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A Journal of Science Communication
VOLUME 4, NO.2, JULY, 2013 & VOLUME 5, NO.1, JANUARY, 2014

Presenting Abstract Ideas in Science Museums/Centres : Mathematics Gallery of BITM, Kolkata - A Case Study

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Abstract

Mathematical concepts and techniques allow us to see hidden patterns in the various phenomena happening around us in the natural world and in human societies. As a curricular subject, mathematics evokes strong emotions from its learners – loved by some and hated by most. The general perception is that mathematics is very abstract, hard to visualize and difficult to connect to physical realities. The new gallery on 'Mathematics' at BITM was established in the year 2010 to change this notion about mathematics. This article provides a brief overview of the gallery and how it explains various abstract mathematical concepts through interactive models and hands-on activities to help young learners enjoy mathematics in a playful manner and appreciate its inner order and beauty.

Introduction

Communicating a difficult concept in a simple manner is an art, and it becomes all the more challenging if these concepts relate to high school mathematics curriculum. The Mathematics Gallery, which opened in the year 2010 at the Birla Industrial & Technological Museum (BITM), responded to this challenge by offering an enjoyable encounter with mathematics through a number of three-dimensional interactive models and hands-on activities. The physicality and interactivity of the gallery encourages young learners to explore and discover the underlying ideas behind some of the abstract formulations they often stumble over in high school mathematics.

Figure 1. Entrance to the Mathematics Gallery at BITM, Kolkata

The entrance to the gallery is adorned with motifs that symbolically link the subject matter and an inspiring quotation from Einstein which says: *"Do not worry about your difficulties in Mathematics; I can assure you that mine are greater"*.

Historical Background

The gallery begins with a brief introduction of the history of mathematics. Two scaled down dioramas (Figure 2) have been used to show how ancient people used to count and keep record of their counts in their daily life activities like farming. These are followed by fibre glass plaques depicting the ancient Roman and Hindu-Arabic numeral systems.

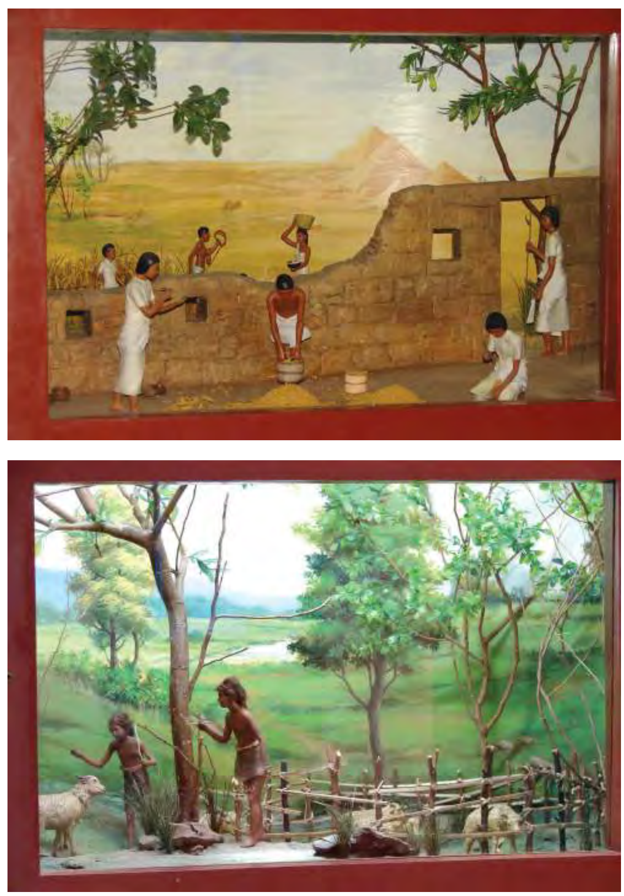


Figure 2. Above two dioramas depict how ancient people used to count and keep record of their counts and measurements.

Pioneering works by ancient mathematicians finds a special place in this section. An innovative multimedia presentation (Figure 3) on the life and works of some of the early pioneers of mathematics allows the visitors to explore and appreciate how mathematical ideas and formulations evolved in different cultures of the world in the early part of its history.

Figure 3. Visitors interact with the exhibit on 'Ancient Mathematicians'

Numbers and Number Systems

Many abstract concepts in mathematics namely, Number System, Series & Progression, Plane and Solid Geometry, Algebra, Functions and Variables, and non-Euclidean Spherical Geometry are elucidated in the gallery with the help of 3-dimensional interactive exhibits which the students can play with and understand the underlying concepts. For example, a Number Line is shown as a vertical line in a mock well in which the surface of water is considered '0' (zero). Any value above this level is positive, while those below are taken as negative numbers. Using an electro-mechanical circuit, a toy frog (used as an indicator) can be made to move along this number line and stop at a number which is the algebraic sum of any two numbers, one positive and one negative. For example, when a visitor inputs +5 and -3, the 'frog' moves and stops above the water surface at + 2 level. But when one inputs +3 and -5, that is, taking away more than what one has, the 'frog' goes below the water surface and stops at the -2 level. When the inputs are +3 and -3, the 'frog' stops at the water surface. This simulated number line thus helps the visitor get the concepts of zero and negative numbers which are otherwise very difficult to appreciate by young learners.

Figure 4. Demonstrating the concept of Number Line physically where a sliding indicator can be programmed to indicate the algebraic sum of 2 numerical input values – one positive and the other negative.

The number systems which we use everyday – the decimal and the binary - are explained in a pair of interactive exhibits (Figures 5 & 6). Through interaction, a visitor understands that Decimal Number System has ten digits, 0 through 9, using which one can form any number, big or small, and that the system uses a base 10.

