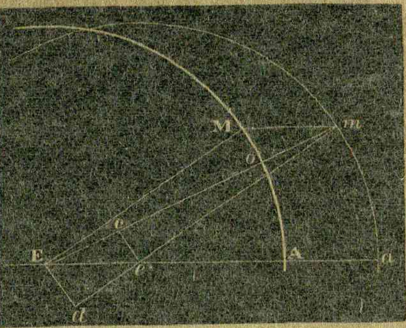
Sun's mean longitude, midnight, at Ujjayinî (i. 53),	8ˢ 17° 48′ 7″
add for difference of meridian (i. 60, 61),	25 6
Sun's mean longitude at required time,	8 18 13 13
Longitude of sun's apsis (i. 41),	2 17 17 24
Sun's mean anomaly (ii. 29),	5 29 4 11
subtract from two quadrants (ii. 30),	6
Arc determining base-sine,	55′ 49″
Base-sine (*bhujajyâ*),	56′
Dimensions of epicycle (ii. 38),	14°
Result from base-sine (*bhujajyàphala*), or sine of equation (ii. 39),	2′
Equation (*mânda phala*, ii. 45),	+2′
Sun's true longitude,	8ˢ 18° 15′

In making these calculations, we have neglected the seconds, rejecting the fraction of a minute, or counting it as a minute, according as it was less or greater than a half. For, considering that this method is followed in the table of sines, which lies at the foundation of the whole process, and considering that the sine of the arc in the epicycle is assumed to be equal to that of the equation, it would evidently be a waste of labor, and an affectation of an exactness greater than the process contemplates, or than its general method renders practicable, to carry into seconds the data employed.

As stated below, in verse 43, the equation thus found is the only one required in determining the true longitude of the sun and of the moon: in the case of the other planets, however, of which the apparent place is affected by the motion of the earth, a much longer and more complicated process is necessary, of which the explanation commences with the next following passage.

The Ptolemaic method of making the calculation of the equation of the centre for the sun and moon is illustrated by the annexed figure (Fig. 4). The points E, A, M, *a*, *m*, and *o*, correspond with those similarly marked in the last figure (Fig. 3). The centre of the eccentric circle is at *e*, and E *e*, which equals A *a*, is the eccentricity, which is given. Join *e m*; the angle *m e a* equals M E A, the mean anomaly, and E *m e* equals M E *o*, the equation. Extend *m e* to *d*, where it meets E *d*, a perpendicular let fall upon it from E. Then, in the right-angled triangle E *e d*, the side E *e* and the angles —since E *e d* equals *m e a*—are given, to find the other sides, *e d* and *d* E. Add *e d* to *e m*, the radius; add the square of the sum to that of E *d*; the square root of their sum is E *m*: then, in the right-angled triangle *m* E *d*, all the sides and the right angle are given, to find the angle E *m e*, the equation.

Fig. 4.



This process is equivalent to a transfer of the epicycle from M to E; E *d* becomes the result from the base-sine (*bhujajyâphala*), and *d e* that from the perpendicular-sine (*kotijyâphala*), and the angle of the equation is found in the same manner as its sine, *e c*, is found in the Hindu process next to be explained; while, in that which we have been considering, E *d* is assumed to be equal to *e c*.

Ptolemy also adds to the moon's orbit an epicycle, to account for her second inequality, the evection, the discovery of which does him so much honor. Of this inequality the Hindus take no notice.

40. The result from the perpendicular-sine (*koṭiphala*) of the distance from the conjunction is to be added to radius, when the distance (*kendra*) is in the half-orbit beginning with Capricorn; but when in that beginning with Cancer, the result from the perpendicular-sine is to be subtracted.

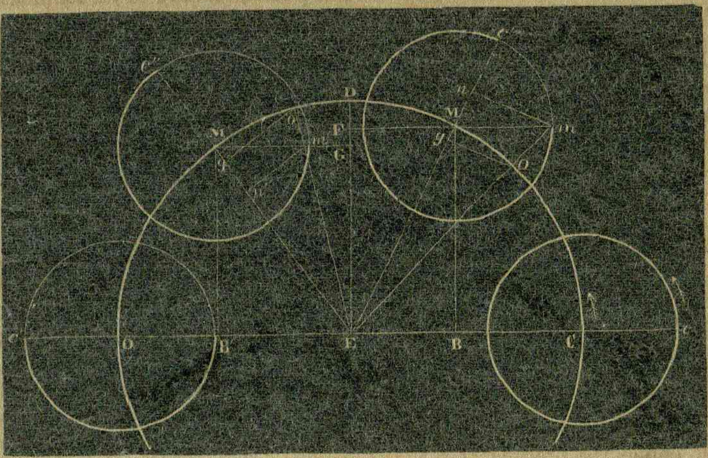
41. To the square of this sum or difference add the square of the result from the base-sine (*bâhuphala*); the square root of their sum is the hypothenuse (*karṇa*) called variable (*cala*). Multiply the result from the base-sine by radius, and divide by the variable hypothenuse:

42. The arc corresponding to the quotient is, in minutes, etc., the equation of the conjunction (*çâighrya phala*); it is employed in the first and in the fourth process of correction (*karman*) for Mars and the other planets.

The process prescribed by this passage is essentially the same with that explained and illustrated under the preceding verse, the only difference being that here the sine of the required equation, instead of being assumed equal to that of the arc traversed by the planet in the epicycle, is obtained by calculation from it. The annexed figure (Fig. 5) will exhibit the method pursued.

The larger circle, C M M′ O, represents, as before, the orbit in which any one of the planets, as also the being at its conjunction (*çighrocca*) are

Fig. 5.



making the circuit of the heavens about E, the earth, as a centre, in the direction indicated by the arrow, from C through M and M′ to O, and so on. But since, in every case, the conjunction moves more rapidly eastward than the planet, overtaking and passing it, if we suppose the conjunction stationary at C, the virtual motion of the planet relative to that point is backward, or from O through M′ and M to C, its mean rate of approach toward C being the difference between the mean motion of the planet and that of the sun. As before, the amount to which the planet is drawn away from its mean place toward the conjunction is calculated by means of an epicycle. The circles drawn in the figure to represent the epicycle are of the relative dimensions of that assigned to Mercury, or a little more than half that of Mars. The direction of the planet's motion in the epicycle is the reverse of that in the epicycle of the apsis, as regards the actual motion of the planet in its orbit, being eastward at the conjunction; as regards the motion of the planet relative to the conjunction, it is the same as in the former case, being in the contrary direction at the conjunction: its effect, of course, is to increase the rate of the eastward movement at that point. The time of the planet's revolution about the centre of the epicycle is the interval between two successive passages through the point C, the conjunction: that is to say, it is equal to the period of synodical revolution of each planet. These periods are, according to the elements presented in the text of this Siddhânta, as follows:

Mercury,	115^{d}	21^{h}	42^{m}
Venus,	583	21	37
Mars,	779	22	11
Jupiter,	398	21	20
Saturn,	378	2	4

The arc of the epicycle traversed by the planet, at any point in its revolution, is equal to its distance from the conjunction, when reckoned forward from the planet, according to the method prescribed in verse 29.

Suppose, now, the mean place of the planet, relative to its conjunction (*çîghrocca*) at C, to be at M: its place in the epicycle is at *m*, as far from *c'''*, in either direction, as M from C. The arc of the epicycle already traversed is indicated in this figure, as in Fig. 3, by the heavier line. Draw E*m*, cutting the orbit in *o*; then *o* is the planet's true place, and *o*M is the equation, or the amount of removal from the mean place by the attraction of the being at C.

The sine and cosine of the distance from the conjunction, the dimensions of the epicycle, and the value of the correspondents in the epicycle to the sine and cosine, are found as in the preceding process. Add *n*M, the result from the cosine (*koṭijyâphala*), to M E, the radius: the result is the perpendicular, E*n*, of the triangle E*nm*. To the square of E*n* add that of the base *nm*, the result from the sine (*bhujajyâphala*); the square root of the sum is the line E*m*, the hypothenuse: it is termed the variable hypothenuse (*cala karṇa*) from its constantly changing its length. We have now the two similar triangles E*mn* and E*og*, a comparison of the corresponding parts of which gives us the proportion E*m* : *mn* : : E*o* : *og*; that is to say, *og*, which is the sine of the equation *o*M, equals the product of E*o*, the radius, into *mn*, the result from the base-sine, divided by the variable hypothenuse, E*m*.

When the planet's mean place is in the quadrant D O, as at M', the result from the perpendicular-sine (*koṭijyâphala*), or M'*n'*, is subtracted from radius, and the remainder, E*n'*, is employed as before to find the value of E*m'*, the variable hypothenuse: and the comparison of the similar triangles E*m'n'* and E*o'g'* gives *o'g'*, the sine of the equation, *o'*M'.

It is obvious that when the mean distance of a planet from its conjunction is less than a quadrant in either direction, as at M, the base E*n* is greater than radius; when that distance is more than a quadrant, as at M', the base E*n'* is less than radius: the cosine is to be added to radius in the one case, and subtracted from it in the other. This is the meaning of the rule in verse 40: compare the notes to i. 58 and ii. 30.

In illustration of the process, we will calculate the equation of the conjunction of Mercury for the given time, or for midnight preceding January 1st, 1860, at Washington.

Since the Hindu system, like the Greek, interchanges in the case of the two inferior planets the motion and place of the planet itself and of the sun, giving to the former as its mean motion that which is the mean apparent motion of the sun, and assigning to the conjunction (*çîghrocca*) a revolution which is actually that of the planet in its orbit, the mean position of Mercury at the given time is that found above (under v. 39) to be that of the sun at the same time, while to find that of its conjunction we have to add the equation for difference of meridian (*deçântaraphala*, i. 60, 61), to the longitude given under i. 53 as that of the planet.

Longitude of Mercury's conjunction (*çîghrocca*), midnight, at Ujjayinî,	4ˢ	15°	13′	8″
add for difference of meridian,		1	44	14
Longitude of conjunction at required time,	4	16	57	22
Mean longitude of Mercury,	8	18	13	13
Mean commutation (*çîghrakendra*,),	7	28	44	9

The position of Mercury with reference to the conjunction is accordingly very nearly that of M′, in Fig. 5. The arc which determines the base-sine (*bhujajyâ*), or O M′, is 58° 44′, while M′ D, its complement, from which the perpendicular-sine (*koṭijyâ*) is taken, is 31° 16′. The corresponding sines, M′ B′ and M′ G, are 2938′ and 1784′ respectively.

The epicycle of Mercury is one degree less at D than at O. Hence the proportion

$$3438 : 60 :: 2938 : 51$$

gives 51′ as the diminution at M′: the circumference of the epicyle at M, then, is 132° 9′. The two proportions

$$360° : 132° \; 9' :: 2938 : 1078, \quad \text{and} \quad 360° : 132° \; 9' :: 1784 : 655,$$

give us the value of $m' n'$ as 1078′, and that of n' M′ as 655′. The commutation being more than three and less than nine signs, or in the half-orbit beginning with Cancer, the fourth sign, n' M′ is to be subtracted from EM′, or radius, 3438′; the remainder, 2783′, is the perpendicular En'.

To the square of En',	7,745,089
add the square of $n' m'$,	1,162,084
of their sum,	8,907,173
the square root,	2984

is the variable hypothenuse (*cala karṇa*), Em'. The comparison of the triangles E$m' n'$ and E$o' g'$ gives the proportion Em' : $m' n'$ :: Eo' : $o' g'$, or

$$2984 : 1078 :: 3438 : 1242$$

The value of $o' q'$, the sine of the equation, is accordingly 1242′: the corresponding arc, o' M, is found by the process prescribed in verse 33 to be 21° 12′. The figure shows the equation to be subtractive.

The annexed table presents the results of the calculation of the equation of the conjunction (*çîghrakarman*) for the five planets.

Results of the First Process for finding the True Places of the Planets.

Planet.	Mean Longitude.	Longitude of Conjunction.	Mean Commutation.	Base-sine.	Corr. Epicycle.	Result from B.-sine.	Result from P.-sine.	Va abl Hyp.	Equat'n of Conj.
	s ° ′ ″	s ° ′ ″	s ° ′ ″	′	° ′	′	′	′	° ′
Mercury,	8 18 13 13	4 16 57 22	7 28 44 9	2938	132 9	1078	655	2984	−21 12
Venus,	8 18 13 13	10 21 49 47	2 3 36 34	3080	260 13	2226	1104	5058	+26 7
Mars,	5 24 30 57	8 18 13 13	2 23 42 16	3416	232 1	2202	225	4274	+31 1
Jupiter,	2 26 2 14	8 18 13 13	5 22 10 59	468	70 16	91	665	2774	+ 1 53
Saturn,	3 20 12 3	8 18 13 13	4 28 1 10	1820	39 32	200	320	3124	+ 3 40

This is, however, only a first step in the whole operation for finding the true longitudes of these five planets, as is laid down in the next passage.

43. The process of correction for the apsis (*mânda karman*) is the only one required for the sun and moon: for Mars and the other planets are prescribed that for the conjunction (*çâighrya*), that for the apsis (*mânda*), again that for the apsis, and that for the conjunction—four, in succession.

44. To the mean place of the planet apply half the equation of the conjunction (*çîghraphala*), likewise half the equation of the apsis; to the mean place of the planet apply the whole equation of the apsis (*mandaphala*), and also that of the conjunction.

45. In the case of all the planets, and both in the process of correction for the conjunction and in that for the apsis, the equation is additive (*dhana*) when the distance (*kendra*) is in the half-orbit beginning with Aries; subtractive (*ṛṇa*), when in the half-orbit beginning with Libra.

The rule contained in the last verse is a general one, applying to all the processes of calculation of the equations of place, and has already been anticipated by us above. Its meaning is, that when the anomaly, (*mandakendra*), or commutation (*çîghrakendra*), reckoned always forward from the planet to the apsis or conjunction, is less than six signs, the equation of place is additive; when the former is more than six signs, the equation is subtractive. The reason is made clear by the figures given above, and by the explanations under verses 1–5 of this chapter.

It should have been mentioned above, under verse 29, where the word *kendra* was first introduced, that, as employed in this sense by the Hindus, it properly signifies the position (see note to i. 53) of the "centre" of the epicycle—which coincides with the mean place of the planet itself—relative to the apsis or conjunction respectively. In the text of the Sûrya-Siddhânta it is used only with this signification: the commentary employs it also to designate the centre of any circle.

Since the sun and moon have but a single inequality, according to the Hindu system, the calculation of their true places is simple and easy. With the other planets the case is different, on account of the existence of two causes of disturbance in their orbits, and the consequent necessity both of applying two equations, and also of allowing for the effect of each cause in determining the equation due to the other. For, to the apprehension of the Hindu astronomer, it would not be proper to calculate the two equations from the mean place of the planet; nor, again, to calculate either of the two from the mean place, and, having applied it, to take the new position thus found as a basis from which to calculate the other; since the planet is virtually drawn away from its mean place by the divinity at either apex (*ucca*) before it is submitted to the action of the other. The method adopted in this Siddhânta of balancing the two influences, and arriving at their joint effect upon the planet, is stated in verses 43 and 44. The phraseology of the text is not entirely explicit, and would bear, if taken alone, a different interpretation from that which the commentary puts upon it, and which the rules to be given later show to be its true meaning; this is as follows: first calculate from the mean place of the planet the equation of the conjunction, and apply the half of it to the mean place; from the position thus obtained calculate the equation of the apsis, and apply half of it to the longitude as already once equated; from this result find once more the equation of the apsis, and apply it to the original mean place of the planet; and finally, calculate from, and apply to, this last place the whole equation of the conjunction.

We have calculated by this method the true places of the five planets, and present the results of the processes in the following tables. Those of the first process have been already given under the preceding passage: the application of half the equations there found to the mean longitude gives us the longitude once equated as a basis for the next process.

Results of the Second Process for finding the True Places of the Planets.

Planet.	Equated Longitude.	Longitude of Apsis.	Equated Anomaly.	Base-sine.	Corrected Epicycle.	Equation of Apsis.
	s ° '	s ° ' "	s ° '	'	° '	° '
Mercury,	8 7 37	7 10 28 20	11 2 51	1568	29 5	− 2 7
Venus,	9 1 17	2 19 52 17	5 18 35	681	11 48	+ 0 22
Mars,	6 10 1	4 10 2 40	10 0 2	2977	72 24	−10 2
Jupiter,	2 26 59	5 21 22 19	2 24 23	3420	32 0	+ 5 5
Saturn,	3 22 1	7 26 37 34	4 4 37	2829	48 11	+ 6 20

Again, the application of half these equations to the longitudes as once equated furnishes the data for the third process. The longitudes of the apsides, being the same as in the second operation, are not repeated in this table.

Results of the Third Process for finding the True Places of the Planets.

Planet.	Equated Longitude.	Equated Anomaly.	Base-sine.	Corrected Epicycle.	Equation of Apsis.
	s ° '	s ° '	'	° '	° '
Mercury,	8 6 34	11 3 54	1512	29 7	−2 2
Venus,	9 1 28	5 18 24	691	11 48	+0 23
Mars,	6 5 0	10 5 3	2814	72 33	−9 30
Jupiter,	2 29 30	2 21 52	3403	32 1	+5 4
Saturn,	3 25 11	4 1 27	2932	48 9	+6 33

The original mean longitudes are now corrected by the results of the third process, to obtain a position from which shall be once more calculated the equation of the conjunction; and the application of this to the position which furnished it yields, as a final result, the true place of each planet.

Results of the Fourth Process for finding the True Places of the Planets.

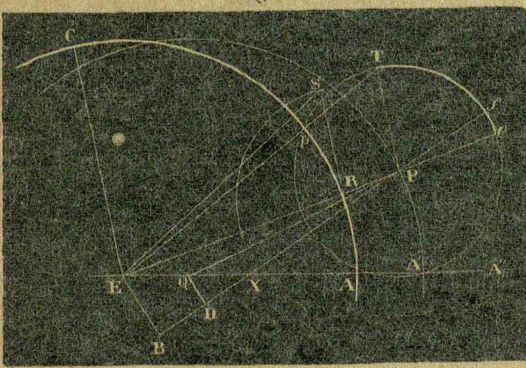
Planet.	Equated Longitude.	Equated Commutation.	Base-sine.	Corr. Epicycle.	Result from B.-sine.	Result from P.-sine.	Variable Hypoth.	Equation of Conj.	True Longitude.
	s ° '	s ° '	'	° '	'	'	'	° '	s ° '
Mercury,	8 16 11	8 0 46	3000	132 8	1101	616	3029	−21 20	7 24 51
Venus,	8 18 36	2 3 14	3069	260 13	2218	1118	5067	+25 59	9 14 35
Mars,	5 15 1	3 3 12	3432	232 0	2212	124	3984	+33 44	6 18 45
Jupiter,	3 1 6	5 17 7	766	70 27	150	656	2786	+ 3 5	3 4 11
Saturn,	3 26 45	4 21 28	2141	39 37	236	296	3151	+ 4 17	4 1 2

We cannot furnish a comparison of the Hindu determinations of the true places of the planets with their actual positions as ascertained by our modern methods, until after the subject of the latitude has been dealt with: see below, under verses 56–58.

The Hindu method of finding the true longitudes of the five planets whose apparent position is affected by the parallax of the earth's motion having thus been fully explained, we will proceed to indicate, as succinctly as possible, the way in which the same problem is solved by the great Greek astronomer. The annexed figure (Fig. 6) will illustrate his method: it is taken from those presented in the Syntaxis, but with such modifications of form as to make it correspond with the figures previously given here: the conditions which it represents are only hypothetical, not according with the actual elements of any of the planetary orbits.

Let E be the earth's place, and let the circle A p C, described about E as a centre, represent the mean orbit of any planet, E A being the direction of its line of apsides, and E C that of its conjunction (*çîghra*), called by Ptolemy the apogee of its epicycle. Let E X be the double eccentricity, or the equivalent to the radius of the Hindu epicycle of the apsis; and let E X be bisected in Q. Then, as regards the influence of the eccentricity of the orbit upon the place of the planet, the centre of equable angular motion is at X, but the centre of equal distance is at Q: the planet virtually describes the circle A′ P, of which Q is the centre, but at the same rate as if it were moving equably upon the dotted circle, of which the centre is at X. The angle of mean anomaly, accordingly, which increases proportionally to the time, is x X A″, but P is the planet's place, P E A the true anomaly, and E P X the equation of place. The value of E P X is obtained by a process analogous to that described above, under verse 39 (pp. 210, 211); E B and B X, and Q D and D X, are first found; then D P, which, by subtracting D X, gives X P; X P added to B X gives B P; and from B P and B E is derived E P B, the equation required; subtract this from P X A, and the remainder is R E A, the planet's true distance from the apsis. About P describe the epicycle of the conjunction, and draw the radius P T parallel to E C: then T is the planet's place in the epicycle, p its apparent position in the mean orbit, and T E P the equation of the epicycle, or of the conjunction. In order to arrive at the value of this equation, Ptolemy first finds that of S E R, the corresponding angle when the centre of the epicycle is placed at R, at the mean distance E R, or radius, from E: he then diminishes it by a complicated process, into the details of which it is not necessary here to enter, and which, as he himself acknowledges, is not strictly accurate, but yields results sufficiently near to the truth. The application of the equation thus obtained

Fig. 6.



to the place of the planet as already once equated gives the final result sought for, its geocentric place.

In the case of Mercury, Ptolemy introduces the additional supposition that the centre of equal distances, instead of being fixed at Q, revolves in a retrograde direction upon the circumference of a circle of which X is the centre, and X Q the radius.

After a thorough discussion of the observations upon which his data and his methods are founded, and a full exposition of the latter, Ptolemy proceeds himself to construct tables, which are included in the body of his work, from which the true places of the planets at any given time may be found by a brief and simple process. The Hindus are also accustomed to employ such tables, although their construction and use are nowhere alluded to in this treatise. Hindu tables, in part professing to be calculated according to the Sûrya-Siddhânta, have been published by Bailly (Traité de l'Astr. Ind. et Or., p. 335, etc.), by Bentley (Hind. Ast., p. 219, etc.), by Warren (Kâla Sankalita, Tables), by Mr. Hoisington (Oriental Astronomer, p. 61, etc.), and, for the sun and moon, by Davis (As. Res., ii. 255, 256).

We are now in a condition to compare the planetary system of the Hindus with that of the Greeks, and to take note of the principal resemblances and differences between them. And it is evident, in the first place, that in all their grand features the two are essentially the same. Both alike analyze, with remarkable success, the irregularities of the apparent motions of the planets into the two main elements of which they are made up, and both adopt the same method of representing and calculating those irregularities. Both alike substitute eccentric circles for the true elliptic orbits of the planets. Both agree in assigning to Mercury and Venus the same mean orbit and motion as to the sun, and in giving them epicycles which in fact correspond to their heliocentric orbits, making the centre of those epicycles, however, not the true, but the mean place of the sun, and also applying to the latter the correction due to the eccentricity of the orbit. Both transfer the centre of the orbits of the superior planets from the sun to the earth, and then assign to each, as an epicycle, the earth's orbit; not, however, in the form of an ellipse, nor even of an eccentric, but in that of a true circle; and here, too, both make the place of the centre of the epicycle to depend upon the mean, instead of the true, place of the sun. The key to the whole system of the Greeks, and the determining cause both of its numerous accordances with the actual conditions of things in nature, and of its inaccuracies, is the principle, distinctly laid down and strictly adhered to by them, that the planetary movements are to be represented by a combination of equable circular motions alone, none other being deemed suited to the dignity and perfection of the heavenly bodies. By the Hindus, this principle is nowhere expressly recognized, so far as we are aware, as one of binding influence, and although their whole system, no less than that of the Greeks, seems in other respects inspired by it, it is in one point, as we shall note more particularly hereafter, distinctly abandoned and violated by them (see below, under vv. 50, 51). We cannot but regard with the highest admiration the acuteness and industry, the power of observation, analysis, and deduction of the Greeks, that, hampered by false assumptions, and imperfectly provided with instruments, they were able to construct a science containing so much of truth, and serving as a secure basis for the improvements of after time: whether we pay the same tribute to the genius of the Hindu will depend upon whether we consider him also, like all the rest of the world, to have been the pupil of the Greek in astronomical science, or whether we shall believe him to have arrived independently at a system so closely the counterpart of that of the West.

The differences between the two systems are much less fundamental and important. The assumption of a centre of equal distance different from that of equal angular motion—and, in the case of Mercury, itself also movable—is unknown to the Hindus: this, however, appears to be an innovation introduced into the Greek system by Ptolemy, and unknown before his time; it was adopted by him, in spite of its seeming arbitrariness, because it gave him results according more nearly with his observations. The moon's evection, the discovery of Ptolemy, is equally wanting in the Hindu astronomy. As regards the combined application of the equations of the apsis and the conjunction, the two systems are likewise at variance. Ptolemy follows the truer, as well as the simpler, method: he applies first the whole correction for the eccentricity of the orbit, obtaining as a result, in the case of the superior planets, the planet's true heliocentric place; and this he then corrects for the parallax of the earth's position. Here, too, ignorant as he was of the actual relation between the two equations, we may suppose him to have been guided by the better coincidence with observation of the results of his processes when thus conducted. The Hindus, on the other hand, not knowing to which of the two supernatural beings at the apsis and conjunction should be attributed the priority of influence, conceived them to act simultaneously, and adopted the method stated above, in verse 44, of obtaining an average place whence their joint effect should be calculated. This is the only point where they forsook the geometrical method, and suffered their theory respecting the character of the forces producing the inequalities of motion to modify their processes and results. The change of dimensions of the epicycles is also a striking peculiarity of the Hindu system, and to us, thus far, its most enigmatical feature. The virtual effect of the alteration upon the epicycles themselves is to give them a form approximating to the elliptical. But, although the epicycles of the conjunction of the inferior planets represent the proper orbits of those planets, and those of the superior the orbit of the earth, it is not possible to see in this alteration an unconscious recognition of the principle of ellipticity, because the major axis of the quasi-ellipse—or, in the case of Jupiter and Saturn, the minor axis—is constantly pointed toward the earth. Its effect upon the orbit described by the planet is, as concerns the epicycle of the apsis, to give to the eccentric circle an ovoid shape, flattened in the first and fourth quadrants, bulging in the second and third: this is, so far as it goes, an approximation toward Ptolemy's virtual orbit, a circle described about a centre distant from the earth's place by only half the equivalent of the radius of the Hindu epicycle (the circle A′P in figure 6): but the approximation seems too distant to furnish any hint of an explanation. A diminution of the epicycle also effects a corresponding diminution of the equation, carrying the planet forward where the equation is subtractive, and backward where it is additive: but we hardly feel justified in assuming that it is to be regarded as an empirical correction, applied to make the results of calculation agree more nearly with those of observation, because its amount and place stand in no relation which we have been able to trace to the true elements of the planetary orbits, nor is the accuracy of either the Hindu calculations or observations so great as to make such slight corrections of appreciable importance. We are compelled to leave the solution of this difficulty, if it shall prove soluble, to later investigation, and a more extended comparison of the different text-books of Hindu astronomical science.

As regards the numerical value of the elements adopted by the two systems—their mutual relation, and their respective relations to the true elements established by modern science, are exhibited in the annexed table. The first part of it presents the comparative dimensions of the planetary orbits, or the value of the radius of each in terms of that of the earth's orbit. In the case of Mercury and Venus, this is represented by the relation of the radius of the epicycle (of the conjunction) to that of the orbit; in the case of the superior planets, by that of the radius of the orbit to the radius of the epicycle. For the Hindu system it was necessary to give two values in every case, derived respectively from the greatest and least dimensions of the epicycles. Such a relative determination of the moon's orbit, of course, could not be obtained: its absolute dimensions will be found stated later (see under iv. 3 and xii. 84). The second part of the table gives, as the fairest practicable comparison of the values assigned by each system to the eccentricities, the greatest equations of the centre. For Mercury and Venus, however, the ancient and modern determinations of these equations are not at all comparable, the latter giving their actual heliocentric amount, the former their apparent value, as seen from the earth.

Relative Dimensions and Eccentricities of the Planetary Orbits, according to Different Authorities.

Planet.	Radius of the Orbit.				Greatest Equation of the Centre.		
	Sûrya-Siddhânta even quad.	Sûrya-Siddhânta odd quad	Ptolemy.	Moderns	Sûrya-Sidddhânta.	Ptolemy.	Moderns.
					° ′ ″	° ′	° ′ ″
Sun,	1.0000	1.0000	1.0000	1.0000	2 10 31	2 23	1 55 27
Moon,	.....	.....	.....	.....	5 2 46	5 1	6 17 13
Mercury,	.3694	.3667	.3750	.3871	4 27 35	2 52	23 40 43
Venus,	.7278	.7222	.7194	.7233	1 45 3	2 23	0 47 11
Mars,	1.5139	1.5513	1.5190	1.5237	11 32 3	11 32	10 41 33
Jupiter,	5.1429	5.0000	5.2174	5.2028	5 5 58	5 16	5 31 14
Saturn,	9.2308	9.0000	9.2308	9.5389	7 39 32	6 32	6 26 12

46. Multiply the daily motion (*bhukti*) of a planet by the sun's result from the base-sine (*bâhuphala*), and divide by the number of minutes in a circle (*bhacakra*); the result, in minutes, apply to the planet's true place, in the same direction as the equation was applied to the sun.

By this rule, allowance is made for that part of the equation of time, or of the difference between mean and apparent solar time, which is due to the difference between the sun's mean and true places. The instruments employed by the Hindus in measuring time are described, very briefly and insufficiently, in the thirteenth chapter of this work: in all probability the gnomon and shadow was that most relied upon; at any rate, they can have had no means of keeping mean time with any accuracy, and it appears from this passage that apparent time alone is regarded as ascertainable directly. Now if the sun moved in the equinoctial instead of in the ecliptic, the interval between the passage of his mean and his true place across the meridian would be the same part of a day, as the difference of the two places is of a circle: hence the proportion upon which the rule in the text is founded: as the number of minutes in a circle is to that in the sun's equation (which is the same with his "result from the base-sine:" see above, v. 39), so is the whole daily motion of any planet to its motion during the interval. And since, when the sun is in advance of his true place, he comes later to the meridian, the planet moving on during the interval, and the reverse, the result is additive to the planet's place, or subtractive from it, according as the sun's equation is additive or subtractive.

The other source of difference between true and apparent time, the difference in the daily increment of the arcs of the ecliptic, in which the sun moves, and of those of the equinoctial, which are the measures of time, is not taken account of in this treatise. This is the more strange, as that difference is, for some other purposes, calculated and allowed for.

At the time for which we have ascertained above the true places of the planets, the sun is so near the perigee, and his equation of place is so small, that it renders necessary no modification of the places as given: even the moon moves but a small fraction of a second during the interval between mean and apparent midnight.

By *bhukti*, as used in this verse, we are to understand, of course, not the mean, but the actual, daily motion of the planet: the commentary also gives the word this interpretation. How the actual rate of motion is found at any given time, is taught in the next passage.

47. From the mean daily motion of the moon subtract the daily motion of its apsis (*manda*), and, having treated the difference in the manner prescribed by the next rule, apply the result, as an additive or subtractive equation, to the daily motion.

48. The equation of a planet's daily motion is to be calculated like the place of the planet in the process for the apsis: multiply the daily motion by the difference of tabular sines corresponding to the base-sine (*dorjyâ*) of anomaly, and then divide by two hundred and twenty-five;

49. Multiply the result by the corresponding epicycle of the apsis (*mandaparidhi*), and divide by the number of degrees in a circle (*bhagaṇa*); the result, in minutes, is additive when in the half-orbit beginning with Cancer, and subtractive when in that beginning with Capricorn.

Only the effect of the apsis upon the daily rate of motion is treated of in these verses; the farther modification of it by the conjunction is the subject of those which succeed.

Verse 47 is a separate specification under the general rule given in the following verse, applying to the moon alone. The rate of a planet's motion in its epicycle being equal to its mean motion from the apsis, or its anomalistic motion, it is necessary in the case of the moon, whose apsis has a perceptible forward movement, to subtract the daily amount of this movement from that of the planet in order to obtain the daily rate of removal from the apsis.

In the first half of verse 48 the commentary sees only an intimation that, as regards the apsis, the equation of motion is found in the same general method as the equations of place, a certain factor being multiplied by the circumference of the epicycle and divided by that of the orbit. Such a direction, however, would be altogether trifling and superfluous, and not at all in accordance with the usual compressed style of the treatise; and moreover, were it to be so understood, we should lack any direction as to which of the several places found for a planet in the process for ascertaining its true place should be assumed as that for which this first equation of motion is to be calculated. The true meaning of the line, beyond all reasonable question, is, that the equation is to be derived from the same data from which the equation of place for the apsis was finally obtained, to be applied to the planet's mean position, as this is applied to its mean motion; from the data, namely, of the third process, as given above.

The principle upon which the rule is founded may be explained as follows. The equation of motion for any given time is evidently equal to the amount of acceleration or of retardation effected during that time by the influence of the apsis. Thus, in Fig. 3 (p. 208), *m n*, the sine of *a′ m*, is the equation of motion for the whole time during which the centre of the epicycle has been traversing the arc A M. If that arc, and the arc *a′ m*, be supposed to be divided into any number of equal portions, each equal to a day's motion, the equation of motion for each successive day will be equal to the successive increments of the sines of the increasing arcs in the epicycle; and these will be equal to the successive increments from day to day of the sines of mean anomaly, reduced to the dimensions of the epicycle. But the rate at which the sine is increasing or decreasing at any point in the quadrant is approximately measured by the difference of the tabular sines at that point: and as the arcs of mean daily motion are generally quite small—being, except in the case of the moon, much less than 3° 45′, the unit of the table—we may form this proportion: if, at the point in the orbit occupied by the planet, a difference of 3° 45′ in arc produces an increase or decrease of a given amount in sine, what increase or decrease of sine will be produced by a difference of arc equal to the planet's daily motion? or, 225 : diff. of tab. sines : : planet's daily motion : corresponding diff. of sine. The reduction of the result of this proportion to the dimensions of the epicycle gives the equation sought.

We will calculate by this method the true daily motion of the moon at the time for which her true longitude has been found above.

Moon's mean daily motion (i. 30),	790′ 35″
deduct daily motion of apsis (i. 33),	6 41
Moon's mean anomalistic motion,	783 54

From the process of calculation of the moon's true place, given above, we take

Moon's mean anomaly,	10ˢ 18° 46′ 15″
Sine of anomaly (*bhujajyâ*),	2266′

From the table of sines (ii. 15–27), we find

Corresponding difference of tabular sines,	174′

Hence the proportion

$$225' : 174' :: 783'\ 54'' : 606'\ 13''$$

shows the increase of the sine of anomaly in a day at this point to be 606′ 13″. The dimensions of the epicycle were found to be 31° 47′. Hence the proportion

$$360° : 31°\ 47' :: 606'\ 13'' : 53'\ 31''$$

give us the desired equation of motion, as 53′ 31″. By verse 49 it is subtractive, the planet being less than a quadrant from the apsis, or its anomaly being more than nine and less than three signs. Therefore, from the

Moon's mean daily motion,	790′ 35″
subtract the equation,	53 31
Moon's true daily motion at given time,	737 4

The roughness of the process is well illustrated by this example. Had the sine of anomaly been but 2′ greater, the difference of sines would have been 10′ less, and the equation only about 50′.

The equation of the sun's motion, calculated in a similar manner, is found to be +2′ 18″, and his true motion 61′ 26″.

The corrected rate of motion of the other planets will be given under the next following passage.

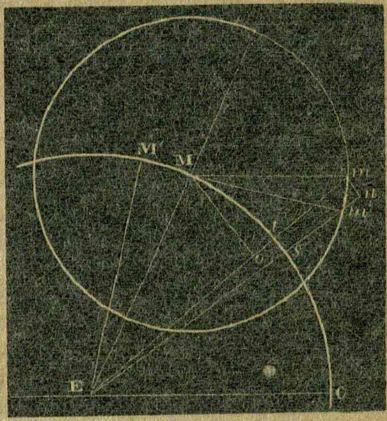
50. Subtract the daily motion of a planet, thus corrected for the apsis (*manda*), from the daily motion of its conjunction (*çîghra*); then multiply the remainder by the difference between the last hypothenuse and radius,

51. And divide by the variable hypothenuse (*cala karṇa*): the result is additive to the daily motion when the hypothenuse is greater than radius, and subtractive when this is less; if, when subtractive, the equation is greater than the daily motion, deduct the latter from it, and the remainder is the daily motion in a retrograde (*vakra*) direction.

