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Glimpses of cosmic menagerie through S. Chandrasekhar's eyes

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Abstract

This article provides an exposition to Subrahmanyan Chandrasekhar's seminal contributions to wide ranging topics in astrophysics - from white dwarf mass limit to blackholes and gravitational waves, in the light of observational astronomy. Most of the results obtained by Chandrasekhar led to far reaching consequences in astrophysics, many of them experimentally validated.

Chandrasekhar Limit

Subrahmanyan Chandrasekhar's astrophysical journey had begun in the year 1928 with the publication of his first paper when he was barely 18 years old[1]. His impactful voyage across oceans began on July 31, 1930, from Gateway of India at Bombay. The Italian liner, the Lloyd Triestino, on its way through the stormy seas onward to Venice, had as its passenger, 19 year old Chandra on his way to England, with a government scholarship, for higher studies. During that momentous voyage, he (who had chewed and digested Fowler's work on white dwarfs[2]) was struggling with the mathematical equations concerning very dense stars, known as white dwarfs, whose beauty and elegance moved and motivated young Chandra to unravel the underlying physical consequences.

What are white dwarfs? Before answering this question, let us have a quick glance at the stellar evolution theory. Nuclear fusion of hydrogen to helium is the main fuel store of a main sequence star like Sun that provides necessary thermal energy to stall the gravitational contraction, and helps the star in attaining a quasi-hydrostatic equilibrium. As the star advances in its life cycle, a further sequence of nuclear fusion reactions play themselves out in its core - helium burning to carbon and oxygen, carbon burning to sodium and magnesium and so on, if the star is massive enough, till the eventual formation of iron rich core. Iron nucleus being the most stable one, further nuclear burning cease to take place. As the star cools, it collapses under its own weight, till the electron density becomes so high that electron degeneracy pressure prevents subsequent contraction.

What is electron degeneracy pressure? In quantum mechanics, Pauli exclusion principle (PEP) states that no two identical fermions can have the same state. Electrons, being spin half particles, are fermions. According to PEP, in a gravitationally bound system like the iron-rich core of an evolved star, all the electrons cannot occupy the lowest energy level (unlike the bosons in Bose-Einstein condensates). Therefore, the energy levels have to be filled up with two electrons per orbital (one spin up and other spin down, so that PEP is maintained). Hence, more the density of electrons, higher is the energy level that gets to be occupied.

Gravitational contraction of such a dense core leads to an increase in electron density, thereby facing a resistance since the contraction implies putting electrons at higher energy levels. So, in such a degenerate system, gravitational collapse instead of lowering the energy of the star tends to increase it. The resulting pressure against contraction, arising out of PEP in such electron rich dense matter is called electron degeneracy pressure (EDP). A white dwarf is a star that is in hydrostatic equilibrium not because of thermal pressure but due to the EDP that counteracts gravitational contraction. Assuming that electrons could be treated non-relativistically, Fowler had shown that the EDP in the core of a white dwarf is proportional to $\rho^{5/3}$, where ρ is the density of the core[2].

On board Lloyd Triestino, while studying the white dwarf problem, Chandra had realized that at such incredible densities inside a white dwarf, electrons would be occupying very high energy levels, whizzing around with velocities close to that of light. He proceeded to incorporate special relativity in the analysis, and found that the EDP is now proportional to $\rho^{4/3}$.

Performing an accurate study of the relativistic problem of a dense star ruled by a polytropic equation of state, in which gravity was countered by the EDP, he arrived at the celebrated Chandrasekhar mass limit[3],

$$M_{Ch} = \frac{0.2}{(m_p \mu_e)^2} \left(\frac{\hbar c}{G}\right)^{3/2}$$

where $\hbar$, G, c, m_p and μ_e are the reduced Plank's constant, Newton's gravitational constant, speed of light, mass of a proton and mean molecular weight per electron, respectively. It is remarkable that a significant result concerning stars should be expressible in terms of fundamental quantities (except for μ_e). In white dwarfs, the value of μ_e is such that the above expression leads to

$$M_{Ch} \approx 1.4\ M_{\odot}, \text{ where } M_{\odot} = 2 \times 10^{30}\ \text{kg},$$

is the sun's mass.

Chandra was unaware initially that Anderson in 1929 and Stoner in 1930 had independently applied special relativity to obtain mass limits for a degenerate, dense star of uniform density without taking into account the condition of hydrostatic equilibrium[4,5,7]. Fowler pointed this out to him when Chandra reached Cambridge, and he added these references to his papers on relativistic degeneracy in white dwarf stars[6].