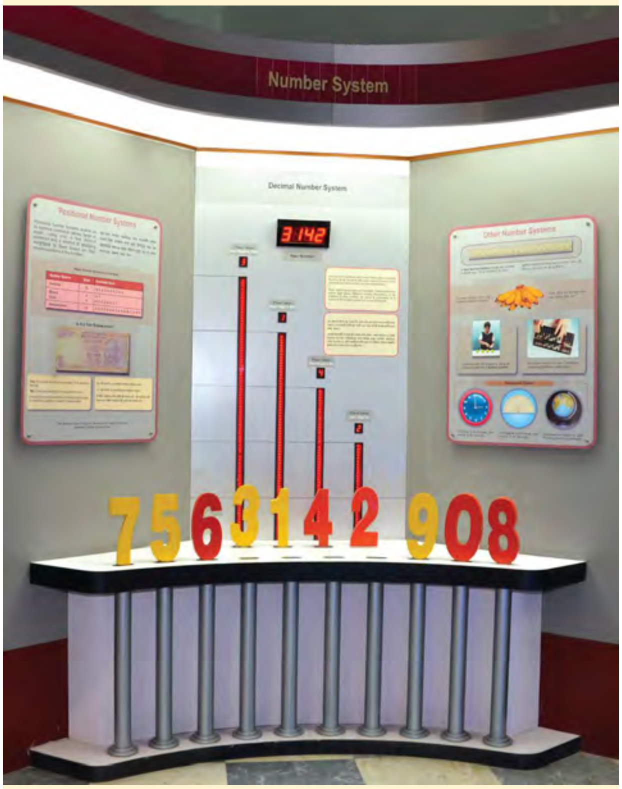


Figure 5a. Exhibit on Decimal Number System and Place Value

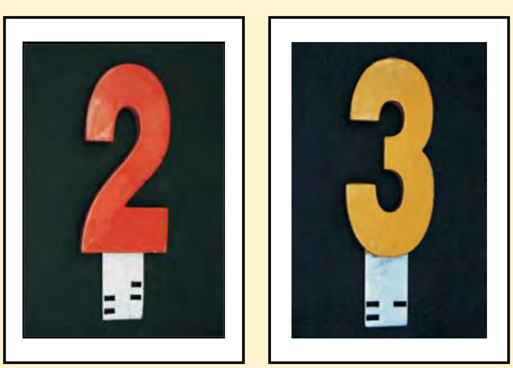


Figure 5b. View of two Digits

In the exhibit, each of the ten numerical symbols or digits carries metal legs (Figure 5b) of the same dimension that allows the user to put any digit in any of the ten slots provided on the table. These metal legs also work as 'codes' that help the electronic sensor fitted inside the slots detect the position of the inserted digits. The display over each slot shows the intrinsic value of the digits wherever they are. But the total value of the number generated by positioning of digits in different slots is given by the big display on top of all. What the users easily understand from playing with the same set of digits is that while their intrinsic values do not change, the value of the number they represent as a sequence changes as they change positions with respect to each other. The idea of place values of digits in the decimal system is thus clearly appreciated by the users.

The binary number system exhibit (Figure 6) works in the same way as the decimal system, except that it contains only two digits, 0 and 1 and each move to the left increases the value by a power of 2. Here also, the visitor can insert any one of the two digits 0 and 1 into the slots to form a binary number and see the equivalent decimal number in the display panel.

Figure 6. Exhibit explaining the Binary Number System

Series and Progression

The concept of Arithmetic and Geometric Progression is presented in a very lucid way. The Arithmetic Progression is shown like a standard staircase with equal height steps such that the railing is a straight line. Geometric Progression on the other hand has unequal steps with heights of each varying in a multiplicative manner bearing a constant ratio. For example, if the height of the first step is 2 units, the next ones are 4 units, 8 units and so on. The railing of such a staircase is therefore curved.

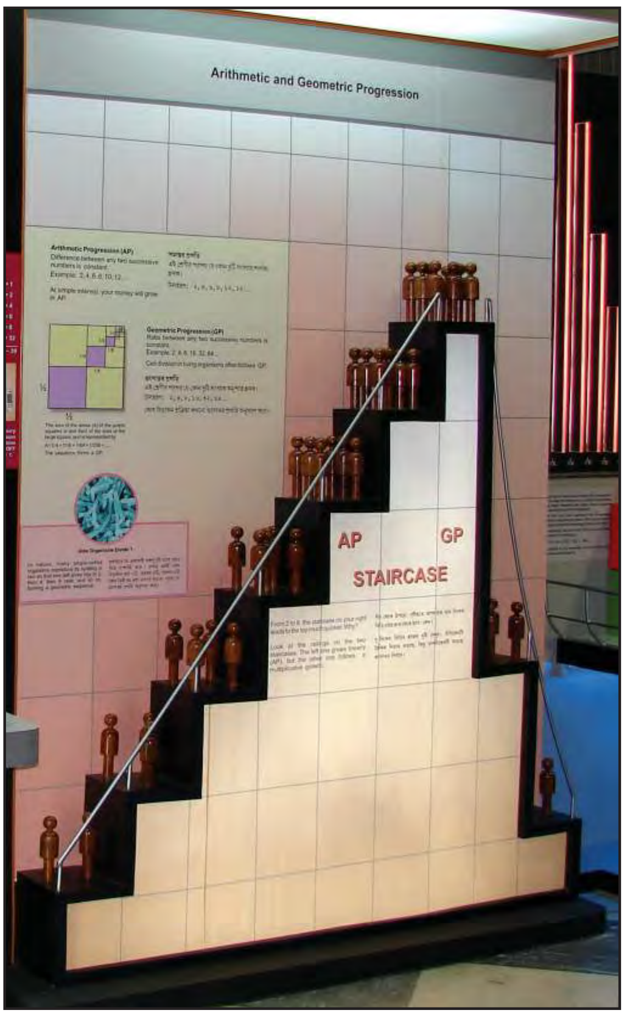
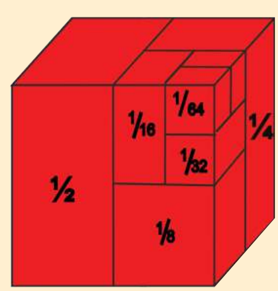


Figure 7. Exhibit on Arithmetic Progression (AP) and Geometric Progression (GP). The growth in AP is uniform and linear, while that in GP is multiplicative and follows a curve as evident from the railings of AP & GP staircases.