The commentary gives no demonstration of the rule by which we are here taught to calculate the variation of the rate of motion of a planet occasioned by the action of its conjunction: the following figure, however (Fig. 7), will illustrate the principle upon which it is founded.

As in a previous figure (Fig. 5, p. 212), C M M′ represents the mean orbit of a planet, E the earth, and M the planet's mean position, at a given time, relative to its conjunction, C: the circle described about M is the epicycle of the conjunction: it is drawn, in the figure, of the relative dimensions of that assumed for Mars. Suppose M′ M to be the amount of motion of the centre of the epicycle, or the (equated) mean synodical motion of the planet, during one day; *m′ m* is the arc of the epicycle traversed by the planet in the same time. As the amount of daily synodical motion is in every case small, these arcs are necessarily greatly exaggerated in the figure, being made about twenty-four times too great for Mars. Had the planet remained stationary in the epicycle at *m′* while the centre of the epicycle moved from M′ to M, its place at the given time would be at *s*; having moved to *m*, it is seen at *t*: hence *s t* is the equation of daily motion, of which it is required to ascertain the value. Produce E *m′* to *n*, making E *n* equal to E *m*, and join *m n*; from M draw M *o* at right angles to E *m*. Then, since the arc *m m′* is very small, the angles E *m n* and E *n m*, as also M *m m′* and M *m′ m*, may be regarded as right angles; M *m o* and *n m m′* are therefore equal, each being the complement of E *m m′*, and the triangles *m n m′* and M *m o* are similar. Hence

Fig. 7.



	M *m* : *m o* :: *m m′* : *m n*
But	E M : M *m* :: M M′ : *m m′*
Hence, by combining terms,	E M : *m o* :: M M′ : *m n*
But	*t s* : E *t* :: *m n* : E *m*
therefore, since E M equals E *t*, by again combining,	*t s* : *m o* :: M M′ : E *m*

and, reducing the proportion to an equation, *t s*, the required equation of motion, equals M M′, the equated mean synodical motion in a day, multiplied by *m o*, and divided by E *m*, the variable hypothenuse. This, however, is not precisely the rule given above; for in the text of this Siddhânta, *m t*, the difference between the variable hypothenuse and radius, is substituted for *m o*, as if the two were virtually equivalent: a highly inaccurate assumption, since they differ from one another by the versed sine, *o t*, of the equation of the conjunction, M *t*, which equation is sometimes as much as 40°: and indeed, the commentary, contrary to its usual habit of obsequiousness to the inspired text with which it has to deal, rejects this assumption, and says, without even an apology for the liberty it is taking, that by the word "radius" in verse 50 is to be understood the cosine (*koṭijyâ*) of the second equation of the conjunction.

In illustration of the rule, we will calculate the true rate of daily motion of the planet Mars, at the same time for which the previous calculations have been made.

By the process already illustrated under the preceding passage, the equation of Mars's daily motion for the effect of the apsis, as derived from the data of the third process for ascertaining his true place, is found to be −3′ 41″, the difference of tabular sines being 131′. Accordingly,

from the mean daily motion of Mars (i. 34),	31′ 26″
deduct the equation for the apsis,	3 41
Mars's equated daily motion,	27 45

Now, to find the equated daily synodical motion,

from the daily motion of Mars's conjunction (the sun),	59′ 8″
deduct his equated daily motion,	27 45
Mars's equated daily synodical motion,	31 23

The variable hypothenuse used in the last process for finding the true place was 3984′; its excess above radius is 546′. The proportion

3984′ : 546′ :: 31′ 23″ : 4′ 18″

shows, then, that the equation of motion due to the conjunction at the given time is 4′ 18″. Since the hypothenuse is greater than radius—that is to say, since the planet is in the half-orbit in which the influence of the conjunction is accelerative—the equation is additive. Therefore,

to Mars's equated daily motion,	27′ 45″
add the equation for the conjunction,	4 18
Mars's true daily motion at the given time,	32 3

In this calculation we have followed the rule stated in the text: had we accepted the amendment of the commentary, and, in finding the second term of our proportion, substituted for radius the cosine of 33° 44′, the resulting equation would have been more than doubled, becoming 8′ 51″, instead of 4′ 18″; this happening to be a case where the difference is nearly as great as possible. We have deemed it best, however, in making out the corresponding results for all the five planets, as presented in the annexed table, to adhere to the directions of the text itself. The inaccuracy, it may be observed, is greatest when the equation of motion is least, and the contrary; so that, although sometimes very large relatively to the equation, it never comes to be of any great importance absolutely.

Results of the Processes for finding the True Daily Motion of the Planets.

Planet.	Diff. of sines.	Equation of Apsis.		Equated Motion.		Equated Synod. Motion.		Equation of Conjunction.		True Motion.	
	′	′	″	′	″	′	″	′	″	′	″
Mercury,	205	−4	21	54	47	190	45	−25	45	+29	2
Venus,	219	+1	53	61	1	35	7	+11	17	+72	18
Mars,	131	−3	41	27	45	31	23	+ 4	18	+32	3
Jupiter,	37	−0	4	4	55	54	13	−12	41	− 7	46
Saturn,	119	+0	8	2	8	57	0	− 5	11	− 3	3

The final abandonment by the Hindus of the principle of equable circular motion, which lies at the foundation of the whole system of eccentrics and epicycles, is, as already pointed out above (under vv. 43–45), distinctly exhibited in this process: $m'm$ (Fig. 7), the arc in the epicycle traversed by the planet during a given interval of time, is no fixed and equal quantity, but is dependent upon the arc M′M, the value of which, having suffered correction by the result of a triply complicated process, is altogether irregular and variable. This necessarily follows from the assumption of simultaneous and mutual action on the part of the beings at the apsis and conjunction, and the consequent impossibility of constructing a single connected geometrical figure which shall represent the joint effect of the two disturbing influences. By the Ptolemaic method the principle is consistently preserved: the fixed axis of the epicycle (see Fig. 6, p. 217), to the revolution of which that of the epicycle itself is bound, is xPX; and as the angle xPT, like xXA″, increases equably, the planet traverses the circumference of the epicycle with an unvarying motion relative to the fixed point x; although the equation is derived, not from the arc xT, but from eT, the equivalent of CR, its part ex varying with the varying angle EPX.

In case the reverse motion of the planet upon the half-circumference of the epicycle within the mean orbit is, when projected upon the orbit, greater than the direct motion of the centre of the epicycle, the planet will appear to move backward in its orbit, at a rate equal to the excess of the former over the latter motion. This is, as the last table shows, the case with Jupiter and Saturn at the given time. The subject of the retrogradation of the planets is continued and completed in the next following passage.

52. When at a great distance from its conjunction (*çîghrocca*), a planet, having its substance drawn to the left and right by slack cords, comes then to have a retrograde motion.

53. Mars and the rest, when their degrees of commutation (*kendra*), in the fourth process, are, respectively, one hundred and sixty-four, one hundred and forty-four, one hundred and thirty, one hundred and sixty-three, one hundred and fifteen,

54. Become retrograde (*vakrin*): and when their respective commutations are equal to the number of degrees remaining after subtracting those numbers, in each several case, from a whole circle, they cease retrogradation.

55. In accordance with the greatness of their epicycles of the conjunction (*çîghraparidhi*), Venus and Mars cease retrograding in the seventh sign, Jupiter and Mercury in the eighth, Saturn in the ninth.

The subject of the stations and retrogradations of the planets is rather briefly and summarily disposed of in this passage, although treated with as much fullness, perhaps, as is consistent with the general method of the Siddhânta. Ptolemy devotes to it the greater part of the twelfth book of the Syntaxis.

The first verse gives the theory of the physical cause of the phenomenon: it is to be compared with the opening verses of the chapter, particularly verse 2. We note here, again, the entire disavowal of the system of epicycles as a representation of the actual movements of the planets. How the slackness of the cords by which each planet is attached to, and attracted by, the supernatural being at its conjunction, furnishes an explanation of its retrogradation which should commend itself as satisfactory to the mind even of one who believed in the supernatural being and the cords, we find it very hard to see, in spite of the explanation of the commentary: it might have been better to omit verse 52 altogether, and to suffer the phenomenon to rest upon the simple and intelligible explanation given at the end of the preceding verse, which is a true statement of its cause, expressed in terms of the Hindu system. The actual reason of the apparent retrogradation is, indeed, different in the case of the inferior and of the superior planets. As regards the former, when they are traversing the inferior portion of their orbits, or are nearly between the sun and the earth, their heliocentric eastward motion becomes, of course, as seen from the earth, westward, or retrograde; by the parallax of the earth's motion in the same direction this apparent retrogradation is diminished, both in rate and in continuance, but is not prevented, because the motion of the inferior planets is more rapid than that of the earth. The retrogradation of the superior planets, on the other hand, is due to the parallax of the earth's motion in the same direction when between them and the sun, and is lessened by their own motion in their orbits, although not done away with altogether, because their motion is less rapid than that of the earth. But, in the Hindu system, the revolution of the planet in the epicycle of the conjunction represents in the one case the proper motion of the planet, in the other, that of the earth, reversed; hence, whenever its apparent amount, in a contrary direction, exceeds that of the movement of the centre of the epicycle—which is, in the one case, that of the earth, in the other, that of the planet itself—retrogradation is the necessary consequence.

Verses 53–55 contain a statement of the limits within which retrogradation takes place. The data of verse 53 belong to the different planets in the order, Mars, Mercury, Jupiter, Venus, and Saturn (see above, under i. 51, 52). That is to say, Mercury retrogrades, when his equated commutation, as made use of in the fourth process for finding his true place (see above, under vv. 43–45), is more than 144° and less than 216°; Venus, when her commutation, in like manner, is between 163° and 197°; Mars, between 164° and 196°; Jupiter, between 130° and 230°; Saturn, between 115° and 245°. These limits ought not, however, even according to the theory of this Siddhânta, to be laid down with such exactness; for the precise point at which the subtractive equation of motion for the conjunction will exceed the proper motion of the planet must depend, in part, upon the varying rate of the latter as affected by its eccentricity, and must accordingly differ a little at different times. We have not thought it worth while to calculate the amount of this variation, nor to draw up a comparison of the Hindu with the Greek and the modern determinations of the limits of retrogradation, since these are dependent for their correctness upon the accuracy of the elements assumed, and the processes employed, both of which have been already sufficiently illustrated.

The last verse of the passage adds little to what had been already said, being merely a repetition, in other and less precise terms, of the specifications of the preceding verse, together with the assertion of a relation between the limits of retrogradation and the dimensions of the respective epicycles; a relation which is only empirical, and which, as regards Venus and Mars, does not quite hold good.

56. To the nodes of Mars, Saturn, and Jupiter, the equation of the conjunction is to be applied, as to the planets themselves respectively; to those of Mercury and Venus, the equation of the apsis, as found by the third process, in the contrary direction.

57. The sine of the arc found by subtracting the place of the node from that of the planet—or, in the case of Venus and Mercury, from that of the conjunction—being multiplied by the extreme latitude, and divided by the last hypothenuse—or, in the case of the moon, by radius—gives the latitude (*vikshepa*).

58. When latitude and declination (*apakrama*) are of like direction, the declination (*krânti*) is increased by the latitude; when of different direction, it is diminished by it, to find the true (*spashṭa*) declination: that of the sun remains as already determined.

How to find the declination of a planet at any given point in the ecliptic, or circle of declination (*krântivṛtta*), was taught us in verse 28 above, taken in connection with verses 9 and 10 of the next chapter: here we have stated the method of finding the actual declination of any planet, as modified by its deviation in latitude from the ecliptic.

The process by which the amount of a planet's deviation in latitude from the ecliptic is here directed to be found is more correct than might have been expected, considering how far the Hindus were from comprehending the true relations of the solar system. The three quantities employed as data in the process are, first, the angular distance of the planet from its node; second, the apparent value, as latitude, of its greatest removal from the ecliptic, when seen from the earth at a mean distance, equal to the radius of its mean orbit; and lastly, its actual distance from the earth. Of these quantities, the second is stated for each planet in the concluding verses of the first chapter; the third is correctly represented by the variable hypothenuse (*cala karṇa*) found in the fourth process for determining the planet's true place (see above, under vv. 43–45); the first is still to be obtained, and verse 56 with the first part of verse 57 teach the method of ascertaining it. The principle of this method is the same for all the planets, although the statement of it is so different; it is, in effect, to apply to the mean place of the planet, before taking its distance from the node, only the equation of the apsis, found as the result of the third process. In the case of the superior planets, this method has all the correctness which the Hindu system admits; for by the first three processes of correction is found, as nearly as the Hindus are able to find it, the true heliocentric place of the planet, the distance from which to the node determines, of course, the amount of removal from the ecliptic. Instead, however, of taking this distance directly, rejecting altogether the fourth equation, that for the parallax of the earth's place, the Hindus apply the latter both to the planet and to the node; their relative position thus remains the same as if the other method had been adopted.

Thus, for instance, the position of Jupiter's node upon the first of January, 1860, is found from the data already given above (see i. 41–44) to be 2^s 19° 40′; his true heliocentric longitude, employed as a datum in the fourth process (see p. 216), is 3^s 1° 6′; Jupiter's heliocentric distance from the node is, accordingly, 11° 26′. Or, by the Hindu method, the planet's true geocentric place is 3^s 4° 11′, and the corrected longitude of its node is 2^s 22° 45′; the distance remains, as before, 11° 26′.

In the case of the inferior planets, as the assumptions of the Hindus respecting them were farther removed from the truth of nature, so their method of finding the distance from the node is more arbitrary and less accurate. In their system the heliocentric position of the planet is represented by the place of its conjunction (*çîghra*), and they had, as is shown above (see ii. 8), recognized the fact that it was the distance of the latter from the node which determined the amount of deviation from the ecliptic. Now, in ascertaining the heliocentric distance of an inferior planet from its node, allowance needs to be made, of course, for the effect upon its position of the eccentricity of its orbit. But the Hindu equation of the apsis is no true representative of this effect: it is calculated in order to be applied to the mean place of the sun, the assumed centre of the epicycle—that is, of the true orbit; its value, as found, is geocentric, and, as appears by the table on p. 220, is widely different from its heliocentric value; and its sign is plus or minus according as its influence is to carry the planet, as seen from the earth, eastward or westward; while, in either case, the true heliocentric effect may be at one time to bring the planet nearer to, at another time to carry it farther from, the node. The Hindus, however, overlooking these incongruities, and having, apparently, no distinct views of the subject to guide them to a correcter method, follow with regard to Venus and Mercury what seems to them the same rule as was employed in the case of the other planets—they apply the equation of the apsis, the result of the third process, to the mean place of the conjunction; only here, as before, by an indirect process: instead of applying it to the conjunction itself, they apply it with a contrary sign to the node, the effect upon the relative position of the two being the same.

Thus, for instance, the longitude of Mercury's conjunction at the given time is (see p. 214) 4^s 16° 57′; from this subtract 2° 2′, the equation of the apsis found by the third process, and its equated longitude is 4^s 14° 55′: now deducting the longitude of the node at the same time, which is 20° 41′, we ascertain the planet's distance from the node to be 3^s 24° 14′. Or, by the Hindu method, add the same equation to the mean position of the node, and its equated longitude is 22° 43′; subtract this from the mean longitude of the conjunction, and the distance is, as before, 3^s 24° 14′.

The planet's distance from the node being determined, its latitude would be found by a process similar to that prescribed in verse 28 of this chapter, if the earth were at the centre of motion; and that rule is accordingly applied in the case of the moon; the proportion being, as radius is to the sine of the distance from the node, so is the sine of extreme latitude (or the latitude itself, the difference between the sine and the arc being of little account when the arc is so small) to the latitude at the given point. In the case of the other planets, however, this proportion is modified by combination with another, namely: as the last variable hypothenuse (*cala karna*), which is the line drawn from the earth to the finally determined place of the planet, or its true distance, is to radius, its mean distance, so is its apparent latitude at the mean distance to its apparent latitude at its true distance. That is, with

	R : sin nod. dist.	: : extreme lat.	: actual lat. at dist. R
combining	var. hyp : R	: : lat. at dist. R	: lat. at true dist.
we have	var. hyp : sin nod. dist.	: : extreme lat.	: actual lat. at true dist.

which, turned into an equation, is the rule in the latter half of v. 57.

The latitude, as thus found, is measured, of course, upon a secondary to the ecliptic. By the rule in verse 58, however, it is treated as if measured upon a circle of declination, and is, without modification, added to or subtracted from the declination, according as the direction of the two is the same or different. The commentary takes note of this error, but explains it, as in other similar cases, as being, "for fear of giving men trouble, and on account of the very slight inaccuracy, overlooked by the blessed Sun, moved with compassion."

We present in the annexed table the results of the processes for calculating the latitude, the declination, and the true declination as affected by latitude, of all the planets, at the time for which their longitude has already been found. The declination is calculated by the rule in verse 28 of this chapter, the precession at the given time being, as found under verses 9–12 of the next chapter, 20° 24′ 39″. Upon the line for the sun in the table are given the results of the process for calculating his declination, the equinox itself being accounted as a "node": it is, in fact, styled, in modern Hindu astronomy, *krântipâta*, "node of declination," although that term does not occur in this treatise.

Results of the Process for finding the Latitude and Declination of the Planets.

Planet.	Longitude of Node.	do. corrected.	Distance from Node.	Sine.	Latitude.	Declination.	Corrected Declination.
	s ° ′ ″	s ° ′	s ° ′	′	° ′	° ′	° ′
Sun,	0 20 24 38	....	9 8 40	3397	....	23 41 S.	....
Moon,	9 24 24 43	....	1 23 14	2754	3 36 N.	4 56 N.	8 32 N.
Mercury,	0 20 40 41	0 22 43	3 24 14	3134	2 4 N.	23 10 S.	21 6 S.
Venus,	1 29 39 22	1 29 16	8 22 34	3409	1 21 S.	20 27 S.	21 48 S.
Mars,	1 10 3 5	2 13 47	4 4 58	2816	1 4 N.	14 52 S.	13 48 S.
Jupiter,	2 19 40 5	2 22 45	0 11 26	682	0 15 N.	21 42 N.	21 57 N.
Saturn,	3 10 20 45	3 14 38	0 16 24	970	0 37 N.	14 40 N.	15 17 N.

We are now able to compare the Hindu determinations of the true places and motions of the planets with their actual positions and motions, as obtained by modern science. The comparison is made in the annexed table. As the longitudes given by the Sûrya-Siddhânta contain a constant error of 2° 20′, owing to the incorrect rate of precession adopted by the treatise, and the false position thence assigned to the equinox, we give, under the head of longitude, the distance of each planet both from the Hindu equinox, and from the true vernal equinox of Jan. 1, 1860. The Hindu daily motions are reduced from longitude to right ascension by the rule given in the next following verse (v. 59). The modern data are taken from the American Nautical Almanac.

True Places and Motions of the Planets, Jan. 1st, 1860, midnight, at Washington, according to the Sûrya-Siddhânta and to Modern Science.

Planet.	True Longitude. Sûrya Siddhânta: from Hindu eq.	from true eq.	Moderns.	Declination. Sûrya-Siddhânta.	Moderns.	Daily Motion in Right Ascension. Sûrya-Siddhânta.	Moderns.
	° ′	° ′	° ′	° ′	° ′	′ ″	′ ″
Sun,	278 40	276 20	280 5	23 41 S.	23 5 S.	+ 66 2	+ 66 18
Moon,	8 4	5 44	7 27	8 32 N.	6 56 N.	+683 50	+655 14
Mercury,	255 16	252 56	257 25	21 6 S.	20 42 S.	+ 31 13	+ 52 39
Venus,	305 0	302 40	303 25	21 48 S.	20 58 S.	+ 72 59	+ 78 6
Mars,	219 10	216 50	221 33	13 48 S.	14 23 S.	+ 31 58	+ 36 19
Jupiter,	114 36	112 16	111 34	21 57 N.	22 1 N.	− 8 21	− 8 17
Saturn,	141 27	139 7	145 32	15 17 N.	14 15 N.	− 3 3	− 2 29

The proper subject of the second chapter, the determination of the true places of the planets, being thus brought to a close, we should expect to see the chapter concluded here, and the other matters which it contains put off to that which follows, in which they would seem more properly to belong. The treatise, however, is nowhere distinguished for its orderly and consistent arrangement.

59. Multiply the daily motion of a planet by the time of rising of the sign in which it is, and divide by eighteen hundred; the quotient add to, or subtract from, the number of respirations in a revolution: the result is the number of respirations in the day and night of that planet.

In the first half of this verse is taught the method of finding the increment or decrement of right ascension corresponding to the increment or decrement of longitude made by any planet during one day. For the "time of rising" (*udayaprânâs*, or, more commonly, *udayâsavas*, literally "respirations of rising") of the different signs, or the time in respirations (see i. 11), occupied by the successive signs of the ecliptic in passing the meridian—or, at the equator, in rising above the horizon—see verses 42–44 of the next chapter. The statement upon which the rule is founded is as follows: if the given sign, containing 1800′ of arc (each minute of arc corresponding, as remarked above, under i. 11–12, to a respiration of sidereal time), occupies the stated number of respirations in passing the meridian, what number of respirations will be occupied by the arc traversed by the planet on a given day? The result gives the amount by which the day of each planet, reckoned in the manner of this Siddhânta, or from transit to transit across the inferior meridian, differs from a sidereal day: the difference is additive when the motion of the planet is direct; subtractive, when this is retrograde.

Thus, to find the length of the sun's day, or the interval between two successive apparent transits, at the time for which his true longitude and rate of motion have already been ascertained. The sun's longitude, as corrected by the precession, is $9^s\ 8^\circ\ 40'$; he is accordingly in the tenth sign, of which the time of rising (*udayâsavas*), or the equivalent in right ascension, is 1935^p. His rate of daily motion in longitude is $61'\ 26''$. Hence the proportion

$$1800' : 1935^p :: 61'\ 26'' : 66^p.04$$

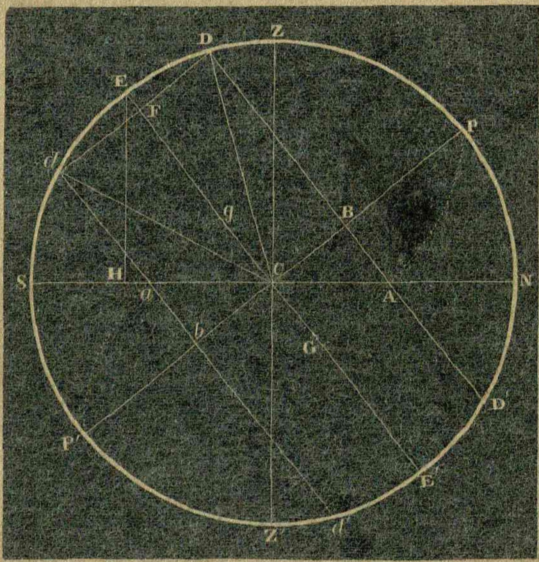
shows that his day differs from the true sidereal day by $11^v\ 0^p.04$. As his motion is direct, the difference is additive: the length of the apparent day is therefore $60^n\ 11^v\ 0^p.04$, which is equivalent to $24^h\ 0^m\ 27^s.5$, mean solar time. According to the Nautical Almanac, it is $24^h\ 0^m\ 28^s.6$. By a similar process, the length of Jupiter's day at the same time is found to be $59^n\ 58^v\ 4^p$, or $23^h\ 55^m\ 30^s.8$; by the Nautical Almanac, it is $23^h\ 55^m\ 30^s$.

60. Calculate the sine and versed sine of declination: then radius, diminished by the versed-sine, is the day-radius: it is either south or north.

The quantities made use of, and the processes prescribed, in this and the following verses, may be explained and illustrated by means of the annexed figure (Fig. 8).

Let the circle Z S Z′ N represent the meridian of a given place, C being the centre, the place of the observer, S N the section of the plane of his horizon—S being the south, and N the north point—Z and Z′ the zenith and its opposite point, the nadir, P and P′ the north and south poles, and E and E′ the points on the meridian cut by the equator. Let E D be the declination of a planet at a given time; then D D′ will be the diameter of the circle of diurnal revolution described by the planet, and B D the radius of that circle: B D is the line which in verse 60 is called the "day-radius." Draw D F perpendicular to E C:

Fig. 8.



then it is evident that B D is equal to E C diminished by E F, which is the versed sine of E D, the declination.

For "radius" we have hitherto had only the term *trijyâ* (or its equivalents, *trijîvâ*, *tribhajîvâ*, *tribhajyâ*, *tribhamâurvikâ*), literally "the sine of three signs," that is, of 90°. That term, however, is applicable only to the radius of a great circle, or to tabular radius. In this verse, accordingly, we have for "day-radius" the word *dinavyâsadala*, "half-diameter of the day;" and other expressions synonymous with this are found used instead of it in other passages. A more frequent name for the same quantity in modern Hindu astronomy is *dyujyâ*, "day-sine:" this, although employed by the commentary, is not found anywhere in our text.

It is a matter for surprise that we do not find the day-radius declared equal simply to the cosine (*koṭijyâ*) of declination.

In illustration of the rule, it will be sufficient to find the radius of the diurnal circle described by the sun at the time for which his place has been determined. His declination, E *d* (Fig. 8) was found to be 23° 41′: of this the versed sine, E F, is, by the table given above (ii. 22–27), 290′: the difference between this and radius, E C, or 3438′, is 3148′, which is the value of C F or *b d*, the day-radius. The declination in this case being south, the day-radius is also south of the equator.

61. Multiply the sine of declination by the equinoctial shadow, and divide by twelve; the result is the earth-sine (*kshitijyâ*); this, multiplied by radius and divided by the day-radius, gives the sine of the ascensional difference (*cara*): the number of respirations due to the ascensional difference

62. Is shown by the corresponding arc. Add these to, and subtract them from, the fourth part of the corresponding day and night, and the sum and remainder are, when declination is north, the half-day and half-night;

63. When declination is south, the reverse; these, multiplied by two, are the day and the night. The day and the night of the asterisms (*bha*) may be found in like manner, by means of their declination, increased or diminished by their latitude.

We were taught in verse 59 how to find the length of the entire day of a planet at any given time; this passage gives us the method of ascertaining the length of its day and of its night, or of that part of the day during which the planet is above, and that during which it is below, the horizon.

In order to this, it is necessary to ascertain, for the planet in question, its ascensional difference (*cara*), or the difference between its right and oblique ascension, the amount of which varies with the declination of the planet and the latitude of the observer. The method of doing this is stated in verse 61: it may be explained by means of the last figure (Fig. 8). First, the value of the line A B, which is called the "earth-sine" (*kshitijyâ*), is found, by comparing the two triangles A B C and C H E, which are similar, since the angles A C B and C E H are each equal to the latitude of the observer. The triangle C H E is represented here by a triangle of which a gnomon of twelve digits is the perpendicular, and its equinoctial shadow, cast when the sun is in the equator and on the meridian (see the next chapter, verse 7, etc.), is the base. Hence the proportion E H : H C : : B C : A B is equivalent—since B C equals D F, the sine of declination—to gnom. : eq. shad. : : sin decl. : earth-sine. But the arc of which A B is sine is the same part of the circle of diurnal revolution as the ascensional difference is of the equator; hence the reduction of A B to the dimensions of a great circle, by the proportion B D : A B : : C E : C G, gives the value of C G, the sine of the ascensional difference. The corresponding arc is the measure in time of the amount by which the part of the diurnal circle intercepted between the meridian and the horizon differs from a quadrant, or by which the time between sun-rise or sun-set and noon or midnight differs from a quarter of the day.

In illustration of the process, we will calculate the respective length of the sun's day and night at Washington at the time for which our previous calculations have been made.

The latitude of Washington being 38° 54′, the length of the equinoctial shadow cast there by a gnomon twelve digits long is found, by the rule given below (iii. 17), to be $9^d.68$. The sine, *d* F or *b* C, of the sun's declination at the given time, 23° 41′ S, is 1380′. Hence the proportion

12 : 9.68 : : 1380 : 1113

gives us the value of the earth-sine, *a b*, as 1113′. This is reduced to the dimensions of a great circle by the proportion

3148 : 3438 : : 1113 : 1216

The value of C *g*, the sine of ascensional difference, is therefore 1216′: the corresponding arc is 20° 44′, or 1244′, which, as a minute of arc equals a respiration of time, is equivalent to $3^n\ 27^v\ 2^p$. The total length of the day was found above (under v. 59) to be $60^n\ 11^v$; increase and diminish the quarter of this by the ascensional difference, and double the sum and remainder, and the length of the night is found to be $37^n\ 0^v\ 1^p$, and that of the day $23^n\ 10^v\ 5^p$, which are equivalent respectively to $14^h\ 45^m\ 38^s.6$ and $9^h\ 14^m\ 48^s.9$, mean solar time.

Of course, the respective parts of a sidereal day during which each of the lunar mansions, as represented by its principal star, will remain above and below the horizon of a given latitude, may be found in the same manner, if the declination of the star is known; and this is stated in the chapter (ch. viii) which treats of the asterisms.

Why A B is called *kshitijyâ* is not easy to see. One is tempted to understand the term as meaning rather "sine of situation" than "earth-sine," the original signification of *kshiti* being "abode, residence": it might then indicate a sine which, for a given declination, varies with the situation of the observer. But that *kshiti* in this compound is to be taken in its other acceptation, of "earth," is at least strongly indicated by the other and more usual name of the sine in question, *kujyâ*, which is used by the commentary, although not in the text, and which can only mean "earth-sine." The word *cara*, used to denote the ascensional difference, means simply "variable"; we have elsewhere *carakhanda*, *caradala*, "variable portion"; that is to say, the constantly varying amount by which the apparent day and night differ from the equatorial day and night of one half the whole day each. The gnomon, the equinoctial shadow, etc., are treated of in the next chapter.

64. The portion (*bhoga*) of an asterism (*bha*) is eight hundred minutes; of a lunar day (*tithi*), in like manner, seven hundred and twenty. If the longitude of a planet, in minutes, be divided by the portion of an asterism, the result is its position in asterisms: by means of the daily motion are found the days, etc.

The ecliptic is divided (see ch. viii) into 27 lunar mansions or asterisms, of equal amount; hence the portion of the ecliptic occupied by each asterism is 13° 20′, or 800′. In order to find, accordingly, in which asterism, at a given time, the moon or any other of the planets is, we have only to reduce its longitude, not corrected by the precession, to minutes, and divide by 800: the quotient is the number of asterisms traversed, and the remainder the part traversed of the asterism in which the planet is. The last clause of the verse is very elliptical and obscure; according to the commentary, it is to be understood thus: divide by the planet's true daily motion the part past and the part to come of the current asterism, and the quotients are the days and fractions of a day which the planet has passed, and is to pass, in that asterism. This interpretation is supported by the analogy of the following verses, and is doubtless correct.

The true longitude of the moon was found above (under v. 39) to be 11^s 17° 39′, or 20,859′. Dividing by 800, we find that, at the given time, the moon is in the 27th, or last, asterism, named Revatî, of which it has traversed 59′, and has 741′ still to pass over. Its daily motion being 737′, it has spent 28^v 4^p, and has yet to continue 1^d 0^n 19^v 3^p, in the asterism.

The latter part of this process proceeds upon the assumption that the planet's rate of motion remains the same during its whole continuance in the asterism. A similar assumption, it will be noticed, is made in all the processes from verse 59 onward; its inaccuracy is greatest, of course, where the moon's motion is concerned.

Respecting the lunar day (*tithi*) see below, under verse 66.

65. From the number of minutes in the sum of the longitudes of the sun and moon are found the *yogas*, by dividing that sum by the portion (*bhoga*) of an asterism. Multiply the minutes past and to come of the current yoga by sixty, and divide by the sum of the daily motions of the two planets: the result is the time in nâḍîs.

What the *yoga* is, is evident from this rule for finding it; it is the period, of variable length, during which the joint motion in longitude of the sun and moon amounts to 13° 20′, the portion of a lunar mansion. According to Colebrooke (As. Res., ix. 365; Essays, ii. 362, 363), the use of the yogas is chiefly astrological; the occurrence of certain movable festivals is, however, also regulated by them, and they are so frequently employed that every Hindu almanac contains a column specifying the yoga for each day, with the time of its termination. The names of the twenty-seven yogas are as follows:

1. Vishkambha.
2. Prîti.
3. Âyushmant.
4. Sâubhâgya.
5. Çobhana.
6. Atiganda.
7. Sukarman.
8. Dhrti.
9. Çûla.
10. Ganda.
11. Vrddhi.
12. Dhruva.
13. Vyâghâta.
14. Harshana.
15. Vajra.
16. Siddhi.
17. Vyatîpâta.
18. Varîyas.
19. Parigha.
20. Çiva.
21. Siddha.
22. Sâdhya.
23. Çubha.
24. Çukla.
25. Brahman.
26. Indra.
27. Vâidhrti.

There is also in use in India (see Colebrooke, as above) another system of yogas, twenty-eight in number, having for the most part different names from these, and governed by other rules in their succession. Of this system the Sûrya-Siddhânta presents no trace.

We will find the time in yogas corresponding to that for which the previous calculations have been made.

The longitude of the moon at that time is $11^s\ 17°\ 39'$, that of the sun is $8^s\ 18°\ 15'$; their sum is $8^s\ 5°\ 54'$, or $14,754'$. Dividing by 800, we find that eighteen yogas of the series are past, and that the current one is the nineteenth, Parigha, of which $354'$ are past, and $446'$ to come. To ascertain the time at which the current yoga began and that at which it is to end, we divide these parts respectively by $798'\frac{1}{2}$, the sum of the daily motions of the sun and moon at the given time, and multiply by 60 to reduce the results to nâdîs: and we find that Parigha began $26^n\ 36^v$ before, and will end $33^n\ 30^v\ 4^p$ after the given time.

The name *yoga*, by which this astrological period is called, is applied to it, apparently, as designating the period during which the "sum" (*yoga*) of the increments in longitude of the sun and moon amounts to a given quantity. It seems an entirely arbitrary device of the astrologers, being neither a natural period nor a subdivision of one, not being of any use that we can discover in determining the relative position, or aspect, of the two planets with which it deals, nor having any assignable relation to the asterisms, with which it is attempted to be brought into connection. Were there thirty yogas, instead of twenty-seven, the period would seem an artificial counterpart to the lunar day, which is the subject of the next verse; being derived from the sum, as the other from the difference, of the longitudes of the sun and moon.

66. From the number of minutes in the longitude of the moon diminished by that of the sun are found the lunar days (*tithi*), by dividing the difference by the portion (*bhoga*) of a lunar day. Multiply the minutes past and to come of the current lunar day by sixty, and divide by the difference of the daily motions of the two planets: the result is the time in nâḍîs.

The *tithi*, or lunar day, is (see i. 13) one thirtieth of a lunar month, or of the time during which the moon gains in longitude upon the sun a whole revolution, or 360°: it is, therefore, the period during which the difference of the increment of longitude of the two planets amounts to 12°, or 720′, which arc, as stated in verse 64, is its portion (*bhoga*). To find the current lunar day, we divide by this amount the whole excess of the longitude of the moon over that of the sun at the given time; and to find the part past and to come of the current day, we convert longitude into time in a manner analogous to that employed in the case of the yoga.

Thus, to find the date in lunar time of the midnight preceding the first of January, 1860, we first deduct the longitude of the sun from that of the moon; the remainder is 2^s 29° 24′, or 5364′: dividing by 720, it appears that the current lunar day is the eighth, and that 324′ of its portion are traversed, leaving 396′ to be traversed. Multiplying these numbers respectively by 60, and dividing by 675′ 38″, the difference of the daily motions at the time, we find that 28^n 46^v 2^p have passed since the beginning of the lunar day, and that it still has 35^n 10^v 8^p to run.

The lunar days have, for the most part, no distinctive names, but those of each half month (*paksha*—see above, under i. 48–51) are called first, second, third, fourth, etc., up to fourteenth. The last, or fifteenth, of each half has, however, a special appellation: that which concludes the first, the light half, ending at the moment of opposition, is called *pâurnamâsî*, *pûrnimâ*, *pûrṇamâ*, "day of full moon;" that which closes the month, and ends with the conjunction of the two planets, is styled *amâvâsyâ*, "the day of dwelling together."