The Chandrasekhar mass limit implies that no white dwarf with mass greater than this limit can hold out against gravitational collapse. So far, all the white dwarfs discovered (like Sirius B, the companion star to Sirius) in the cosmos, have mass less than M_{Ch}. For masses beyond this limit, Eddington had found the idea of a star shrinking to a point absurd. After about three decades, Penrose and Hawking, making use of Raychaudhuri equation proved the celebrated singularity theorems according to which collapse of normal matter indeed lead to point singularities, namely, the blackholes[8-10]. We shall discuss towards the end, Chandra's contribution to the mathematical theory of blackholes.

Dynamical Friction

Chandra played a significant role in the research area of stellar dynamics from 1939 to 1944 that culminated in the publication of his celebrated papers on dynamical friction[11,12].

Cosmos is filled with gravitationally bound systems of massive objects like globular clusters, galaxies, clusters of galaxies, etc. Objects that make up these bound systems, apart from moving in gravitational potential wells, also suffer two-body gravitational encounters, resulting in exchange of energy and momentum. It was Chandra who showed for the first time that a massive body in motion, surrounded by a swarm of other less massive objects, suffers deceleration that is proportional to its mass[11].

Dynamical friction arises out of cumulative gravitational encounters that the massive body suffers due to the presence of other objects in the background. The origin of dynamical friction can be intuitively understood by going to the reference frame in which the body is at rest. In this frame, the swarm of background objects while moving past the massive body get gravitationally focused behind the body, forming a wake of higher mass density. Now, switching back to the frame in which the massive body is moving, we find that the mass density of the wake behind is greater than the density ahead. Consequently, because of a greater gravitational pull from behind, the massive body experiences a gravitational drag force whose magnitude is proportional to the square of its mass and inversely proportional to the square of its speed[13-15].

Observational consequences of dynamical friction include sinking of globular clusters towards the central regions of galaxies and galactic cannibalism in which the orbit of a satellite galaxy decays, leading eventually to its merger with the bigger galaxy[14].

Negative Hydrogen Ion

Around the same time, Chandra was also involved with the quantum mechanics of negative hydrogen ion. Can a proton capture two electrons to form a charged bound state? How is it relevant to astrophysics? The first issue had, once for all, been settled by Bethe in a 1929 paper, that according to quantum theory, H^- ion can indeed form[16]. Coming to the second question, it has been found over the years that H^- is a weakly bound system with a binding energy of

$$\approx 0.75\ \text{eV}.$$

As it takes only about 0.75 eV to knock off the extra electron from H^-, its life-time under terrestrial conditions is small but in thin and tenuous plasma where the collision frequency is low, one expects negative hydrogen ions to survive for longer duration.

So, early on, Wildt had foreseen that because of copious presence of hydrogen atoms and electrons in the upper atmosphere of Sun, H^- would form and photo-detachment of H^- would contribute majorly to solar opacity[17-19].

At this juncture, Chandra and his collaborators played an important role in calculating H^- photo-absorption matrix element, so crucial for opacity estimation[20-26].

In 1943, Chandrasekhar and Krogdahl drew attention to the fact that dominant contribution to this matrix element came from the wavefunction at large distances (several times Bohr radius), and therefore an accurate knowledge of electronic wavefunction of H^- was required[20].

The negative hydrogen ion has only the ground state as a bound state with the possibility of singly excited states ruled out. As a result, photons with energy above 0.75 eV, executing random walks out of Sun due to multiple scatterings, would be absorbed by H^- ions after detaching their extra electrons to the continuum. This is the dominant cause for solar opacity in the infra-red to visible range.

Chandra and his collaborators made seminal contributions towards calculating the continuous absorption coefficient κ_λ of H^- as a function of the photon wavelength λ, incorporating dipole-length and dipole-velocity formulae, that provided a solid theoretical foundation for the characteristic κ_λ- λ plot which exhibits a rise in the range 4000 to 9000 angstroms, then dropping to a minimum at 16000 angstroms, with a subsequent rise[27].

Interestingly, the negatively charged hydrogen ion has been in great demand for cyclotrons and particle accelerators[28]. Fascination for H^- arises out of the possibility of using it for heating neutral beams in Tokamaks (like in ITER), because of the relative ease in detaching its extra electron when these ions are present in gas cells[29].