What are the values of the series 1+2+4+8+16+32+…. & 1/2+1/4+1/8+1/16+1/32 + …… ? In order to explain a series, we pose this question to a visitor. While one observes that both of them are G. P series, one also finds one special characteristic in them, that is, they are both endless or infinite. The value of the first series mentioned above becomes infinitely large as the value of the term increases and hence it is an example of a divergent series. But the second series is interestingly different. Here, the sum of the series tends to assume a finite value as the series progresses. Hence it is an example of a convergent series.

Figure 8 . The exhibit on 'Divergent and Convergent Series'

This feature of a convergent series is explained in the form of a simple activity.



1/2 + 1/4 + 1/8 + 1/16 + 1/32 +…... = 1

Figure 9. Explaining a Convergent Series

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A big wooden cube is divided into a number of parts, namely 1/2 cube, 1/4 cube 1/8 cube, 1/16 cube, 1/32 cube and so on as shown in Figure 9. Assembling all the parts together, one can get the original cube, thus verifying that $1/2+1/4+1/8+1/16+1/32 + \ldots\ldots = 1$. It is easy to understand from this activity that a series is a converging one when an infinitely large number of its terms add up to a finite sum. Not only that, once a student gets the concept of infinite converging series, he himself will be able to extend the above observation further, like

$$1/3 + 1/3^2 + 1/3^3 + 1/3^4 + 1/3^5 + \ldots\ldots \ldots\ldots \ldots\ldots = 1/2$$

$$1/4 + 1/4^2 + 1/4^3 + 1/4^4 + 1/4^5 + \ldots\ldots \ldots\ldots \quad \ldots\ldots = 1/3$$

……………. ………….. …………….

……………. ………….. …………….

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$$\mathbf{1/n + 1/n^2 + 1/n^3 + 1/n^4 + 1/n^5 + \ldots\ldots \quad \ldots\ldots = 1/(n\text{-}1)}$$

Concepts in Geometry and Algebra

Simple activities have been designed on algebraic formulae, properties of triangles, polygons and polyhedrons, which help beginners to get an insightful experience. In Algebra, a student can verify the following important algebraic identities through activities by using some wooden and plastic plates and blocks (Figure 10a, 10b & 10c) :

(1) $(a + b)^2 = a^2 + 2ab + b^2$
(2) $(a - b)^2 = a^2 - 2ab + b^2$
(3) $a^2 - b^2 = (a + b)(a - b)$
(4) $(a + b)^3 = a^3 + 3ab^2 + 3a^2b + b^3$