Each lunar day is farther divided into two halves, called *karaṇa*, as appears from the next following passage.

67. The fixed (*dhruva*) *karaṇas*, namely *çakuni*, *nâga*, *catushpada* the third, and *kinstughna*, are counted from the latter half of the fourteenth day of the dark half-month.

68. After these, the karaṇas called movable (*cara*), namely *bava*, etc., seven of them: each of these karaṇas occurs eight times in a month.

69. Half the portion (*bhoga*) of a lunar day is established as that of the karaṇas . . . .

Of the eleven *karaṇas*, four occur only once in the lunar month, while the other seven are repeated each of them eight times to fill out the remainder of the month. Their names, and the numbers of the half lunar days to which each is applied, are presented below:

1. Kinstughna.	1st.
2. Bava.	2nd, 9th, 16th, 23rd, 30th, 37th, 44th, 51st.
3. Bâlava.	3rd, 10th, 17th, 24th, 31st, 38th, 45th, 52nd.
4. Kâulava.	4th, 11th, 18th, 25th, 32nd, 39th, 46th, 53rd.
5. Tâitila.	5th, 12th, 19th, 26th, 33rd, 40th, 47th, 54th.
6. Gara.	6th, 13th, 20th, 27th, 34th, 41st, 48th, 55th.
7. Baṇij.	7th, 14th, 21st, 28th, 35th, 42nd, 49th, 56th.
8. Vishṭi.	8th, 15th, 22nd, 29th, 36th, 43rd, 50th, 57th.
9. Çakuni.	58th.
10. Nâga.	59th.
11. Catushpada.	60th.

Most of these names are very obscure: the last three mean "hawk," "serpent," and "quadruped." *Karana* itself is, by derivation, "factor, cause:" in what sense it is applied to denote these divisions of the month, we do not know. Nor have we found anywhere an explanation of the value and use of the karanas in Hindu astronomy or astrology.

The time which we have had in view in our other calculations being, as is shown under the preceding passage, in the first half of the eighth lunar day, is, of course, in the fifteenth karaṇa, which is named Vishṭi.

The remaining half-verse is simply a winding-up of the chapter.

69. . . . . Thus has been declared the corrected (*sphuṭa*) motion of the sun and the other planets.

The following chapter is styled the "chapter of the three inquiries" (*tripraçnâdhikâra*). According to the commentary, this means that it is intended by the teacher as a reply to his pupil's inquiries respecting the three subjects of direction (*diç*), place (*deça*), and time (*kâla*).

CHAPTER III.

OF DIRECTION, PLACE, AND TIME.

CONTENTS:—1–6, construction of the dial, and description of its parts; 7, the measure of amplitude; 8, of the gnomon, hypothenuse, and shadow, any two being given, to find the third; 9–12, precession of the equinoxes; 12–13, the equinoctial shadow; 13–14, to find, from the equinoctial shadow, the latitude and co-latitude; 14–17, the sun's declination being known, to find, from a given shadow at noon, his zenith-distance, the latitude, and its sine and cosine; 17, latitude being given, to find the equinoctial shadow; 17–20, to find, from the latitude and the sun's zenith-distance at noon, his declination and his true and mean longitude; 20–22, latitude and declination being given, to find the noon-shadow and hypothenuse; 22–23, from the sun's declination and the equinoctial shadow, to find the measure of amplitude; 23–25, to find, from the equinoctial shadow and the measure of amplitude at any given time, the base of the shadow; 25–27, to find the hypothenuse of the shadow when the sun is upon the prime vertical; 27–28, the sun's declination and the latitude being given, to find the sine and the measure of amplitude; 28–33, to find the sines of the altitude and zenith-distance of the sun, when upon the south-east and south-west vertical circles; 33–34, to find the corresponding shadow and hypothenuse; 34–36, the sun's ascensional difference and the hour-angle being given, to find the sines of his altitude and zenith-distance, and the corresponding shadow and hypothenuse; 37–39, to find, by a contrary process, from the shadow of a given time, the sun's altitude and zenith-distance, and the hour-angle; 40–41, the latitude and the sun's amplitude being known, to find his declination and true longitude; 41–42, to draw the path described by the extremity of the shadow; 42–45, to find the arcs of right and oblique ascension corresponding to the several signs of the ecliptic; 46–48, the sun's longitude and the time being known, to find the point of the ecliptic which is upon the horizon; 49, the sun's longitude and the hour-angle being known, to find the point of the ecliptic which is upon the meridian; 50–51, determination of time by means of these data.

1. On a stony surface, made water-level, or upon hard plaster, made level, there draw an even circle, of a radius equal to any required number of the digits (*angula*) of the gnomon (*çanku*).

2. At its centre set up the gnomon, of twelve digits of the measure fixed upon; and where the extremity of its shadow touches the circle in the former and after parts of the day,

3. There fixing two points upon the circle, and calling them the forenoon and afternoon points, draw midway between them, by means of a fish-figure (*timi*), a north and south line.

4. Midway between the north and south directions draw, by a fish-figure, an east and west line: and in like manner also, by fish-figures (*matsya*) between the four cardinal directions, draw the intermediate directions.

5. Draw a circumscribing square, by means of the lines going out from the centre; by the digits of its base-line (*bhujasûtra*) projected upon that is any given shadow reckoned.

In this passage is described the method of construction of the Hindu dial, if that can properly be called a dial which is without hour-lines, and does not give the time by simple inspection. It is, as will be at once remarked, a horizontal dial of the simplest character, with a vertical gnomon. This gnomon, whatever may be the length chosen for it, is regarded as divided into twelve equal parts called digits (*angula*, "finger"). The ordinary digit is one twelfth of a span (*vitasti*), or one twenty-fourth of a cubit (*hasta*): if made according to this measure, then, the gnomon would be about nine inches long. Doubtless the first gnomons were of such a length, and the rules of the gnomonic science were constructed accordingly, "twelve" and "the gnomon" being used, as they are used everywhere in this treatise, as convertible terms: thus twelve digits became the unvarying conventional length of the staff, and all measurements of the shadow and its hypothenuse were made to correspond. How the digit was subdivided, we have nowhere any hint. In determining the directions, the same method was employed which is still in use; namely, that of marking the points at which the extremity of the shadow, before and after noon, crosses a circle described about the base of the gnomon; these points being, if we suppose the sun's declination to have remained the same during the interval, at an equal distance upon either side from the meridian line. In order to bisect the line joining these points by another at right angles to it, which will be the meridian, the Hindus draw the figure which is called here the "fish" (*matsya* or *timi*); that is to say, from the two extremities of the line in question as centres, and with a radius equal to the line itself, arcs of circles are described, cutting one another in two points. The lenticular figure formed by the two arcs is the "fish;" through the points of intersection, which are called (in the commentary) the "mouth" and "tail," a line is drawn, which is the one required. The meridian being thus determined, the east and west line, and those for the intermediate points of directions, are laid down from it, by a repetition of the same process. A square (*caturasra*, "having four corners") is then farther described about the general centre, or about a circle drawn about that centre, the eastern and western sides of which are divided into digits; its use is, to aid in ascertaining the "base" (*bhuja*) of any given shadow, which is the value of the latter when projected upon a north and south line (see below, vv. 23–25); the square is drawn, as explained by the commentary, in order to insure the correctness of the projection, by a line strictly parallel to the east and west line.

The figure (Fig. 9) given below, under verse 7, will illustrate the form of the Hindu dial, as described in this passage.

The term used for "gnomon" is *çanku*, which means simply "staff." For the shadow, we have the common word *châyâ*, "shadow," and also, in many places, *prabhâ* and *bhâ*, which properly signify the very opposite of shadow, namely "light, radiance:" it is difficult to see how they should come to be used in this sense; so far as we are aware, they are applied to no other shadow than that of the gnomon.

6. The east and west line is called the prime vertical (*samamaṇḍala*); it is likewise denominated the east and west hour circle (*unmaṇḍala*) and the equinoctial circle (*vishuvanmaṇḍala*).

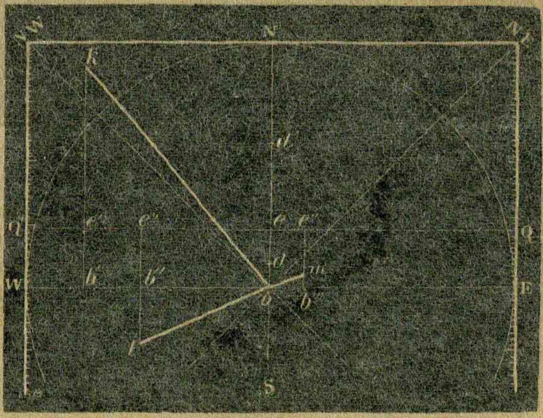
The line drawn east and west through the base of the gnomon may be regarded as the line of common intersection at that point of three great circles, as being a diameter to each of the three, and as thus entitled to represent them all. These circles are the ones which in the last figure (Fig. 8, p. 232) are shown projected in their diameters Z Z′, P P′, and E E′; the centre C, in which the diameters intersect, is itself the projection of the line in question here: Z Z′ represents the prime vertical, which is styled *samamaṇḍala*, literally "even circle:" P P′ is the hour circle, or circle of declination, which passes through the east and west points of the observer's horizon; it is called *unmaṇḍala*, "up-circle"—that is to say, the circle which in the oblique sphere is elevated; E E′ finally, the equator, has the name of *vishuvanmaṇḍala*, or *vishuvadvṛtta*, "circle of the equinoxes;" the equinoctial points themselves being denominated *vishuvat*, or *vishuva*, which may be rendered "point of equal separation." The same line of the dial might be regarded as the representative in like manner of a fourth circle, that of the horizon (*kshitija*), projected, in the figure, in S N: hence the commentary adds it also to the other three; it is omitted in the text, perhaps, because it is represented by the whole circle drawn about the base of the gnomon, and not by this diameter alone.

The specifications of this verse, especially of the latter half of it, are of little practical importance in the treatise, for there hardly arises a case, in any of its calculations, in which the east and west axis of the dial comes to be taken as standing for these circles, or any one of them. In drawing the base (*bhuja*) of the shadow, indeed, it does represent the plane of the prime vertical (see below, under vv. 23–25); but this is not distinctly stated, and the name of the prime vertical (*samamaṇḍala*) occurs in only one other passage (below, v. 26): the east and west hour-circle (*unmaṇḍala*) is nowhere referred to again: and the equator, as will be seen under the next verse, is properly represented on the dial, not by its east and west axis, but by the line of the equinoctial shadow.

7. Draw likewise an east and west line through the extremity of the equinoctial shadow (*vishuvadbhâ*); the interval between any given shadow and the line of the equinoctial shadow is denominated the measure of amplitude (*agrâ*).

The equinoctial shadow is defined in a subsequent passage (vv. 12, 13); it is, as we have already had occasion to notice (under ii. 61–63), the shadow cast at mid-day when the sun is at either equinox—that is to say, when he is in the plane of the equator. Now as the equator is a circle of diurnal revolution, the line of intersection of its plane with that of the horizon will be an east and west line; and since it is also a great circle, that line will pass through the centre, the place of the observer: if, therefore, we draw through the extremity of the equinoctial shadow a line parallel to the east and west axis of the dial, it will represent the intersection with the dial of an equinoctial plane passing through the top of the gnomon, and in it will terminate the lines drawn through that point from any point in the plane of the equator; and hence, it will also coincide with the path of the extremity of the shadow on the day of the equinox. Thus, let the following figure (Fig. 9) represent the plane of the dial, N S and E W being its two axes, and *b* the base of the gnomon: and let the shadow cast at noon when the sun is upon the equator be, in a given latitude, *b e*: then *b e* is the equinoctial shadow, and Q Q′, drawn through *e* and parallel to E W, is the path of the equinoctial shadow, being the line in which a ray of the sun, from any point in the plane of the equator, passing through the top of the gnomon, will meet the face of the dial. In the figure as given, the circle is supposed to be described about the base of the gnomon with a radius of forty digits, and the graduation of the eastern and western sides of the circumscribing square, used in measuring the base (*bhuja*) of the shadow, is indicated: the length given to the equinoctial shadow corresponds to that which it has in the latitude of Washington.

Fig. 9.

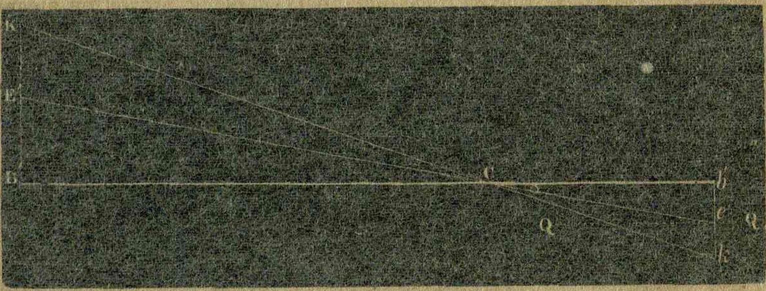


It is not, however, on account of the coincidence of Q Q′ with the path of the equinoctial shadow that it is directed to be permanently drawn upon the dial-face: its use is to determine for any given shadow its *agrâ*, or measure of amplitude. Thus, let *b d*, *b d′*, *b k*, *b l*, *b m*, be shadows cast by the gnomon, under various conditions of time and declination: then the distance from the extremity of each of them to the line of the equinoctial shadow, or *d e*, *d' e*, *k e'*, *l e''*, *m e'''* respectively, is denominated the *agrâ* of that shadow or of that time.

The term *agrâ* we have translated "measure of amplitude," because it does in fact represent the sine of the sun's amplitude—understanding by "amplitude" the distance of the sun at rising or setting from the east or west-point of the horizon—varying with the hypothenuse of the shadow, and always maintaining to that hypothenuse the fixed ratio of the sine of amplitude to radius. That this is so, is assumed by the text in its treatment of the *agrâ*, but is nowhere distinctly stated, nor is the commentator at the pains of demonstrating the principle. Since, however, it is not an immediately obvious one, we will take the liberty of giving the proof of it.

In the annexed figure (Fig. 10) let C represent the top of the gnomon, and let K be any given position of the sun in the heavens. From K draw K B' at right angles to the plane of the prime vertical, meeting that

Fig. 10.



plane in B', and let the point of its intersection with the plane of the equator be in E'. Join K C, E' C, and B' C. Then K C is radius, and E' K is equal to the sine of the sun's amplitude: for if, in the sun's daily revolution, the point K is brought to the horizon, E' B' will disappear, K E' C will become a right angle, K C E' will be the amplitude, and E' K its sine; but, with a given declination, the value of E' K remains always the same, since it is a line drawn in a constant direction between two parallel planes, that of the circle of declination and that of the equator. Now conceive the three lines intersecting in C to be produced until they meet the plane of the dial in *b'*, *e'*, and *k* respectively; these three points will be in the same straight line, being in the line of intersection with the horizon of the plane K B' C produced, and this line, *b' k*, will be at right angles to B' *b'*, since it is the line of intersection of two planes, each of which is at right angles to the plane of the prime vertical, in which B' *b'* lies. K B' and *k b'* are therefore parallel, and the triangles C E' K and C *e' k* are similar, and *e' k* : C *k* : : E' K : C K. But C *k* is the hypothenuse of the shadow at the given time, and *e' k* is the *agrâ*, or measure of amplitude, since *e'*, by what was said above, is in the line of the equinoctial shadow; therefore meas. ampl. : hyp. shad. : : sin ampl. : R. Hence, if the declination and the latitude, which together determine the sine of amplitude, be given, the measure of amplitude will vary with the hypothenuse of the shadow, and the measure of amplitude of any given shadow will be to that of any other, as the hypothenuse of the former to that of the latter.

The lettering of the above figure is made to correspond, as nearly as may be, with that of the one preceding, and also with that of the one given later, under verses 13 and 14, in either of which the relations of the problem may be farther examined.

There are other methods of proving the constancy of the ratio borne by the measure of amplitude to the hypothenuse of the shadow, but we have chosen to give the one which seemed to us most likely to be that by which the Hindus themselves deduced it. Our demonstration is in one respect only liable to objection as representing a Hindu process—it is founded, namely, upon the comparison of oblique-angled triangles, which elsewhere in this treatise are hardly employed at all. Still, although the Hindus had no methods of solving problems excepting in right-angled trigonometry, it is hardly to be supposed that they refrained from deriving proportions from the similarity of oblique-angled triangles. The principle in question admits of being proved by means of right-angled triangles alone, but these would be situated in different planes.

Why the line on the dial which thus measures the sun's amplitude is called the *agrâ*, we have been unable thus far to discover. The word, a feminine adjective (belonging, probably, to *rekhâ*, "line," understood), literally means "extreme, first, chief." Possibly it may be in some way connected with the use of *antyâ*, "final, lowest," to designate the line E*g* or E G (Fig. 8, p. 232): see below, under v. 35. The sine of amplitude itself, *a* C or A C (Fig. 8), is called below (vv. 27–30) *agrajyâ*.

8. The square root of the sum of the squares of the gnomon and shadow is the hypothenuse: if from the square of the latter the square of the gnomon be subtracted, the square root of the remainder is the shadow: the gnomon is found by the converse process.

This is simply an application of the familiar rule, that in a right-angled triangle the square of the hypothenuse is equal to the sum of the squares of the other two sides, to the triangle produced by the gnomon as perpendicular, the shadow as base, and the hypothenuse of the shadow, the line drawn from the top of the gnomon to the extremity of the shadow, as hypothenuse.

The subject next considered is that of the precession of the equinoxes.

9. In an Age (*yuga*), the circle of the asterisms (*bha*) falls back eastward thirty score of revolutions. Of the result obtained after multiplying the sum of days (*dyugaṇa*) by this number, and dividing by the number of natural days in an Age,

10. Take the part which determines the sine, multiply it by three, and divide by ten; thus are found the degrees called those of the precession (*ayana*). From the longitude of a planet as corrected by these are to be calculated the declination, shadow, ascensional difference (*caradala*), etc.

11. The circle, as thus corrected, accords with its observed place at the solstice (*ayana*) and at either equinox; it has moved eastward, when the longitude of the sun, as obtained by calculation, is less than that derived from the shadow,

12. By the number of degrees of the difference; then, turning back, it has moved westward by the amount of difference, when the calculated longitude is greater. . . .

Nothing could well be more awkward and confused than this mode of stating the important fact of the precession of the equinoxes, of describing its method and rate, and of directing how its amount at any time is to be found. The theory which the passage, in its present form, is actually intended to put forth is as follows: the vernal equinox librates westward and eastward from the fixed point, near ζ Piscium, assumed as the commencement of the sidereal sphere—the limits of the libratory movement being 27° in either direction from that point, and the time of a complete revolution of libration being the six-hundredth part of the period called the Great Age (see above, under i. 15–17), or 7200 years; so that the annual rate of motion of the equinox is 54″. We will examine with some care the language in which this theory is conveyed, as important results are believed to be deducible from it.

The first half of verse 9 professes to teach the fundamental fact of the motion in precession. The words *bhânâm cakram*, which we have rendered "circle of the asterisms," i. e., the fixed zodiac, would admit of being translated "circle of the signs," i. e., the movable zodiac, as reckoned from the actual equinox, since *bha* is used in this treatise in either sense. But our interpretation is shown to be the correct one by the directions given in verses 11 and 12, which teach that when the sun's calculated longitude—which is his distance from the initial point of the fixed sphere—is less than that derived from the shadow by the process to be taught below (vv. 17–19)—which is his distance from the equinox—the circle has moved eastward, and the contrary: it is evident, then, that the initial point of the sphere is regarded as the movable point, and the equinox as the fixed one. Now this is no less strange than inconsistent with the usage of the rest of the treatise. Elsewhere ζ Piscium is treated as the one established limit, from which all motion commenced at the creation, and by reference to which all motion is reckoned, while here it is made secondary to a point of which the position among the stars is constantly shifting, and which hardly has higher value than a node, as which the Hindu astronomy in general treats it (see p. 230). The word used to express the motion (*lambate*) is the same with that employed in a former passage (i. 25) to describe the eastward motion of the planets, and derivatives of which (as *lamba*, *lambana*, etc.) are not infrequent in the astronomical language; it means literally to "lag, hang back, fall behind:" here we have it farther combined with the prefix *pari*, "about, round about," which seems plainly to add the idea of a complete revolution in the retrograde direction indicated by it, and we have translated the line accordingly. This verse, then, contains no hint of a libratory movement, but rather the distinct statement of a continuous eastward revolution. It should be noticed farther, although the circumstance is one of less significance, that the form in which the number of revolutions is stated, *trinçatkṛtyas*, "thirty twenties," has no parallel in the usage of this Siddhânta elsewhere.

We may also mention in this connection that Bhâskara, the great Hindu astronomer of the twelfth century, declares in his Siddhânta-Çiromaṇi (Golâdh., vi. 17) that the revolutions of the equinox are given by the Sûrya-Siddhânta as thirty in an Age (see Colebrooke, As. Res., xii. 209, etc.; Essays, ii. 374, etc., for a full discussion of this passage and its bearings); thus not only ignoring the theory of libration, but giving a very different number of revolutions from that presented by our text. As regards this latter point, however, the change of a single letter in the modern reading (substituting *trinçatkṛtvas*, "thirty times," for *trinçatkṛtyas*, "thirty twenties") would make it accord with Bhâskara's statement. We shall return again to this subject.

The number of revolutions, of whatever kind they may be, being 600 in an Age, the position at any given time of the initial point of the sphere with reference to the equinox is found by a proportion, as follows: as the number of days in an Age is to the number of revolutions in the same period, so is the given "sum of days" (see above, under i. 48–51) to the revolutions and parts of a revolution accomplished down to the given time. Thus, let us find, in illustration of the process, the amount of precession on the first of January, 1860. Since the number of years elapsed before the beginning of the present Iron Age (*kali yuga*) is divisible by 7200, it is unnecessary to make our calculation from the commencement of the present order of things: we may take the sum of days since the current Age began, which is (see above, under i. 53) 1,811,945. Hence the proportion

$$1{,}577{,}917{,}828^{d} : 600^{rev} :: 1{,}811{,}945^{d} : 0^{rev}\ 248^{\circ}\ 2'\ 8''.9$$

gives us the portion accomplished of the current revolution. Of this we are now directed (v. 10) to take the part which determines the sine (*dos*, or *bhuja*—for the origin and meaning of the phrase, see above, under ii. 29, 30). This direction determines the character of the motion as libratory. For a motion of 91°, 92°, 93°, etc., gives, by it, a precession of 89°, 88°, 87°, etc.; so that the movable point virtually returns upon its own track, and, after moving 180°, has reverted to its starting-place. So its farther motion, from 180° to 270°, gives a precession increasing from 0° to 90° in the opposite direction; and this, again, is reduced to 0° by the motion from 270° to 360°. It is as if the second and third quadrants were folded over upon the first and fourth, so that the movable point can never, in any quadrant of its motion, be more than 90° distant from the fixed equinox. Thus, in the instance taken, the *bhuja* of 248° 2′ 8″.9 is its supplement, or 68° 2′ 8″.9; the first 180° having only brought the movable point back to its original position, its present distance from that position is the excess over 180° of the arc obtained as the result of the first process. But this distance we are now farther directed to multiply by three and divide by ten: this is equivalent to reducing it to the measure of an arc of 27°, instead of 90°, as the quadrant of libration, since 3 : 10 :: 27 : 90. This being done, we find the actual distance of the initial point of the sphere from the equinox on the first of January, 1860, to be 20° 24′ 38″.67.

The question now arises, in which direction is the precession, thus ascertained, to be reckoned? And here especially is brought to light the awkwardness and insufficiency, and even the inconsistency, of the process as taught in the text. Not only have we no rule given which furnishes us the direction, along with the amount, of the precessional movement, but it would even be a fair and strictly legitimate deduction from verse 9, that that movement is taking place at the present time in an opposite direction from the actual one. We have already remarked above that the last complete period of libratory revolution closed with the close of the last Brazen Age, and the process of calculation has shown that we are now in the third quarter of a new period, and in the third quadrant of the current revolution. Therefore, if the revolution is an eastward one, as taught in the text, only taking place upon a folded circle, so as to be made libratory, the present position of the movable point, ζ Piscium, ought to be to the west of the equinox, instead of to the east, as it actually is. It was probably on account of this unfortunate flaw in the process, that no rule with regard to the direction was given, excepting the experimental one contained in verses 11 and 12, which, moreover, is not properly supplementary to the preceding rules, but rather an independent method of determining, from observation, both the direction and the amount of the precession. In verse 12, it may be remarked, in the word *âvṛtya*, "turning back," is found the only distinct intimation to be discovered in the passage of the character of the motion as libratory.

We have already above (under ii. 28) hinted our suspicions that the phenomenon of the precession was made no account of in the original composition of the Sûrya-Siddhânta, and that the notice taken of it by the treatise as it is at present is an afterthought: we will now proceed to expose the grounds of those suspicions.

It is, in the first place, upon record (see Colebrooke, As. Res., xii. 215; Essays, ii. 380, etc.) that some of the earliest Hindu astronomers were ignorant of, or ignored, the periodical motion of the equinoxes; Brahmagupta himself is mentioned among those whose systems took no account of it; it is, then, not at all impossible that the Sûrya-Siddhânta, if an ancient work, may originally have done the same. Among the positive evidences to that effect, we would first direct attention to the significant fact that, if the verses at the head of this note were expunged, there would not be found, in the whole body of the treatise besides, a single hint of the precession. Now it is not a little difficult to suppose that a phenomenon of so much consequence as this, and which enters as an element into so many astronomical processes, should, had it been borne distinctly in mind in the framing of the treatise, have been hidden away thus in a pair of verses, and unacknowledged elsewhere—no hint being given, in connection with any of the processes taught, as to whether the correction for precession is to be applied or not, and only the general directions contained in the latter half of verse 10, and ending with an "etc.," being even here presented. It has much more the aspect of an after-thought, a correction found necessary at a date subsequent to the original composition, and therefore inserted, with orders to "apply it wherever it is required." The place where the subject is introduced looks the same way: as having to do with a revolution, as entering into the calculation of mean longitudes, it should have found a place where such matters are treated of, in the first chapter; and even in the second chapter, in connection with the rule for finding the declination, it would have been better introduced than it is here. Again, in the twelfth chapter, where the orbits of the heavenly bodies are given, in terms dependent upon their times of revolution, such an orbit is assigned to the asterisms (v. 88) as implies a revolution once in sixty years: it is, indeed, very difficult to see what can have been intended by such a revolution as this; but if the doctrine respecting the revolution of the asterisms given in verse 9 of this chapter had been in the mind of the author of the twelfth chapter, he would hardly have added another and a conflicting statement respecting the same or a kindred phenomenon. It appears to us even to admit of question whether the adoption by the Hindus of the sidereal year as the unit of time does not imply a failure to recognize the fact that the equinox was variable. We should expect, at any rate, that if, at the outset, the ever-increasing discordance between the solar and the sidereal year had been fully taken into account by them, they would have more thoroughly established and defined the relations of the two, and made the precession a more conspicuous feature of their general system than they appear to have done. In the construction of their cosmical periods they have reckoned by sidereal years only, at the same time assuming (as, for instance, above, i. 13, 14) that the sidereal year is composed of the two *ayanas*, "progresses" of the sun from solstice to solstice. The supposition of an after-correction likewise seems to furnish the most satisfactory explanation of the form given to the theory of the precession. The system having been first constructed on the assumption of the equality of the tropical and sidereal years, when it began later to appear, too plainly to be disregarded, that the equinox had changed its place, the question was how to introduce the new element. Now to assign to the equinox a complete revolution would derange the whole system, acknowledging a different number of solar from sidereal years in the chronological periods; if, however, a libratory motion were assumed, the equilibrium would be maintained, since what the solar year lost in one part of the revolution of libration it would gain in another, and so the tropical and sidereal years would coincide, in number and in limits, in each great period. The circumstance which determined the limit to be assigned to the libration we conceive to have been, as suggested by Bentley (Hind. Ast., p. 132), that the earliest recorded Hindu year had been made to begin when the sun entered the asterism Kṛttikâ, or was 26° 40′ west of the point fixed upon as the commencement of the sidereal sphere for all time (see above, under i. 27), on which account it was desirable to make the arc of libration include the beginning of Kṛttikâ.

Besides these considerations, drawn from the general history of the Hindu astronomy, and the position of the element of the precession in the system of the Sûrya-Siddhânta, we have still to urge the blind and incoherent, as well as unusual, form of statement of the phenomenon, as fully exposed above. There is nothing to compare with it in this respect in any other part of the treatise, and we are unwilling to believe that in the original composition of the Siddhânta a clearer explanation, and one more consistent in its method and language with those of the treatise generally, would not have been found for the subject. We even discover evidences of more than one revision of the passage. The first half of verse 9 so distinctly teaches, if read independently of what follows it, a complete revolution of the equinoxes, that, especially when taken in connection with Bhâskara's statement, as cited above, it almost amounts to proof that the theory put forth in the Sûrya-Siddhânta was at one time that of a complete revolution. The same conclusion is not a little strengthened, farther, by the impossibility of deducing from verse 9, through the processes prescribed in the following verses, a true expression for the direction of the movement at present: we can see no reason why, if the whole passage came from the same hand, at the same time, this difficulty should not have been avoided; while it is readily explainable upon the supposition that the libratory theory of verse 10 was added as an amendment to the theory of verse 9, while at the same time the language of the latter was left as nearly unaltered as possible.

There seems, accordingly, sufficient ground for suspecting that in the Sûrya-Siddhânta, as originally constituted, no account was taken of the precession; that its recognition is a later interpolation, and was made at first in the form of a theory of complete revolution, being afterward altered to its present shape. Whether the statement of Bhâskara truly represents the earlier theory, as displayed in the Sûrya-Siddhânta of his time, we must leave an undetermined question. The very slow rate assigned by it to the movement of the equinox—only 9″ a year—throws a doubt upon the matter: but it must be borne in mind that, so far as we can see, the actual amount of the precession since about A. D. 570 (see above, under i. 27) might by that first theory have been distributed over the whole duration of the present Age, since B. C. 3102.

In his own astronomy, Bhâskara teaches the complete revolution of the equinoxes, giving the number of revolutions in an Æon (of 4,320,000,000 years) as 199,669; this makes the time of a single revolution to be 21,635.8073 years, and the yearly rate of precession 59″.9007. It is not to be supposed that he considered himself to have determined the rate with such exactness as would give precisely the odd number of 199,669 revolutions to the Æon; the number doubtless stands in some relation which we do not at present comprehend to the other elements of his astronomical system. Bhâskara's own commentators, however, reject his theory, and hold to that of a libration, which has been and is altogether the prevailing doctrine throughout India, and seems to have made its way thence into the Arabian, and even into the early European astronomy (see Colebrooke, as above).

Bentley, it may be remarked here, altogether denies (Hind. Ast., p. 130, etc.) that the libration of the equinoxes is taught in the Sûrya-Siddhânta, maintaining, with arrogant and unbecoming depreciation of those who venture to hold a different opinion, that its theory is that of a continuous revolution in an epicycle, of which the circumference is equal to 108° of the zodiac. In truth, however, Bentley's own theory derives no color of support from the text of the Siddhânta, and is besides in itself utterly untenable. It is not a little strange that he should not have perceived that, if the precession were to be explained by a revolution in an epicycle, its rate of increase would not be equable, but as the increment of the sine of the arc in the epicycle traversed by the movable point, farther varied by the varying distance at which it would be seen from the centre in different parts of the revolution; and also that, the dimensions of the epicycle being 108°, the amount of precession would never come to equal 27°, but would, when greatest, fall short of 18°, being determined by the radius of the epicycle. Bentley's whole treatment of the passage shows a thorough misapprehension of its meaning and relations: he even commits the blunder of understanding the first half of verse 9 to refer to the motion of the equinox, instead of to that of the initial point of the sidereal sphere.

Among the Greek astronomers, Hipparchus is regarded as the first who discovered the precession of the equinoxes; their rate of motion, however, seems not to have been confidently determined by him, although he pronounces it to be at any rate not less than 36″ yearly. For a thorough discussion of the subject of the precession in Greek astronomy see Delambre's History of Ancient Astronomy, ii. 247, etc. From the observations reported as the data whence Hipparchus made his discovery, Delambre deduces very nearly the true rate of the precession. Ptolemy, however, was so unfortunate as to adopt for the true rate Hipparchus's minimum, of 36″ a year: the subject is treated of by him in the seventh book of the Syntaxis. The actual motion of the equinox at the present time is 50″.25; its rate is slowly on the increase, having been, at the epoch of the Greek astronomy, somewhat less than 50″. How the Hindus succeeded in arriving at a determination of it so much more accurate than was made by the great Greek astronomer, or whether it was anything more than a lucky hit on their part, we will not attempt here to discuss.

The term by which the precession is designated in this passage is *ayanânça*, "degrees of the *ayana*." The latter word is employed in different senses: by derivation, it means simply "going, progress," and it seems to have been first introduced into the astronomical language to designate the half-revolutions of the sun, from solstice to solstice; these being called respectively (see xiv. 9) the *uttarâyana* and *dakshinâyana*, "northern progress" and "southern progress." From this use the word was transferred to denote also the solstices themselves, as we have translated it in the first half of verse 11. In the latter sense we conceive it to be employed in the compound *ayanânça;* although why the name of the precession should be derived from the solstice we are unable clearly to see. The term *krântipâtagati*, "movement of the node of declination," which is often met with in modern works on Hindu astronomy, does not occur in the Sûrya-Siddhânta.

12. . . . In like manner, the equatorial shadow which is cast at mid-day at one's place of observation

13. Upon the north and south line of the dial—that is the equinoctial shadow (*vishuvatprabhâ*) of that place. . . .

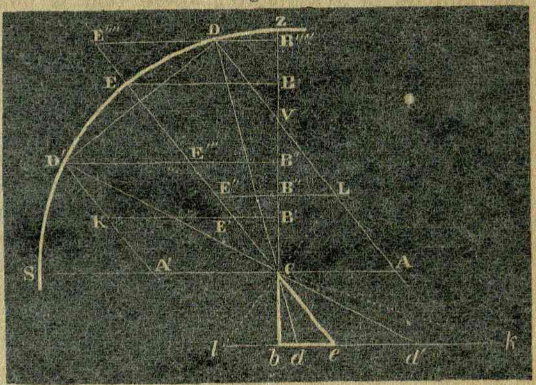
The equinoctial shadow has been already sufficiently explained, in connection with a preceding passage (above, v. 7). In this treatise it is known only by names formed by combining one of the words for shadow (*châyâ*, *bhâ*, *prabhâ*), with *vishuvat*, "equinox" (see above, under v. 6). In modern Hindu astronomy it is also called *akshabhâ*, "shadow of latitude"—i. e., which determines the latitude—and *pala-bhâ*, of which, as used in this sense, the meaning is obscure.

13. . . . Radius, multiplied respectively by gnomon and shadow, and divided by the equinoctial hypothenuse,

14. Gives the sines of co-latitude (*lamba*) and of latitude (*aksha*): the corresponding arcs are co-latitude and latitude, always south. . . .

The proportions upon which these rules are founded are illustrated by the following figure (Fig. 11), in which, as in a previous figure (Fig. 8, p. 232), Z S represents a quadrant of the meridian, Z being the zenith and S the south point, C being the centre, and E C the projection of the plane of the equator. In order to illustrate the corresponding relations of the dial, we have conceived the gnomon, C *b*, to be placed at the centre. Then, when the sun is on the meridian and in the equator, at E, the shadow cast, which is the equinoctial shadow, is *b e*, while C *e* is the corresponding hypothenuse. But, by similarity of triangles,

Fig. 11.



$$C\,e : b\,e :: C\,E : B\,E$$

and

$$C\,e : C\,b :: C\,E : C\,B$$

and as B E is the sine of E Z, which equals the latitude, and C B the sine of E S, its complement, the reduction of these proportions to the form of equations gives the rules of the text.