Magnetohydrodynamics and Chandrasekhar number

Almost all cosmic entities are threaded with magnetic fields, be it planets like Earth, Jupiter, etc., sunspots, stars, spiral arms of Milky Way, galaxies and so on. Magnetic field in conducting medium like metal or plasma decays due to Ohmic dissipation. So, how does terrestrial magnetic field generated by the electric currents flowing in the molten, conducting and rotating earth's core prevent itself from Ohmic decay?

Dynamo theories involving magnetohydrodynamics strive to provide explanations to this conundrum. In this paradigm, differential rotation and convection in conducting fluid are invoked to generate steady magnetic fields. Cowling had proved that a completely axisymmetric geometry for magnetohydrodynamic flow will always result in a decaying magnetic field[30].

Two decades later, Backus and Chandrasekhar generalized Cowling's theorem[31]. In this context, Chandra tackled the possibility of increasing the decay duration so that an axisymmetric dynamo is still relevant for geomagnetism[32]. But soon Backus showed that the increase was not large enough to be of geophysical interest[33].

Chandra also studied several fluid dynamical stability problems employing variational methods that had far reaching consequences[34,35].

A stellar binary system consisting of an expanding star, spewing out gaseous matter, and a massive compact object (MCO) like a neutron star or a blackhole (BH) going around the centre of mass, is often invoked to explain astronomical sources emitting high energy photons. In such a binary system, gas from the bloated star cannot radially fall on the MCO because it possesses angular momentum. Instead, it spirals inwards and forms an accretion disc around the MCO, with each tiny gaseous element of the disc moving in a circular Keplerian orbit[36]. Fluid elements of the disc rotate differentially so that farther the element from the MCO, lower is its circular speed. Differential rotation leads to viscous rubbing of neighbouring fluid elements, causing the accretion disc to become so hot that it emits copious electromagnetic radiation from visible spectrum to UV and X-rays.

In fact, there are strong observational evidences that the rapidly time varying, intense X-ray sources like Cygnus X-1 are accreting blackholes. Essentially, the gravitational potential energy of the gas falling in, due to MCO's gravity, gets converted into radiative energy. But, for the conversion efficiency to be large, the accretion disc is required to have a high viscosity. The physics of the mechanism responsible for large viscosities in the disc is an active area of research.

Interestingly, as shown by Balbus and Hawley in 1991, the Chandrasekhar instability might be the key to the origin of accretion disc viscosity[37]. Chandra had pointed out that a differentially rotating, conducting and magnetized incompressible fluid in a cylindrical configuration. is unstable with respect to oscillating axisymmetric perturbations[34].

While investigating Rayleigh-Benard convection in conducting and viscous fluids threaded with magnetic field, Chandra studied the onset of convection and its dependence on a dimensionless number Q, representing the square of the ratio of magnetic force to viscous force[34]. Today, this number Q is referred to as Chandrasekhar number (or, also as the square of Hartmann number).

Chandra made several other contributions in the field of plasma physics and magnetohydrodynamics that had far reaching consequences[38].

Rotating self-gravitating fluids and Chandrasekhar-Friedman-Schutz instability

In the late 1960s, Chandra turned his attention to incompressible fluids in gravitational equilibrium[39]. While studying self-gravitating and rotating fluid configurations, Chandra showed that a uniformly dense and uniformly rotating incompressible spheroid is unstable because of non-radial perturbations emitting gravitational radiation[40]. Later, Friedman and Schutz in 1978 demonstrated the existence of this gravitational wave driven instability in the general case of rotating and self-gravitating stars made of perfect fluid[41].

A physically intuitive way to understand this Chandrasekhar-Friedman-Schutz (CFS) instability is to look at a perturbation mode in a rotating star that is retrograde, i.e. moving in the backward sense relative to the fluid element going around. According to general relativity, the space-time geometry around a rotating body is such that inertial frames are dragged along the direction of rotation (This has been recently verified by the Gravity Probe B satellite-borne experiment[42]). This frame dragging, therefore, would make the retrograde mode appear prograde to an inertial observer far away from the star. Gravitational waves emitted by this mode will carry positive angular momentum (i.e. having the same sense as the angular momentum of the fluid element) as measured in the distant inertial frame. Since, the total angular momentum is conserved, gravitational radiation carrying away positive angular momentum from the mode, makes the retrograde mode go around more rapidly in the opposite direction, leading to an instability.

Andersson in 1998 showed that a class of toroidal perturbations (the so called r-modes) in a rotating star are generically unstable because of the gravitational wave driven CFS instability[43]. Close on heels, it was demonstrated that the r-mode instability would put brakes on the rotation of a newly born and rapidly spinning neutron star[44, 45]. Consequently, as the neutron star spins down, a substantial amount of its rotational energy is radiated away as gravitational waves, making it a likely candidate for future detection by the laser interferometric gravitational wave detectors, namely, the LIGOs[46]. Thanks to LIGOs, the CFS instability, discovered by Chandra for the first time, may soon be put to experimental tests.