Figure 10a . Verifying the standard Algebraic Identities

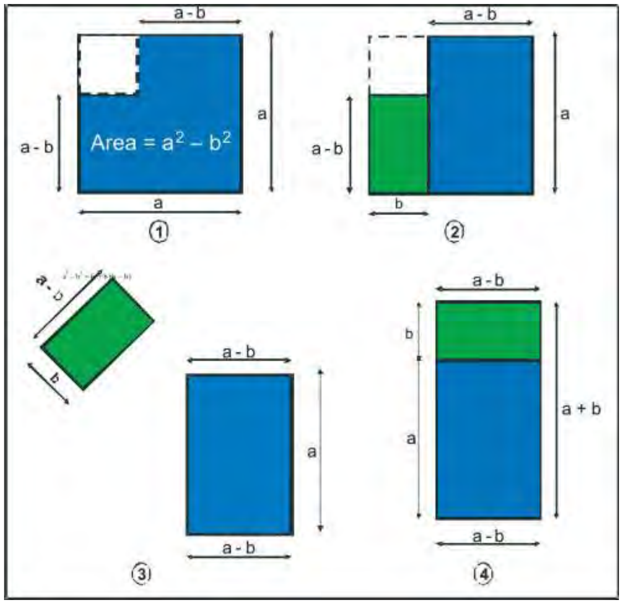

Figure 10b . Showing $a^2 - b^2 = (a + b)(a - b)$

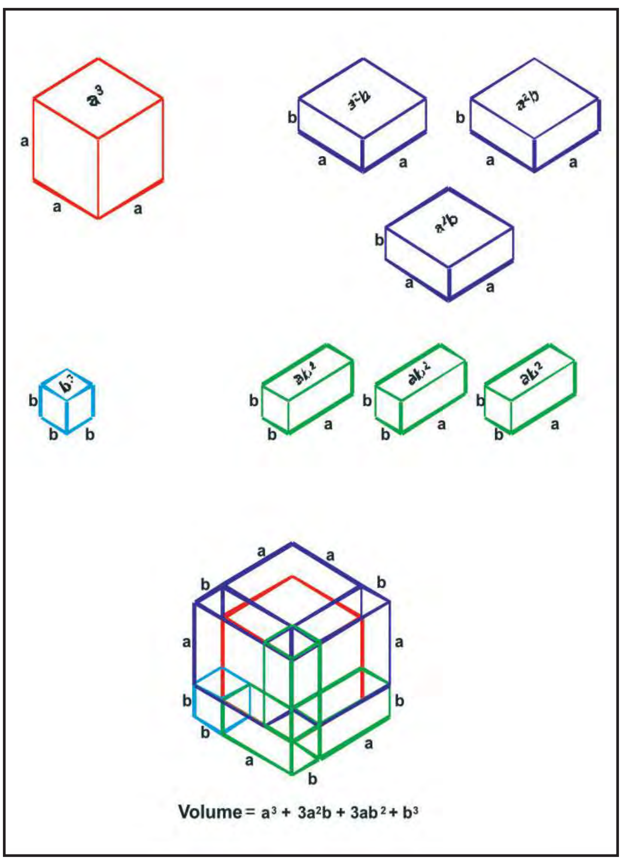

Figure 10c . Showing $(a + b)^3 = a^3 + 3ab^2 + 3a^2b + b^3$

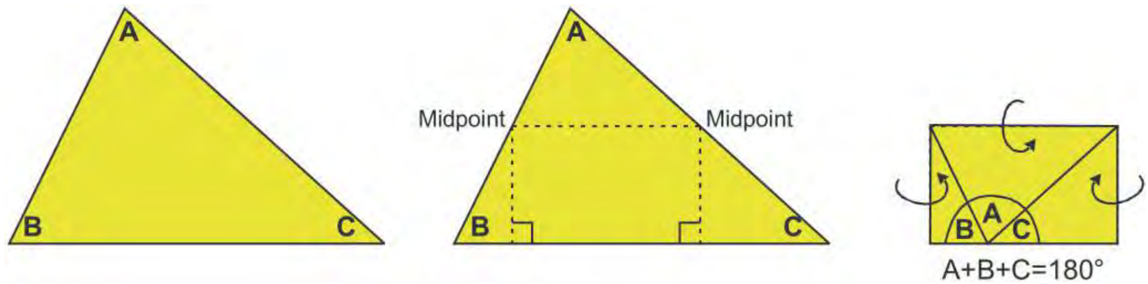

Figure 11a . Showing sum of the angles of a plane triangle = 180°

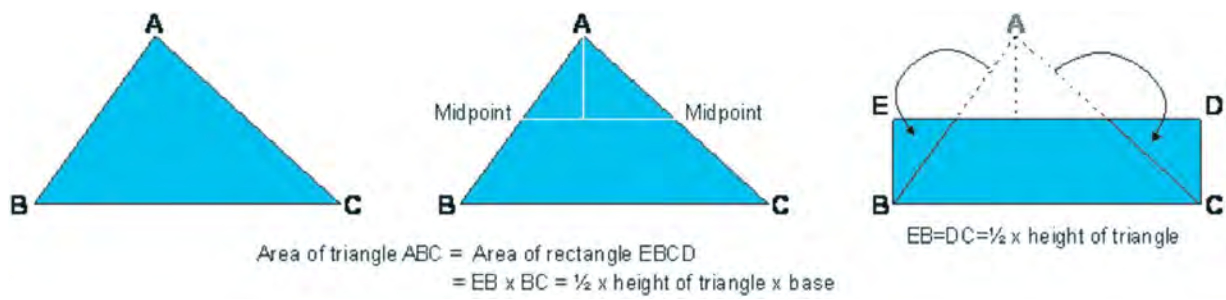

Figure 11b . Finding the area of a Plane Triangle

Similarly in Geometry, in the exhibit 'Plane Geometry' (Figure 11c), by folding or arranging parts of triangular laminar sheets in definite manners as shown in Figures 11a and 11b, a visitor can physically prove that the sum of the angles of a triangle is 180° or the area of a triangle is ½ x (base) x (height). Continuing the experiments with triangles, one can easily find out the sum of the angles or the area of any plane polygon, because a polygon is actually composed of a number of triangles.

'Platonic Solids' or Polyhedrons of Plato is another challenging concept of Solid Geometry. The exhibit provides the students with scopes for experimentation that help them grasp the idea of these unique solid figures. The polygonal faces of all the Plato's polyhedrons are placed on a table in form of wooden plates. These plates have the shapes of regular triangle, square and regular pentagon. The students are given the challenge to rearrange these plates (Figure 12a & 12b) in definite manners and build the polyhedrons.

Figure 11c . Learning the properties of basic Geometrical Figures

Figure 12a . Students experimenting with the Platonic Solid exhibit

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Figure 12b . Students interacting with the Platonic Solid exhibit

This activity makes them understand the unique features of Plato's polyhedrons, like (i) Polygonal faces are all regular, for instance, *equilateral triangles* for Tetrahedron, Octahedron and Icosahedron, *squares* for Hexahedron or Cube and *regular pentagons* for Dodecahedron (ii) polygonal faces are all equal (iii) face-to-face angles are equal (iv) edge-to-face angles are equal, and (v) there are only 5 such polyhedrons possible, namely, *Tetrahedron, Octahedron, Hexahedron (or Cube), Dodecahedron and Icosahedron.*

Students visiting the Mathematics Gallery often get to know something more than what their curriculum exposes them to. For example, Pythagoras Theorem is known to them as *"The square on the hypotenuse of a right angled triangle is the sum of the squares on the other two sides"*. However, on interacting with 'Pythagoras Theorem' exhibit in the gallery, they come to know that the theorem is true not only for the squares, but also for any similar figures drawn on the sides of the right angled triangle. In the exhibit (Figure 13), very thin square shaped chambers are made on the three sides of a right angled triangle. A certain volume of a liquid that fills the chamber on the hypotenuse is also found to completely fill the square shaped chambers on the other two sides of the triangle, thus proving the theorem.

Figure 13 . Exhibit showing Pythagoras Theorem

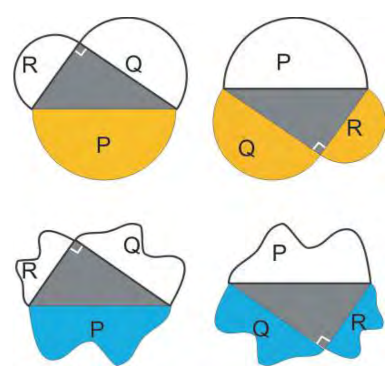

Figure 13a . Pythagoras Theorem with semi-circular and similar shaped compartments. Here, P = Q + R

Here, one can verify that the Pythagoras Theorem holds good also for semi-circular and similar shaped chambers that are built on three sides of a right angled triangle (Figure 13a).

Spherical Geometry

A visitor gets an idea of non-Euclidean spherical geometry when he or she observes the difference between a plane triangle and a spherical triangle. In the exhibit (Figure 14 & 14a), one can see that any three non-collinear points on the earth surface form a Spherical Triangle, and unlike a plane triangle, the sides of a spherical triangle are curved and its internal angles add up to more than 180°. The exhibit also shows that three places on the earth surface, although in the same line, are found to lie on a curved line in the map developed on plain paper. This is because non-Euclidean spherical geometry, and not plane Euclidean geometry, applies on the curved earth surface. This fact further makes him understand and appreciate why the air travel paths in Air-route maps are curved.