14. . . . The mid-day shadow is the base (*bhuja*); if radius be multiplied by that,

15. And the product divided by the corresponding hypothenuse, the result, converted to arc, is the sun's zenith-distance (*nata*), in minutes: this, when the base is south, is north, and when the base is north, is south. Of the sun's zenith-distance and his declination, in minutes,

16. Take the sum, when their direction is different—the difference, when it is the same; the result is the latitude, in minutes. From this find the sine of latitude; subtract its square from the square of radius, and the square-root of the remainder

17. Is the sine of co-latitude. . . .

This passage applies to cases in which the sun is not upon the equator, but has a certain declination, of which the amount and direction are known. Then, from the shadow cast at noon, may be derived his zenith-distance when upon the meridian, and the latitude. Thus, supposing the sun, having north declination ED (Fig. 11), to be upon the meridian, at D: the shadow of the gnomon will be *b d*, and the proportion

$$Cd : db :: CD : DB''''$$

gives DB'''', the sine of the sun's zenith-distance, ZD, which is found from it by the conversion of sine into arc by a rule previously given (ii. 33). ZD in this case being south, and ED being north, their sum, EZ, is the latitude: if, the declination being south, the sun were at D', the difference of ED' and ZD' would be EZ, the latitude. The figure does not give an illustration of north zenith-distance, being drawn for the latitude of Washington, where that is impossible. The latitude being thus ascertained, it is easy to find its sine and cosine: the only thing which deserves to be noted in the process is that, to find the cosine from the sine, resort is had to the laborious method of squares, instead of taking from the table the sine of the complementary arc, or the *koṭijyâ*.

The sun's distance from the zenith when he is upon the meridian is called *natâs*, "deflected," an adjective belonging to the noun *liptâs*, "minutes," or *bhâgâs*, *ançâs*, "degrees." The same term is also employed, as will be seen farther on (vv. 34–36), to designate the hour-angle. For zenith-distance off the meridian another term is used (see below, v. 33).

17. . . . The sine of latitude, multiplied by twelve, and divided by the sine of co-latitude, gives the equinoctial shadow. . . .

That is (Fig. 11),

$$BC : BE :: Cb : be$$

the value of the gnomon in digits being substituted in the rule for the gnomon itself.

17. . . . The difference of the latitude of the place of observation and the sun's meridian zenith-distance in degrees (*nata-bhâgâs*), if their direction be the same, or their sum,

18. If their direction be different, is the sun's declination: if the sine of this latter be multiplied by radius and divided by the sine of greatest declination, the result, converted to arc, will be the sun's longitude, if he is in the quadrant commencing with Aries;

19. If in that commencing with Cancer, subtract from a half-circle; if in that commencing with Libra, add a half-circle; if in that commencing with Capricorn, subtract from a circle: the result, in each case, is the true (*sphuṭa*) longitude of the sun at mid-day.

20. To this if the equation of the apsis (*mânda phala*) be repeatedly applied, with a contrary sign, the sun's mean longitude will be found. . . .

This passage teaches how, when the latitude of the observer is known, the sun's declination, and his true and mean longitudes, may be found by observing his zenith-distance at noon. The several parts of the process are all of them the converse of processes previously given, and require no explanation. To find the sun's declination from his meridian zenith-distance and the latitude (reckoned as south, by v. 14), the rule given above, in verses 15 and 16, is inverted; the true longitude is found from the declination by the inversion of the method taught in ii. 28, account being taken of the quadrant in which the sun may be according to the principle of ii. 30: and finally, the mean may be derived from the true longitude by a method of successive approximation, applying in reverse the equation of the centre, as calculated by ii. 39.

It is hardly necessary to remark that this is a very rough process for ascertaining the sun's longitude, and could give, especially in the hands of Hindu observers, results only distantly approaching to accuracy.

20. . . . The sum of the latitude of the place of observation and the sun's declination, if their direction is the same, or, in the contrary case, their difference,

21. Is the sun's meridian zenith-distance (*natânçâs*); of that find the base-sine (*bahujyâ*) and the perpendicular-sine (*koṭijyâ*). If, then, the base-sine and radius be multiplied respectively by the measure of the gnomon in digits,

22. And divided by the perpendicular-sine, the results are the shadow and hypothenuse at mid-day. . . .

The problem here is to determine the length of the shadow which will be cast at mid-day when the sun has a given declination, the latitude of the observer being also known. First, the sun's meridian zenith-distance is found, by a process the converse of that taught in verses 15 and 16; then, the corresponding sine and cosine having been calculated, a simple proportion gives the desired result. Thus, let us suppose the sun to be at D′ (Fig. 11, p. 250); the sum of his south declination, E D′, and the north latitude, E Z, gives the zenith-distance, Z D′: its sine (*bhujajyâ*) is D′ B‴, and its cosine (*koṭijyâ*) is C B‴. Then

$$CB''' : B'''D' :: Cb : bd'$$

and

$$CB''' : CD' :: Cb : Cd'$$

which proportions, reduced to equations, give the value of bd', the shadow, and Cd', its hypothenuse.

22. . . . The sine of declination, multiplied by the equinoctial hypothenuse, and divided by the gnomon-sine (*çankujîvâ*),

23. Gives, when farther multiplied by the hypothenuse of any given shadow, and divided by radius (*madhyakarṇa*), the sun's measure of amplitude (*arkâgrâ*) corresponding to that shadow. . .

In this passage we are taught, the declination being known, how to find the measure of amplitude (*agrâ*) of any given shadow, as preparatory to determining, by the next following rule, the base (*bhuja*) of the shadow, by calculation instead of measurement. The first step is to find the sine of the sun's amplitude: in order to this, we compare the triangles A B C and C E H (Fig. 13, p. 254), which are similar, since the angles A C B and C E H are each equal to the latitude of the observer. Hence E H : E C : : B C : A C

But the triangles C E H (Fig. 13) and C *b e* (Fig. 11) are also similar; and E H : E C : : C *b* : C *e*

Hence, by equality of ratios, C *b* : C *e* : : B C : A C

and A C, the sine of the sun's amplitude, equals B C—which is the sine of declination, being equal to D F—multiplied by C *e*, the equinoctial hypothenuse, and divided by C *b*, the gnomon. The remaining part of the process depends upon the principle which we have demonstrated above, under verse 7, that the measure of amplitude is to the hypothenuse of the shadow as the sine of amplitude to radius.

Why the gnomon is in this passage called the "gnomon-sine," it is not easy to discover. Verse 23 presents also a name for radius, *madhya-karṇa*, "half-diameter," which is not found again; nor is *karṇa* often employed in the sense of "diameter" in this treatise.

23. . . . The sum of the equinoctial shadow and the sun's measure of amplitude (*arkâgrâ*), when the sun is in the southern hemisphere, is the base, north;

24. When the sun is in the northern hemisphere, the base is found, if north, by subtracting the measure of amplitude from the equinoctial shadow; if south, by a contrary process—according to the direction of the interval between the end of the shadow and the east and west axis.

25. The mid-day base is invariably the midday shadow. . . .

We have already had occasion to notice, in connection with the first verses of this chapter, that the base (*bhuja*) of the shadow is its projection upon a north and south line, or the distance of its extremity from the east and west axis of the dial. It is that line which, as shown above (under v. 7), corresponds to and represents the perpendicular let fall from the sun upon the plane of the prime vertical. Thus, if (Fig. 11, p. 250) K, L, D′, D be different positions of the sun—K and L being conceived to be upon the surface of the sphere—the perpendiculars KB′, LB″, D′B‴, DB⁗ are represented upon the dial by *k b*, *l b*, *d′ b*, *d b*, or, in Fig. 9 (p. 241), by *kb′*, *lb″*, *d′ b*, *d b*. Of these, the two latter coincide with their respective shadows, the shadow cast at noon being always itself upon a north and south line. The base of any shadow may be found by combining its measure of amplitude (*agrâ*) with the equinoctial shadow. When the sun is in the southern hemisphere, as at D′ or K (Fig. 11), the measure of amplitude, *e d′* or *ek*, is to be added always to the equinoctial shadow, *b e*, in order to give the base, *b d′* or *b k*. If, on the contrary, the sun's declination be north, a different method of procedure will be necessary, according as he is north or south from the prime vertical. If he be south, as at D, the shadow, *b d*, will be thrown northward, and the base will be found by subtracting the measure of amplitude, *d e*, from the equinoctial shadow, *b e*: if he be north, as at L, the extremity of the shadow, *l*, will be south from the east and west axis, and the base, *b l*, will be obtained by subtracting the equinoctial shadow, *b e*, from the measure of amplitude, *l e*.

25\. . . . Multiply the sines of co-latitude and of latitude respectively by the equinoctial shadow and by twelve,

26\. And divide by the sine of declination; the results are the hypothenuse when the sun is on the prime vertical (*samamaṇḍala*). When north declination is less than the latitude, then the mid-day hypothenuse (*çrava*),

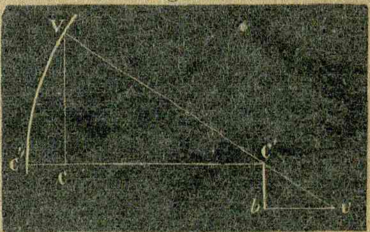
27\. Multiplied by the equinoctial shadow, and divided by the mid-day measure of amplitude (*agrâ*), is the hypothenuse. . . .

Here we have two separate and independent methods of finding the hypothenuse of the east and west shadow cast by the sun at the moment when he is upon the prime vertical. In connection with the second of the two are stated the circumstances under which alone a transit of the sun across the prime vertical will take place: if his declination is south, or if, being north, it is greater than the latitude, his diurnal revolution will be wholly to the south, or wholly to the north, of that circle.

The first method is illustrated by the following figures. Let V C″ (Fig. 12) be an arc of the prime vertical, V being the point at which the sun crosses it in his daily revolution; and let C′ be the centre; then V C′ is radius, and V C the sine of the sun's altitude; and, C′ *b* being the gnomon, *b v* will be the shadow, and C′ *v* its hypothenuse. But, by similarity of triangles,

V C : V C′ : : C′ *b* : C′ *v*

Fig. 12.

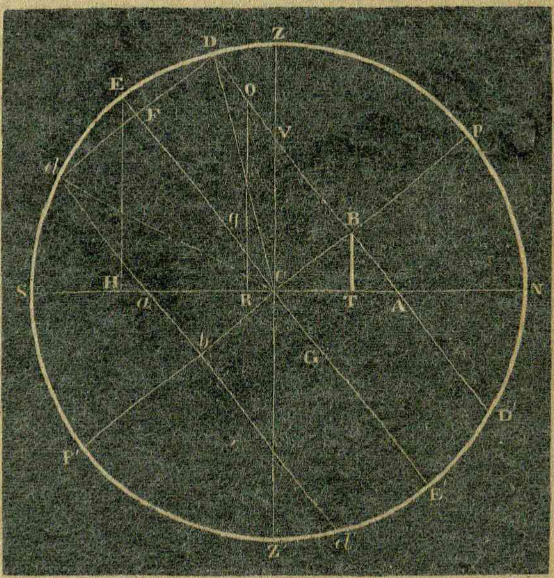


Again, in the other figure (Fig. 13)—of which the general relations are those of Fig. 8 (p. 232)—A D being the projection of the circle of the sun's diurnal revolution, and the point at which it crosses the prime vertical being seen projected in V, V C is the sine of the sun's altitude at that point. But V C B and E C H are similar triangles, the angles B V C and C E H being each equal to the latitude; hence

V C : E C : : B C : C H

Now the first of these ratios is—since E C equals V C′, both being radius—the same with the first

Fig. 13.



in the former proportion; and therefore

BC : CH : : C′b : C′v

or sin decl. : sin lat. : : gnom. : hyp. pr. vert. shad.

but sin lat. : cos. lat. : : eq. shad. : gnom.

therefore, by combining terms,

sin. decl. : cos. lat. : : eq. shad. : hyp. pr. vert. shad.

and the reduction of the first and third of these proportions to the form of equations gives the rules of the text.

The other method of finding the same quantity is an application of the principle demonstrated above, under verse 7, that, with a given declination, the measure of amplitude of any shadow is to that of any other shadow as the hypothenuse of the former to that of the latter. Now when the sun is upon the prime vertical, the shadow falls directly eastward or directly westward, and hence its extremity lies in the east and west axis of the dial, and its measure of amplitude is equal to the equinoctial shadow. The noon measure of amplitude is, accordingly, to the hypothenuse of the noon shadow as the equinoctial shadow to the hypothenuse of the shadow cast when the sun is upon the prime vertical.

27. . . . If the sine of declination of a given time be multiplied by radius and divided by the sine of co-latitude, the result is the sine of amplitude (*agramâurvikâ*).

28. And this, being farther multiplied by the hypothenuse of a given shadow at that time, and divided by radius, gives the measure of amplitude (*agrâ*), in digits (*angula*), etc. . . .

The sine of the sun's amplitude is found—his declination and the latitude being known—by a comparison of the similar triangles ABC and CEH (Fig. 13), in which

HE : EC : : BC : CA

or cos. lat. : R : : sin. decl. : sin. ampl.

And the proportion upon which is founded the rule in verse 28—namely, that radius is to the sine of amplitude as the hypothenuse of a given shadow to the corresponding measure of amplitude—has been demonstrated under verse 7, above.

28. . . . If from half the square of radius the square of the sine of amplitude (*agrajyâ*) be subtracted, and the remainder multiplied by twelve,

29. And again multiplied by twelve, and then farther divided by the square of the equinoctial shadow increased by half the square of the gnomon—the result obtained by the wise

30. Is called the "surd" (*karanî*): this let the wise man set down in two places. Then multiply the equinoctial shadow by twelve, and again by the sine of amplitude,

31. And divide as before: the result is styled the "fruit" (*phala*). Add its square to the "surd," and take the square root of their sum; this, diminished and increased by the "fruit," for the southern and northern hemispheres,

32. Is the sine of altitude (*çanku*) of the southern intermediate directions (*vidiç*); and equally, whether the sun's revolution take place to the south or to the north of the gnomon (*çanku*)—only, in the latter case, the sine of altitude is that of the northern intermediate directions.

33. The square root of the difference of the squares of that and of radius is styled the sine of zenith-distance (*dṛç.*) If, then, the sine of zenith-distance and radius be multiplied respectively by twelve, and divided by the sine of altitude,

34. The results are the shadow and hypothenuse at the angles (*koṇa*), under the given circumstances of time and place. . . .

The method taught in this passage of finding, with a given declination and latitude, the sine of the sun's altitude at the moment when he crosses the south-east and south-west vertical circles, or when the shadow of the gnomon is thrown toward the angles (*koṇa*) of the circumscribing square of the dial, is, when stated algebraically, as follows:

$$\frac{(\tfrac{1}{2}\text{R}^2 - \sin^2 \text{ampl.}) \times \text{gn.}^2}{\tfrac{1}{2}\text{gn.}^2 + \text{eq. sh.}^2} = \text{surd.}$$

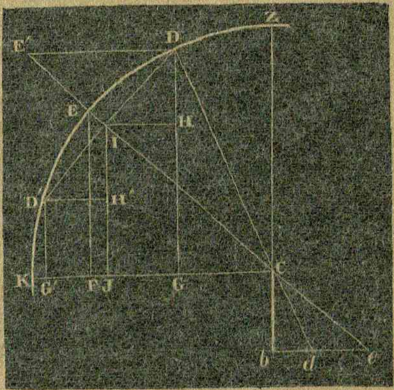
$$\frac{\text{eq. sh.} \times \text{gn.} \times \sin \text{ampl.}}{\tfrac{1}{2}\text{gn.}^2 + \text{eq. sh.}^2} = \text{fruit.}$$

$$\sqrt{\text{surd} + \text{fruit}^2} + \text{fruit} = \sin \text{alt., declination being north.}$$

$$\sqrt{\text{surd} + \text{fruit}^2} - \text{fruit} = \sin \text{alt., declination being south.}$$

It is at once apparent that a problem is here presented more complicated and difficult of solution than any with which we have heretofore had to do in the treatise. The commentary gives a demonstration of it, in which, for the first time, the notation and processes of the Hindu algebra are introduced, and with these we are not sufficiently familiar to be able to follow the course of the demonstration. The problem, however, admits of solution without the aid of mathematical knowledge of a higher character than has been displayed in the processes already explained; by means, namely, of the consideration of right-angled triangles, situated in the same plane, and capable of being represented by a single figure. We give below such a solution, which, we are persuaded, agrees in all its main features with the process by which the formulas of the text were originally deduced.

Fig. 14.



Let Z E K be the south-eastern circle of altitude, from the zenith, Z, to the horizon, K; let E be its intersection with the equator, and D the position of the sun; and let C *b* represent the gnomon.

Since *e* is in the line of the equinoctial shadow (see above, v. 7), and since *be* makes an angle of 45° with either axis of the dial, we have $be^2 = 2$ eq. sh.^{2}, and $Ce^2 = Cb^2 + be^2 =$ gn.$^2 + 2$ eq. sh.^{2}

In like manner, $de^2 = 2$ meas. ampl.^{2} But the similar triangles Cde and CDE' give $Cd^2 : de^2 :: CD^2 : DE'^2$; which, by halving the two consequents, and observing the constant relation of Cd to the measure of amplitude (see above, under v. 7), gives $R^2 : \sin \text{ampl.}^2 :: R^2 : \frac{1}{2} DE'^2$: whence $\frac{1}{2} DE'^2 = \sin \text{ampl.}^2$, or $DE'^2 = 2 \sin \text{ampl.}^2$

Now the required sine of altitude is DG, and $DG = DH + HG = DH + IJ$. And, obviously, the triangles DHI, DIE', EFC, IJC, and Cbe are all similar. Then, from DHI and Cbe, we derive

$$DH : DI :: be : Ce$$

from DIE' and Cbe, $\quad DI : DE' :: Cb : Ce$

and, by combining terms, $DH : DE' :: be \times Cb : Ce^2$

whence $\quad DH = \dfrac{\sqrt{2} \cdot \text{eq. sh.} \times \text{gn.} \times \sqrt{2} \cdot \sin \text{ampl.}}{\text{gn.}^2 + 2\,\text{eq. sh.}^2} = \dfrac{\text{eq. sh.} \times \text{gn.} \times \sin \text{ampl.}}{\frac{1}{2}\text{gn.}^2 + \text{eq. sh.}^2} = \text{fruit}.$

Again, from DHI and EFC, we derive

$$IH^2 : DI^2 :: EF^2 : EC^2$$

from IJC and EFC, $\quad IJ^2 : IC^2 :: EF^2 : EC^2$

whence, by adding the terms of the equal ratios, and observing that $IH^2 + IJ^2 = JH^2$, and $DI^2 + IC^2 = DC^2 = EC^2$, we have

$$JH^2 : EC^2 :: EF^2 : EC^2$$

or $JH^2 = EF^2$. Hence $IJ^2 = JH^2 - IH^2 = EF^2 - IH^2 = EF^2 - DI^2 + DH^2$

But from EFC and Cbe are derived

$$Ce^2 : Cb^2 :: EC^2 : EF^2$$

from DIE' and Cbe, $\quad Ce^2 : Cb^2 :: DE'^2 : DI^2$

whence $EF^2 = \dfrac{EC^2 \cdot Cb^2}{Ce^2}$, and $DI^2 = \dfrac{DE'^2 \cdot Cb^2}{Ce^2}$, and $EF^2 - DI^2 = \dfrac{(EC^2 - DE'^2)Cb^2}{Ce^2}$

that is to say,

$$EF^2 - DI^2 = \frac{(R^2 - 2 \sin \text{ampl.}^2) \times \text{gn.}^2}{\text{gn.}^2 + 2\,\text{eq. sh.}^2} = \frac{(\frac{1}{2}R^2 - \sin \text{ampl.}^2) \times \text{gn.}^2}{\frac{1}{2}\text{gn.}^2 + \text{eq. sh.}^2} = \text{surd}.$$

But, as was shown above, $IJ^2 = EF^2 - DI^2 + DH^2 = \text{surd} + \text{fruit}^2$

and $\sqrt{\text{surd} + \text{fruit}^2} + \text{fruit} = IJ + DH = DG = $ sine of altitude.

When declination is south, so that the sun crosses the circle of altitude at D', IH', the equivalent of DH, is to be subtracted from IJ, to give $D'G'$, the sine of altitude.

The correctness of the Hindu formulas may likewise be briefly and succinctly demonstrated by means of our modern methods. Thus, let PZS (Fig. 15) be a spherical triangle, of which the three angular points are P, the pole, Z, the zenith, and S, the place of the sun when upon the south-east or the south-west vertical circles; PZ, then, is the co-latitude, ZS the zenith-distance, or co-altitude, and PS the co-declination; and the angle PZS is 135°; the problem is, to find the sine of the complement of ZS, or of the sun's altitude. By spherical trigonometry, $\cos SP = \cos ZS \cos ZP + \sin ZS \sin ZP \cos Z$. Dividing by $\sin ZP$, and observing that $\cos SP \div \sin ZP = \sin \text{decl.} \div \cos \text{lat.} =$ sine of amplitude, we have $\sin \text{ampl.} = \sin \text{alt.} \tan \text{lat.} + \cos \text{alt.} \cos 135°$. If, now, we represent sin ampl. by a, tan lat. by b, cos 135° by

Fig. 15.

$-\sqrt{\frac{1}{2}}$, sin alt. by x, and cos alt. by $\sqrt{1-x^2}$, we have $a^2-2abx+b^2x^2=\frac{1}{2}(1-x^2)$; and, by reduction, $x^2-\frac{2ab}{\frac{1}{2}+b^2}x=\frac{\frac{1}{2}-a^2}{\frac{1}{2}+b^2}$. Representing, again, $\frac{ab}{\frac{1}{2}+b^2}$ by f, and $\frac{\frac{1}{2}-a^2}{\frac{1}{2}+b^2}$ by s, and reducing, we have $x=f+\sqrt{f^2+s}$. But f is evidently the same with the "fruit," since b, or tan lat., equals eq. sh. ÷ gnom., and therefore $\frac{ab}{\frac{1}{2}+b^2}=\frac{\text{eq. sh.}\times\text{gn.}\times\text{sin. ampl.}}{\frac{1}{2}\text{ gnom.}^2+\text{eq. sh.}^2}$: and s is also the same with the "surd," for $\frac{\frac{1}{2}-a^2}{\frac{1}{2}+b^2}=\frac{(\frac{1}{2}\text{R}^2-\sin^2\text{ampl.})\times\text{gn.}^2}{\frac{1}{2}\text{gnom.}^2+\text{eq. sh.}^2}$.

If, the latitude being north, we consider the north direction as positive, b will be positive. The value of f, given above, will then evidently be positive or negative as the sign of a is plus or minus. But a, the sine of amplitude, is positive when declination is north, and negative when declination is south. Hence f is to be added to or subtracted from the radical, according as the sun is north or south of the equator, as prescribed by the Hindu rule. A minus sign before the radical would correspond to a second passage of the sun across the south-east and north-west vertical circle; which, except in a high latitude, would take place always below the horizon.

The construction of the last part of verse 32 is by no means clear, yet we cannot question that the meaning intended to be conveyed by it is truly represented by our translation. When declination is greater than north latitude, the sun's revolution is made wholly to the north of the prime vertical, and the vertical circles which he crosses are the north-east and the north-west. The process prescribed in the text, however, gives the correct value for the sine of altitude in this case also. For, in the triangle S Z P (Fig. 15), all the parts remain the same, excepting that the angle P Z S becomes 45°, instead of 135°: but the cosine of the former is the same as that of the latter arc, with a difference only of sign, which disappears in the process, the cosine being squared.

The sine of altitude being found, that of its complement, or of zenith-distance, is readily derived from it by the method of squares (as above, in vv. 16, 17). To ascertain, farther, the length of the corresponding shadow and of its hypothenuse, we make the proportions

sin alt. : sin zen. dist. : : gnom. : shad.

and sin alt. : R : : gnom. : hyp. shad.

In this passage, as in those that follow, the sine of altitude is called by the same name, *çanku*, "staff," which is elsewhere given to the gnomon: the gnomon, in fact, representing in all cases, if the hypothenuse be made radius, the sine of the sun's altitude. The word is frequently used in this sense in the modern astronomical language: thus V C (Fig. 13, p. 254), the sine of the sun's altitude when upon the prime vertical, is called the *samamandalaçanku*, "prime vertical staff," and B T, the sine of altitude when the sun crosses the *unmandala*, or east and west hour-circle, is styled the *unmandalaçanku*: of the latter line, however, the Sûrya-Siddhânta makes no account. We are surprised, however, not to find a distinct name for the altitude, as for its complement, the zenith-distance: the sine of the latter might with very nearly the same propriety be called the "shadow," as that of the former the "gnomon." The particular sine of altitude which is the result of the present process is commonly known as the *koṇaçanku*, from the word *koṇa*, which, signifying originally "angle," is used, in connection with the dial, to indicate the angles of the circumscribing square (see Fig. 9, p. 241), and then the directions in which those angles lie from the gnomon. The word itself is doubtless borrowed from the Greek *γωνία*, the form given to it being that in which it appears in the compounds *τρίγωνον* (Sanskrit *trikoṇa*), etc. Lest it seem strange that the Hindus should have derived from abroad the name for so familiar and elementary a quantity as an angle, we would direct attention to the striking fact that in that stage of their mathematical science, at least, which is represented by the Sûrya-Siddhânta, they appear to have made no use whatever in their calculations of the angle: for, excepting in this passage (v. 34) and in the term for "square" employed in a previous verse (v. 5) of this chapter, no word meaning "angle" is to be met with anywhere in the text of this treatise. The term *dṛç*, used to signify "zenith-distance"—excepting when this is measured upon the meridian; see above, under vv. 14–16—means literally "sight:" in this sense, it occurs here for the first time: we have had it more than once above with the signification of "observed place," as distinguished from a position obtained by calculation. In verse 32, *çanku* might be understood as used in the sense of "zenith," yet it has there, in truth, its own proper signification of "gnomon;" the meaning being, that the sun, in the cases supposed, makes his revolution to the south or to the north of the gnomon itself, or in such a manner as to cast the shadow of the latter, at noon, northward or southward. One of the factors in the calculation is styled *karaṇî*, "surd" (see Colebrooke's Hind. Alg., p. 145), rather, apparently, as being a quantity of which the root is not required to be taken, than one of which an integral root is always impossible; or, it may be, as being the square of a line which is not, and cannot be, drawn. The term translated "fruit" (*phala*) is one of very frequent occurrence elsewhere, as denoting "quotient, result, corrective equation," etc.

The form of statement and of injunction employed in verses 29 and 30, in the phrases "the result obtained by the wise," and "let the wise man set down," etc., is so little in accordance with the style of our treatise elsewhere, while it is also frequent and familiar in other works of a kindred character, that it furnishes ground for suspicion that this passage, relating to the *koṇaçanku*, is a later interpolation into the body of the text; and the suspicion is strengthened by the fact that the process prescribed here is so much more complicated than those elsewhere presented in this chapter.

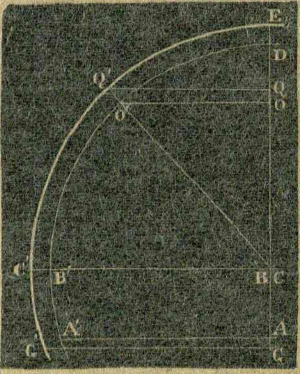
34. . . . If radius be increased by the sine of ascensional difference (*cara*) when declination is north, or diminished by the same, when declination is south,

35. The result is the day-measure (*antyâ*); this, diminished by the versed sine (*utkramajyâ*) of the hour-angle (*nata*), then multiplied by the day-radius and divided by radius, is the "divisor" (*cheda*); the latter, again, being multiplied by the sine of co-latitude (*lamba*), and divided

36. By radius, gives the sine of altitude (*çanku*): subtract its sine from that of radius, and the square root of the remainder is the sine of zenith-distance (*drç*): the shadow and its hypothenuse are found as in the preceding process.

The object of this process is, to find the sine of the sun's altitude at any given hour of the day, when his distance from the meridian, his declination, and the latitude, are known. The sun's angular distance from the meridian, or the hour-angle, is found, as explained by the commentary, by subtracting the time elapsed since sunrise, or which is to elapse before sunset, from the half day, as calculated by a rule previously given (ii. 61–63). From the declination and the latitude the sine of ascensional difference (*carajyâ*) is supposed to have been already derived, by the method taught in the same passage; as also, from the declination (by ii. 60), the radius of the diurnal circle. The successive steps of the process of calculation will be made clear by a reference to the annexed figure (Fig. 16), taken in connection with Fig. 13 (p. 254), with which it corresponds in dimensions and lettering. Let G G′ C′ E represent a portion of the plane of the equator, C being its centre, and G E its intersection with the plane of the meridian; and let A A′ B′ D represent a corresponding portion of the plane of the diurnal circle, as seen projected upon the other, its centre and its line of intersection with the meridian coinciding with those of the latter. Let C G equal the sine of ascensional difference, and A B its correspondent in the lesser circle, or the earth-sine (*kujyâ* or *kshitijyâ*; see above, ii. 61). Now let O′ be the place of the sun at a given time; the angle O′ C D, measured by the arc of the equator Q′ E, is the hour-angle: from Q′ draw Q′ Q perpendicular to C E; then Q′ Q is the sine, and Q E is the versed sine, of Q′ E. Add to radius, E C, the sine of ascensional difference, C G; their sum, E G—which is the equivalent, in terms of a great circle, of D A, that part of the diameter of the circle of diurnal revolution which is above the horizon, and which consequently measures the length of the day—is the day-measure (*antyâ*). From E G deduct E Q, the versed sine of the hour-angle; the remainder, G Q, is the same quantity in terms of a great circle which A O is in terms of the diurnal circle: hence the reduction of G Q to the dimensions of the lesser circle, by the proportion

Fig. 16.



$$CE : BD :: GQ : AO$$

gives us the value of A O; to this the text gives the technical name of "divisor" (*cheda*). But, by Fig. 13,

$$CE : EH :: AO : OR$$

hence O R, which is the sine of the sun's altitude at the given time, equals A O, the "divisor," multiplied by E H, the cosine of latitude, and divided by C E, or radius.

The processes for deriving from the sine of altitude that of zenith-distance, and from both the length of the corresponding shadow and its hypothenuse, are precisely the same as in the last problem.

For the meaning of *antyâ*—which, for lack of a better term, we have translated "day-measure"—see above, under verse 7. The word *nata*, by which the hour-angle is designated, is the same with that employed above with the signification of "meridian zenith-distance;" see the note to verses 14–17.

37. If radius be multiplied by a given shadow, and divided by the corresponding hypothenuse, the result is the sine of zenith-distance (*dṛc*): the square root of the difference between the square of that and the square of radius

38. Is the sine of altitude (*çanku*); which, multiplied by radius and divided by the sine of co-latitude (*lamba*), gives the "divisor" (*cheda*); multiply the latter by radius, and divide by the radius of the diurnal circle,

39. And the quotient is the sine of the sun's distance from the horizon (*unnata*); this, then, being subtracted from the day-measure (*antyâ*), and the remainder turned into arc by means of the table of versed sines, the final result is the hour-angle (*nata*), in respirations (*asu*), east or west.

The process taught in these verses is precisely the converse of the one described in the preceding passage. The only point which calls for farther remark in connection with it is, that the line G Q (Fig. 16) is in verse 39 called the "sine of the *unnata*." By this latter term is designated the opposite of the hour-angle (*nata*)—that is to say, the sun's angular distance from the horizon upon his own circle, O′ A′, reduced to time, or to the measure of a great circle. Thus, when the sun is at O′, his hour-angle (*nata*), or the time till noon, is Q′ E; his distance from the horizon (*unnata*), or the time since sunrise, is Q′ G′. But G Q is with no propriety styled the sine of G′ Q′; it is not itself a sine at all, and the actual sine of the arc in question would have a very different value.

40. Multiply the sine of co-latitude by any given measure of amplitude (*agrâ*), and divide by the corresponding hypothenuse in digits; the result is the sine of declination. This, again, is to be multiplied by radius, and divided by the sine of greatest declination;

41. The quotient, converted into arc, is, in signs, etc., the sun's place in the quadrant; by means of the quadrants is then found the actual longitude of the sun at that point. . . .

By the method taught in this passage, the sun's declination, and, through that, his true and mean longitude, may, the latitude of the observer being known, be found from a single observation upon the shadow at any hour in the day. The declination is obtained from the measure of amplitude and the hypothenuse of the shadow, in the following

manner: first, as was shown in connection with verse 7 of this chapter,
hyp. shad. : meas. ampl. : : E C : C A (Fig. 13, p. 254)
but E C : C A : : E H : B C
therefore hyp. shad. : meas. ampl. : : E H : B C
B C, the sine of declination, being thus ascertained, the longitude is deduced from it as in a previous process (see above, vv. 17–20).

41. . . . Upon a given day, the distances of three bases, at noon, in the forenoon, and in the afternoon, being laid off,

42. From the point of intersection of the lines drawn between them by means of two fish-figures, (*matsya*), and with a radius touching the three points, is described the path of the shadow. . .

This method of drawing upon the face of the dial the path which will be described by the extremity of the shadow upon a given day proceeds upon the assumption that that path will be an arc of a circle—an erroneous assumption, since, excepting within the polar circles, the path of the shadow is always a hyperbola, when the sun is not in the equator. In low latitudes, however, the difference between the arc of the hyperbola, at any point not too far from the gnomon, and the arc of a circle, is so small, that it is not very surprising that the Hindus should have overlooked it. The path being regarded as a true circle, of course it can be drawn if any three points in it can be found by calculation: and this is not difficult, since the rules above given furnish means of ascertaining, if the sun's declination and the observer's latitude be known, the length of the shadow and the length of its base, or the distance of its extremity from the east and west axis of the dial, at different times during the day. One part of the process, however, has not been provided for in the rules hitherto given. Thus (Fig. 9, p. 241), supposing d, m, and l to be three points in the same daily path of the shadow, we require, in order to lay down l and m, to know not only the bases $l\,b''$, $m\,b'''$, but also the distances $b\,b''$, $b\,b'''$. But these are readily found when the shadow and the base corresponding to each are known, or they may be calculated from the sines of the respective hour-angles.

The three points being determined, the mode of describing a circle through them is virtually the same with that which we should employ: lines are drawn from the noon-point to each of the others, which are then, by fish-figures (see above, under vv. 1–5), bisected by other lines at right angles to them, and the intersection of the latter is the centre of the required circle.

42. . . . Multiply by the day-radius of three signs, and divide by their own respective day-radii,

43. In succession, the sines of one, of two, and of three signs; the quotients, converted into arc, being subtracted, each from the one following, give, beginning with Aries, the times of rising (*udayâsavas*) at Lankâ;

44. Namely sixteen hundred and seventy, seventeen hundred and ninety-five, and nineteen hundred and thirty-five respirations. And these, diminished each by its portion of ascensional

difference (*carakhanda*), as calculated for a given place, are the times of rising at that place.

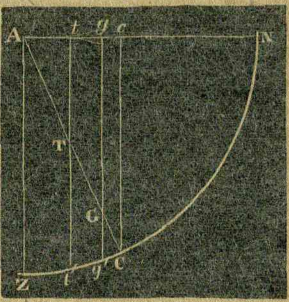
45. Invert them, and add their own portions of ascensional difference inverted, and the sums are the three signs beginning with Cancer: and these same six, in inverse order, are the other six, commencing with Libra.

The problem here is to determine the "times of rising" (*udayâsavas*) of the different signs of the ecliptic—that is to say, the part of the 5400 respirations (*asavas*) constituting a quarter of the sidereal day, which each of the three signs making up a quadrant of the ecliptic will occupy in rising (*udaya*) above the horizon. And in the first place, the times of rising at the equator, or in the right sphere—which are the equivalents of the signs in right ascension—are found as follows:

Let Z N (Fig. 17) be a quadrant of the solstitial colure, A N the projection upon its plane of the equinoctial colure, A Z of the equator, and A C of the ecliptic; and let A, T, G, and C be the projections upon A C of the initial points of the first four signs; then A T is the sine of one sign, or of 30°, A G of two signs, or of 60°, and A C, which is radius, the sine of three signs, or of 90°. From T, G, and C, draw T t, G g, C c, perpendicular to A N. Then A T t and A C c are similar triangles, and, since A C equals radius,

$$R : Cc :: AT : Tt$$

Fig. 17.