Blackholes

In his book on blackholes (BHs), Chandra called the astrophysical BHs the most perfect macroscopic objects[47]. Things macroscopic - like chairs, books, computers, etc. around us, require an astronomically large number of characteristics each for their description. For instance, just to specify a sugar cube would need not only its mass, density, temperature, but also amount and nature of trace compounds present, the stacking of sugar molecules, porosity, surface granularities, etc. On the other hand, a BH is characterized by just three real parameters - its mass, charge and angular momentum.

But do BHs exist? As classical BHs by themselves do not emit any radiation (Hawking radiation, which is of quantum mechanical origin, from astrophysical BHs, is too miniscule in amount to be of any observational significance[48]), how does one find them? In conventional astronomy, their detection relies on the presence of gas or stars in their vicinity and the ensuing stellar or dissipative gas dynamics around an accreting MCO.

As discussed earlier, if the MCO has an accretion disc around it like in galactic X-ray sources, quasars, blazars or radio-galaxies, the swirling and inward spiralling gas gets heated up, emitting radio, optical, UV and X-ray photons, often accompanied by large scale jets[49]. From causality arguments applied to time variability (on the scale of few hours in blazars) and from spectral nature (e.g. presence of the big blue bump in quasar spectrum) of the radiation emanating from such an accretion disc, one can infer the presence of a BH. The models that provide the best explanations for the observational data about quasars, blazars, radio-galaxies, etc. involve an accreting supermassive blackhole, having a mass in excess of $10^6 M_\odot$ [49].

Similarly, by monitoring stellar dynamics around the central region of Milky Way for decades, one infers that the Galactic nucleus contains a heavy and compact object, most likely to be a supermassive BH with a mass of about $4 \times 10^6 M_\odot$, within a radius of 10^{18} cm from the Galactic Centre[50]. It is interesting to note that the Chandra X-ray observatory (launched on July 23, 1999, and named after S. Candrasekhar) revealed the presence of a X-ray source as well as hot gas with high pressure and strong magnetic field in the vicinity of the Galactic Centre.

However, these are indirect detections, implying strictly speaking the presence of a very compact, massive central object. Inference of an astrophysical BH, although very likely, relies on theoretical interpretation. BHs are characterized by a fictitious spherical surface called the event horizon centred around the point singularity created by the collapse of matter. Nothing can escape from regions enclosed within the event horizon. What happens when a BH is perturbed by incident gravitational waves or electromagnetic radiation or Dirac waves describing electrons or neutrinos? Does a perturbed BH have a signature emission like a 'ringing', analogous to the case of a struck bell? To answer such questions, Chandra devoted himself to studying BH perturbations from 1970s onwards[47,53-57].

When a BH is perturbed, the curved space-time geometry around the BH will be subjected to metric fluctuations. For sufficiently small perturbations, a linear analysis of the metric fluctuations can be carried out in terms of normal modes except that dissipation due to both emission of gravitational waves as well as energy loss caused by BH absorption makes the mode frequencies complex, with the decay reflected in the imaginary parts. In the case of a perturbed BH, such quasi-normal modes (QNMs) correspond to a characteristic ringing that eventually decays due to

QNMs were discovered by Vishveshwara[51] and Press[52] while studying gravitational wave perturbations of BHs. Chandra and Detweiler suggested for the first time numerical methods for calculating the QNM frequencies[55]. Such investigations throw light on methods for direct detection of BHs. For example, matter falling into a Schwarzschild BH would lead to excitation of QNMs, resulting in emission of gravitational waves with a characteristic frequency that is inversely proportional to the BH mass. A supermassive BH with mass $10^6 M_\odot$ would ring with a frequency of about 10^{-2} Hz. Because of seismic noise, LIGOs cannot detect gravitational waves having such low frequencies. Only a space-based gravitational wave detector like LISA (Laser Interferometer Space Antenna) can pick up such low frequency signals from supermassive BHs.

In 1983, Chandra was awarded the Nobel prize in Physics. The lesson we learn from his way of researching astrophysical topics is that one can make accurate and far reaching predictions concerning cosmic entities, by using standard laws of physics applicable to these problems, and then subjecting them to rigorous mathematical analysis. Looking at Chandra's achievements, one is overwhelmed by the diversity of his research contributions, most of which have experimental validation.

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