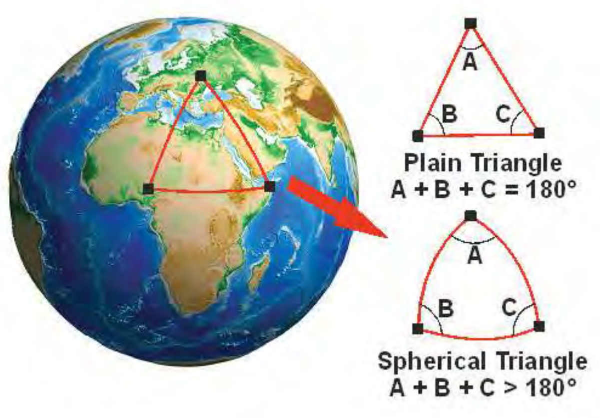

Figure 14. Plain Triangle and Spherical Triangle

Figure 14a . Elucidating why the air travel paths in air-route maps are curved

Mathematical Function

The exhibit on 'Functions' in the gallery (Fig.15b) is an attempt to elucidate the concept of mathematical functions in physical terms. The exhibit explains what a linear and a quadratic (non-linear) function physically imply. Three containers of different shapes as shown in Fig. 15a are filled by means of equally rated pumps so as to ensure that the volume of water entering each container per unit time are equal. The objective is to study the rise of water levels in these containers with time. To achieve this, special electronic tapes (pressure dependant e-tapes) have been used to sense the water levels and subsequently feed these values to a computer, which outputs the results (water level vs. time) on the monitors placed above the respective containers (Fig. 15a). Water level in the container with uniform cross-section evidently shows a linear relationship with time, while water levels for the other containers exhibit parabolic relationship with time.

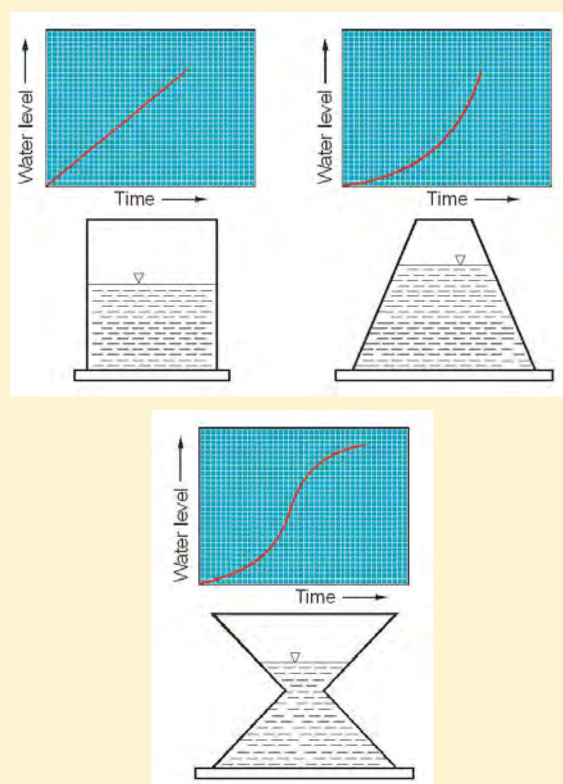

Figure 15a . Real time plot of Water Level vs. Time, while uniform flow of water fills three different containers

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Figure 15b. Showing water level as different functions of time while uniform flow of water fills up containers of uniform and non-uniform cross-section. Here, functions are linear and quadratic.

Law of Average

The exhibit titled 'Law of Average' introduces the concept and usefulness of statistical techniques through a simple measurement activity. A visitor stretches his or her forearm fully on a flat bed lined with a number of switches and presses the furthest switch he or she can reach. This in fact measures the length of their forearms (a cubit) which is statistically found to bear a relationship with the individual's height. The pressing of the switch lights up a vertical row of LEDs to corresponding heights, thus converting the length of the user's forearm to his or her height. One finds that on an average, our heights are roughly equal to 3.8 to 4 times the cubit length.

Concepts of Calculus: Differentiation & Integration

Calculus as a mathematical tool is difficult to grasp by young learners. There are a few exhibits in the gallery that were designed to illustrate the basic concepts of calculus like limit, differentiation and integration etc. in a simple manner. *Differentiation* is a rate measurement process and *Integration* means a summation process – these ideas are made quite clear to the students by two exhibits in the gallery (Figures 16b & 16c).

A stick is made to move such that its mid-point slides in a curved slot (Figure 16a). Its slope or gradient changes as indicated by the angle ? it makes with x-axis. The rate at which ordinate (y-coordinate) of a point in the curve $y = f(x)$ changes with respect to its abscissa (x-coordinate) is determined by dy/dx or tan? at that point, and is represented by a tangent drawn to the curve at that point. The stick physically takes the position of the tangent, and hence as the stick moves in x-y plane along $y = f(x)$, the slope of the stick or dy/dx changes. Thus, the stick moving along the curved slot is the visual and physical representation of dy/dx or differentiation of $y = f(x)$.

Once differentiation shows us how to measure the rate at which a variable quantity changes with respect to another, the concept is applied in computing the exact

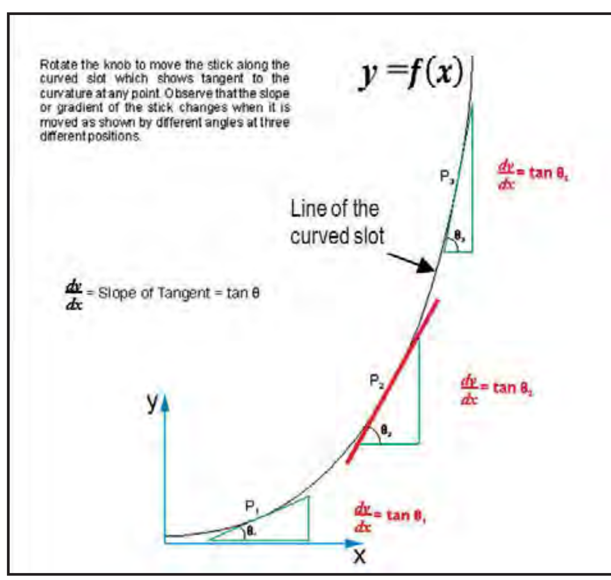

Figure 16a. Explaining the concept of Differentiation

length of a curved line or exact area of a curved surface shown in Figure 16b, which is otherwise not possible using standard geometrical methods. The surface area of the inverted funnel (Figure 16b) could be very approximately calculated by adding up the peripheral areas of the circular plates the funnel is made up of. However, differentiation technique allows us to calculate the rate of change of height with change in its radius, and hence the curved surface area can be perfectly computed using the technique of integration.