But the arc of which T t is sine, is the same part of the circle of diurnal revolution of which the radius is tt', as the required ascensional equivalent of one sign is of the equator: hence the sine of the latter, which we may call x, is found by reducing T t to the measure of a great circle, which is done by the proportion

$$tt' : R :: Tt : \sin x$$

Combining this with the preceding proportion, we have,

$$tt' : Cc :: AT : \sin x$$

Again, to find the ascensional equivalent of two signs, which we will call y, we have first, by comparison of the triangles A G g and A C c,

$$R : Cc :: AG : Gg$$

and $\quad gg' : R :: Gg : \sin y$

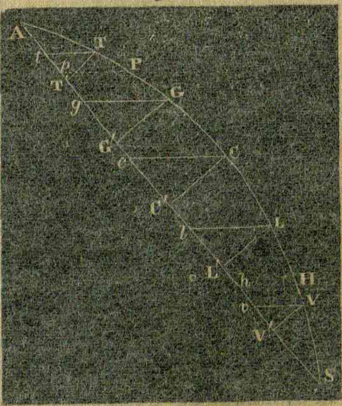
therefore, as before, $\quad gg' : Cc :: AG : \sin y$

Hence, the sines of the ascensional equivalents of one and of two signs respectively are equal to the sines of one and of two signs, A T and A G, multiplied by the day-radius of three signs, C c, and divided each by its own day-radius, tt' and gg'; and the conversion of the sines thus obtained into arc gives the ascensional equivalents themselves. The rule of the text includes also the equivalent of three signs, but this is so obviously equal to a quadrant that it is unnecessary to draw out the process, all the terms in the proportions disappearing except radius.

Upon working out the process, by means of the table of sines given in the second chapter (vv. 15–22), and assuming the inclination of the plane of the ecliptic to be 24° (ii. 28), we find, by the rule given above (ii. 60), that the day-radii of one, of two, and of three sines, or $t\,t'$, $g\,g'$, $C\,c$, are 3366′, 3216′, and 3140′ respectively, and that the sines of x and y are 1604′ and 2907′, to which the corresponding arcs are 27° 50′ and 57° 45′, or 1670′ and 3465′. The former is the ascensional equivalent of the first sign; subtracting it from the latter gives that of the second sign, which is 1795′, and subtracting 3465′ from a quadrant, 5400′, gives the equivalent of the third sign, which is 1935′—all as stated in the text.

These, then, are the periods of sidereal time which the first three signs of the ecliptic will occupy in rising above the horizon at the equator, or in passing the meridian of any latitude. It is obvious that the same quantities, in inverse order, will be the equivalents in right ascension of the three following signs also, and that the series of six equivalents thus found will belong also to the six signs of the other half of the ecliptic. In order, now, to ascertain the equivalents of the signs in oblique ascension, or the periods of sidereal time which they will occupy in rising above the horizon of a given latitude, it is necessary first to calculate, for that latitude, the ascensional difference (*cara*) of the three points T, G, and C (Fig. 17), which is done by the rule given in the last chapter (vv. 61, 62). We have calculated these quantities, in the Hindu method, for the latitude of Washington, 38° 54′, and find the ascensional difference of T to be 578′, that of G 1061′, and that of C 1263′. The manner in which these are combined with the equivalents in right ascension to produce the equivalents in oblique ascension may be explained by the following figure (Fig. 18), which, although not a true projection, is sufficient for the purpose of illustration. Let A C S be a semicircle of the ecliptic, divided into its successive signs, and A S a semicircle of the equator, upon which A T′, T′ G′, etc., are the equivalents of those signs in right ascension; and let t, g, etc., be the points which rise simultaneously with T, G, etc. Then t T′ and v V′, the ascensional difference of T and V, are 578′, g G′ and l L′ are 1061′, and c C′ is 1263′. Then A t, the equivalent in oblique ascension of A T, equals A T′ − t T′, or 1092′. To find, again, the value of $t\,g$, the second equivalent, the text directs to subtract from T′ G′ the difference between t T′ and g G′, which is called the *carakhanda*, "portion of ascensional difference"—that is to say, the increment or decrement of ascensional difference at the point G as compared with T. Thus

Fig. 18.



$$tg = T'G' - (gG' - tT') = T'G' + tT' - gG' = tG' - gG' = 1312'$$

and

$$gc = G'C' - (cC' - gG') = G'C' + gG' - cC' = gC' - cC' = 1733'$$

Farther, to find the oblique equivalents in the second quadrant, we are directed to invert the right equivalents, and to add to each its own *carakhaṇḍa*, decrement of ascensional difference. Thus

$$cl = C'L' + (cC' - lL') = cL' - lL' = 2137'$$
$$lv = L'V' + (lL' - vV') = lV' - vV' = 2278'$$

and finally, $$vS = V'S + vV' = 2248'.$$

It is obvious without particular explanation that the arcs of oblique ascension thus found as the equivalents, in a given latitude, of the first six signs of the ecliptic, are likewise, in inverse order, the equivalents of the other six. We have, then, the following table of times of rising (*udayâsavas*), for the equator and for the latitude of Washington, of all the divisions of the ecliptic:

Equivalents in Right and Oblique Ascension of the Signs of the Ecliptic.

Sign.		Equivalent in Right Ascension.	Lat. of Washington.		Sign.	
No.	Name.		Ascens. Diff.	Equiv. in Obl. Ascension.	Name.	No.
		′ or p.	′ or p.	′ or p.		
1.	Aries, *mesha*,	1670	578	1092	Pisces, *mîna*,	12.
2.	Taurus, *vṛshan*,	1795	1061	1312	Aquarius, *kumbha*,	11.
3.	Gemini, *mithuna*,	1935	1263	1733	Capricornus, *makara*,	10.
4.	Cancer, *karkaṭa*,	1935	1061	2137	Sagittarius, *dhanus*,	9.
5.	Leo, *sinha*,	1795	578	2278	Scorpio, *âli*,	8.
6.	Virgo, *kanyâ*,	1670		2248	Libra, *tulâ*,	7.

For the expression "at Lankâ," employed in verse 43 to designate the equator, see above, under i. 62.

46. From the longitude of the sun at a given time are to be calculated the ascensional equivalents of the parts past and to come of the sign in which he is: they are equal to the number of degrees traversed and to be traversed, multiplied by the ascensional equivalent (*udayâsavas*) of the sign, and divided by thirty;

47. Then, from the given time, reduced to respirations, subtract the equivalent, in respirations, of the part of the sign to come, and also the ascensional equivalents (*lagnâsavas*) of the following signs, in succession—so likewise, subtract the equivalents of the part past, and of the signs past, in inverse order;

48. If there be a remainder, multiply it by thirty and divide by the equivalent of the unsubtracted sign; add the quotient, in degrees, to the whole signs, or subtract it from them: the result is the point of the ecliptic (*lagna*) which is at that time upon the horizon (*kshitija*).

49. So, from the east or west hour-angle (*nata*) of the sun, in nâḍîs, having made a similar calculation, by means of the equivalents in right ascension (*lankodayâsavas*), apply the result as an additive or subtractive equation to the sun's longitude: the result is the point of the ecliptic then upon the meridian (*madhyalagna*).

The word *lagna* means literally "attached to, connected with," and hence, "corresponding, equivalent to." It is, then, most properly, and likewise most usually, employed to designate the point or the arc of the equator which corresponds to a given point or arc of the ecliptic. In such a sense it occurs in this passage, in verse 47, where *lagnâsavas* is precisely equivalent to *udayâsavas*, explained in connection with the next preceding passage; also below, in verse 50, and in several other places. In verses 48 and 49, however, it receives a different signification, being taken to indicate the point of the ecliptic which, at a given time, is upon the meridian or at the horizon; the former being called *lagnam kshitije*, "*lagna* at the horizon"—or, in one or two cases elsewhere, simply *lagna*—the other receiving the name of *madhyalagna*, "meridian-*lagna*."

The rules by which, the sun's longitude and the hour of the day being known, the points of the ecliptic at the horizon and upon the meridian are found, are very elliptically and obscurely stated in the text; our translation itself has been necessarily made in part also a paraphrase and explication of them. Their farther illustration may be best effected by means of an example, with reference to the last figure (Fig. 18).

At a given place of observation, as Washington, let the moment of local time—reckoned in the usual Hindu manner, from sunrise—be 18^n 12^v 3^p, and let the longitude of the sun, as corrected by the precession, be, by calculation, 42°, or 1^s 12°: it is required to know the longitude of the point of the ecliptic (*lagna*) then upon the eastern horizon.

Let P (Fig. 18) be the place of the sun, and H *h* the line of the horizon, at the given time; and let *p* be the point of the equator which rose with the sun; then the arc *p h* is equivalent to the time since sunrise, 18^n 12^v 3^p, or 6555^p. The value of *t g*, the equivalent in oblique ascension of the second sign T G, in which the sun is, is given in the table presented at the end of the note upon the preceding passage as 1312^p. To find the value of the part of it *p g* we make the proportion

$$TG : PG :: tg : pg$$

or

$$30° : 18° :: 1312^p : 787^p$$

From *p h*, or 6555^p, we now subtract *p g*, 787^p, and then, in succession, the ascensional equivalents of the following signs, G C and C L—that is, *g c*, or 1733^p, and *c l*, or 2137^p—until there is left a remainder, *l h*, or 1898^p, which is less than the equivalent of the next sign. To this remainder of oblique ascension the corresponding arc of longitude is then found by a proportion the reverse of that formerly made, namely

$$lv : lh :: LV : LH$$

or

$$2278^p : 1898^p :: 30° : 25°$$

The result thus obtained being added to A L, or 4^s, the sum, 4^s 25°, or 145°, is the longitude of H.

The arc *p g* is called in the text *bhogyâsavas*, "the equivalent in respirations of the part of the sign to be traversed," while *t p* is styled *bhuktâsavas*, "the respirations of the part traversed."

If, on the other hand, it were desired to arrive at the same result by reckoning in the opposite direction from the sun to the horizon, either on account of the greater proximity of the two in that direction, or for any other reason, the manner of proceeding would be somewhat different. Thus, if A H (Fig. 18) were the sun's longitude, and *p* P the line of the eastern horizon, we should first find *h p*, by subtracting the part of the day already elapsed from the calculated length of the day (this step is, in the text, omitted to be specified); from it we should then subtract the *bhuktâsavas*, *l h*, and then the equivalents of the signs through which the sun has already passed, in inverse order, until there remained only the part of an equivalent, *p g*, which would be converted into the corresponding arc of longitude, P G, in the same manner as before: and the subtraction of P G from A G would give A P, the longitude of the point P.

But again, if it be required to determine the point of the ecliptic which is at any given time upon the meridian, the general process is the same as already explained, excepting that for the time from sunrise is substituted the time until or since noon, and also for the equivalents in oblique ascension those in right ascension, or, in the language of the text, the "times of rising at Lankâ" (*lankodâyasavas*); since the meridian, like the equatorial horizon, cuts the equator at right angles.

It will be observed that all these calculations assume the increments longitude of to be proportional to those of ascension throughout each sign: in a process of greater pretensions to accuracy, this would lead to errors of some consequence.

The use and value of the methods here taught, and of the quantities found as their results, will appear in the sequel (see ch. v. 1–9; vii. 7; ix. 5–11; x. 2).

The term *kshitija*, by which the horizon is designated, may be understood, according to the meaning attributed to *kshiti* (see above, under ii. 61–63), either as the "circle of situation"—that is, the one which is dependent upon the situation of the observer, varying with every change of place on his part—or as the "earth-circle," the one produced by the intervention of the earth below the observer, or drawn by the earth upon the sky. Probably the latter is its true interpretation.

50. Add together the ascensional equivalents, in respirations, of the part of the sign to be traversed by the point having less longitude, of the part traversed by that having greater longitude, and of the intervening signs—thus is made the ascertainment of time (*kâlasâdhana*).

51. When the longitude of the point of the ecliptic upon the horizon (*lagna*) is less than that of the sun, the time is in the latter part of the night; when greater, it is in the day-time; when greater than the longitude of the sun increased by half a revolution, it is after sunset.

The process taught in these verses is, in a manner, the converse of that which is explained in the preceding passage, its object being to find the instant of local time when a given point of the ecliptic will be upon the horizon, the longitude of the sun being also known. Thus (Fig. 18), supposing the sun's longitude, A P, to be, at a given time, $1^s\ 12°$; it is required to know at what time the point H, of which the longitude is

4^s 25°, will rise. The problem, is, virtually, to ascertain the arc of the equator intercepted between *p*, the point which rose with the sun, and *h*, which will rise with H, since that arc determines the time elapsed between sunrise and the rise of H, or the time in the day at which the latter will take place. In order to this, we ascertain, by a process similar to that illustrated in connection with the last passage, the *bhogyâsavas*, "ascensional equivalent of the part of the sign to be traversed," of the point having less longitude—or *p g*—and the *bhuktâsavas*, "ascensional equivalent of the part traversed," belonging to H, the point having greater longitude—or *l h*—and add the sum of both to that of the ascensional equivalents of the intervening whole signs, *g c* and *c l*, which the text calls *antaralagnâsavas*, "equivalent respirations of the interval;" the total is, in respirations of time, corresponding to minutes of arc, the interval of time required: it will be found to be 6555^p, or 18^n 12^v 3^p: and this, in the case assumed, is the time in the day at which the rise of H takes place: were H, on the other hand, the position of the sun, 18^n 12^v 3^p would be the time before sunrise of the same event, and would require to be subtracted from the calculated length of the day to give the instant of local time.

It is evident that the main use of this process must be to determine the hour at which a given planet, or a star of which the longitude is known, will pass the horizon, or at which its "day" (see above, ii. 59–63) will commence. A like method—substituting only the equivalents in right for those in oblique ascension—might be employed in determining at what instant of local time the complete day, *ahorâtra*, of any of the heavenly bodies, reckoned from transit to transit across the lower meridian, would commence: and this is perhaps to be regarded as included also in the terms of verse 50; even though the following verse plainly has reference to the time of rising, and the word *lagna*, as used in it, means only the point upon the horizon.

The last verse we take to be simply an obvious and convenient rule for determining at a glance in which part of the civil day will take place the rising of any given point of the ecliptic, or of a planet occupying that point. If the longitude of a planet be less than that of the sun, while at the same time they are not more than three signs apart—this and the other corresponding restrictions in point of distance are plainly implied in the different specifications of the verse as compared with one another, and are accordingly explicitly stated by the commentator—the hour when that planet comes to assume the position called in the text *lagna*, or to pass the eastern horizon, will evidently be between midnight and sunrise, or in the after part (*çesha*, literally "remainder") of the night: if, again, it be more than three and less than six signs behind the sun, or, which is the same thing, more than six and less than nine signs in advance of him, its time of rising will be between sunset and midnight: if, once more, it be in advance of the sun by less than six signs, it will rise while the sun is above the horizon.

The next three chapters treat of the eclipses of the sun and moon, the fourth being devoted to lunar eclipses, and the fifth to solar, and the sixth containing directions for projecting an eclipse.

The Journal of the American Oriental Society is published, at present, in annual numbers of about 250 pages each, of which two constitute a Volume. The Second Number of this Volume, which is expected to appear early in 1860, will contain, besides other Articles, the continuation and completion of the Translation of the Sûrya-Siddhânta, commenced in this Number, with Additional Notes, Indexes, etc.; also, the usual Miscellanies. Of the Sûrya-Siddhânta a small separate edition is struck off, which will be for sale by the Society's Agents when the work is completed.

Each Number of the Journal is forwarded, as soon as published, to all the Corporate Members of the Society. It is also sent, by the Corresponding Secretary, to such of the Corresponding Members as have aided the Society by contributions, correspondence, donations to the Library or Cabinet, etc.

Twenty copies of each Article published will be forwarded to the author. A larger number will be furnished at cost.

A number of fonts of oriental type are at the Society's command, for use in the printing of the Journal, and others will be procured from time to time, as they are needed.

CONTENTS.

CHAPTER IV.

OF ECLIPSES, AND ESPECIALLY OF LUNAR ECLIPSES.

CONTENTS:—1, dimensions of the sun and moon; 2–3, measurement of their apparent dimensions; 4–5, measurement of the earth's shadow; 6, conditions of the occurrence of an eclipse; 7–8, ascertainment of longitude at the time of conjunction or of opposition; 9, causes of eclipses; 10–11, to determine whether there will be an eclipse, and the amount of obscuration; 12–15, to find half the time of duration of the eclipse, and half that of total obscuration; 16–17, to ascertain the times of contact and of separation, and, in a total eclipse, of immersion and emergence; 18–21, to determine the amount of obscuration at a given time; 22–23, to find the time corresponding to a given amount of obscuration; 24–25, measurement of the deflection of the ecliptic, at the point occupied by the eclipsed body, from an east and west line; 26, correction of the scale of projection for difference of altitude.

1. The diameter of the sun's disk is six thousand five hundred yojanas; of the moon's, four hundred and eighty.

We shall see, in connection with the next passage, that the diameters of the sun and moon, as thus stated, are subject to a curious modification, dependent upon and representing the greater or less distance of those bodies from the earth; so that, in a certain sense, we have here only their mean diameters. These represent, however, in the Hindu theory—which affects to reject the supposition of other orbits than such as are circular, and described at equal distances about the earth—the true absolute dimensions of the sun and moon.

Of the two, only that for the moon is obtained by a legitimate process, or presents any near approximation to the truth. The diameter of the earth being, as stated above (i. 59), 1600 yojanas, that of the moon, 480 yojanas, is .3 of it: while the true value of the moon's diameter in terms of the earth's is .2716, or only about a tenth less. An estimate so nearly correct supposes, of course, an equally correct determination of the moon's horizontal parallax, distance from the earth, and mean apparent diameter. The Hindu valuation of the parallax may be deduced from the value given just below (v. 3), of a minute on the moon's orbit, as 15 yojanas. Since the moon's horizontal parallax is equal to the angle subtended at her centre by the earth's radius, and since, at the moon's mean distance, 1′ of arc equals 15 yojanas, and the earth's radius, 800 yojanas, would accordingly subtend an angle of 53′ 20″—the latter angle, 53′ 20″, is, according to the system of the Sûrya-Siddhânta, the moon's parallax, when in the horizon and at her mean distance. This is considerably less than the actual value of the quantity, as determined by modern science, namely 57″ 1′; and it is practically, in the calculation of solar eclipses, still farther lessened by 3′ 51″, the excess of the value assigned to the sun's horizontal parallax, as we shall see farther on. Of the variation in the parallax, due to the varying distance of the moon, the Hindu system makes no account: the variation is actually nearly 8′, being from 53′ 48″, at the apogee, to 61′ 24″, at the perigee.

How the amount of the parallax was determined by the Hindus—if, indeed, they had the instruments and the skill in observation requisite for making themselves an independent determination of it—we are not informed. It is not to be supposed, however, that an actual estimate of the mean horizontal parallax as precisely 53′ 20″ lies at the foundation of the other elements which seem to rest upon it; for, in the making up of the artificial Hindu system, all these elements have been modified and adapted to one another in such a manner as to produce certain whole numbers as their results, and so to be of more convenient use.

From this parallax the moon's distance may be deduced by the proportion

sin 53′ 20″ : R : : earth's rad. : moon's dist.

or 53′⅓ : 3438′ : : 800y : 51,570y

The radius of the moon's orbit, then, is 51,570 yojanas, or, in terms of the earth's radius, 64.47. The true value of the moon's mean distance is 59.96 radii of the earth.

The farther proportion

3438′ : 5400′ : : 57,570y : 81,000y

would give, as the value of a quadrant of the moon's orbit, 81,000 yojanas, and, as the whole orbit, 324,000 yojanas. This is, in fact, the circumference of the orbit assumed by the system, and stated in another place (xii. 85). Since, however, the moon's distance is nowhere assumed as an element in any of the processes of the system, and is even directed (xii. 84) to be found from the circumference of the orbit by the false ratio of 1 : $\sqrt{10}$, it is probable that it was also made no account of in constructing the system, and that the relations of the moon's parallax and orbit were fixed by some such proportion as

53′ 20″ : 360° : : 800y : 324,000y

The moon's orbit being 324,000 yojanas, the assignment of 480 yojanas as her diameter implies a determination of her apparent diameter at her mean distance as 32′; since

360° : 32′ : : 324,000y : 480y

The moon's mean apparent diameter is actually 31′ 7″.

In order to understand, farther, how the dimensions of the sun's orbit and of the sun himself are determined by the Hindus, we have to notice that, the moon's orbit being 324,000 yojanas, and her time of sidereal revolution $27^d.32167416$, the amount of her mean daily motion is $11,858^y.717$. The Hindu system now assumes that this is the precise amount of the actual mean daily motion, in space, of all the planets, and ascertains the dimensions of their several orbits by multiplying it by the periodic time of revolution of each (see below, xii. 80–90). The length of the sidereal year being $365^d.25875648$, the sun's orbit is, as stated elsewhere (xii. 86), 4,331,500 yojanas. From a quadrant of this, by the ratio 5400′ : 3438′, we derive the sun's distance from the earth, 689,430 yojanas, or 861.8 radii of the earth. This is vastly less than his true distance, which is about 24,000 radii. His horizontal parallax is, of course, proportionally over-estimated, being made to be nearly 4′ (more exactly, 3′ 59″.4), instead of 8″.6, its true value, an amount so small that it should properly have been neglected as inappreciable.

It is an important property of the parallaxes of the sun and moon, resulting from the manner in which the relative distances of the latter from the earth are determined, that they are to one another as the mean daily motions of the planets respectively: that is to say,

$$53' \, 20'' : 3' \, 59'' :: 790' \, 35'' : 59' \, 8''$$

Each is likewise very nearly one fifteenth of the whole mean daily motion, or equivalent to the amount of arc traversed by each planet in 4 nâḍîs; the difference being, for the moon, about 38″, for the sun, about 3″. We shall see that, in the calculations of the next chapter, these differences are neglected, and the parallax taken as equal, in each case, to the mean motion during 4 nâḍîs.

The circumference of the sun's orbit being 4,331,500 yojanas, the assignment of 6500 yojanas as his diameter implies that his mean apparent diameter was considered to be 32′ 24″.8; for

$$360^{\circ} : 32' \, 24''.8 :: 4{,}331{,}500^{y} : 6500^{y}$$

The true value of the sun's apparent diameter at his mean distance is 32′ 3″.6.

The results arrived at by the Greek astronomers relative to the parallax, distance, and magnitude of the sun and moon are not greatly discordant with those here presented. Hipparchus found the moon's horizontal parallax to be 57′: Aristarchus had previously, by observation upon the angular distance of the sun and moon when the latter is half-illuminated, made their relative distances to be as 19 to 1; this gave Hipparchus 3′ as the sun's parallax. Ptolemy makes the mean distances of the sun and moon from the earth equal to 1210 and 59 radii of the earth, and their parallaxes 2′ 51″ and 58′ 14″ respectively: he also states the diameter of the moon, earth, and sun to be as 1, $3\frac{2}{5}$, $18\frac{4}{5}$, while the Hindus make them as 1, $3\frac{1}{3}$, and $18\frac{1}{2}\frac{3}{4}$, and their true values, as determined by modern science, are as 1, $3\frac{2}{3}$, and $412\frac{3}{4}$, nearly.

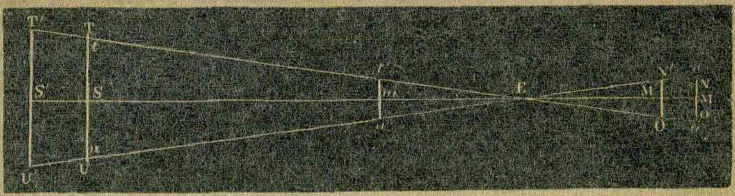
2. These diameters, each multiplied by the true motion, and divided by the mean motion, of its own planet, give the corrected (*sphuṭa*) diameters. If that of the sun be multiplied by the number of the sun's revolutions in an Age, and divided by that of the moon's,

3. Or if it be multiplied by the moon's orbit (*kakshâ*), and divided by the sun's orbit, the result will be its diameter upon the moon's orbit: all these, divided by fifteen, give the measures of the diameters in minutes.

The absolute values of the diameters of the sun and moon being stated in yojanas, it is required to find their apparent values, in minutes of arc. In order to this, they are projected upon the moon's orbit, or upon a circle described about the earth at the moon's mean distance, of which circle—since $324{,}000 \div 21{,}600 = 15$—one minute is equivalent to fifteen yojanas.

The method of the process will be made clear by the annexed figure (Fig. 19). Let E be the earth's place, E M or E *m* the mean distance of

Fig. 19.



the moon, and E S the mean distance of the sun. Let T U equal the sun's diameter, 6500^y. But now let the sun be at the greater distance E S′: the part of his mean orbit which his disk will cover will no longer be T U, but a less quantity, *t u*, and *t u* will be to T U, or T′ U′, as E S to E S′. But the text is not willing to acknowledge here, any more than in the second chapter, an actual inequality in the distance of the sun from the earth at different times, even though that inequality be most unequivocally implied in the processes it prescribes: so, instead of calculating E S′ as well as E S, which the method of epicycles affords full facilities for doing, it substitutes, for the ratio of E S to E S′, the inverse ratio of the daily motion at the mean distance E S to that at the true distance E S′. The ratios, however, are not precisely equal. The arc *a m* (Fig. 4, p. 211) of the eccentric circle is supposed to be traversed by the sun or moon with a uniform velocity. If, then, the motion at any given point, as *m*, were perpendicular to E *m*, the apparent motion would be inversely as the distance. But the motion at *m* is perpendicular to *e m* instead of E *m*. The resulting error, it is true, and especially in the case of the sun, is not very great. It may be added that the eccentric circle which best represents the apparent motions of the sun and moon in their elliptic orbits, gives much more imperfectly the distances and apparent diameters of those bodies. The value of *t u*, however, being thus at least approximately determined, *t′ u′*, the arc of the moon's mean orbit subtended by it, is then found by the proportion E S : E *m* (or E M) : : *t u* : *t′ u′*—excepting that here, again, for the ratio of the distances, E S and E M, is substituted either that of the whole circumferences of which they are respectively the radii, or the inverse ratio of the number of revolutions in a given time of the two planets, which, as shown in the note to the preceding passage, is the same thing. Having thus ascertained the value of *t′ u′* in yojanas, division by 15 gives us the number of minutes in the arc *t′ u′*, or in the angle *t′* E *u′*.

In like manner, if the moon be at less than her mean distance from the earth, as E M′, she will subtend an arc of her mean orbit *n o*, greater than N O, her true diameter; the value of *n o*, in yojanas and in minutes, is found by a method precisely similar to that already described.

There is hardly in the whole treatise a more curious instance than this of the mingling together of true theory and false assumption in the same process, and of the concealment of the real character of a process by substituting other and equivalent data for its true elements.

We meet for the first time, in this passage, the term employed in the treatise to designate a planetary orbit, namely *kakshâ*, literally "border, girdle, periphery." The value finally obtained for the apparent diameter of the sun or moon, as later of the shadow, is styled its *mâna*, "measure."

In order to furnish a practical illustration of the processes taught in this chapter, we have calculated in full, by the methods and elements of the Sûrya-Siddhânta, the lunar eclipse of Feb. 6th, 1860. Rather, however, than present the calculation piecemeal, and with its different processes severed from their natural connection, and arranged under the passages to which they severally belong, we have preferred to give it entire in the Appendix, whither the reader is referred for it.

4. Multiply the earth's diameter by the true daily motion of the moon, and divide by her mean motion: the result is the earth's corrected diameter (*sûcî*). The difference between the earth's diameter and the corrected diameter of the sun

5. Is to be multiplied by the moon's mean diameter, and divided by the sun's mean diameter: subtract the result from the earth's corrected diameter (*sûcî*), and the remainder is the diameter of the shadow; which is reduced to minutes as before.

The method employed in this process for finding the diameter of the earth's shadow upon the moon's mean orbit may be explained by the aid of the following figure (Fig. 20).

As in the last figure, let E represent the earth's place, S and M points in the mean orbits of the sun and moon, and M′ the moon's actual place. Let tu be the sun's corrected diameter, or the part of his mean orbit which his disk at its actual distance covers, ascertained as directed in the preceding passage, and let F G be the earth's diameter. Through

Fig. 20.

F and G draw vFf and wGg parallel to S M, and also tFh and uGk: then hk will be the diameter of the shadow where the moon actually enters it. The value of hk evidently equals fg (or F G) $-(fh+gk)$; and the value of $fh+gk$ may be found by the proportion

$$Fv \text{ (or ES)} : tv+wu \text{ (or } tu-\text{FG)} :: Ff \text{ (or EM}') : fh+gk$$

But the Hindu system provides no method of measuring the angular value of quantities at the distance E M′, nor does it ascertain the value of E M′ itself: and as, in the last process, the diameter of the moon was reduced, for measurement, to its value at the distance E M′, so, to be made commensurate with it, all the data of this process must be similarly modified. That is to say, the proportion

$$EM' : EM :: fg : f'g'$$

—substituting, as before, the ratio of the moon's mean to her true motion for that of E M′ to E M—gives $f'g'$, which the text calls the *sûcî*: the word means literally "needle, pyramid; we do not see precisely how it comes to be employed to designate the quantity $f'g'$, and have translated it, for lack of a better term, and in analogy with the language of the text respecting the diameters of the sun and moon, "corrected diameter of the earth." It is also evident that

$$EM' : fh+gk :: EM : f'h'+g'k'$$

hence, substituting the latter of these ratios for the former in our first proportion, and inverting the middle terms, we have

$$ES : EM :: tu-FG : f'h'+g'k'$$

Once more, now, we have a substitution of ratios, E S : E M being replaced by the ratio of the sun's mean diameter to that of the moon. In this there is a slight inaccuracy. The substitution proceeds upon the assumption that the mean apparent values of the diameters of the sun and moon are precisely equal, in which case, of course, their absolute diameters would be as their distances; but we have seen, in the note to the first verse of this chapter, that the moon's mean angular diameter is made a little less than the sun's, the former being 32′, the latter 32′ 24″.8. The error is evidently neglected as being too small to impair sensibly the correctness of the result obtained: it is not easy to see, however, why we do not have the ratio of the mean distances represented here, as in verses 2 and 3, by that of the orbits, or by that of the revolutions in an Age taken inversely. The substitution being made, we have the final proportion on which the rule in the text is based, viz., the sun's mean diameter is to the moon's mean diameter as the excess of the sun's corrected diameter over the actual diameter of the earth is to a quantity which, being subtracted from the *sûcî*, or corrected diameter of the earth, leaves as a remainder the diameter of the shadow as projected upon the moon's mean orbit: it is expressed in yojanas, but is reduced to minutes, as before, by dividing by fifteen. The earth's penumbra is not taken into account in the Hindu process of calculation of an eclipse.

The lines fg, $f'g'$, etc., are treated here as if they were straight lines, instead of arcs of the moon's orbit: but the inaccuracy never comes to be of any account practically, since the value of these lines always falls inside of the limits within which the Hindu methods of calculation recognize no difference between an arc and its sine.

6. The earth's shadow is distant half the signs from the sun: when the longitude of the moon's node is the same with that of the shadow, or with that of the sun, or when it is a few degrees greater or less, there will be an eclipse.

To the specifications of this verse we need to add, of course, "at the time of conjunction or of opposition."

It will be noticed that no attempt is made here to define the lunar and solar ecliptic limits, or the distances from the moon's node within which eclipses are possible. Those limits are, for the moon, nearly 12°; for the sun, more than 17°.

The word used to designate "eclipse," *grahana*, means literally "seizure": it, with other kindred terms, to be noticed later, exhibits the influence of the primitive theory of eclipses, as seizures of the heavenly bodies by the monster Râhu. In verses 17 and 19, below, instead of *grahaṇa* we have *graha*, another derivative from the same root *grah* or *grabh*, "grasp, seize." Elsewhere *graha* never occurs except as signifying "planet," and it is the only word which the Sûrya-Siddhânta employs with that signification: as so used, it is an active instead of a passive derivative, meaning "seizer," and its application to the planets is due to the astrological conception of them, as powers which "lay hold upon" the fates of men with their supernatural influences.

7. The longitudes of the sun and moon, at the moment of the end of the day of new moon (*amâvâsyâ*), are equal, in signs, etc.; at the end of the day of full moon (*paurṇamâsî*) they are equal in degrees, etc., at a distance of half the signs.

8. When diminished or increased by the proper equation of motion for the time, past or to come, of opposition or conjunction, they are made to agree, to minutes: the place of the node at the same time is treated in the contrary manner.

The very general directions and explanations contained in verses 6, 7, and 9 seem out of place here in the middle of the chapter, and would have more properly constituted its introduction. The process prescribed in verse 8, also, which has for its object the determination of the longitudes of the sun, moon, and moon's node, at the moment of opposition or conjunction, ought no less, it would appear, to precede the ascertainment of the true motions, and of the measures of the disks and shadow, already explained. Verse 8, indeed, by the lack of connection in which it stands, and by the obscurity of its language, furnishes a striking instance of the want of precision and intelligibility so often characteristic of the treatise. The subject of the verse, which requires to be supplied, is, "the longitudes of the sun and moon at the instant of midnight next preceding or following the given opposition or conjunction"; that being the time for which the true longitudes and motions are first calculated, in order to test the question of the probability of an eclipse. If there appears to be such a probability, the next step is to ascertain the interval between midnight and the moment of opposition or conjunction, past or to come: this is done by the method taught in ii. 66, or by some other analogous process: the instant of the occurrence of opposition or conjunction, in local time, counted from sunrise of the place of observation, must also be determined, by ascertaining the interval between mean and apparent midnight (ii. 46), the length of the complete day (ii. 59), and of its parts (ii. 60–63), etc.; the whole process is sufficiently illustrated by the two examples of the calculation of eclipses given in the Appendix. When we have thus found the interval between midnight and the moment of opposition or conjunction, verse 8 teaches us how to ascertain the true longitudes for that moment: it is by calculating—in the manner taught in i. 67, but with the true daily motions—the amount of motion of the sun, moon, and node during the interval, and applying it as a corrective equation to the longitude of each at midnight, subtracting in the case of the sun and moon, and adding in the case of the node, if the moment was then already past; and the contrary, if it was still to come. Then, if the process has been correctly performed, the longitudes of the sun and moon will be found to correspond, in the manner required by verse 7.

For the days of new and full moon, and their appellations, see the note to ii. 66, above. The technical expression employed here, as in one or two other passages, to designate the "moment of opposition or conjunction" is *parvanâdyas*, "nâḍis of the *parvan*," or "time of the *parvan* in nâḍis, etc.:" *parvan* means literally "knob, joint," and is frequently applied, as in this term, to denote a conjuncture, the moment that distinguishes and separates two intervals, and especially one that is of prominence and importance.

9. The moon is the eclipser of the sun, coming to stand underneath it, like a cloud: the moon, moving eastward, enters the earth's shadow, and the latter becomes its eclipser.

The names given to the eclipsed and eclipsing bodies are either *châdya* and, as here, *châdaka*, "the body to be obscured" and "the obscurer," or *grâhya* and *grâhaka*, "the body to be seized" and "the seizer." The latter terms are akin with *grahaṇa* and *graha*, spoken of above (note to v. 6), and represent the ancient theory of the phenomena, while the others are derived from their modern and scientific explanation, as given in this verse.

10. Subtract the moon's latitude at the time of opposition or conjunction from half the sum of the measures of the eclipsed and eclipsing bodies: whatever the remainder is, that is said to be the amount obscured.

11. When that remainder is greater than the eclipsed body, the eclipse is total; when the contrary, it is partial; when the latitude is greater than the half sum, there takes place no obscuration (*grâsa*).

It is sufficiently evident that when, at the moment of opposition, the moon's latitude—which is the distance of her centre from the ecliptic, where is the centre of the shadow—is equal to the sum of the radii of her disk and of the shadow, the disk and the shadow will just touch one another; and that, on the other hand, the moon will, at the moment of opposition, be so far immersed in the shadow as her latitude is less than the sum of the radii: and so in like manner for the sun, with due allowance for parallax. The Hindu mode of reckoning the amount eclipsed is not by digits, or twelfths of the diameter of the eclipsed body, which method we have inherited from the Greeks, but by minutes.

The word *grâsa*, used in verse 11 for obscuration or eclipse, means literally "eating, devouring," and so speaks more distinctly than any other term we have had of the old theory of the physical cause of eclipses.