Figure 16b. Using differentiation for determining the profile of Curved Surface
Figure 16c. Exhibiting the concept of Integration

The exhibit on 'Integration' presents the concept in a comprehensible form (Figure 16c). On three circular discs that can be rotated in a vertical plane, two equal circular compartments are made and interconnected (Figure 16d). One of these compartments in each has a number of rectangular areas forming the circular shape, while the other is perfectly circular. Rectangular areas in disc 2 are narrower and more in number than in disc 1 and in disc 3, these are much narrower.

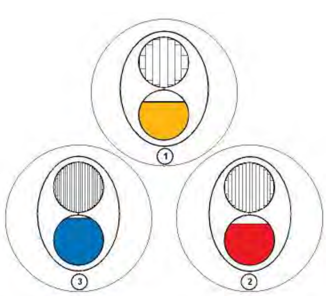

Figure 16d. Finding the Area of a Circle

On turning the three discs upside down to allow the coloured liquid from the chambers having rectangular peripheral walls to trickle down to the circular chambers, it is observed that the liquid does not fill the circular compartments completely. Narrower is the size of rectangular area that make up the inner periphery of the upper chambers, smaller is the unfilled space in the circular compartments and vice versa. This clearly shows that if there were infinitely large numbers of rectangular areas with infinitesimally narrow width, then in the limit, this width tends to zero leaving no unfilled space in the circular compartment, thus enabling us to compute the exact area of the circle.

Maxima – Minima : Application of Differentiation

Proper realization of the physical significance of Differentiation and Integration sensitizes students to explore how Calculus is applied to solve physical problems. The exhibit on 'Maxima – Minima' addresses the students on this issue. A challenge is given to them to find out which of the three shapes of the containers of equal volume (Figure 17a) has the minimum surface area ?

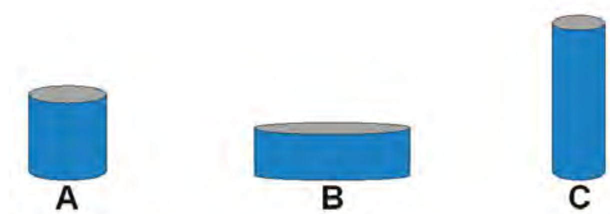

Figure 17a . Three containers of equal volume. Which shape has the minimum surface area ?

Sometimes solving such problems becomes very important. For example, soft drink cans are manufactured in huge numbers and the manufacturer would like to save on material used for making the cans. Differentiation can help one find out the exact solution instead of going into cumbersome trials and errors. The relation between the surface area and the radius of the cylindrical container shows that the container's surface area is minimum when differentiation of its surface area (*S*) with respect to its radius (*r*), i.e. dS/dr is zero (Figure 17b). This is possible only when the height of the container is equal to twice the radius (or diameter) of the container.

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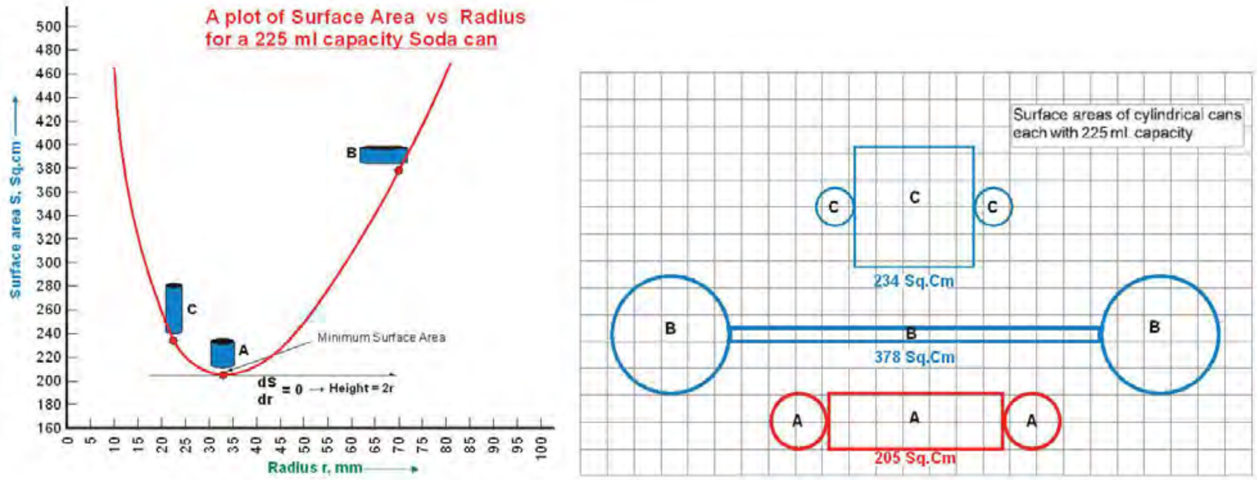


Figure 17b. Calculating the minimum surface area of a cylindrical container having equal volume but different shapes. Surface area (S) is a function of Radius (r), S = f(r)

Students can verify this by taking a container and comparing its surface area with those drawn on the table. They can observe that the container with height equal to its diameter has the minimum surface area.

Mathematical Activities

The activity hall of Mathematics Gallery is about mathematical challenges and brain teasers. Puzzles, mazes, juggling with shapes and figures keep visitors busy and engrossed for hours.