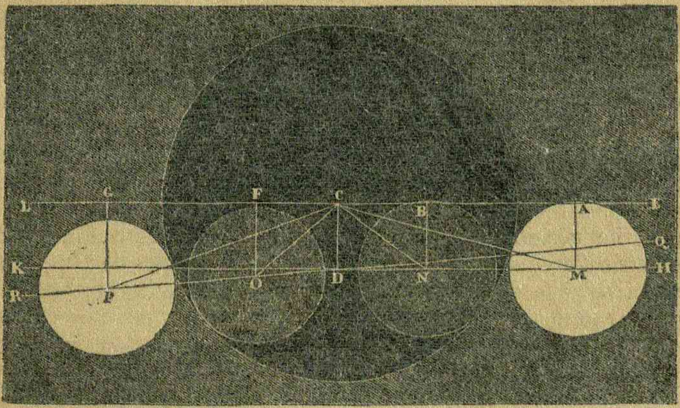
12. Divide by two the sum and difference respectively of the eclipsed and eclipsing bodies: from the square of each of the resulting quantities subtract the square of the latitude, and take the square roots of the two remainders.

13. These, multiplied by sixty and divided by the difference of the daily motions of the sun and moon, give, in nâḍîs, etc., half the duration (*sthiti*) of the eclipse, and half the time of total obscuration.

These rules for finding the intervals of time between the moment of opposition or conjunction in longitude, which is regarded as the middle of the eclipse, and the moments of first and last contact, and, in a total eclipse, of the beginning and end of total obscuration, may be illustrated by help of the annexed figure (Fig. 21).

Let E C L represent the ecliptic, the point C being the centre of the shadow, and let C D be the moon's latitude at the moment of opposi-

Fig. 21.



tion; which, for the present, we will suppose to remain unchanged through the whole continuance of the eclipse. It is evident that the first contact of the moon with the shadow will take place when, in the triangle C A M, A C equals the moon's distance in longitude from the centre of the shadow, A M her latitude, and C M the sum of her radius and that of the shadow. In like manner, the moon will disappear entirely within the shadow when B C equals her distance in longitude from the centre of the shadow, B N her latitude, and C N the difference of the two radii. Upon subtracting, then, the square of A M or B N from those of C M and C N respectively, and taking the square roots of the remainders, we shall have the values of A C and B C in minutes. These may be reduced to time by the following proportion: as the excess at the given time of the moon's true motion in a day over that of the sun is to a day, or sixty nâḍîs, so are A C and B C, the amounts which the moon has to gain in longitude upon the sun between the moments of contact and immersion respectively and the moment of opposition, to the corresponding intervals of time.

But the process, as thus conducted, involves a serious error: the moon's latitude, instead of remaining constant during the eclipse, is constantly and sensibly changing. Thus, in the figure above, of which the conditions are those found by the Hindu processes for the eclipse of Feb. 6th, 1860, the moon's path, instead of being upon the line H K, parallel to the ecliptic, is really upon Q R. The object of the process next taught is to get rid of this error.

14. Multiply the daily motions by the half-duration, in nâḍîs, and divide by sixty: the result, in minutes, subtract for the time of contact (*pragraha*), and add for that of separation (*moksha*), respectively;

15. By the latitudes thence derived, the half-duration, and likewise the half-time of total obscuration, are to be calculated anew, and the process repeated. In the case of the node, the proper correction, in minutes, etc., is to be applied in the contrary manner.

This method of eliminating the error involved in the supposition of a constant latitude, and of obtaining another and more accurate determination of the intervals between the moment of opposition and those of first and last contact, and of immersion and emergence, is by a series of successive approximations. For instance: A C, as already determined, being assumed as the interval between opposition and first contact, a new calculation of the moon's longitude is made for the moment A, and, with this and the sum of the radii, a new value is found for A C. But now, as the position of A is changed, the former determination of its latitude is vitiated and must be made anew, and made to furnish anew a corrected value of A C; and so on, until the position of A is fixed with the degree of accuracy required. The process must be conducted separately, of course, for each of the four quantities affected; since, where latitude is increasing, as in the case illustrated, the true values of A C and B C will be greater than their mean values, while G C and F C, the true intervals in the after part of the eclipse, will be less than A C and B C: and the contrary when latitude is decreasing.

We have illustrated these processes by reference only to a lunar eclipse: their application to the conditions of a solar eclipse requires the introduction of another element, that of the parallax, and will be explained in the notes upon the next chapter.

The first contact of the eclipsed and eclipsing bodies is styled in this passage *pragraha*, "seizing upon, laying hold of;" elsewhere it is also called *grâsa*, "devouring," and *sparça*, "touching:" the last contact, or separation, is named *moksha*, "release, letting go." The whole duration of the eclipse, from contact to separation, is the *sthiti*, "stay, continuance;" total obscuration is *vimarda*, "crushing out, entire destruction."

16. The middle of the eclipse is to be regarded as occurring at the true close of the lunar day: if from that time the time of half-duration be subtracted, the moment of contact (*grâsa*) is found; if the same be added, the moment of separation.

17. In like manner also, if from and to it there be subtracted and added, in the case of a total eclipse, the half-time of total obscuration, the results will be the moments called those of immersion and emergence.

The instant of true opposition, or of apparent conjunction (see below, under ch. v. 9), in longitude, of the sun and moon, is to be taken as the middle of the eclipse, even though, owing to the motion of the moon in latitude, and also, in a solar eclipse, to parallax, that instant is not midway between those of contact and separation, or of immersion and emergence. To ascertain the moment of local time of each of these phases of the eclipse, we subtract and add, from and to the local time of opposition or conjunction, the true intervals found by the processes described in verses 12 to 15.

The total disappearance of the eclipsed body within, or behind, the eclipsing body, is called *nimîlana*, literally the "closure of the eyelids, as in winking:" its first commencement of reappearance is styled *unmîlana*, "parting of the eyelids, peeping." We translate the terms by "immersion" and "emergence" respectively.

18. If from half the duration of the eclipse any given interval be subtracted, and the remainder multiplied by the difference of the daily motions of the sun and moon, and divided by sixty, the result will be the perpendicular (*koṭi*) in minutes.

19. In the case of an eclipse (*graha*) of the sun, the perpendicular in minutes is to be multiplied by the mean half-duration, and divided by the true (*sphuṭa*) half-duration, to give the true perpendicular in minutes.

20. The latitude is the base (*bhuja*): the square root of the sum of their squares is the hypothenuse (*çrava*): subtract this from half the sum of the measures, and the remainder is the amount of obscuration (*grâsa*) at the given time.

21. If that time be after the middle of the eclipse, subtract the interval from the half-duration on the side of separation, and treat the remainder as before: the result is the amount remaining obscured on the side of separation.

The object of the process taught in this passage is to determine the amount of obscuration of the eclipsed body at any given moment during the continuance of the eclipse. It, as well as that prescribed in the following passage, is a variation of that which forms the subject of verses 12 and 13 above, being founded, like the latter, upon a consideration of the right-angled triangle formed by the line joining the centres of the eclipsed and eclipsing bodies as hypothenuse, the difference of their longitudes as perpendicular, and the moon's latitude as base. And whereas, in the former problem, we had the base and hypothenuse given to find the perpendicular, here we have the base and perpendicular given to find the hypothenuse. The perpendicular is furnished us in time, and the rule supposes it to be stated in the form of the interval between the given moment and that of contact or of separation: a form to which, of course, it may readily be reduced from any other mode of statement. The interval of time is reduced to its equivalent as difference of longitude by a proportion the reverse of that given in verse 13, by which difference of longitude was converted into time; the moon's latitude is then calculated; from the two the hypothenuse is deduced; and the comparison of this with the sum of the radii gives the measure of the amount of obscuration.

Verse 21 seems altogether superfluous: it merely states the method of proceeding in case the time given falls anywhere between the middle and the end of the eclipse, as if the specifications of the preceding verses applied only to a time occurring before the middle: whereas they are general in their character, and include the former case no less than the latter.

When the eclipse is one of the sun, allowance needs to be made for the variation of parallax during its continuance; this is done by the process described in verse 19, of which the explanation will be given in the notes to the next chapter (vv. 14–17).

In verse 20, for the first and only time, we have latitude called *kshepa*, instead of *vikshepa*, as elsewhere. In the same verse, the term employed for "hypothenuse" is *çrava*, "hearing, organ of hearing;" this, as well as the kindred *çravaṇa*, which is also once or twice employed, is a synonym of the ordinary term *karṇa*, which means literally "ear." It is difficult to see upon what conception their employment in this signification is founded.

22. From half the sum of the eclipsed and eclipsing bodies subtract any given amount of obscuration, in minutes: from the square of the remainder subtract the square of the latitude at the time, and take the square root of their difference.

23. The result is the perpendicular (*koṭi*) in minutes—which, in an eclipse of the sun, is to be multiplied by the true, and divided by the mean, half-duration—and this, converted into time by the same manner as when finding the duration of the eclipse, gives the time of the given amount of obscuration (*grâsa*).

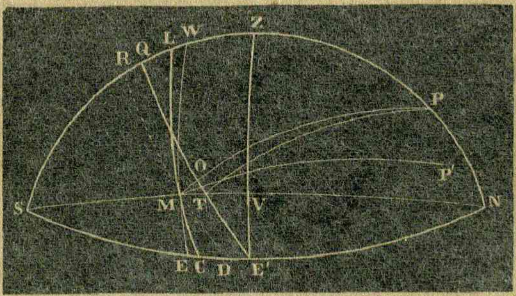
The conditions of this problem are precisely the same with those of the problem stated above, in verses 12–15, excepting that here, instead of requiring the instant of time when obscuration commences, or becomes total, we desire to know when it will be of a certain given amount. The solution must be, as before, by a succession of approximative steps, since, the time not being fixed, the corresponding latitude of the moon cannot be otherwise determined.

24. Multiply the sine of the hour-angle (*nata*) by the sine of the latitude (*aksha*), and divide by radius: the arc corresponding to the result is the degrees of deflection (*valanânçâs*), which are north and south in the eastern and western hemispheres (*kapâla*) respectively.

25. From the position of the eclipsed body increased by three signs calculate the degrees of declination: add them to the degrees of deflection, if of like direction; take their difference, if of different direction: the corresponding sine is the deflection (*valana*)—in digits, when divided by seventy.

This process requires to be performed only when it is desired to project an eclipse. In making a projection according to the Hindu method, as will be seen in connection with the sixth chapter, the eclipsed body is represented as fixed in the centre of the figure, with a north and south line, and an east and west line, drawn through it. The absolute position of these lines upon the disk of the eclipsed body is, of course, all the time changing: but the change is, in the case of the sun, not observable, and in the case of the moon it is disregarded: the Sûrya-Siddhânta takes no notice of the figure visible in the moon's face as determining any fixed and natural directions upon her disk. It is desired to represent to the eye, by the figure drawn, where, with reference to the north, south, east, and west points of the moment, the contact, immersion, emergence, separation, or other phases of the eclipse, will take place. In order to this, it is necessary to know what is, at each given moment, the direction of the ecliptic, in which the motions of both eclipsed and eclipsing bodies are made. The east and west direction is represented by a small circle drawn through the eclipsed body, parallel to the prime vertical; the north and south direction, by a great circle passing through the body and through the north and south points of the horizon: and the direction of the ecliptic is determined by ascertaining the angular amount of its deflection from the small east and west circle at the point occupied by the eclipsed body. Thus, in the annexed figure (Fig. 22), if M be the place of the eclipsed body upon the ecliptic, C L, and if E W be the small east and west circle drawn through M parallel with E′ Z, the prime vertical, then the deflection will be the angle made at M by C M and E M, which is equal to P′ M N, the angle made by perpendiculars to the two circles drawn from their respective poles. In order to find the value of this angle, a double process is adopted: first, the angle made at M by the two small circles E M and D M, which is equivalent to P M N, is approximately determined: as this depends for its amount upon the observer's latitude, being nothing in a right sphere, it is called by the commentary *âksha valana*, "the deflection due to latitude:" the text calls it simply *valanânçâs*, "degrees of deflection," since it does not, like the net result of the whole operation, require to be expressed in terms of its sine. Next, the angle made at M by the ecliptic,

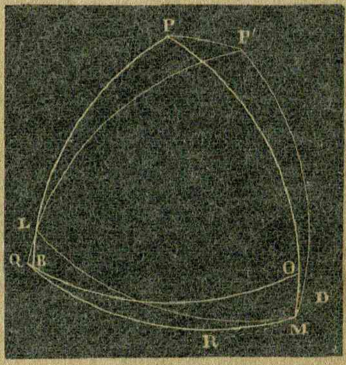
Fig. 22.



C L, and the circle of daily revolution, D R, which angle is equal to P M P′, is also measured: this the commentary calls *âyana valana,* "the deflection due to the deviation of the ecliptic from the equator;" the text has no special name for it. The sum of these two results, or their difference, as the case may be, is the *valana*, or the deflection of the ecliptic from the small east and west circle at M, or the angle P′ M N.

In explaining the method and value of these processes, we will commence with the second one, or with that by which P M P′, the *âyana valana*, is found. In the following figure (Fig. 23), let O Q be the equator, and M L the ecliptic, P and P′ being their respective poles. Let M be the point at which the amount of deflection of M L from the circle of diurnal revolution, D R, is sought. Let M L equal a quadrant; draw P′ L, cutting the equator at Q; as also P L, cutting it at B; then draw P M and Q M. Now P′ M L is a triquadrantal triangle, and hence M Q is a quadrant; and therefore Q is a pole of the circle P O M, and Q O is also a quadrant, and Q M O is a right angle. But D R also makes right angles at M with P M; hence Q M and D R are tangents to one another at M, and the spherical angle Q M L is equal to that which the ecliptic makes at M with the circle of declination, or to P M P′: and Q M L is measured by Q L. The rule given in the text produces a result which is a near approach to this, although not entirely accordant with it excepting at the solstice and equinox, the points where the deflection is greatest and where it is nothing. We are directed to reckon forward a quadrant from the position of the eclipsed body—that is, from M to L, in the figure—and then to calculate the declination at that point, which will be the amount of deflection. But the declination at L is B L, and since L B Q is a right-angled triangle, having a right angle at B, and since L Q and L B are always less than quadrants, L B must be less than L Q. The difference between them, however, can never be of more than trifling amount; for, as the angle Q L B increases, Q L diminishes; and the contrary.

Fig. 23.



In order to show how the Hindus have arrived at a determination of this part of the deflection so nearly correct, and yet not quite correct, we will cite the commentator's explanation of the process. He says: "The 'east' (*prâcî*) of the equator [i. e., apparently, the point of the equator eastward toward which the small circle must be considered as pointing at M] is a point 90° distant from that where a circle drawn from the pole (*dhruva*) through the planet cuts the equator:" that is to say, it is the point Q (Fig. 23), a quadrant from O: "and the interval by which this is separated from the 'east' of the ecliptic at 90° from the planet, that is the *âyana valana*." This is entirely correct, and would give us Q L, the true measure of the deflection. But the commentator goes on farther to say that since this interval, when the planet is at the solstice, is nothing, and when at the equinox is equal to the greatest declination, it is therefore always equal to the declination at a quadrant's distance from the planet. This is, as we have seen, a false conclusion, and leads to an erroneous result: whether they who made the rule were aware of this, but deemed the process a convenient one, and its result a sufficiently near approximation to the truth, we will not venture to say.

The other part of the operation, to determine the amount of deflection of the circle of declination from the east and west small circle, is considerably more difficult, and the Hindu process correspondingly defective. We will first present the explanation of it which the commentator gives. He states the problem thus: "by whatever interval the directions of the equator are deflected from directions corresponding to those of the prime vertical, northward or southward, that is the deflection due to latitude (*âksha valana*). Now then: if a movable circle be drawn through the pole of the prime vertical (*sama*) and the point occupied by the planet [i. e., the circle N M S, Fig. 22], then the interval of the 'easts,' at the distance of a quadrant upon each of the two circles, the equator and the prime vertical, from the points where they are respectively cut by that circle [i. e., from T and V] will be the deflection. . . . Now when the planet is at the horizon [as at D, referred to E'], then that interval is equal to the latitude [Z Q]; when the planet is upon the meridian (*yâmyottaravṛtta*, "south and north circle") [i. e., when it is at R, referred to Q and Z], there is no interval [as at E']. Hence, by the following proportion—with a sine of the hour-angle which is equal to radius the sine of deflection for latitude is equal to the sine of latitude; then with any given sine of the hour-angle what is it?—a sine of latitude is found, of which the arc is the required deflection for latitude." This is, in the Hindu form of statement, the proportion represented by the rule in verse 24, viz. R : sin lat. : : sin hour-angle : sin deflection.

It seems to us very questionable, at least, whether the Hindus had any more rigorous demonstration than this of the process they adopted, or knew wherein lay the inaccuracies of the latter. These we will now proceed to point out. In the first place, instead of measuring the angle made at the point in question, M, by the two small circles, the east and west circle and that of daily revolution—which would be the angle P M N—they refer the body to the equator by a circle passing through the north and south points of the horizon, and measure the deflection of the equator from a small east and west circle at its intersection with that circle—which is the angle P T N. Or, if we suppose that, in the process formerly explained, no regard was had to the circle of daily revolution, D R, the intention being to measure the difference in direction of the ecliptic at M and the equator at O, then the two parts of the process are inconsistent in this, that the one takes as its equatorial point of measurement O, and the other T, at which two points the direction of the equator is different. But neither is the value of P T N correctly found. For, in the spherical triangle P N T, to find the angle at T, we should make the proportion

sin P T (or R) : sin P N : : sin P N T : sin P T N

But, as the third term in this proportion, the Hindus introduce the sine of the hour-angle, Z P M or M P N, although with a certain modification which the commentary prescribes, and which makes of it something very near the angle T P N. The text says simply *natajyâ*, "the sine of the hour-angle" (for *nata*, see notes to iii. 34–36, and 14–16), but the commentary specifies that, to find the desired angle in degrees, we must multiply the hour-angle in time by 90, and divide by the half-day of the planet. This is equivalent to making a quadrant of that part of the circle of diurnal revolution which is between the horizon and the meridian, or to measuring distances upon D R as if they were proportional parts of E′ Q. To make the Hindu process correct, the product of this modification should be the angle P N T, with which, however, it only coincides at the horizon, where both T P N and T N P become right-angles, and at the meridian, where both are reduced to nullity. The error is closely analogous to that involved in the former process, and is of slight account when latitude is small, as is also the error in substituting T for O or M when neither the latitude nor the declination is great.

The direction of the ecliptic deflection (*âyana valana*) is the same, evidently, with that of the declination a quadrant eastward from the point in question; thus, in the case illustrated by the figure, it is south. The direction of the equatorial deflection (*âksha valana*) depends upon the position of the point considered with reference to the meridian, being—in northern latitudes, which alone the Hindu system contemplates—north when that point is east of the meridian, and south when west of it, as specified in verse 24: since, for instance, E′ being the east point of the horizon, the equator at any point between E′ and Q points, eastward, toward a point north of the prime vertical. In the case for which the figure is drawn, then, the difference of the two would be the finally resulting deflection. Since, in making the projection of the eclipse, it is laid off as a straight line (see the illustration given in connection with chapter vi), it must be reduced to its value as a sine; and moreover, since it is laid down in a circle of which the radius is 49 digits (see below, vi. 2), or in which one digit equals 70′—for 3438′÷49 = 70′, nearly—that sine is reduced to its value in digits by dividing it by 70.

The general subject of this passage, the determination of directions during an eclipse, for the purpose of establishing the positions, upon the disk of the eclipsed body, of the points of contact, immersion, emergence, and separation, also engaged the attention of the Greeks; Ptolemy devotes to it the eleventh and twelfth chapters of the sixth book of his Syntaxis: his representation of directions, however, and consequently his method of calculation also, are different from those here exposed.

26. To the altitude in time (*unnata*) add a day and a half, and divide by a half-day; by the quotient divide the latitudes and the disks; the results are the measures of those quantities in digits (*angula*).

By this process due account is taken, in the projection of an eclipse, of the apparent increase in magnitude of the heavenly bodies when near the horizon. The theory lying at the foundation of the rule is this:

that three minutes of arc at the horizon, and four at the zenith, are equal to a digit, the difference between the two, or the excess above three minutes of the equivalent of a digit at the zenith, being one minute. To ascertain, then, what will be, at any given altitude, the excess above three minutes of the equivalent of a digit, we ought properly, according to the commentary, to make the proportion

R : 1′ : : sin altitude : corresp. excess

Since, however, it would be a long and tedious process to find the altitude and its sine, another and approximative proportion is substituted for this "by the blessed Sun," as the commentary phrases it, "through compassion for mankind, and out of regard to the very slight difference between the two." It is assumed that the scale of four minutes to the digit will be always the true one at the noon of the planet in question, or whenever it crosses the meridian, although not at the zenith; and so likewise, that the relation of the altitude to 90° may be measured by that of the time since rising or until setting (*unnata*—see above, iii. 37–39) to a half-day. Hence the proportion becomes

half-day : 1′ : : altitude in time : corresp. excess

and the excess of the digital equivalent above 3′ equals $\frac{\text{alt. in time}}{\text{half-day}}$.

Adding, now, the three minutes, and bringing them into the fractional expression, we have

$$\text{equiv. of digit in minutes at given time} = \frac{\text{alt. in time} + 3 \text{ half-days}}{\text{half-day}}$$

The title of the fourth chapter is *candragrahaṇâdhikâra*, "chapter of lunar eclipses," as that of the fifth is *sûryagrahaṇâdhikâra*, "chapter of solar eclipses." In truth, however, the processes and explanations of this chapter apply not less to solar than to lunar eclipses, while the next treats only of parallax, as entering into the calculation of a solar eclipse. We have taken the liberty, therefore, of modifying accordingly the headings which we have prefixed to the chapters.

CHAPTER V.

OF PARALLAX IN A SOLAR ECLIPSE.

CONTENTS:—1, when there is no parallax in longitude, or no parallax in latitude; 2, causes of parallax; 3, to find the orient-sine; 4–5, the meridian-sine; 5–7, and the sines of ecliptic zenith-distance and altitude; 7–8, to find the amount, in time, of the parallax in longitude; 9, its application in determining the moment of apparent conjunction; 10–11, to find the amount, in arc, of the parallax in latitude; 12–13, its application in calculating an eclipse; 14–17, application of the parallax in longitude in determining the moments of contact, of separation, etc.

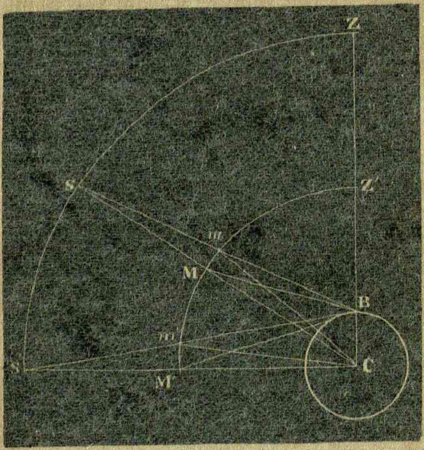
1. When the sun's place is coincident with the meridian ecliptic-point (*madhyalagna*), there takes place no parallax in

longitude (*hariya*): farther, when terrestrial latitude (*aksha*) and north declination of the meridian ecliptic-point (*madhyabha*) are the same, there takes place no parallax in latitude (*avanati*).

The latter of these specifications is entirely accurate: when the north declination of that point of the ecliptic which is at the moment upon the meridian (*madhyalagna*; see iii. 49) is equal to the observer's latitude—regarded by the Hindus as always north—the ecliptic itself passes through the zenith, and becomes a vertical circle; of course, then, the effect of parallax would be only to depress the body in that circle, not to throw it out of it. The other is less exact: when the sun is upon the meridian, there is, indeed, no parallax in right ascension, but there is parallax in longitude, unless the ecliptic is also bisected by the meridian. Here, as below, in verses 8 and 9, the text commits the inaccuracy of substituting the meridian ecliptic-point (L in Fig. 26) for the central or highest point of the ecliptic (B in the same figure). The latter point, although we are taught below (vv. 5–7) to calculate the sine and cosine of its zenith-distance, is not once distinctly mentioned in the text; the commentary calls it *tribhonalagna*, "the orient ecliptic-point (*lagna*—see above, iii. 46–48; it is the point C in Fig. 26) less three signs." The commentary points out this inaccuracy on the part of the text.

In order to illustrate the Hindu method of looking at the subject of parallax, we make the following citation from the general exposition of it given by the commentator under this verse: "At the end of the day of new moon (*amâvâsyâ*) the sun and moon have the same longitude; if, now, the moon has no latitude, then a line drawn from the earth's centre [C in the accompanying figure] to the sun's place [S] just touches the moon [M]: hence, at the centre, the moon becomes an eclipsing, and the sun an eclipsed, body. Since, however, men are not at the earth's centre, (*garbha*, "womb") but upon the earth's surface (*pṛshṭha*, "back"), a line drawn from the earth's surface [B] up to the sun does not just touch the moon; but it cuts the moon's sphere above the point occupied by the moon [at *m*], and when the moon arrives at this point, then is she at the earth's surface the eclipser of the sun. But when the sun is at the zenith (*khamadhya*, "mid-heaven"), then the lines drawn up to the sun from the earth's centre and surface, being one and the same, touch the moon, and so the moon becomes an eclipsing body at the end of the day of new moon. Hence, too, the interval [M *m*] of the lines from the earth's centre and surface is the parallax (*lambana*)."

Fig. 24.

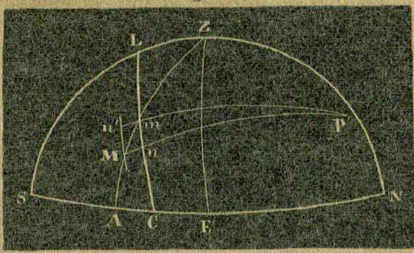


It is evident from this explication how far the Hindu view of parallax is coincident with our own. The principle is the same, but its application is somewhat different. Instead of taking the parallax absolutely, determining that for the sun, which is B S C, and that for the moon, which is B M C, the Hindus look at the subject practically, as it must be taken account of in the calculation of an eclipse, and calculate only the difference of the two parallaxes, which is *m* B M, or, what is virtually the same thing, M C *m*. The Sûrya-Siddhânta, however, as we shall see hereafter more plainly, takes no account of any case in which the line C S would not pass through M, that is to say, the moon's latitude is neglected, and her parallax calculated as if she were in the ecliptic.

We cite farther from the commentary, in illustration of the resolution of the parallax into parallax in longitude and parallax in latitude.

"Now by how many degrees, measured on the moon's sphere (*gola*), the line drawn from the earth's surface up to the sun cuts the moon's vertical circle (*dṛgvṛtta*) above the point occupied by the moon—this is, when the vertical circle and the ecliptic coincide, the moon's parallax in longitude (*lambana*). But when the ecliptic deviates from a vertical circle, then, to the point where the line from the earth's surface cuts the moon's sphere on the moon's vertical circle above the moon [i. e., to *m*, Fig. 25], draw through the pole of the ecliptic (*kadamba*) a circle [P′ *m n′*] north and south to the ecliptic on the moon's sphere [M *n′*]: and then the east and west interval [M *n′*] on the ecliptic between the point occupied by the moon [M] and the point where the circle as drawn cuts the ecliptic on the moon's sphere [*n′*] is the moon's true (*sphuṭa*) parallax in longitude, in minutes, and is the perpendicular (*koṭi*). And since the moon moves along with the ecliptic, the north and south interval, upon the circle we have drawn, between the ecliptic and the vertical circle [*m n′*] is, in minutes, the parallax in latitude (*nati*); which is the base (*bhuja*). The interval, in minutes, on the vertical circle [Z A], between the lines from the earth's centre and surface [*m* M], is the vertical parallax (*dṛglambana*), and the hypothenuse."

Fig. 25.



The conception here presented, it will be noticed, is that the moon's path, or the "ecliptic on the moon's sphere," is depressed away from C L, which might be called the "ecliptic on the sun's sphere," to an amount measured as latitude by *m n′*, and as longitude by *n′* M. To our apprehension, *m n* M, rather than *m n′* M, would be the triangle of resolution: the two are virtually equal.

The commentary then goes on farther to explain that when the vertical circle and the secondary to the ecliptic coincide, the parallax in longitude disappears, the whole vertical parallax becoming parallax in latitude: and again, when the vertical circle and the ecliptic coincide, the parallax in latitude disappears, the whole vertical parallax becoming parallax in longitude.

The term uniformly employed by the commentary, and more usually by the text, to express parallax in longitude, namely *lambana*, is from the same root which we have already more than once had occasion to notice (see above, under i. 25, 60), and means literally "hanging downward." In this verse, as once or twice later (vv. 14, 16), the text uses *harija*, which the commentary explains as equivalent to *kshitija*, "produced by the earth:" this does not seem very plausible, but we have nothing better to suggest. For parallax in latitude the text presents only the term *avanati*, "bending downward, depression;" the commentary always substitutes for it *nati*, which has nearly the same sense, and is the customary modern term.

2. How parallax in latitude arises by reason of the difference of place (*deça*) and time (*kâla*), and also parallax in longitude (*lambana*) from direction (*diç*) eastward or the contrary—that is now to be explained.

This distribution of the three elements of direction, place, and time, as causes respectively of parallax in longitude and in latitude, is somewhat arbitrary. The verse is to be taken, however, rather as a general introduction to the subject of the chapter, than as a systematic statement of the causes of parallax.

3. Calculate, by the equivalents in oblique ascension (*udayâsavas*) of the observer's place, the orient ecliptic-point (*lagna*) for the moment of conjunction (*parvavinâḍyas*): multiply the sine of its longitude by the sine of greatest declination, and divide by the sine of co-latitude (*lamba*): the result is the quantity known as the orient-sine (*udaya*).

The object of this first step in the rather tedious operation of calculating the parallax is to find for a given moment—here the moment of true conjunction—the sine of amplitude of that point of the ecliptic which is then upon the eastern horizon. In the first place the longitude of that point (*lagna*) is determined, by the data and methods taught above, in iii. 46–48, and which are sufficiently explained in the note to that passage: then its sine of amplitude is found, by a process which is a combination of that for finding the declination from the longitude, and that for finding the amplitude from the declination. Thus, by ii. 28,

R : sin gr. decl. : : sin long. : sin decl.

and, by iii. 22–23,

sin co-lat. : R : : sin decl. : sin ampl.

Hence, by combining terms, we have

sin co-lat. : sin gr. decl. : : sin long. : sin ampl.

This sine of amplitude receives the technical name of *udaya*, or *udayajyâ*: the literal meaning of *udaya* is simply "rising."

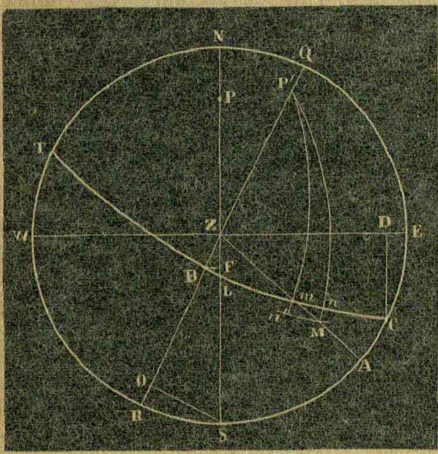
4. Then, by means of the equivalents in right ascension (*lankodayâsavas*), find the ecliptic-point (*lagna*) called that of the meridian (*madhya*): of the declination of that point and the lati-

tude of the observer take the sum, when their direction is the same: otherwise, take their difference.

5. The result is the meridian zenith-distance, in degrees (*natânçâs*): its sine is denominated the meridian-sine (*madhyajyâ*). . . .

The accompanying figure (Fig. 26) will assist the comprehension of this and the following processes. Let N E S W be a horizontal plane, N S the projection upon it of the meridian, and E W that of the prime vertical, Z being the zenith. Let C L T be the ecliptic. Then C is the orient ecliptic-point (*lagna*), and C D the sine of its amplitude (*udayajyâ*), found by the last process. The meridian ecliptic point (*madhyalagna*) is L: it is ascertained by the method prescribed in iii. 49, above. Its distance from the zenith is found from its declination and the latitude of the place of observation, as taught in iii. 20–22; and the sine of that distance, by which, in the figure, it is seen projected, is Z L: it is called by the technical name *madhyajyâ*, which we have translated "meridian-sine."

Fig. 26.



5. . . . Multiply the meridian-sine by the orient-sine, and divide by radius: square the result,

6. And subtract it from the square of the meridian-sine: the square root of the remainder is the sine of ecliptic zenith-distance (*dṛkkshepa*); the square root of the difference of the squares of that and radius is the sine of ecliptic-altitude (*dṛggati*).

Here we are taught how to find the sines of the zenith-distance and altitude respectively of that point of the ecliptic which has greatest altitude, or is nearest to the zenith, and which is also the central point of the portion of the ecliptic above the horizon: it is called by the commentary, as already noticed (see note to v. 1), *tribhonalagna*. Thus, in the last figure, if Q R be the vertical circle passing through the pole of the ecliptic, P', and cutting the ecliptic, C T, in B, B is the central ecliptic-point (*tribhonalagna*), and the arcs seen projected in Z B and B R are its zenith-distance and altitude respectively. In order, now, to find the sine of Z B, we first find that of B L, and by the following process. C D is the orient-sine, already found. But since C Z and C P' are quadrants, C is a pole of the vertical circle Q R, and C R is a quadrant. E S is also a quadrant: take away their common part C S, and C E remains equal to S R, and the sine of the latter, S O, is equal to that of the former, C D, the "orient-sine." Now, then, Z B L is treated as if it were a plane horizontal triangle, and similar to Z O S, and the proportion is made

ZS : SO : ZL : BL

or R : or.-sine : : mer.-sine : B L

This is so far a correct process, that it gives the true sine of the arc B L: for, by spherical trigonometry, in the spherical triangle Z B L, right-angled at B,

sin Z B L : sin B Z L : : sin arc Z L : sin arc B L

or R : S O : : Z L : sin B L

But the third side of a plane right-angled triangle of which the sines of the arcs Z B and Z L are hypothenuse and perpendicular, is not the sine of B L. If we conceive the two former sines to be drawn from Z, meeting in *b* and *l* respectively the lines drawn from B and L to the centre, then the line joining *b l* will be the third side, being plainly less than sin B L. Hence, on subtracting $\sin^2$B L from $\sin^2$Z L, and taking the square root of the remainder, we obtain, not sin Z B, but a less quantity, which may readily be shown, by spherical trigonometry, to be sin Z B cos B L. The value, then, of the sine of ecliptic zenith-distance (*dṛkkshepa*) as determined by this process, is always less than the truth, and as the corresponding cosine (*dṛggati*) is found by subtracting the square of the sine from that of radius, and taking the square root of the remainder, its value is always proportionally greater than the truth. This inaccuracy is noticed by the commentator, who points out correctly its reason and nature: probably it was also known to those who framed the rule, but disregarded, as not sufficient to vitiate the general character of the process: and it may, indeed, well enough pass unnoticed among all the other inaccuracies involved in the Hindu calculations of the parallax.

As regards the terms employed to express the sines of ecliptic zenith-distance and altitude, we have already met with the first member of each compound, *dṛç*, literally "sight," in other connected uses: as in *dṛgjyâ*, "sine of zenith-distance" (see above, iii. 33), *dṛgvṛtta*, "vertical-circle" (commentary to the first verse of this chapter): here it is combined with words which seem to be rather arbitrarily chosen, to form technical appellations for quantities used only in this process: the literal meaning of *kshepa* is "throwing, hurling;" of *gati*, "gait, motion."

7. The sine and cosine of meridian zenith-distance (*natânçâs*) are the approximate (*asphuṭa*) sines of ecliptic zenith-distance and altitude (*dṛkkshepa*, *dṛggati*). . . .

This is intended as an allowable simplification of the above process for finding the sines of ecliptic zenith-distance and altitude, by substituting for them other quantities to which they are nearly equivalent, and which are easier of calculation. These are the sines of zenith-distance and altitude of the meridian ecliptic-point (*madhyalagna*—L in Fig. 26) the former of which has already been made an element in the other process, under the name of "meridian-sine" (*madhyajyâ*). It might, indeed, from the terms of the text, be doubtful of what point the altitude and zenith-distance were to be taken; a passage cited by the commentator from Bhâskara's Siddhânta-Çiromaṇi (found on page 221 of the published edition of the Gaṇitâdhyâya) directs the sines of zenith-distance and altitude of B (*tribhonalagna*) when upon the meridian—that is to say, the sine and cosine of the arc ZF—to be substituted for those of ZB in a hasty process: but the value of the sine would in this case be too small, as in the other it was too great: and as the text nowhere directly recognizes the point B, and as directions have been given in verse 5 for finding the meridian zenith-distance of L, it seems hardly to admit of a doubt that the latter is the point to which the text here intends to refer.

Probably the permission to make this substitution is only meant to apply to cases where ZL is of small amount, or where C has but little amplitude.

7. . . . Divide the square of the sine of one sign by the sine called that of ecliptic-altitude (*dṛggatijîvâ*); the quotient is the "divisor" (*cheda*).

8. By this "divisor" divide the sine of the interval between the meridian ecliptic-point (*madhyalagna*) and the sun's place: the quotient is to be regarded as the parallax in longitude (*lambana*) of the sun and moon, eastward or westward, in nâḍîs, etc.

The true nature of the process by which this final rule for finding the parallax in longitude is obtained is altogether hidden from sight under the form in which the rule is stated. Its method is as follows:

We have seen, in connection with the first verse of the preceding chapter, that the greatest parallaxes of the sun and moon are quite nearly equivalent to the mean motion of each during 4 nâḍîs. Hence, were both bodies in the horizon, and the ecliptic a vertical circle, the moon would be depressed in her orbit below the sun to an amount equal to her excess in motion during 4 nâḍîs. This, then, is the moon's greatest horizontal parallax in longitude. To find what it would be at any other point in the ecliptic, still considered as a vertical circle, we make the proportion

R : 4 (hor. par.) : : sin zen.-dist. : vert. parallax

This proportion is entirely correct, and in accordance with our modern rule that, with a given distance, the parallax of a body varies as the sine of its zenith-distance: whether the Hindus had made a rigorous demonstration of its truth, or whether, as in so many other cases, seeing that the parallax was greatest when the sine of zenith-distance was greatest, and nothing when this was nothing, they assumed it to vary in the interval as the sine of zenith-distance, saying "if, with a sine of zenith-distance which is equal to radius, the parallax is four nâḍîs, with a given sine of zenith-distance what is it?"—this we will not venture to determine.

But now is to be considered the farther case in which the ecliptic is not a vertical circle, but is depressed below the zenith a certain distance, measured by the sine of ecliptic zenith-distance (*dṛkkshepa*), already found. Here again, noting that the parallax is all to be reckoned as parallax in longitude when the ecliptic is a vertical circle, or when the sine of ecliptic-altitude is greatest, and that it would be only parallax in latitude when the ecliptic should be a horizontal circle, or when the sine of ecliptic-altitude should be reduced to nothing, the Hindus assume it to vary in the interval as that sine, and accordingly make the proportion: "if, with a sine of ecliptic-altitude that is equal to radius, the parallax in longitude is equal to the vertical parallax, with any given sine of ecliptic-altitude what is it?"—or, inverting the middle terms,

R : sin ecl.-alt. : : vert. parallax : parallax in long.

But we had before

R : 4 : : sin zen.-dist. : vert. parallax

hence, by combining terms,

R^2 : 4 sin ecl.-alt. : : sin zen.-dist. : parallax in long.

For the third term of this proportion, now, is substituted the sine of the distance of the given point from the central ecliptic-point: that is to say, B*m* (Fig. 26) is substituted for Z*m*; the two are in fact of equal value only when they coincide, or else at the horizon, when each becomes a quadrant; but the error involved in the substitution is greatly lessened by the circumstance that, as it increases in proportional amount, the parallax in longitude itself decreases, until at B the latter is reduced to nullity, as is the vertical parallax at Z. The text, indeed, as in verses 1 and 9, puts *madhyalagna*, L, for *tribhonalagna*, B, in reckoning this distance: but the commentary, without ceremony or apology, reads the latter for the former. These substitutions being made, and the proportion being reduced to the form of an equation, we have

$$\text{par. in long.} = \frac{\text{sin dist.} \times 4 \text{ sin ecl.-alt.}}{R^2}$$

which reduces to

$$\frac{\text{sin dist.}}{R^2 \div 4 \text{ sin ecl.-alt.}} \quad \text{or} \quad \frac{\text{sin dist.}}{\frac{1}{4}R^2 \div \text{sin ecl.-alt.}}$$

and since $\frac{1}{4}R^2 = (\frac{1}{2}R)^2$, and $\frac{1}{2}R = \sin 30°$, we have finally

$$\text{par. in long.} = \frac{\text{sin dist.}}{\sin^2 30° \div \text{sin ecl.-alt.}}$$

which is the rule given in the text. To the denominator of the fraction, in its final form, is given the technical name of *cheda*, "divisor," which word we have had before similarly used, to designate one of the factors in a complicated operation (see above, iii. 35, 38).

We will now examine the correctness of the second principal proportion from which the rule is deduced. It is, in terms of the last figure (Fig. 26),

R : sin Z P′ (=B R) : : *m* M : *m n*

Assuming the equality of the little triangles M *m n* and M *m n′*, and accordingly that of the angles *m* M *n* and M *m n′*, which latter equals Z *m* P′, we have, by spherical trigonometry, as a true proportion,

sin *m n′* M : sin M *m n′* : : *m* M : *m n′*

or R : sin Z *m* P′ : : *m* M : *m n*

Hence the former proportion is correct only when sin Z P′ and sin Z *m* P′ are equal; that is to say, when Z P′ measures the angle Z *m* P′;

and this can be the case only when Zm, as well as $P'm$, is a quadrant, or when m is on the horizon. Here again, however, precisely as in the case last noticed, the importance of the error is kept within very narrow limits by the fact that, as its relative consequence increases, the amount of the parallax in longitude affected by it diminishes.

9. When the sun's longitude is greater than that of the meridian ecliptic-point (*madhyalagna*), subtract the parallax in longitude from the end of the lunar day; when less, add the same: repeat the process until all is fixed.

The text so pertinaciously reads "meridian ecliptic-point" (*madhyalagna*) where we should expect, and ought to have, "central ecliptic-point" (*tribhonalagna*), that we are almost ready to suspect it of meaning to designate the latter point by the former name. It is sufficiently clear that, whenever the sun and moon are to the eastward of the central ecliptic-point, the effect of the parallax in longitude will be to throw the moon forward on her orbit beyond the sun, and so to cause the time of apparent to precede that of real conjunction; and the contrary. Hence, in the eastern hemisphere, the parallax, in time, is subtractive, while in the western it is additive. But a single calculation and application of the correction for parallax is not enough; the moment of apparent conjunction must be found by a series of successive approximations: since if, for instance, the moment of true conjunction is $25^{n}\ 2^{v}$, and the calculated parallax in longitude for that moment is $2^{n}\ 21^{v}$, the apparent end of the lunar day will not be at $27^{n}\ 23^{v}$, because at the latter time the parallax will be greater than $2^{n}\ 21^{v}$, deferring accordingly still farther the time of conjunction; and so on. The commentary explains the method of procedure more fully, as follows: for the moment of true conjunction in longitude calculate the parallax in longitude, and apply it to that moment: for the time thus found calculate the parallax anew, and apply it to the moment of true conjunction: again, for the time found as the result of this process, calculate the parallax, and apply it as before; and so proceed, until a moment is arrived at, at which the difference in actual longitude, according to the motions of the two planets, will just equal and counterbalance the parallax in longitude.

The accuracy of this approximative process cannot but be somewhat impaired by the circumstance that, while the parallax is reckoned in difference of mean motions, the corrections of longitude must be made in true motions. Indeed, the reckoning of the horizontal parallax in time as 4 nâḍîs, whatever be the rate of motion of the sun and moon, is one of the most palpable among the many errors which the Hindu process involves.

To ascertain the moment of apparent conjunction in longitude, only the parallax in longitude requires to be known; but to determine the time of occurrence of the other phases of the eclipse, it is necessary to take into account the parallax in latitude, the ascertainment of which is accordingly made the subject of the next rule.

10. If the sine of ecliptic zenith-distance (*dṛkkshepa*) be multiplied by the difference of the mean motions of the sun and

moon, and divided by fifteen times radius, the result will be the parallax in latitude (*avanati*).

As the sun's greatest parallax is equal to the fifteenth part of his mean daily motion, and that of the moon to the fifteenth part of hers (see note to iv. 1, above), the excess of the moon's parallax over that of the sun is equal, when greatest, to one fifteenth of the difference of their respective mean daily motions. This will be the value of the parallax in latitude when the ecliptic coincides with the horizon, or when the sine of ecliptic zenith-distance becomes equal to radius. On the other hand, the parallax in latitude disappears when this same sine is reduced to nullity. Hence it is to be regarded as varying with the sine of ecliptic zenith-distance, and, in order to find its value at any given point, we say "if, with a sine of ecliptic zenith-distance which is equal to radius, the parallax in latitude is one fifteenth of the difference of mean daily motions, with a given sine of ecliptic zenith-distance what is it?" or

R : diff. of mean m. ÷ 15 : : sin ecl. zen.-dist. : parallax in lat.

This proportion, it is evident, would give with entire correctness the parallax at the central ecliptic-point (B in Fig. 26), where the whole vertical parallax is to be reckoned as parallax in latitude. But the rule given in the text also assumes that, with a given position of the ecliptic, the parallax in latitude is the same at any point in the ecliptic. Of this the commentary offers no demonstration, but it is essentially true. For, regarding the little triangle Mmn as a plane triangle, right-angled at n, and with its angle nmM equal to the angle ZmB, we have

R : sin ZmB : : Mm : Mn

But, in the spherical triangle ZmB, right-angled at B,

R : sin ZmB : : sin Zm : sin ZB

Hence, by equality of ratios,

sin Zm : sin ZB : : Mm : Mn

But, as before shown,

R : sin Zm : : gr. parallax : Mm

Hence, by combining terms,

R : sin ZB : : gr. parallax : Mn

That is to say, whatever be the position of m, the point for which the parallax in latitude is sought, this will be equal to the product of the greatest parallax into the sine of ecliptic zenith-distance, divided by radius: or, as the greatest parallax equals the difference of mean motions divided by fifteen,

$$\text{par. in lat.} = \frac{\text{sin ecl. zen.-dist.} \times \text{diff. of m. m.} \div 15}{\text{R}} \quad \text{or} \quad \frac{\text{sin ecl. zen.-dist.} \times \text{diff. of m. m.}}{\text{R} \times 15}$$

The next verse teaches more summary methods of arriving at the same quantity.

11. Or, the parallax in latitude is the quotient arising from dividing the sine of ecliptic zenith-distance (*dṛkkshepa*) by sev-

enty, or, from multiplying it by forty-nine, and dividing it by radius.

In the expression given above for the value of the parallax in latitude, all the terms are constant excepting the sine of ecliptic zenith-distance. The difference of the mean daily motions is 731′ 27″, and fifteen times radius is 51,570′. Now 731′ 27″ ÷ 51,570′ equals $\frac{1}{70.50}$ or 48.77 ÷ R; to which the expressions given in the text are sufficiently near approximations.

12. The parallax in latitude is to be regarded as south or north according to the direction of the meridian-sine (*madhyajyâ*). When it and the moon's latitude are of like direction, take their sum; otherwise, their difference:

13. With this calculate the half-duration (*sthiti*), half total obscuration (*vimarda*), amount of obscuration (*grâsa*), etc., in the manner already taught; likewise the scale of projection (*pramâṇa*), the deflection (*valana*), the required amount of obscuration, etc., as in the case of a lunar eclipse.

In ascertaining the true time of occurrence of the various phases of a solar eclipse, as determined by the parallax of the given point of observation, we are taught first to make the whole correction for parallax in latitude, and then afterward to apply that for parallax in longitude. The former part of the process is succinctly taught in verses 12 and 13: the rules for the other follow in the next passage. The language of the text, as usual, is by no means so clear and explicit as could be wished. Thus, in the case before us, we are not taught whether, as the first step in this process of correction, we are to calculate the moon's parallax in latitude for the time of true conjunction (*tithyanta*, "end of the lunar day"), or for that of apparent conjunction (*madhyagrahaṇa*, "middle of the eclipse"). It might be supposed that, as we have thus far only had in the text directions for finding the sine and cosine of ecliptic zenith-distance at the moment of true conjunction, the former of them was to be used in the calculations of verses 10 and 11, and the result from it, which would be the parallax at the moment of true conjunction, applied here as the correction needed. Nor, so far as we have been able to discover, does the commentator expound what is the true meaning of the text upon this point. It is sufficiently evident, however, that the moment of apparent conjunction is the time required. We have found, by a process of successive approximation, at what time (see Fig. 25), the moon (her latitude being neglected) being at *m* and the sun at *n*, the parallax in longitude and the difference of true longitude will both be the same quantity, *m n*, and so, when apparent conjunction will take place. Now, to know the distance of the two centres at that moment, we require to ascertain the parallax in latitude, *n* M, for the moon at *m*, and to apply it to the moon's latitude when in the same position, taking their sum when their direction is the same, and their difference when their direction is different, as prescribed by the text; the net result will be the distance required. The commentary, it may be remarked, expressly states that the moon's latitude is to be calculated in this operation for the time of apparent conjunction (*madhyagrahana*). The distance thus found will determine the amount of greatest obscuration, and the character of the eclipse, as taught in verse 10 of the preceding chapter. It is then farther to be taken as the foundation of precisely such a process as that described in verses 12–15 of the same chapter, in order to ascertain the half-time of duration, or of total obscuration: that is to say, the distance in latitude of the two centres being first assumed as invariable through the whole duration of the eclipse, the half-time of duration, and the resulting moments of contact and separation are to be ascertained: for these moments the latitude and parallax in latitude are to be calculated anew, and by them a new determination of the times of contact and separation is to be made, and so on, until these are fixed with the degree of accuracy required. If the eclipse be total, a similar operation must be gone through with to ascertain the moments of immersion and emergence. No account is made, it will be noticed, of the possible occurrence of an annular eclipse.

The intervals thus found, after correction for parallax in latitude only, between the middle of the eclipse and the moments of contact and separation respectively, are those which are called in the last chapter (vv. 19, 23), the "mean half-duration" (*madhyasthityardha*).

In this process for finding the net result, as apparent latitude, of the actual latitude and the parallax in latitude, is brought out with distinctness the inaccuracy already alluded to; that, whatever be the moon's actual latitude, her parallax is always calculated as if she were in the ecliptic. In an eclipse, however, to which case alone the Hindu processes are intended to be applied, the moon's latitude can never be of any considerable amount.

The propriety of determining the direction of the parallax in latitude by means of that of the meridian-sine (Z L in Fig. 26), of which the direction is established as south or north by the process of its calculation, is too evident to call for remark.

In verse 13 is given a somewhat confused specification of matters which are, indeed, affected by the parallax in latitude, but in different modes and degrees. The amount of greatest obscuration, and the (mean) half-times of duration and total obscuration, are the quantities directly dependent upon the calculation of that parallax, as here presented: to find the amount of obscuration at a given moment—as also the time corresponding to a given amount of obscuration—we require to know also the true half-duration, as found by the rules stated in the following passage: while the scale of projection and the deflection are affected by parallax only so far as this alters the time of occurrence of the phases of the eclipse.

14. For the end of the lunar day, diminished and increased by the half-duration, as formerly, calculate again the parallax in longitude for the times of contact (*grâsa*) and of separation (*moksha*), and find the difference between these and the parallax in longitude (*harija*) for the middle of the eclipse.

15. If, in the eastern hemisphere, the parallax in longitude for the contact is greater than that for the middle, and that for

the separation less; and if, in the western hemisphere, the contrary is the case—

16. Then the difference of parallax in longitude is to be added to the half-duration on the side of separation, and likewise on that of contact (*pragrahaṇa*); when the contrary is true, it is to be subtracted.

17. These rules are given for cases where the two parallaxes are in the same hemisphere: where they are in different hemispheres, the sum of the parallaxes in longitude is to be added to the corresponding half-duration. The principles here stated apply also to the half-time of total obscuration.

We are supposed to have ascertained, by the preceding process, the true amount of apparent latitude at the moments of first and last contact of the eclipsed and eclipsing bodies, and consequently to have determined the dimensions of the triangle—corresponding, in a solar eclipse, to C G P, Fig. 21, in a lunar—made up of the latitude, the distance in longitude, and the sum of the two radii. The question now is how the duration of the eclipse will be affected by the parallax in longitude. If this parallax remained constant during the continuance of the eclipse, its effect would be nothing; and, having once determined by it the time of apparent conjunction, we should not need to take it farther into account. But it varies from moment to moment, and the effect of its variation is to prolong the duration of every part of a visible eclipse. For, to the east of the central ecliptic-point, it throws the moon's disk forward upon that of the sun, thus hastening the occurrence of all the phases of the eclipse, but by an amount which is all the time decreasing, so that it hastens the beginning of the eclipse more than the middle, and the middle more than the close: to the west of that same point, on the other hand, it depresses the moon's disk away from the sun's, but by an amount constantly increasing, so that it retards the end of the eclipse more than its middle, and its middle more than its beginning. The effect of the parallax in longitude, then, upon each half-duration of the eclipse, will be measured by the difference between its retarding and accelerating effects upon contact and conjunction, and upon conjunction and separation, respectively: and the amount of this difference will always be additive to the time of half-duration as otherwise determined. If, however, contact and conjunction, or conjunction and separation, take place upon opposite sides of the point of no parallax in longitude, then the sum of the two parallactic effects, instead of their difference, will be to be added to the corresponding half-duration: since the one, on the east, will hasten the occurrence of the former phase, while the other, on the west, will defer the occurrence of the latter phase. The amount of the parallax in longitude for the middle of the eclipse has already been found; if, now, we farther determine its amount—reckoned, it will be remembered, always in time—for the moments of contact and separation, and add the difference or the sum of each of these and the parallax for the moment of conjunction to the corresponding half-duration as previously determined, we shall have the true times of half-duration. In order to find the parallax for contact and separation, we repeat the same process (see above, v. 9) by which that for conjunction was found: as we then started from the moment of true conjunction, and, by a series of successive approximations, ascertained the time when the difference of longitude would equal the parallax in longitude, so now we start from two moments removed from that of true conjunction by the equivalents in time of the two distances in longitude obtained by the last process, and, by a similar series of successive approximations, ascertain the times when the differences of longitude, together with the parallax, will equal those distances in longitude.

In the process, as thus conducted, there is an evident inaccuracy. It is not enough to apply the whole correction for parallax in latitude, and then that for parallax in longitude, since, by reason of the change effected by the latter in the times of contact and separation, a new calculation of the former becomes necessary, and then again a new calculation of the latter, and so on, until, by a series of doubly compounded approximations, the true value of each is determined. This was doubtless known to the framers of the system, but passed over by them, on account of the excessively laborious character of the complete calculation, and because the accuracy of such results as they could obtain was not sensibly affected by its neglect.

The question naturally arises, why the specifications of verse 15 are made hypothetical instead of positive, and why, in the latter half of verse 16, a case is supposed which never arises. The commentator anticipates this objection, and takes much pains to remove it: it is not worth while to follow his different pleas, which amount to no real explanation, saving to notice his last suggestion, that, in case an eclipse begins before sunrise, the parallax for its earlier phase or phases, as calculated according to the distance in time from the lower meridian, may be less than for its later phases—and the contrary, when the eclipse ends after sunset. This may possibly be the true explanation, although we are justly surprised at finding a case of so little practical consequence, and to which no allusion has been made in the previous processes, here taken into account.

The text, it may be remarked, by its use of the terms "eastern and western hemispheres" (*kapâla*, literally "cup, vessel"), repeats once more its substitution of the meridian ecliptic-point (*madhyalagna*) for the central ecliptic-point (*tribhonalagna*), as that of no parallax in longitude; the meridian forming the only proper and recognized division of the heavens into an eastern and a western hemisphere.

We are now prepared to see the reason of the special directions given in verses 19 and 23 of the last chapter, respecting the reduction, in a solar eclipse, of distance in time from the middle of the eclipse to distance in longitude of the two centres. The "mean half-duration" (*madhyasthityardha*) of the eclipse is the time during which the true distance of the centres at the moments of contact or separation, as found by the process prescribed in verses 12 and 13 of this chapter, would be gained by the moon with her actual excess of motion, leaving out of account the variation of parallax in longitude: the "true half-duration" (*sphutasthityardha*) is the increased time in which, owing to that variation, the same distance in longitude is actually gained by the moon;

the effect of the parallax being equivalent either to a diminution of the moon's excess of motion, or to a protraction of the distance of the two centers—both of them in the ratio of the true to the mean half-duration. If then, for instance, it be required to know what will be the amount of obscuration of the sun half an hour after the first contact, we shall first subtract this interval from the true half-duration before conjunction; the remainder will be the actual interval to the middle of the eclipse: this interval, then, we shall reduce to its value as distance in longitude by diminishing it, either before or after its reduction to minutes of arc, in the ratio of the true to the mean half-duration. The rest of the process will be performed precisely as in the case of an eclipse of the moon.

Notwithstanding the ingenuity and approximate correctness of many of the rules and methods of calculation taught in this chapter, the whole process for the ascertainment of parallax contains so many elements of error that it hardly deserves to be called otherwise than cumbrous and bungling. The false estimate of the difference between the sun's and moon's horizontal parallax—the neglect, in determining it, of the variation of the moon's distance—the estimation of its value in time made always according to mean motions, whatever be the true motions of the planets at the moment—the neglect, in calculating the amount of parallax, of the moon's latitude—these, with all the other inaccuracies of the processes of calculation which have been pointed out in the notes, render it impossible that the results obtained should ever be more than a rude approximation to the truth.

In farther illustration of the subject of solar eclipses, as exposed in this and the preceding chapters, we present, in the Appendix, a full calculation of the eclipse of May 26th, 1854, mainly as made for the translator, during his residence in India, by a native astronomer.

CHAPTER VI.

OF THE PROJECTION OF ECLIPSES.

CONTENTS:—1, value of a projection; 2-4, general directions; 5-6, how to lay off the deflection and latitude for the beginning and end of the eclipse; 7, to exhibit the points of contact and separation; 8-10, how to lay off the deflection and latitude for the middle of the eclipse; 11, to show the amount of greatest obscuration; 12, reversal of directions in the western hemisphere; 13, least amount of obscuration observable; 14-16, to draw the path of the eclipsing body; 17-19, to show the amount of obscuration at a given time; 20-22, to exhibit the points of immersion and emergence in a total eclipse; 23, color of the part of the moon obscured; 24, caution as to communicating a knowledge of these matters.

1. Since, without a projection (*chedyaka*), the precise (*sphuṭa*) differences of the two eclipses are not understood, I shall proceed to explain the exalted doctrine of the projection.

The term *chedyaka* is from the root *chid*, "split, divide, sunder," and indicates, as here applied, the instrumentality by which distinctive differences are rendered evident. The name of the chapter, *parilekhâdhikâra*, is not taken from this word, but from *parilekha*, "delineation, figure," which occurs once below, in the eighth verse.

2. Having fixed, upon a well prepared surface, a point, describe from it, in the first place, with a radius of forty-nine digits (*angula*), a circle for the deflection (*valana*):

3. Then a second circle, with a radius equal to half the sum of the eclipsed and eclipsing bodies; this is called the aggregate-circle (*samâsa*); then a third, with a radius equal to half the eclipsed body.

4. The determination of the directions, north, south, east, and west, is as formerly. In a lunar eclipse, contact (*grahaṇa*) takes place on the east, and separation (*moksha*) on the west; in a solar eclipse, the contrary.

The larger circle, drawn with a radius of about three feet, is used solely in laying off the deflection (*valana*) of the ecliptic from an east and west circle. We have seen above (iv. 24, 25) that the sine of this deflection was reduced to its value in a circle of forty-nine digits' radius, by dividing by seventy its value in minutes. The second circle is employed (see below, vv. 6, 7) in determining the points of contact and separation. The third represents the eclipsed body itself, always maintaining a fixed position in the centre of the figure, even though, in a lunar eclipse, it is the body which itself moves, relatively to the eclipsing shadow. For the scale by which the measures of the eclipsed and eclipsing bodies, the latitudes, etc., are determined, see above, iv. 26.

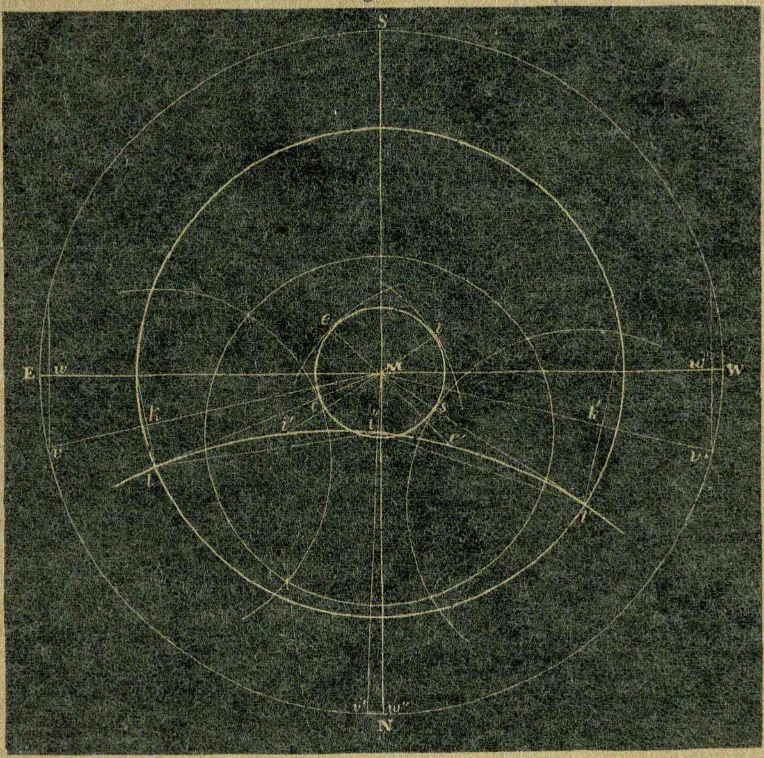
The method of laying down the cardinal directions is the same with that used in constructing a dial; it is described in the first passage of the third chapter (iii. 1–4).

The specifications of the latter half of verse 4 apply to the eclipsed body, designating upon which side of it obscuration will commence and terminate.

5. In a lunar eclipse, the deflection (*valana*) for the contact is to be laid off in its own proper direction, but that for the separation in reverse; in an eclipse of the sun, the contrary is the case.

The accompanying figure (Fig. 27) will illustrate the Hindu method of exhibiting, by a projection, the various phases of an eclipse. Its conditions are those of the lunar eclipse of Feb. 6th, 1860, as determined by the data and methods of this treatise: for the calculation see the Appendix. Let M be the centre of the figure and the place of the moon, and let N S and E W be the circles of direction drawn through the moon's centre; the former representing (see above, under iv. 24, 25) a great circle drawn through the north and south points of the horizon, the latter a small circle parallel to the prime vertical. In explanation of the manner in which these directions are presented by the figure, we would remark that we have adapted it to a supposed position of the observer on the north side of his projection, as at N, and looking southward—a position which, in our latitude, he would naturally assume, for

Fig. 27.



the purpose of comparing the actual phases of the eclipse, as they occurred, with his delineation of them. The heavier circle, *l l'*, is that drawn with the sum of the semi-diameters, or the "aggregate-circle;" while the outer one, N E S W, is that for the deflection. This, in order to reduce the size of the whole figure, we have drawn upon a scale very much smaller than that prescribed; its relative dimensions being a matter of no consequence whatever, provided the sine of the deflection be made commensurate with its radius. In our own, or the Greek, method of laying off an arc, by its angular value, the radius of the circle of deflection would also be a matter of indifference: the Hindus, ignoring angular measurements, adopt the more awkward and bungling method of laying off the arc by means of its sine. Let *v w* equal the deflection, calculated for the moment of contact, expressed as a sine, and in terms of a circle in which E M is radius. Now, as the moon's contact with the shadow takes place upon her eastern limb, the deflection for the contact must be laid off from the east point of the circle; and, as the calculated direction of the deflection indicates in what way the ecliptic is pointing eastwardly, it must be laid off from E in its own proper direction. In the case illustrated, the deflection for the contact is north: hence we lay it off northward from E, and then the line drawn from M to *v*, its extremity—which line represents the direction of the ecliptic at the moment—points northward. Again, upon the side of separation—which, for the moon, is the western side—we lay off the deflection for the moment of separation: but we lay it off from W in the reverse of its true direction, in order that the line from its extremity to the centre may truly represent the direction of the ecliptic. Thus, in the eclipse figured, the deflection for separation is south; we lay it off northward from W, and then the line *v'* M points, toward M, southward. In a solar eclipse, in which, since the sun's western limb is the first eclipsed, the deflection for contact must be laid off from W, and that for separation from E, the direction of the former requires to be reversed, and that of the latter to be maintained as calculated.

6. From the extremity of either deflection draw a line to the centre: from the point where that cuts the aggregate-circle (*samâsa*) are to be laid off the latitudes of contact and of separation.

7. From the extremity of the latitude, again, draw a line to the central point: where that, in either case, touches the eclipsed body, there point out the contact and separation.

8. Always, in a solar eclipse, the latitudes are to be drawn in the figure (*parilekha*) in their proper direction; in a lunar eclipse, in the opposite direction. . . .

The lines *v* M and *v'* M, drawn from *v* and *v'*, the extremities of the sines or arcs which measure the deflection, to the centre of the figure, represent, as already noticed, the direction of the ecliptic with reference to an east and west line at the moments of contact and separation. From them, accordingly, and at right angles to them, are to be laid off the values of the moon's latitude at those moments. Owing, however, to the principle adopted in the projection, of regarding the eclipsed body as fixed in the centre of the figure, and the eclipsing body as passing over it, the lines *v* M and *v'* M do not, in the case of a lunar eclipse, represent the ecliptic itself, in which is the centre of the shadow, but the small circle of latitude, in which is the moon's centre: hence, in laying off the moon's latitude to determine the centre of the shadow, we reverse its direction. Thus, in the case illustrated, the moon's latitude is always south: we lay off, then, the lines *k l* and *k' l'*, representing its value at the moments of contact and separation, northward: they are, like the deflection, drawn as sines, and in such manner that their extremities, *l* and *l'*, are in the aggregate-circle: then, since *l* M and *l'* M are each equal to the sum of the two semi-diameters, and *l k* and *l' k'* to the latitudes, *k* M and *k'* M will represent the distances of the centres in longitude, and *l* and *l'* the places of the centre of the shadow, at contact and separation: and upon describing circles from *l* and *l'*, with radii equal to the semi-diameter of the shadow, the points *c* and *s*, where these touch the disk of the moon, will be the points of first and last contact: *c* and *s* being also, as stated in the text, the points where *l* M and *l'* M meet the circumference of the disk of the eclipsed body.

8\. . . . In accordance with this, then, for the middle of the eclipse,

9\. The deflection is to be laid off—eastward, when it and the latitude are of the same direction; when they are of different directions, it is to be laid off westward: this is for a lunar eclipse; in a solar, the contrary is the case.

10\. From the end of the deflection, again, draw a line to the central point, and upon this line of the middle lay off the latitude, in the direction of the deflection.

11\. From the extremity of the latitude describe a circle with a radius equal to half the measure of the eclipsing body: whatever of the disk of the eclipsed body is enclosed within that circle, so much is swallowed up by the darkness (*tamas*).

The phraseology of the text in this passage is somewhat intricate and obscure; it is fully explained by the commentary, as, indeed, its meaning is also deducible with sufficient clearness from the conditions of the problem sought to be solved. It is required to represent the deflection of the ecliptic from an east and west line at the moment of greatest obscuration, and to fix the position of the centre of the eclipsing body at that moment. The deflection is this time to be determined by a secondary to the ecliptic, drawn from near the north or south point of the figure. The first question is, from which of these two points shall the deflection be laid off, and the line to the centre drawn. Now since, according to verse 10, the latitude itself is to be measured upon the line of deflection, the latter must be drawn southward or northward according to the direction in which the latitude is to be laid off. And this is the meaning of the last part of verse 8; "in accordance," namely, with the direction in which, according to the previous part of the verse, the latitude is to be drawn. But again, in which direction from the north or south point, as thus determined, shall the deflection be measured? This must, of course, be determined by the direction of the deflection itself: if south, it must obviously be measured east from the north point and west from the south point; if north, the contrary. The rules of the text are in accordance with this, although the determining circumstance is made to be the agreement or non-agreement, in respect to direction, of the deflection with the moon's latitude—the latter being this time reckoned in its own proper direction, and not, in a lunar eclipse, reversed. Thus, in the case for which the figure is drawn, as the moon's latitude is south, and must be laid off northward from M, the deflection, $v'' w''$, is measured from the north point; as deflection and latitude are both south, it is measured east from N. In an eclipse of the sun, on the other hand, the moon's latitude would, if north, be laid off northward, as in the figure, and hence also, the deflection would be measured from the north point: but it would be measured eastward, if its own direction were south, or disagreed with that of the latitude.

The line of deflection, which is M v'' in the figure, being drawn, and having the direction of a perpendicular to the ecliptic at the moment of opposition, the moon's latitude for that moment, M l'', is laid off directly upon it. The point l'' is, accordingly, the position of the centre of the shadow at the middle of the eclipse, and if from that centre, with a radius equal to the semi-diameter of the eclipsing body, a circle be drawn, it will include so much of the disk of the eclipsed body as is covered when the obscuration is greatest. In the figure the eclipse is shown as total, the Hindu calculations making it so, although, in fact, it is only a partial eclipse.

12. By the wise man who draws the projection (*chedyaka*), upon the ground or upon a board, a reversal of directions is to be made in the eastern and western hemispheres.

This verse is inserted here in order to remove the objection that, in the eastern hemisphere, indeed, all takes place as stated, but, if the eclipse occurs west of the meridian, the stated directions require to be all of them reversed. In order to understand this objection, we must take notice of the origin and literal meaning of the Sanskrit words which designate the cardinal directions. The face of the observer is supposed always to be eastward: then "east" is *prâñc*, "forward, toward the front"; "west" is *paçcât*, "backward, toward the rear": "south" is *dakshina*, "on the right"; "north" is *uttara*, "upward" (i. e., probably, toward the mountains, or up the course of the rivers in north-western India). These words apply, then, in etymological strictness, only when one is looking eastward—and so, in the present case, only when the eclipse is taking place in the eastern hemisphere, and the projector is watching it from the west side of his projection, with the latter before him: if, on the other hand, he removes to E, turning his face westward, and comparing the phenomena as they occur in the western hemisphere with his delineation of them, then "forward" (*prâñc*) is no longer east, but west; "right" (*dakshina*) is no longer south, but north, etc.

It is unnecessary to point out that this objection is one of the most frivolous and hair-splitting character, and its removal by the text a waste of trouble: the terms in question have fully acquired in the language an absolute meaning, as indicating directions in space, without regard to the position of the observer.

13. Owing to her clearness, even the twelfth part of the moon, when eclipsed (*grasta*), is observable; but, owing to his piercing brilliancy, even three minutes of the sun, when eclipsed, are not observable.

The commentator regards the negative which is expressed in the latter half of this verse as also implied in the former, the meaning being that an obscuration of the moon's disk extending over only the twelfth part of it does not make itself apparent. We have preferred the interpretation given above, as being better accordant both with the plain and simple construction of the text and with fact.

14. At the extremities of the latitudes make three points, of corresponding names; then, between that of the contact and

that of the middle, and likewise between that of the separation and that of the middle,

15. Describe two fish-figures (*matsya*): from the middle of these having drawn out two lines projecting through the mouth and tail, wherever their intersection takes place,

16. There, with a line touching the three points, describe an arc: that is called the path of the eclipsing body, upon which the latter will move forward.

The deflection and the latitude of three points in the continuance of the eclipse having been determined and laid down upon the projection, it is deemed unnecessary to take the same trouble with regard to any other points, these three being sufficient to determine the path of the eclipsing body: accordingly, an arc of a circle is drawn through them, and is regarded as representing that path. The method of describing the arc is the same with that which has already been more than once employed (see above, iii. 1–4, 41–42): it is explained here with somewhat more fullness than before. Thus, in the figure, l, l'', and l' are the three extremities of the moon's latitude, at the moments of contact, opposition, and separation, respectively: we join $l\,l''$, $l''\,l'$, and upon these lines describe fish-figures (see note to iii. 1–5); their two extremities ("mouth" and "tail") are indicated by the intersecting dotted lines in the figure: then, at the point, not included in the figure, where the lines drawn through them meet one another, is the centre of a circle passing through l, l'', and l'.