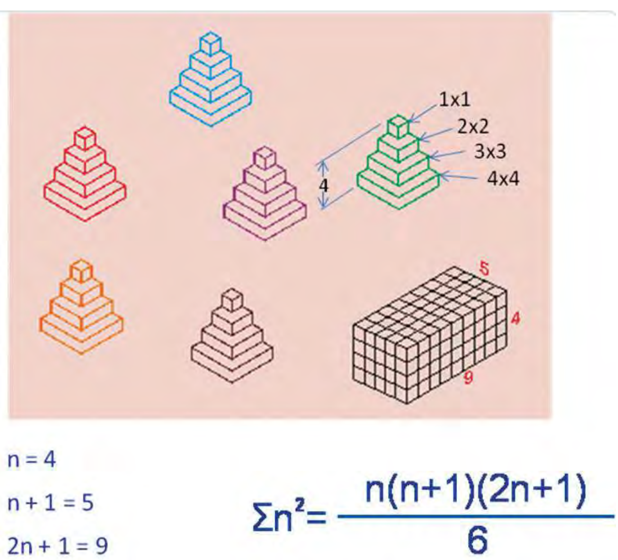


Figure 18a. A 3D model to find out the sum of squares of natural numbers

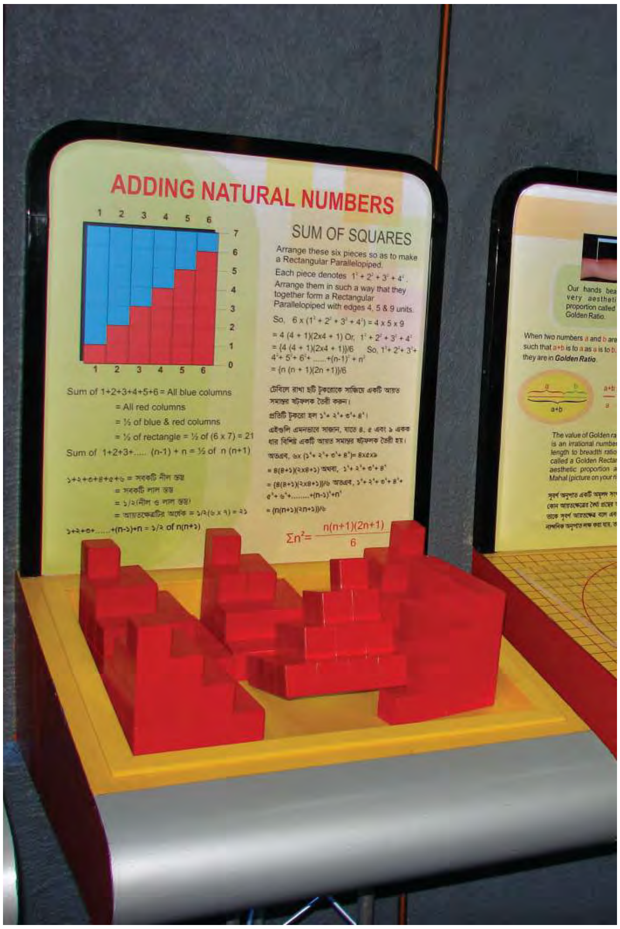


Figure 18b. Another 3D model to find out the sum of squares of natural numbers

There are also other activities in this section that provide opportunities to carry out experiments on a few important mathematical rules and formulae. For example, an experimental model has been designed that would enable a student to find out the sum of the squares of natural numbers, i.e. $1^2 + 2^2 + 3^2 + 4^2 + 5^2 + \ldots \ldots\ldots + n^2$. Six identically stepped blocks, each with four square shaped steps, are provided for this activity. The steps of each block denote 1^2, 2^2, 3^2 and 4^2 according to their sizes, like 1x1x1, 2x2x1, 3x3x1 and 4x4x1 cubic units (Figure 18a & 18b). One has to arrange and combine all the six blocks in such a way that the resultant block is a rectangular parallelepiped, whose edges are 4, 5 and 9 units as shown in the figure. The volume of the rectangular parallelepiped is 4x5x9 cubic units.

We can say that for 4 steps in the block

$$6 \times (1^2 + 2^2 + 3^2 + 4^2) \times 1 = 4 \times 5 \times 9 = 4 \times (4+1) \times (2 \times 4 + 1)$$

$$\rightarrow (1^2 + 2^2 + 3^2 + 4^2) = 1/6 \{4 \times (4+1) \times (2 \times 4 + 1)\}$$

So, for 5 steps in the block $\rightarrow (1^2 + 2^2 + 3^2 + 4^2 + 5^2) = 1/6 \{5 \times (5+1) \times (2 \times 5 + 1)\}$

for 6 steps in the block $\rightarrow (1^2 + 2^2 + 3^2 + 4^2 + 5^2 + 6^2) = 1/6 \{6 \times (6+1) \times (2 \times 6 + 1)\}$

....

....

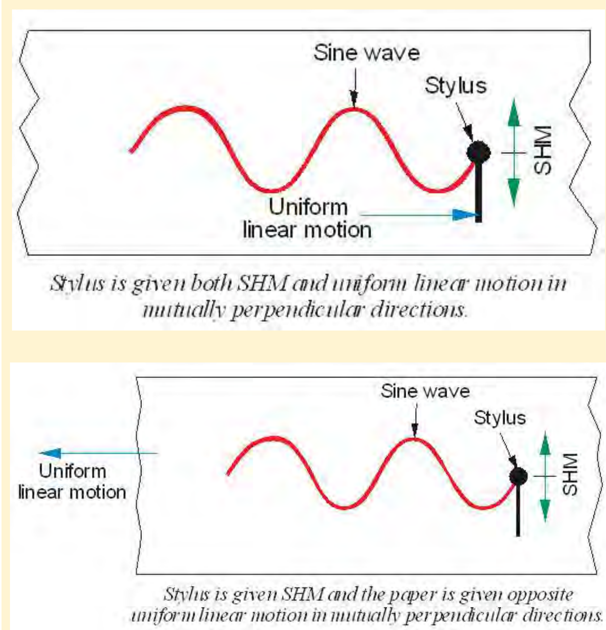
For n steps in the block $\rightarrow \mathbf{(1^2 + 2^2 + 3^2 + \ldots\ldots + n^2)}$

$$\mathbf{= 1/6 \{n (n+1) (2n+1)\}}$$

Apart from applying the standard method of summation of series in school or college, he can thus get to know the physical interpretation of the summation process by doing this experiment.

Another activity most students love to do is generating a Sine wave or a Sinusoidal curve. They normally apply analytical methods in drawing mathematical curves in their school/college curriculum. The basic idea of the activity is to trace a Sine wave using mechanical means by compounding a Simple Harmonic Motion (SHM) and a uniform linear motion in mutually perpendicular directions (Figure 19). A student generally finds problem in generating the SHM. A mechanism has been devised to facilitate him to do the activity in the gallery.