17. From half the sum of the eclipsed and eclipsing bodies subtract the amount of obscuration, as calculated for any given time: take a little stick equal to the remainder, in digits, and, from the central point,

18. Lay it off toward the path upon either side—when the time is before that of greatest obscuration, toward the side of contact; when the obscuration is decreasing, in the direction of separation—and where the stick and the path of the eclipsing body

19. Meet one another, from that point describe a circle with a radius equal to half the eclipsing body: whatever of the eclipsed body is included within it, that point out as swallowed up by the darkness (*tamas*).

20. Take a little stick equal to half the difference of the measures (*mâna*), and lay it off in the direction of contact, calling it the stick of immersion (*nimîlana*): where it touches the path,

21. From that point, with a radius equal to half the eclipsing body, draw a circle, as in the former case: where this meets the circle of the eclipsed body, there immersion takes place.

22. So also for the emergence (*unmîlana*), lay it off in the direction of separation, and describe a circle, as before: it will show the point of emergence in the manner explained.

The method of these processes is so clear as to call for no detailed explanation. The centre of the eclipsing body being supposed to be always in the arc *l l'' l'*, drawn as directed in the last passage, we have only to fix a point in this arc which shall be at a distance from M corresponding to the calculated distance of the centres at the given time, and from that point to describe a circle of the dimensions of the eclipsed body, and the result will be a representation of the then phase of the eclipse. If the point thus fixed be distant from M by the difference of the two semi-diameters, as M *i'*, M *e'*, the circles described will touch the disk of the eclipsed body at the points of immersion and emergence, *i* and *e*.

23. The part obscured, when less than half, will be dusky (*sadhûmra*); when more than half, it will be black; when emerging, it is dark copper-color (*kṛshṇatâmra*); when the obscuration is total, it is tawny (*kapila*).

The commentary adds the important circumstance, omitted in the text, that the moon alone is here spoken of; no specification being added with reference to the sun, because, in a solar eclipse, the part obscured is always black.

A more suitable place might have been found for this verse in the fourth chapter, as it has nothing to do with the projection of an eclipse.

24. This mystery of the gods is not to be imparted indiscriminately: it is to be made known to the well-tried pupil, who remains a year under instruction.

The commentary understands by this mystery, which is to be kept with so jealous care, the knowledge of the subject of this chapter, the delineation of an eclipse, and not the general subject of eclipses, as treated in the past three chapters. It seems a little curious to find a matter of so subordinate consequence heralded so pompously in the first verse of the chapter, and guarded so cautiously at its close.

CHAPTER VII.

OF PLANETARY CONJUNCTIONS.

CONTENTS :—1, general classification of planetary conjunctions; 2-6, method of determining at what point on the ecliptic, and at what time, two planets will come to have the same longitude; 7-10, how to find the point on the ecliptic to which a planet, having latitude, will be referred by a circle passing through the north and south points of the horizon; 11, when a planet must be so referred; 12, how to ascertain the interval between two planets when in conjunction upon such a north and south line; 13-14, dimensions of the lesser planets; 15-18, modes of exhibiting the coincidence between the calculated and actual places of the planets; 18-20, definition of different kinds of conjunction; 20-21, when a planet, in conjunction, is vanquished or victor; 22, farther definition of different kinds of conjunction; 23, usual prevalence of Venus in a conjunction; 23, planetary conjunctions with the moon; 24, conjunctions apparent only; why calculated.

1. Of the star-planets there take place, with one another, encounter (*yuddha*) and conjunction (*samâgama*); with the moon, conjunction (*samâgama*); with the sun, heliacal setting (*astamana*).

The "star-planets" (*târâgraha*) are, of course, the five lesser planets, exclusive of the sun and moon. Their conjunctions with one another and with the moon, with the asterisms (*nakshatra*), and with the sun, are the subjects of this and the two following chapters.

For the general idea of "conjunction" various terms are indifferently employed in this chapter, as *samâgama*, "coming together", *samyoga*, "conjunction," *yoga*, "junction" (in viii. 14, also, *melaka*, "meeting"): the word *yuti*, "union," which is constantly used in the same sense by the commentary, and which enters into the title of the chapter, *graha-yutyadhikâra*, does not occur anywhere in the text. The word which we translate "encounter," *yuddha*, means literally "war, conflict." Verses 18–20, and verse 22, below, give distinctive definitions of some of the different kinds of encounter and conjunction.

2. When the longitude of the swift-moving planet is greater than that of the slow one, the conjunction (*samyoga*) is past; otherwise, it is to come: this is the case when the two are moving eastward; if, however, they are retrograding (*vakrin*), the contrary is true.

3. When the longitude of the one moving eastward is greater, the conjunction (*samâgama*) is past; but when that of the one that is retrograding is greater, it is to come. Multiply the distance in longitude of the planets, in minutes, by the minutes of daily motion of each,

4. And divide the products by the difference of daily motions, if both are moving with direct, or both with retrograde, motion: if one is retrograding, divide by the sum of daily motions.

5. The quotient, in minutes, etc., is to be subtracted when the conjunction is past, and added when it is to come: if the two are retrograding, the contrary: if one is retrograding, the quotients are additive and subtractive respectively.

6. Thus the two planets, situated in the zodiac, are made to be of equal longitude, to minutes. Divide in like manner the distance in longitude, and a quotient is obtained which is the time, in days, etc.

The object of this process is to determine where and when the two planets of which it is desired to calculate the conjunction will have the same longitude. The directions given in the text are in the main so clear as hardly to require explication. The longitude and the rate of motion of the two planets in question is supposed to have been found for some time not far removed from that of their conjunction. Then, in determining whether the conjunction is past or to come, and at what distance, in arc and in time, three separate cases require to be taken into account—when both are advancing, when both are retrograding, and when one is advancing and the other retrograding. In the two former cases, the planets are approaching or receding from one another by the difference of their daily motions; in the latter, by the sum of their daily motions. The point of conjunction will be found by the following proportion: as the daily rate at which the two are approaching or receding from each other is to their distance in longitude, so is the daily motion of each one to the distance which it will have to move before, or which it has moved since, the conjunction in longitude. The time, again, elapsed or to elapse between the given moment and that of the conjunction, will be found by dividing the distance in longitude by the same divisor as was used in the other part of the process, namely the daily rate of approach or separation of the two planets.

The only other matter which seems to call for more special explanation than is to be found in the text is, at what moment the process of calculation, as thus conducted, shall commence. If a time be fixed upon which is too far removed—as, for instance, by an interval of several days—from the moment of actual conjunction, the rate of motion of the two planets will be liable to change in the mean time so much as altogether to vitiate the correctness of the calculation. It is probable that, as in the calculation of an eclipse (see above, note to iv. 7–8), we are supposed, before entering upon the particular process which is the subject of this passage, to have ascertained, by previous tentative calculations, the midnight next preceding or following the conjunction, and to have determined for that time the longitudes and rates of motion of the two planets. If so, the operation will give, without farther repetition, results having the desired degree of accuracy. The commentary, it may be remarked, gives us no light upon this point, as it gave us none in the case of the eclipse.

We have not, however, thus ascertained the time and place of the conjunction. This, to the Hindu apprehension, takes place, not when the two planets are upon the same secondary to the ecliptic, but when they are upon the same secondary to the prime vertical, or upon the same circle passing through the north and south points of the horizon. Upon such a circle two stars rise and set simultaneously; upon such a one they together pass the meridian: such a line, then, determines approximately their relative height above the horizon, each upon its own circle of daily revolution. We have also seen above, when considering the deflection (*valana*—see iv. 24–25), that a secondary to the prime vertical is regarded as determining the north and south directions upon the starry concave. To ascertain what will be the place of each planet upon the ecliptic when referred to it by such a circle is the object of the following processes.

7. Having calculated the measure of the day and night, and likewise the latitude (*vikshepa*), in minutes; having determined the meridian-distance (*nata*) and altitude (*unnata*), in time, according to the corresponding orient ecliptic-point (*lagna*)—

8. Multiply the latitude by the equinoctial shadow, and divide by twelve; the quotient multiply by the meridian-distance in nâḍîs, and divide by the corresponding half-day:

9. The result, when latitude is north, is subtractive in the eastern hemisphere, and additive in the western; when latitude is south, on the other hand, it is additive in the eastern hemisphere, and likewise subtractive in the western.

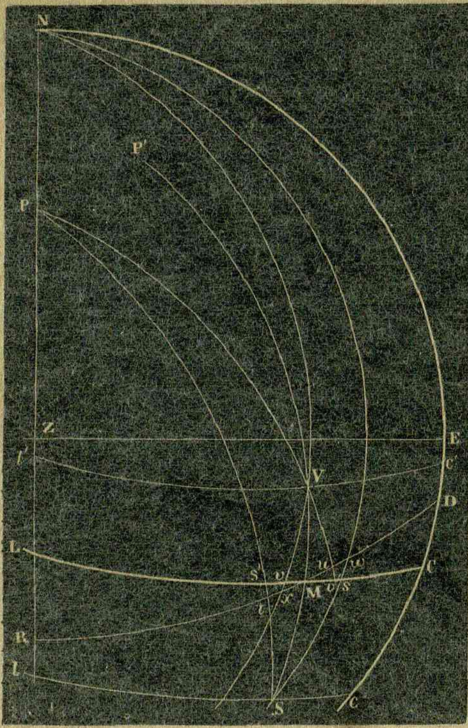
10. Multiply the minutes of latitude by the degrees of declination of the position of the planet increased by three signs: the result, in seconds (*vikalâ*), is additive or subtractive, according as declination and latitude are of unlike or like direction.

11. In calculating the conjunction (*yoga*) of a planet and an asterism (*nakshatra*), in determining the setting and rising of a planet, and in finding the elevation of the moon's cusps, this operation for apparent longitude (*dṛkkarman*) is first prescribed.

12. Calculate again the longitudes of the two planets for the determined time, and from these their latitudes: when the latter are of the same direction, take their difference; otherwise, their sum: the result is the interval of the planets.

The whole operation for determining the point on the ecliptic to which a planet, having a given latitude, will be referred by a secondary to the prime vertical, is called its *dṛkkarman*. Both parts of this compound we have had before—the latter, signifying "operation, process of calculation," in ii. 37, 42, etc.—for the former, see the notes to iii. 28–34, and v. 5–6: here we are to understand it as signifying the "apparent longitude" of a planet, when referred to the ecliptic in the manner stated, as distinguished from its true or actual longitude, reckoned in the usual way: we accordingly translate the whole term, as in verse 11, "operation for apparent longitude." The operation, like the somewhat analogous one by which the ecliptic-deflection (*valana*) is determined (see above, iv. 24–25), consists of two separate processes, which receive in the commentary distinct names, corresponding with those applied to the two parts of the process for calculating the deflection. The whole subject may be illustrated by reference to the next figure (Fig. 28). This represents the projection of a part of the sphere upon a horizontal plane, N and E being the north and east points of the horizon, and Z the zenith. Let C L be the position of the ecliptic at the moment of conjunction in longitude, C being the orient ecliptic-point (*lagna*); and let M be the point at which the conjunction in longitude of the two planets S and V, each upon its parallel of celestial latitude, *c l* and *c′ l′*, and having latitude equal to SM and VM respectively, will take place. Through V and S draw secondaries to the prime vertical, N V and N S, meeting the ecliptic in *v* and *s*: these latter are the points of apparent longitude of the two planets, which are still removed from a true conjunction by the distance *v s*: in order to the ascertainment of the time of that true conjunction, it is desired to know the positions of *v* and *s*, or their respective distances from M. From P, the pole of the equator, draw also circles through the two planets, meeting the ecliptic in *s′* and *v′*: then, in order to find M *s*, we ascertain the values of *s s'* and M *s'* ; and, in like manner, to find M *v*, we ascertain the values of *v v'* and M *v'*. Now at the equator, or in a right sphere, the circles N S and P S would coincide, and the distance *s s'* disappear: hence, the amount of *s s'* being dependent upon the latitude (*aksha*) of the observer, N P, the process by which it is calculated is called the "operation for latitude" (*akshadṛkkarman*, or else *âksha dṛkkarman*). Again, if P and P' were the same point, or if the ecliptic and equator coincided, P S and P' S would coincide, and M *s'* would disappear: hence the process of calculation of M *s'* is called the "operation for ecliptic-deviation" (*ayanadṛkkarman*, or *âyana dṛkkarman*). The latter of the two processes, although stated after the other in the text, is the one first explained by the commentary: we will also, as in the case of the deflection (note to iv. 24–25), give to it our first attention.

Fig. 28.



The point *s'*, to which the planet is referred by a circle passing through the pole P, is styled by the commentary *ayanagraha*, "the planet's longitude as corrected for ecliptic-deviation," and the distance M *s'*, which it is desired to ascertain, is called *ayanakalâs*, "the correction, in minutes, for ecliptic-deviation." Instead, however, of finding M *s'*, the process taught in the text finds M *t*, the corresponding distance on the circle of daily revolution, D R, of the point M—which is then assumed equal to M *s'*. The proportion upon which the rule, as stated in verse 10, is ultimately founded, is

$$R : \sin M S t :: M S : M t$$

the triangle M S *t*, which is always very small, being treated as if it were a plane triangle, right-angled at *t*. But now also, as the latitude M S is always a small quantity, the angle P S P' may be treated as if equal to P M P' (not drawn in the figure); and this angle is, as was shown in connection with iv. 24–25, the deflection of the ecliptic from the equator (*âyana valana*) at M, which is regarded as equal to the declination of the point 90° in advance of M: this point, for convenience's sake, we will call M'. Our proportion becomes, then

R : sin decl. M′ : : M S : M *t*

all the quantities which it contains being in terms of minutes. To bring this proportion, now, to the form in which it appears in the text, it is made to undergo a most fantastic and unscientific series of alterations. The greatest declination (ii. 28) being 24°, and its sine 1397′, which is nearly fifty-eight times twenty-four—since 58×24=1392—it is assumed that fifty-eight times the number of degrees in any given arc of declination will be equal to the number of minutes in the sine of that arc. Again, the value of radius, 3438′, admits of being roughly divided into the two factors fifty-eight and sixty—since 58×60=3480. Substituting, then, these values in the proportion as stated, we have

58×60 : 58×decl. M′ in degr. : : latitude in min. : M *t*

Cancelling, again, the common factor in the first two terms, and transferring the factor 60 to the fourth term, we obtain finally

1 : decl. M′ in degr. : : latitude in min. : M *t*×60

that is to say, if the latitude of the planet, in minutes, be multiplied by the declination, in degrees, of a point 90° in advance of the planet, the result will be a quantity which, after being divided by sixty, or reduced from seconds to minutes, is to be accepted as the required interval on the ecliptic between the real place of the planet and the point to which it is referred by a secondary to the equator.

This explanation of the rule is the one given by the commentator, nor are we able to see that it admits of any other. The reduction of the original proportion to its final form is a process to which we have heretofore found no parallel, and which appears equally absurd and uncalled for. That M *t* is taken as equivalent to M *s*′ has, as will appear from a consideration of the next process, a certain propriety.

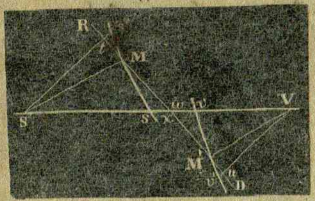
The value of the arc M *s*′ being thus found, the question arises, in which direction it shall be measured from M. This depends upon the position of M with reference to the solstitial colure. At the colure, the lines P S and P′S coincide, so that, whatever be the latitude of a planet, it will, by a secondary to the equator, be referred to the ecliptic at its true point of longitude. From the winter solstice onward to the summer solstice, or when the point M is upon the sun's northward path (*uttarâyaṇa*), a planet having north latitude will be referred backward on the ecliptic by a circle from the pole, and a planet having south latitude will be referred forward. If M, on the other hand, be upon the sun's southward path (*dakshiṇâyana*), a planet having north latitude at that point will be referred forward, and one having south latitude backward: this is the case illustrated by the figure. The statement of the text virtually agrees with this, it being evident that, when M is on the northward path, the declination of the point 90° in advance of it will be north; and the contrary.

We come now to consider the other part of the operation, or the *âksha dṛkkarman*, which forms the subject of verses 7–9. As the first step, we are directed to ascertain the day and the night respectively of the point of the ecliptic at which the two planets are in conjunction in longitude, for the purpose of determining also its distance in time from the horizon and from the meridian. This is accomplished as follows.

Having the longitude of the point in question (M in the last figure), we calculate (by ii. 28) its declination, which gives us (by ii. 60) the radius of its diurnal circle, and (by ii. 61) its ascensional difference; whence, again, is derived (by ii. 62–63) the length of its day and night. Again, having the time of conjunction at M, we easily calculate the sun's longitude at the moment, and this and the time together give us (by iii. 46–48) the longitude of C, the orient ecliptic-point: then (by iii. 50) we ascertain directly the difference between the time when M rose and that when C rises, which is the altitude in time (*unnata*) of M: the difference between this and the half-day is the meridian-distance in time (*nata*) of the same point. If the conjunction takes place when M is below the horizon, or during its night, its distance from the horizon and from the inferior meridian is determined in like manner.

The direct object of this part of the general process being to find the value of *s s′*, we note first that that distance is evidently greatest at the horizon; farther, that it disappears at the meridian, where the lines P S and N S coincide. If, then, it is argued, its value at the horizon can be ascertained, we may assume it to vary as the distance from the meridian. The accompanying figure (Fig. 29) will illustrate the method by which it is attempted to calculate *s s′* at the horizon. Suppose the planet S, being removed in latitude to the distance M S from M, the point of the ecliptic which determines its longitude, to be upon the horizon, and let *s′*, as before, be the point to which it is referred by a circle from the north pole: it is desired to determine the value of *s s′*. Let D R be the circle of diurnal revolution of the point M, meeting S *s′* in *t*, and the horizon in *w*: S *t w* may be regarded as a plane right-angled triangle, having its angles at S and *w* respectively equal to the observer's latitude and co-latitude. In that triangle, to find the value of *t w*, we should make the proportion

Fig. 29.



$$\cos t\,S\,w : \sin t\,S\,w :: t\,S : t\,w$$

Now the first of these ratios, that of the cosine to the sine of latitude, is (see above, iii. 17) the same with that of the gnomon to the equinoctial shadow: again, as the difference of M *t* and M *s′* was in the preceding process neglected, so here the difference of SM and S *t*; and finally, *tw*, the true result of the process, is accepted as the equivalent of *s′ s*, the distance sought. The proportion then becomes

gnom. : eq. shad. : : latitude : required dist. at horizon

The value of the required distance at the horizon having been thus ascertained, its value at any given altitude is, as pointed out above, determined by a proportion, as follows: as the planet's distance in time from the meridian when upon the horizon is to the value of this correction at the horizon, so is any given distance from the meridian (*nata*) to the value at that distance; or

half-day : mer.-dist. in time : : result of last proportion : required distance

The direction in which the distance thus found is to be reckoned, starting in each case from the *áyana graha*, or place of the planet on the ecliptic as determined by a secondary to the equator, which was ascertained by the preceding process, is evidently as the text states it in verse 9. In the eastern hemisphere, which is the case illustrated by the figure, *s′ s* is additive to the longitude of *s′*, while *v′ v* is subtractive from the longitude of *v′*: in the western hemisphere, the contrary would be the case. The final result thus arrived at is the longitude of the two points *s* and *v*, to which S and V are referred by the circles N S and N V, drawn through them from the north and south points of the horizon.

The many inaccuracies involved in these calculations are too palpable to require pointing out in detail. The whole operation is a roughly approximative one, of which the errors are kept within limits, and the result rendered sufficiently correct, only by the general minuteness of the quantity entering into it as its main element—namely, the latitude of a planet—and by the absence of any severe practical test of its accuracy. It may be remarked that the commentary is well aware of, and points out, most of the errors of the processes, excusing them by its stereotyped plea of their insignificance, and the merciful disposition of the divine author of the treatise.

Having thus obtained *s* and *v*, the apparent longitudes of the two planets at the time when their true longitude is M, the question arises, how we shall determine the time of apparent conjunction. Upon this point the text gives us no light at all: according to the commentary, we are to repeat the process prescribed in verses 2–6 above, determining, from a consideration of the rate and direction of motion of the planets in connection with their new places, whether the conjunction sought for is past or to come, and then ascertaining, by dividing the distance *v s* by their daily rate of approach or recession, the time of the conjunction. It is evident, however, that one of the elements of the process of correction for latitude (*akshadṛkkarman*), namely the meridian-distance, is changing so rapidly, as compared with the slow motion of the planets in their orbits, that such a process could not yield results at all approaching to accuracy: it also appears that two slow-moving planets might have more than one, and even several apparent conjunctions on successive days, at different times in the day, being found to stand together upon the same secondary to the prime vertical at different altitudes. We do not see how this difficulty is met by anything in the text or in the commentary. The text, assuming the moment of apparent conjunction to have been, by whatever method, already determined, goes on to direct us, in verse 12, to calculate anew, for that moment, the latitudes of the two planets, in order to obtain their distance from one another. Here, again, is a slight inaccuracy: the interval between the two, measured upon a secondary to the prime vertical, is not precisely equal to the sum or difference of their latitudes, which are measured upon secondaries to the ecliptic. The ascertainment of this interval is necessary, in order to determine the name and character of the conjunction, as will appear farther on (vv. 18–20, 22).

The cases mentioned in verse 11, in which, as well as in calculating the conjunctions of two planets with one another, this operation for apparent longitude (*dṛkkarman*) needs to be performed, are the subjects of the three following chapters.

13. The diameters upon the moon's orbit of Mars, Saturn, Mercury, and Jupiter, are declared to be thirty, increased successively by half the half; that of Venus is sixty.

14. These, divided by the sum of radius and the fourth hypothenuse, multiplied by two, and again multiplied by radius, are the respective corrected (*sphuṭa*) diameters: divided by fifteen, they are the measures (*mâna*) in minutes.

We have seen above, in connection with the calculation of eclipses (iv. 2–5), that the diameters of the sun, moon, and shadow had to be reduced, for measurement in minutes, to the moon's mean distance, at which fifteen yojanas make a minute of arc. Here we find the dimensions of the five lesser planets, when at their mean distances from the earth, stated only in the form of the portion of the moon's mean orbit covered by them, their absolute size being left undetermined. We add them below, in a tabular form, both in yojanas and as reduced to minutes, appending also the corresponding estimates of Tycho Brahe (which we take from Delambre), and the true apparent diameters of the planets, as seen from the earth at their greatest and least distances.

Apparent Diameters of the Planets, according to the Sûrya-Siddhânta, to Tycho Brahe, and to Modern Science.

Planet.	Sûrya-Siddhânta: in yojanas.	Sûrya-Siddhânta: in arc.	Tycho Brahe.	Moderns: least.	Moderns: greatest.
Mars,	30	2′	1′ 40″	4″	27″
Saturn,	37½	2′ 30″	1′ 50″	15″	21″
Mercury,	45	3′	2′ 10″	4″	12″
Jupiter,	52½	3′ 30″	2′ 45″	30″	49″
Venus,	60	4′	3′ 15″	9″	1′ 14″

This table shows how greatly exaggerated are wont to be any determinations of the magnitude of the planetary orbs made by the unassisted eye alone. This effect is due to the well-known phenomenon of the irradiation, which increases the apparent size of a brilliant body when seen at some distance. It will be noticed that the Hindu estimates do not greatly exceed those of Tycho, the most noted and accurate of astronomical observers prior to the invention of the telescope. In respect to order of magnitude they entirely agree, and both accord with the relative apparent size of the planets, except that to Mercury and Venus, whose proportional brilliancy, from their nearness to the sun, is greater, is assigned too high a rank. Tycho also established a scale of apparent diameters for the fixed stars, varying from 2′, for the first magnitude, down to 20″, for the sixth. We do not find that Ptolemy made any similar estimates, either for planets or for fixed stars.

The Hindus, however, push their empiricism one step farther, gravely laying down a rule by which, from these mean values, the true values of the apparent diameters at any given time may be found. The fundamental proportion is, of course,

true dist. : mean dist. : : mean app. diam. : true app. diam.

The second term of this proportion is represented by radius: for the first we have, according to the translation given, one half the sum of radius and the fourth hypothenuse, by which is meant the "variable hypothenuse" (*cala karṇa*) found in the course of the fourth, or last, process for finding the true place of the planet (see above, ii. 43–45). The term, however (*tricatuḥkarṇa*), which is translated "radius and the fourth hypothenuse" is much more naturally rendered "third and fourth hypothenuses"; and the latter interpretation is also mentioned by the commentator as one handed down by tradition (*sâmpradâyika*): but, he adds, owing to the fact that the length of the hypothenuse is not calculated in the third process, that for finding finally the equation of the centre (*mandakarman*), and that that hypothenuse cannot therefore be referred to here as known, modern interpreters understand the first member of the compound (*tri*) as an abbreviation for "radius" (*trijyâ*), and translate it accordingly. We must confess that the other interpretation seems to us to be powerfully supported by both the letter of the text and the reason of the matter. The substitution of *tri* for *trijyâ* in such a connection is quite too violent to be borne, nor do we see why half the sum of radius and the fourth hypothenuse should be taken as representing the planet's true distance, rather than the fourth hypothenuse alone, which was employed (see above, ii. 56–58) in calculating the latitude of the planets. On the other hand, there is reason for adopting, as the relative value of a planet's true distance, the average, or half the sum, of the third hypothenuse, or the planet's distance as affected by the eccentricity of its orbit, and the fourth, or its distance as affected by the motion of the earth in her orbit. There seems to us good reason, therefore, to suspect that verse 14—and with it, probably, also verse 13—is an intrusion into the Sûrya-Siddhânta from some other system, which did not make the grossly erroneous assumption, pointed out under ii. 39, of the equality of the sine of anomaly in the epicycle (*bhujajyâphala*) with the sine of the equation, but in which the hypothenuse and the sine of the equation were duly calculated in the process for finding the equation of the apsis (*mandakarman*), as well as in that for finding the equation of the conjunction (*çîghrakarman*).

15. Exhibit, upon the shadow-ground, the planet at the extremity of its shadow reversed: it is viewed at the apex of the gnomon in its mirror.

As a practical test of the accuracy of his calculations, or as a convincing proof to the pupil or other person of his knowledge and skill, the teacher is here directed to set up a gnomon upon ground properly prepared for exhibiting the shadow, and to calculate and lay off from the base of the gnomon, but in the opposite to the true direction, the shadow which a planet would cast at a given time; upon placing, then, a horizontal mirror at the extremity of the shadow, the reflected image of the planet's disk will be seen in it at the given time by an eye placed at the apex of the gnomon. The principle of the experiment is clearly correct, and the rules and processes taught in the second and third chapters afford the means of carrying it out, since from them the shadow which any star would cast, had it light enough, may be as readily determined as that which the sun actually casts. As no case of precisely this character has hitherto been presented, we will briefly indicate the course of the calculation. The day and night of the planet, and its distance from the meridian, or its hour-angle, are found in the same manner as in the process previously explained (p. 312, above), excepting that here the planet's latitude, and its declination as affected by latitude, must be calculated, by ii. 56–58; and then the hour-angle and the ascensional difference, by iii. 34–36, give the length of the shadow at the given time, together with that of its hypothenuse. The question would next be in what direction to lay off the shadow from the base of the gnomon. This is accomplished by means of the base (*bhuja*) of the shadow, or its value when projected on a north and south line. From the declination is found, by iii. 20–22, the length of the noon-shadow and its hypothenuse, and from the latter, with the declination, comes, by iii. 22–23, the measure of amplitude (*agrâ*) of the given shadow; whence, by iii. 23–25, is derived its base. Having thus both its length and the distance of its extremity from an east and west line running through the base of the gnomon, we lay it off without difficulty.

16. Take two gnomons, five cubits (*hasta*) in height, stationed according to the variation of direction, separated by the interval of the two planets, and buried at the base one cubit.

17. Then fix the two hypothenuses of the shadow, passing from the extremity of the shadow through the apex of each gnomon: and, to a person situated at the point of union of the extremities of the shadow and hypothenuse, exhibit

18. The two planets in the sky, situated at the apex each of its own gnomon, and arrived at a coincidence of observed place (*dr̥ç*). . . .

This is a proceeding of much the same character with that which forms the subject of the preceding passage. In order to make apprehensible, by observation, the conjunction of two planets, as calculated by the methods of this chapter, two gnomons, of about the height of a man, are set up. At what distance and direction from one another they are to be fixed is not clearly shown. The commentator interprets the expression "interval of the two planets" (v. 16), to mean their distance in minutes on the secondary to the prime vertical, as ascertained according to verse 12, above, reduced to digits by the method taught in iv. 26; while, by "according to the variation of direction," he would understand merely, in the direction from the observer of the hemisphere in which the planets at the moment of conjunction are situated. The latter phrase, however, as thus explained, seems utterly nugatory; nor do we see of what use it would be to make the north and south interval of the bases of the gnomons, in digits, correspond with that of the planets in minutes. We do not think it would be difficult to understand the directions given in the text as meaning, in effect, that the two gnomons should be so stationed as to cast their shadows to the same point: it would be easy to do this, since, at the time in question, the extremities of two shadows cast from one gnomon by the two stars would be in the same north and south line, and it would only be necessary to set the second gnomon as far south of the first as the end of the shadow cast by the southern star was north of that cast by the other. Then, if a hole were sunk in the ground at the point of intersection of the two shadows, and a person enabled to place his eye there, he would, at the proper moment, see both the planets with the same glance, and each at the apex of its own gnomon.

In the eighteenth verse also we have ventured to disregard the authority of the commentator: he translates the words *dṛktulyatâm itâs* "come within the sphere of sight," while we understand by *dṛktulyatâ*, as in other cases (ii. 14, iii. 11), the coincidence between observed and computed position.

Such passages as this and the preceding are not without interest and value, as exhibiting the rudeness of the Hindu methods of observation, and also as showing the unimportant and merely illustrative part which observation was meant to play in their developed system of astronomy.

18. . . . When there is contact of the stars, it is styled "depiction" (*ullekha*); when there is separation, "division" (*bheda*);

19. An encounter (*yuddha*) is called "ray-obliteration" (*ançuvimarda*) when there is mutual mingling of rays: when the interval is less than a degree, the encounter is named "dexter" (*apasavya*)—if, in this case, one be faint (*aṇu*).

20. If the interval be more than a degree, it is "conjunction" (*samâgama*), if both are endued with power (*bala*). One that is vanquished (*jita*) in a dexter encounter (*apasavya yuddha*), one that is covered, faint (*aṇu*), destitute of brilliancy,

21. One that is rough, colorless, struck down (*vidhvasta*), situated to the south, is utterly vanquished (*vijita*). One situated to the north, having brilliancy, large, is victor (*jayin*)—and even in the south, if powerful (*balin*).

22. Even when closely approached, if both are brilliant, it is "conjunction" (*samâgama*): if the two are very small, and struck down, it is "front" (*kûṭa*) and "conflict" (*vigraha*), respectively.

23. Venus is generally victor, whether situated to the north or to the south. . . .

In this passage, as later in a whole chapter (chap. xi), we quit the proper domain of astronomy, and trench upon that of astrology. However intimately connected the two sciences may be in practice, they are, in general, kept distinct in treatment—the Siddhântas, or astronomical text-books, furnishing, as in the present instance, only the scientific basis, the data and methods of calculation of the positions of the heavenly bodies, their eclipses, conjunctions, risings and settings, and the like, while the Sanhitâs, Jâtakas, Tâjikas, etc., the astrological treatises, make the superstitious applications of the science to the explanation of the planetary influences, and their determination of human fates. Thus the celebrated astronomer, Varâha-mihira, besides his astronomies, composed separate astrological works, which are still extant, while the former have become lost. It is by no means impossible that these verses may be an interpolation into the original text of the Sûrya-Siddhânta. They form only a disconnected fragment: it is not to be supposed that they contain a complete statement and definition of all the different kinds of conjunction recognized and distinguished by technical appellations; nor do they fully set forth the circumstances which determine the result of a hostile "encounter" between two planets: while a detailed explanation of some of the distinctions indicated—as, for instance, when a planet is "powerful" or the contrary—could not be given without entering quite deeply into the subject of the Hindu astrology. This we do not regard ourselves as called upon to do here: indeed, it would not be possible to accomplish it satisfactorily without aid from original sources which are not accessible to us. We shall content ourselves with following the example of the commentator, who explains simply the sense and connection of the verses, as given in our translation, citing one or two parallel passages from works of kindred subject. We would only point out farther that it has been shown in the most satisfactory manner (as by Whish, in Trans. Lit. Soc. Madras, 1827; Weber, in his Indische Studien, ii. 236 etc.) that the older Hindu science of astrology, as represented by Varâha-mihira and others, reposes entirely upon the Greek, as its later forms depend also, in part, upon the Arab; the latter connection being indicated even in the common title of the more modern treatises, *tâjika*, which comes from the Persian *tâzî*, "Arab." Weber gives (Ind. Stud. ii. 277 etc.) a translation of a passage from Varâha-mihira's lesser treatise, which states in part the circumstances determining the "power" of a planet in different situations, absolute or relative: partial explanations upon the same subject furnished to the translator in India by his native assistant, agree with these, and both accord closely with the teachings of the Tetrabiblos, the astrological work attributed to Ptolemy.

23\. . . . Perform in like manner the calculation of the conjunction (*samyoga*) of the planets with the moon.

This is all that the treatise says respecting the conjunction of the moon with the lesser planets: of the phenomenon, sometimes so striking, of the occultation of the latter by the former, it takes no especial notice. The commentator cites an additional half-verse as sometimes included in the chapter, to the effect that, in calculating a conjunction, the moon's latitude is to be reckoned as corrected by her parallax in latitude (*avanati*), but rejects it, as making the chapter over-full, and as being superfluous, since the nature of the case determines the application here of the general rules for parallax presented in the fifth chapter. Of any parallax of the planets themselves nothing is said: of course, to calculate the moon's parallax by the methods as already given is, in effect, to attribute to them all a horizontal parallax of the same value with that assigned to the sun, or about 4′.

The final verse of the chapter is a caveat against the supposition that, when a "conjunction" of two planets is spoken of, anything more is meant than that they appear to approach one another; while nevertheless, this apparent approach requires to be treated of, on account of its influence upon human fates.

24\. Unto the good and evil fortune of men is this system set forth: the planets move on upon their own paths, approaching one another at a distance.

CHAPTER VIII.

OF THE ASTERISMS.

CONTENTS:—1-9, positions of the asterisms; 10-12, of certain fixed stars; 12, direction to test by observation the accuracy of these positions; 13, splitting of Rohinî's wain; 14-15, how to determine the conjunction of a planet with an asterism; 16-19, which is the junction-star in each asterism; 20-21, positions of other fixed stars.

1. Now are set forth the positions of the asterisms (*bha*), in minutes. If the share of each one, then, be multiplied by ten, and increased by the minutes in the portions (*bhoga*) of the past asterisms (*dhishṇya*), the result will be the polar longitudes (*dhruva*).

The proper title of this chapter is *nakshatragrahayutyadhikâra*, "chapter of the conjunction of asterisms and planets," but the subject of conjunction occupies but a small space in it, being limited to a direction (vv. 14-15) to apply, with the necessary modifications, the methods taught in the preceding chapter. The chapter is mainly occupied with such a definition of the positions of the asterisms—to which are added also those of a few of the more prominent among the fixed stars—as is necessary in order to render their conjunctions capable of being calculated.

Before proceeding to give the passage which states the positions of the asterisms, we will explain the manner in which these are defined. In the accompanying figure (Fig. 30), let E L represent the equator, and C L the ecliptic, P and P′ being their respective poles. Let S be the position of any given star, and through it draw the circle of declination P S *a*. Then *a* is the point on the ecliptic of which the distance from the first of Aries and from the star respectively are here given as its longitude and latitude. So far as the latitude is concerned, this is not unaccordant with the usage of the treatise hitherto. Latitude (*vikshepa*, "disjection") is the amount by which any body is removed from the declination which it ought to have—that is, from the point of the ecliptic which it ought to occupy—declination (*krânti*, *apakrama*) being always, according to the Hindu understanding of the term, in the ecliptic itself. In the case of a planet, whose proper path is in the ecliptic, the point of that circle which it ought to occupy is determined by its calculated longitude: in the case of a fixed star, whose only motion is about the pole of the heavens, its point of declination is that to which it is referred by a

Fig. 30.