When a particle moves in a circular path uniformly, the foot of the perpendicular dropped from it on a diameter of the circle executes an SHM along the diameter – this fact is known to him and the mechanism referred above exactly does this job. He rotates the round plate with the knob to generate a reciprocating motion of the stylus (Figure 19a). Since it is practically not possible to rotate the plate with perfectly uniform



Stylus is given both SHM and uniform linear motion in mutually perpendicular directions.

Stylus is given SHM and the paper is given opposite uniform linear motion in mutually perpendicular directions.

Figure 19

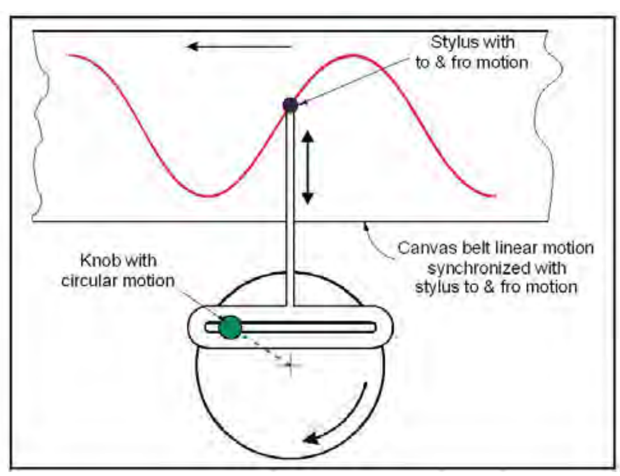


Figure 19a. Mechanism converting Circular Motion into a Reciprocating Motion

motion, the output reciprocating motion of the stylus will not be an SHM. In order to overcome this problem, the circular motion of the plate is mechanically linked with the linear motion of the canvas belt on which the Sine wave is to be drawn. This design ensures synchronization of the reciprocating motion of the stylus with the linear motion of the canvas belt, so that Sinusoidal wave will be traced on the canvas by the stylus whenever there is an input circular motion, no matter whether it is uniform or not (Figure 19b).

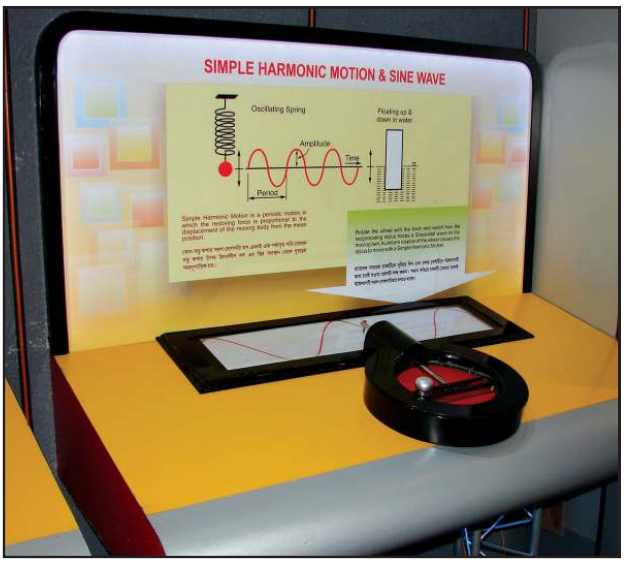


Figure 19b. An interactive exhibit to convert circular motion into a Simple Harmonic Motion and trace a Sine Wave

In the Activity Area, a student can find out various mathematical shapes in nature in the gallery itself. He also enjoys discovering some numerical orders and patterns in nature - in human body, leaf, floral petal arrangement, building architecture, banking and finance and the like. *Golden Ratio* is an appropriate example of nature's mathematical beauty that renders the most beautiful, aesthetic and compact form and shape to the objects in nature. Here one can create a beautiful spiral applying Golden Ratio. This makes one comprehend and appreciate the beauty of mathematics as it helps us find the hidden patterns in the things and events happening in the natural and manmade worlds.

Live Math Demo Corner cum Classroom

But the highlight of the gallery is its 'Live Math Demo Corner cum Classroom' (Figure 20), an area inside the gallery that is fitted with a digital smart board for conducting math demos, especially on Vedic mathematics, and on curricular mathematics. School students accompanied by their mathematics teachers are using this facility round the year. Live demos are conducted by BITM educators on Vedic mathematics regularly for the visiting school children. The advantage for the schools for taking their math classes here is that the teacher can always go back to the respective exhibit in the gallery for experimenting and verifying the key ideas being taught.

Figure 20. Math Demo Corner cum Classroom

Concluding remarks

Ever since its opening in 2010, the Mathematics Gallery has evoked positive responses from visitors, especially from school students and their mathematics teachers. Some schools in Kolkata are using the gallery as their extended practical classroom as they visit the Mathematics Gallery on regular basis and conduct activity sessions by students, organize math workshops and practical demonstrations. The gallery has also generated serious interest among many schools and colleges for setting up mathematics laboratories in their own institutions, for which they are approaching BITM for help and support. For example, BITM has helped set up a Mathematics laboratory in Institute of Education for Women at Hastings, Kolkata and Raja N. L. Khan Girls' College in Medinipur, West Bengal.

Acknowledgement

Author is immensely grateful to Dr. Pradip Kumar Majumdar, Former Professor of Indian Astronomy, Rabindra Bharati University, for providing remarkable and valuable information and guidance on ancient mathematics of India and the World, which was required for developing the section on ancient mathematics in the gallery. Author's sincere thanks also go to Dr. Parthasarathi Mukhopadhyay, Associate Prof., Ramakrishna Mission Residential College, Narendrapur and Dr. Dilip Kr. Sinha, Former Sir Rashbehary Ghose Professor of Applied Mathematics, Calcutta University for extending help with information during development of the gallery. His special thanks go to Sk. E. Islam, Director, BITM, Art section and workshop of BITM for their valuable inputs and support during fabrication of the exhibits of the gallery and for finally shaping the gallery.

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